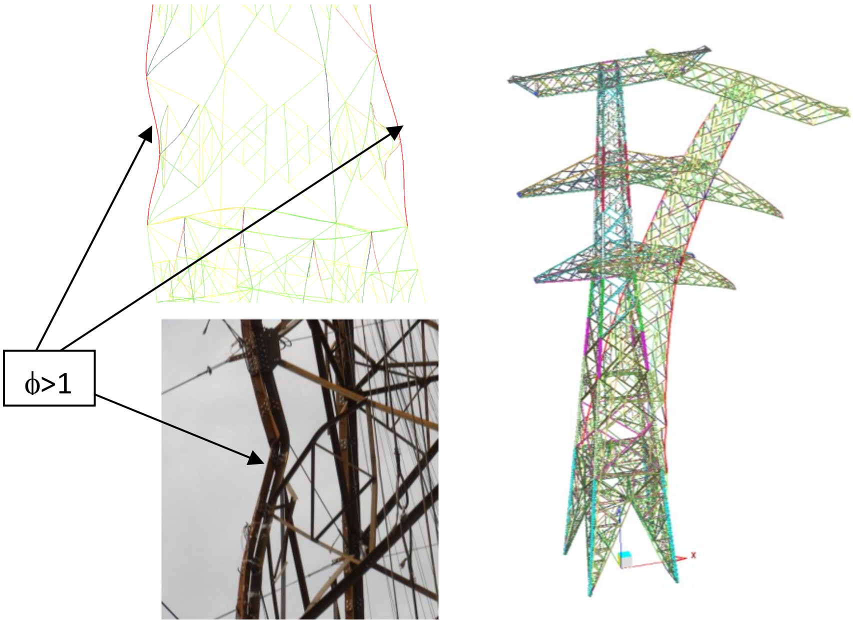
Welcome to NIDA Version 9.0

NIDA - Nonlinear Integrated Design and Analysis

NOTE ON

Second-Order Direct Analysis by NIDA



Second-Order (Direct) Elastic & Plastic Analysis

To

Eurocode 3 (2005), AISC-LRFD (2010) and CoPHK (2011)

Without Assumption of Effective Length

Add: The Hong Kong Polytechnic University,
Hung Hom, Kowloon, Hong Kong, P.R. China

Tel: (852) 2766 6047
Fax: (852) 2334 6389
Email: ceslchan@polyu.edu.hk
Web: http://www.nidacse.com/

Table of Contents

1. Introduction

The second-order direct analysis is a revolutionary approach to the design of not only steel structures, but any other type of structures including steel-concrete composite, reinforced concrete and other structures including bamboo and pre-tensioned steel truss systems. The basic underlying principle is very different from the first-order linear analysis using the effective length. In the new method, the structure is designed by a simulation process, a truly performance-based approach that the safety is directly checked by the section capacity along the length of every member. The section capacity check approach is used for design of steel and concrete members via the elastic modulus with triangular stress blocks, the plastic modulus with rectangular stress blocks or other functions of modulus used with other stress block assumptions.

Unlike the conventional linear analysis method of design, the P-Δ and the P-δ effects are considered during a second-order direct analysis so there is no need to assume any effective length to account for the second-order effects. Despite its convenience, many structural engineers are reluctant to switch to this new design method. One major reason is it requires engineers to learn and get familiar with as mentioned in previous study. Another major reason is the convenience of using this method is rarely demonstrated. The aim of this note is to compare the new design method with the conventional effective length method. Design examples are carried out which include:

1. Simple frames to illustrate the procedures of conventional design and design using second-order direct analysis;

2. Three-dimensional large-scaled structures to demonstrate the advantages and limitation of design using second-order direct analysis over conventional analysis; and

3. A very slender structure which second-order direct analysis must be used.

The second-order direct analysis method of design is a unified and an integrated design and analysis approach that the effect of fire or elevated temperature effects, seismic, effects of accidental member damage and progress collapse can all be modeled in the design process which integrates with the analysis process. To foster the concept, this note is addressed to the conventional and widely exercised design against static loads. While the concept of the method is essentially the same for all applications under various scenarios, they may require different parameters which will be statutory in future. These parameters include member and frame imperfections under these conditions.

2. Background

There exists the P-Δ effect and the P-δ effect in real structures which are due to the global displacement of the structure and the lateral displacement of the member respectively as shown in Figure 2. The consequence of these secondary effects is additional stresses in the member are induced and thus the structure is weakened. A rational design should consider both the P-Δ and P-δ effects. The conventional limit state design method has been used extensively over the past decades. The philosophy of a limit state design can be expressed as follows.

$$\gamma \cdot F \leq \phi \cdot R \tag{1}$$

in which γ is the load factor, F is the applied load, ϕ is the material factor and R is the resistance of the structure. Traditionally, F is obtained from the first order linear analysis in which both geometrical and material nonlinearities are not taken into account while R is calculated based on the specifications so that the second-order P-Δ and the P-δ effects and material yielding are considered. Although the analysis procedure is speed up by the recent rapid development of personal computers, there are still some unavoidable hand calculation processes during the design stage such as calculating the effective length of a compressive column and the amplifications factors for the linear moments. The reliability of the conventional design method depends very much on the accuracy of the assumptions of effective length factors.

In recent years, design method using second-order direct analysis has been developed in which the second-order effects are considered directly during the analysis. There are two major types of second-order analysis, namely second-order elastic analysis and second-order inelastic analysis. The first type does not consider the effect of material yielding therefore section capacity check per member is required to locate the load causing the first plastic moment or first yield moment of the structure. The second type considers the effect of material yielding so the maximum failure load can be directly located by the load deflection plot. The section capacity check is therefore used for assessing the condition of plastic hinge formation. A second-order direct analysis not only facilitates structural design but it also plays a very important role on structural stability problems.

To date, both conventional design method and second-order direct analysis design method are allowed in many national design codes such as Eurocode-3 (2005), Code of Practice for Structural Uses of Steel (2011), BS5950 (2000) and AS4100 (2000). However, despite the convenience of the latter approach, the majority of structural engineers are reluctant to step forward to this state-of-the-art approach. One major reason is most software is programmed for P-Δ-only analysis and extensive manual checking effort is still required. Another major reason is its convenience is rarely illustrated through practical design examples.

Despite this reluctance, the second-order direct inelastic analysis, or the advanced analysis, will be the major trend in structural design in the future together with the second-order direct elastic analysis. Some researchers and codes name the method as advanced analysis. This chapter has two main objectives. The first one is to deliver the idea of how a design can be performed without any effective length. The second objective is to compare the new design method with the conventional method. Design examples are carried out in the hope that through these design examples, engineers will find the merits of design using second-order direct analysis without using effective length.

3. Methods of Analysis

In LRFD (2010), the Eurocode (2005) and the Hong Kong Steel Code (2011), the three methods namely as the first-order linear, second-order indirect analysis and second-order direct analysis methods can be used. But we need to ensure the effects of change of deformed geometry shall be considered with λ_{cr} not less than 5 otherwise the second-order direct analysis must be used. For example, in Eurocode-3 (2005), clause 5.2.2(3) methods a), b) and c) specify respectively the methods of second-order direct analysis, second-order indirect analysis and the linear analysis as in the box below. LRFD (2010) names the methods as first-order analysis or effective length method under Appendix 7, P-Δ-only or simplified second order analysis under Appendix 8 and direct analysis in Chapter C, which shows that the direct analysis or second order direct analysis appears as the principal and preferred method in main text. To this, engineers should certainly need to acquire the skill of such design.

5.2.2 Structural stability of frames

(1) If according to 5.2.1 the influence of the deformation of the structure has to be taken into account (2) to (6) should be applied to consider these effects and to verify the structural stability.

(2) The verification of the stability of frames or their parts should be carried out considering imperfections and second order effects.

(3) According to the type of frame and the global analysis, second order effects and imperfections may be accounted for by one of the following methods:

a) both totally by the global analysis,

b) partially by the global analysis and partially through individual stability checks of members according to 6.3,

c) for basic cases by individual stability checks of equivalent members according to 6.3 using appropriate buckling lengths according to the global buckling mode of the structure.

Extract of Eurocode-3 (2005)

Load factor λ in Figure 1 represents a scalar multiplied to the set of design load in a particular combined load case. To understand the method, we must first appreciate the behaviour of a structure under an increasing load. Various methods provide an answer of the collapse load under its assumptions, such as plastic collapse load which does not consider any buckling effect and P-Δ-only second-order indirect analysis does not consider member imperfection and member buckling.

The results of these methods are compared with the true collapse or ultimate load of a structure, λ_u in the Figure 1 below.

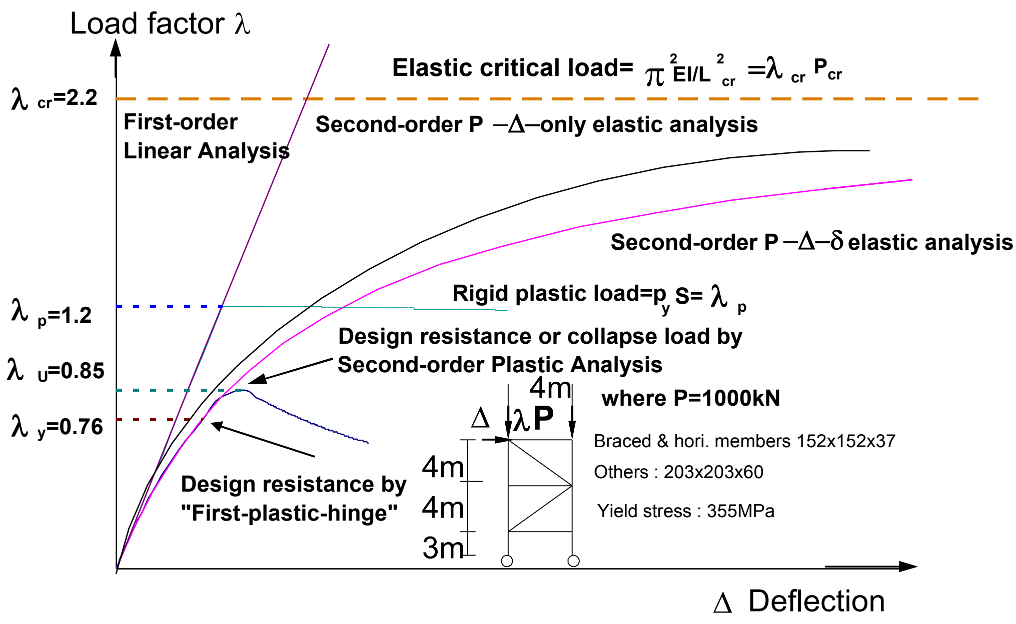


Figure 1 Design methods

Various terms in the above graph are explained below.

Elastic critical load factor λ_{cr} is a factor multiplied to the design load to cause the structure to buckle elastically. The deflection before buckling, large deflection and material yielding effects are not considered here and the factor is an upper bound solution that cannot be used directly for design. λ_{cr} can be used to measure the instability stage of a frame against sway and buckling.

Plastic collapse load factor λ_p is a load factor multiplied to the design load to cause the structure to collapse plastically but buckling and second-order effects are not considered. Because of the ignorance of buckling effects, λ_p cannot be used for direct design and it is an upper bound solution to the true collapse load of the structure. This load factor was widely used in the past for plastic design because of its simplicity to determine.

P-delta effects refer to the second-order effects. There are two types, being P-Δ and P-δ.

P-Δ effect is second-order effect due to change of geometry of the structure

P-δ effect is second-order effect due to member curvature and change of member stiffness under load. A member under tension is stiffer than under compression.

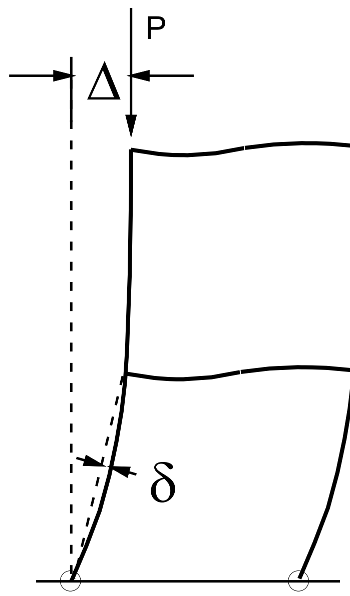


Figure 2 The P-Δ and P-δ effects

Linear analysis or first-order linear analysis is an analysis assuming the deflection and stress are proportional to load. It does not consider buckling nor material yielding.

Notional force is a small force applied horizontally to a structure to simulate lack of verticality and imperfection, see Figure 3. It can also be used to measure the lateral stiffness so that the elastic critical factor can be determined.

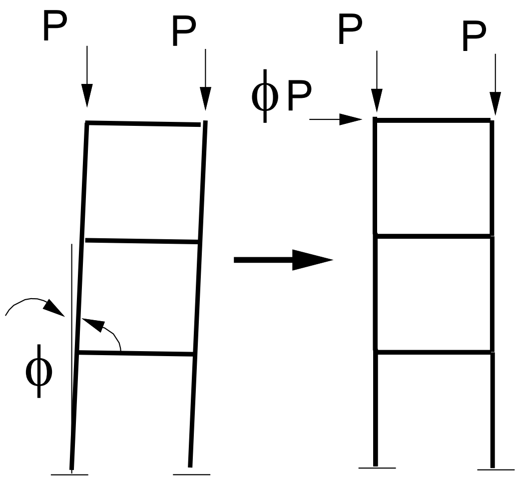


Figure 3 Simulation of out-of-plumbness by the notional force

Second-order indirect analysis or P-Δ-only analysis is an analysis used to plot the bending moment and force diagrams based on the *deformed nodal coordinates*. It does not consider member curvature or the P-δ effect. This method is commonly used in software because of its simplicity. In fact, most software can only do this P-Δ-only analysis which is not qualified for a full second-order analysis accounting for P-Δ and P-δ effects with imperfections at frame and member levels. This method is also named as approximate second-order analysis in LRFD (2010) and as method in clause 5.2.2.3(b) in Eurocode-3 (2005) or in Hong Kong Steel Code (2011) as Second-order indirect analysis.

Second-order direct elastic analysis which allows for section capacity check is an analysis which allows for P-Δ effect and the P-δ effect with their imperfections and stops at first plastic hinge. It needs not assume an effective length for the buckling strength check, but imperfection must be allowed for. Although it allows use of plastic modulus and plastic moment, it does not permit the moment re-distribution so the design load is taken as the load causing the formation of the first plastic hinge.

Second-order direct plastic analysis which allows for section capacity check is an analysis which allows for P-Δ effect and the P-δ effect with their imperfections and stops at first plastic hinge. It needs not assume an effective length for the buckling strength check, but imperfection must be allowed for. It not only allows the use of plastic moment, it further permits the moment re-distribution due to plastic yielding or after formation of plastic hinges and the design load is taken as maximum load causing the structure to form a collapse mechanism or divergence of the iteration process in an incremental-iterative analysis. The load increment should also be taken as a small portion of the design or applied load to prevent early numerical divergence.

The physical meaning of λ_{cr}, named as elastic critical load factor, can be illustrated by the buckling load of a simply supported column of Young's modulus E, second-moment of area I and length L.

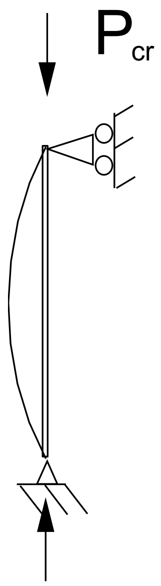


Figure 4 Buckling of a pin-pin column

The Euler buckling load is

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{2}$$

If the calculated buckling load from Equation (2) is 100 kN and the factored design load from self-weight, live, wind and dead load is 20 kN, λ_{cr} is then equal to 100/20

=5. We must remember that λ_{cr} is not for direct design since it does not consider imperfection and material yielding effects. λ_{cr} is only an indicator of stability stage, for calculating effective length factor ($\frac{L_e}{L^2}$) or used for amplification to be discussed.

When using software NIDA, one only needs to use the function of *Vibration and Buckling → Vibration and buckling parameters → eigenvalue buckling* and select the number of mode as 1 or more but only the first buckling mode is used in NIDA. For higher accuracy, we can just select all members and divide them to 2 elements since NIDA uses cubic element to find the buckling load factor. This division is not needed for second-order analysis in NIDA which use curved element to cater for the P-δ effect and imperfections.

We no longer discuss the effective length method which is being phased-out in several codes including the Eurocode-3 (2005), the LRFD (2010) and Code of practice for structural uses of steel Hong Kong (2011). These codes indicate the computer method using second-order analysis (SOA) could be applied to design of steel structures under various scenarios which cover the structures under normal uses and extreme events. Structures with small elastic critical load factors less than 3 in Eurocode-3(2005), 4 in BS5950 (2000) or 5 in AS4100 (1995) and Hong Kong Steel Code 2011 should not be designed by the linear analysis and the use of elastic critical load factor is limited to "**regular**" building frames under dominant gravitational loads. LFRD (2010) moves the linear analysis to appendix with the second-order analysis in the main core of the text and the Eurocode-3 places the chapter for second-order analysis in front of the linear analysis, showing SOA as a preferred method.

In theory, the effective length method and linear analysis cannot consider change of stiffness when a structure is under load and thus the bending moment is, strictly speaking, incorrect and the LFRD (2010) requires use of a reduction factor τ_b for stiffness reduction (see "adjustments to stiffness" on p.24 of LFRD 2010). We can see that a compressive brace takes smaller load than a tension brace and the linear analysis is incorrect in assuming all stiffness is based only on material and geometrical properties but not on initial forces in the members. Some codes increase effective length when members are under eccentric moments but they are actually unrelated. For example, it becomes meaningless to apply this concept when members are in tension and required to increase effective length for eccentric moments. Further and more importantly, assumption of effective length is uncertain and effect of eccentricity on effective length is difficult to quantify in the method etc. So, most modern codes attempt to remove the old effective length approach.

The detailed formulation of our curved stability function with curvature to Table 5.1 of Eurocode-3 (2005) could be found in Chan and Gu (2000) and design application of semi-rigid frames can be referenced to Liu, Chan and Lam (2011).

The design is done by simulating the response of a structure under load, like some examples shown in YOUTUBE© by the author (type "thenidachan" in YOUTUBE© home page) which includes some real towers constructed and now in use.

In the followings, we no longer discuss the linear analysis and the second-order indirect analysis because they are limited in use. For example, the second-order indirect analysis provides meaningless solution in many structures like the following (see Figure 5) as the notional force applied to top level under load is taken by the horizontal support (or braces) and the top of the frame does not sway so it cannot be used to determine any sway or P-Δ moment. In clause 5.2.2(7)b) of Eurocode-3 (2005) further indicates that the second-order indirect analysis requires use of its chapter 6 to find effective length so the method is basically the same as the linear analysis with minor difference that the sway moment, if significant with value of λ between 5 and 10, could be determined in second-order indirect analysis but needs to be calculated by moment amplification factor in the linear analysis.

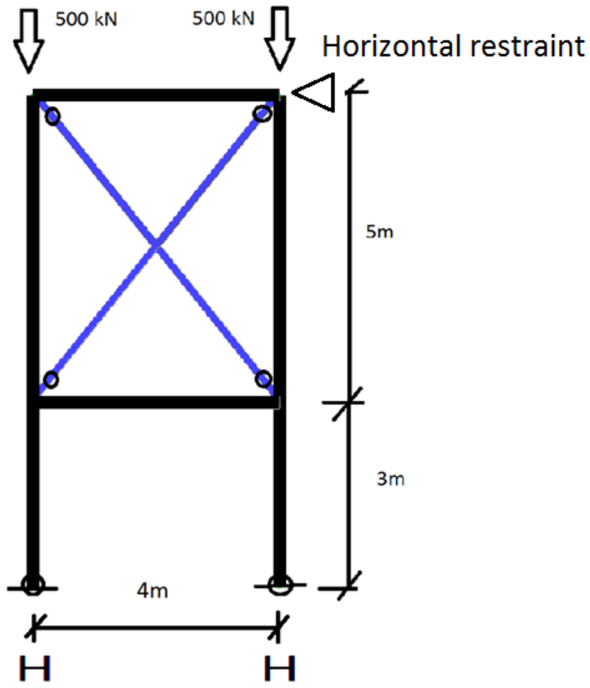


Figure 5 The simple non-sway frame with unknown effective length of its columns

4. Design by Second-Order Direct Analysis

We need to use proper software for second-order analysis indicated in conclusions of this note. With the qualified software, we need to model imperfections as follows.

4.1 Software

Proper software is essential and many programs are not for second-order direct analysis and we must be careful on using suitable software. Most of them are only for P-Δ-only (a term used in LFRD 2010 and Hong Kong Steel Code 2011) or second-order indirect analysis, but not for P-Δ-δ second-order direct analysis allowing for imperfections in member and frame levels. Refer to conclusions for some benchmark example 1.

4.2 Imperfections

Unlike the first-order linear analysis, imperfections must be considered in any second-order analysis since no real structure is perfect and possesses no residual stress and initial crookedness.

The effects of imperfections shall be taken into account for two conditions.

Global analysis : P-Δ effect

Member design : P-δ effect

In Eurocode-3 (2005), a special feature is about the consideration of frame and member imperfections which are not so explicitly expressed in most other outdated codes like BS5950. See below the imperfections required in Eurocode-3(2005).

5.3 Imperfections

5.3.1 Basis

(1) Appropriate allowances should be incorporated in the structural analysis to cover the effects of imperfections, including residual stresses and geometrical imperfections such as lack of verticality, lack of straightness, lack of flatness, lack of fit and any minor eccentricities present in joints of the unloaded structure.

(2) Equivalent geometric imperfections, see 5.3.2 and 5.3.3, should be used, with values which reflect the possible effects of all type of imperfections unless these effects are included in the resistance formulae for member design, see section 5.3.4.

(3) The following imperfections should be taken into account:

a) global imperfections for frames and bracing systems

b) local imperfections for individual members

Some software incorrectly states " *instead of applying global and local imperfections, the real buckling shape itself can be applied as a unique imperfection.*" in its introductory flyer. This is obviously incorrect because, for example, braces with ends pinned are not affected by global buckling mode (since no moment could be transferred from global frame to the braces) and ignoring member imperfections in compression braces is dangerous like forgetting use of buckling curve in design of the braces. Also, buckling of some members may not be dominated by global buckling mode and this assumption could lead to un-conservative design.

4.2.1 Frame Imperfection

We need to model global or frame imperfection due to unavoidable construction tolerance. This can be done by one of the following methods in Eurocode-3.

5.3.2 Imperfections for global analysis of frames

(1) The assumed shape of global imperfections and local imperfections may be derived from the elastic buckling mode of a structure in the plane of buckling considered.

(2) Both in and out of plane buckling including torsional buckling with symmetric and asymmetric buckling shapes should be taken into account in the most unfavourable direction and form.

(3) For frames sensitive to buckling in a sway mode the effect of imperfections should be allowed for in frame analysis by means of an equivalent imperfection in the form of an initial sway imperfection and individual bow imperfections of members. The imperfections may be determined from:

1) Using eigen-buckling (elastic buckling) mode as imperfection mode

The effects of imperfections for typical structures shall be incorporated in frame analysis using an equivalent geometric imperfection as an alternative to the notional horizontal force as,

$$\Delta = \frac{h}{200} \qquad (3)$$

where h is the storey height or largest dimension of a structure, Δ is the initial deformation or out-of-plumbness deflection.

The shape of imperfection may be determined using the notional horizontal force for a regular frame or from the elastic critical mode.

For regular multi-floor building frames, the shape may be simply taken as an inclined straight line.

In many structures, the buckling mode shape is not obvious and we need to use computer program to determine the buckling mode. We can use the buckling mode as imperfection mode as an unfavourable scenario in Eurocode 3 (2005). In software, we can use specify this eigen-buckling mode option and a magnitude equal to 0.5% multiplied by the height or the longest span or an expected value of imperfection for a particular type of structures, see Figure 6. 1% imperfection deflection or notional force is needed for temporary structures and 3% may be needed for structures under demolition.

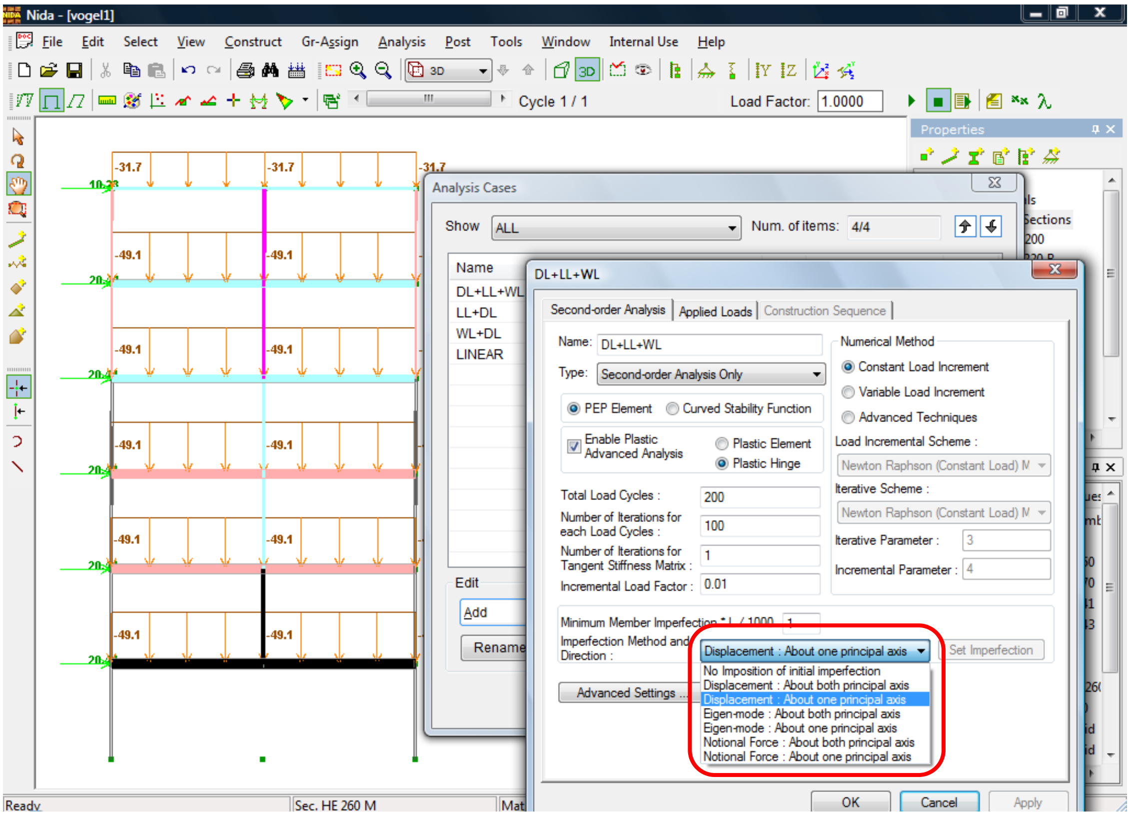


Figure 6 Use of buckling mode as imperfection mode

These initial sway imperfections should be applied in all unfavourable horizontal directions, but need only be considered in one direction at a time. Temporary structures and structures under demolition require greater imperfections.

2) *Method of Notional force*

When structures have irregular shapes, the application of notional forces becomes difficult. For regular frames where the buckling mode is a sway mode and obvious, a 0.5% of the vertical load could be applied horizontally shown in Figure 7. For structures used for other functions and durations, a varied value of notional force is used.

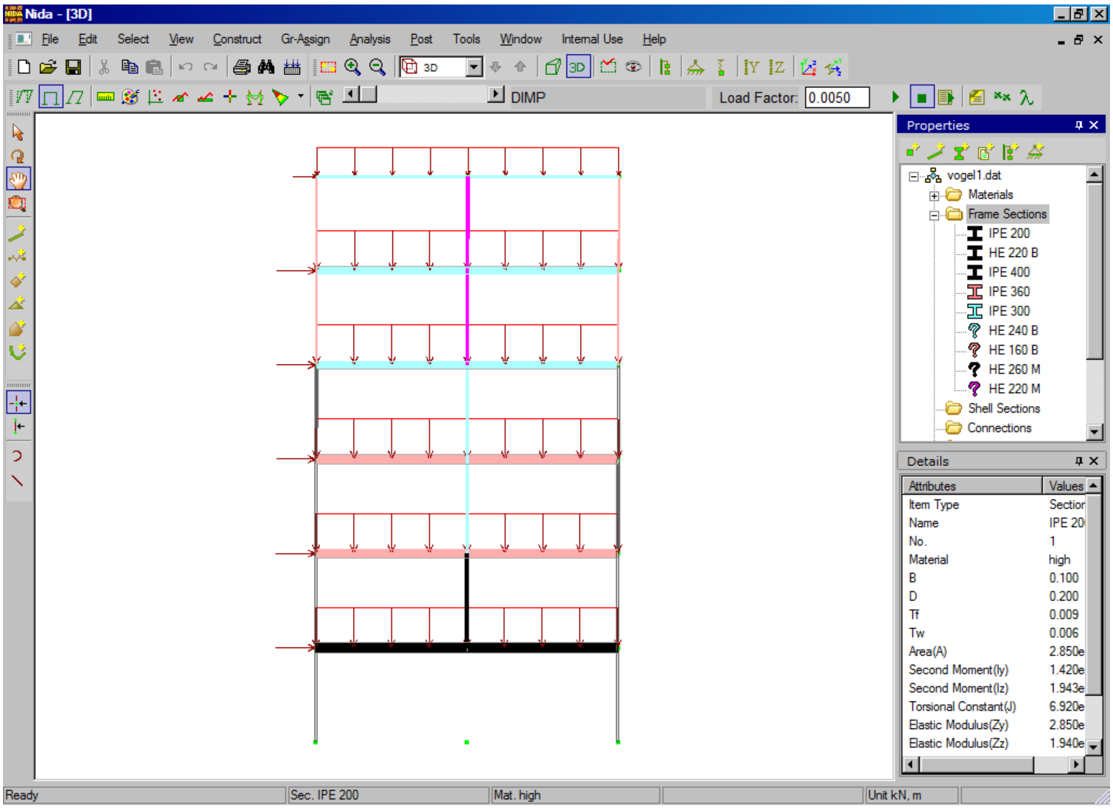


Figure 7 Horizontal notional force as 0.5% of factored vertical loads

The simulation of out-of-plumbness with notional horizontal force is indicated in Figure 8.

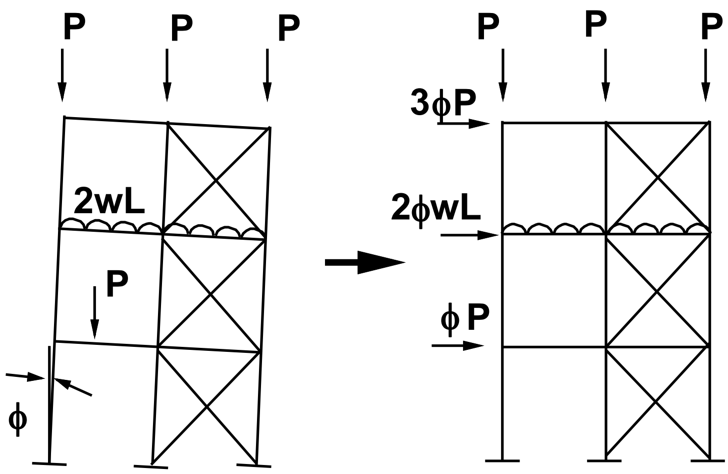


Figure 8 Notional force

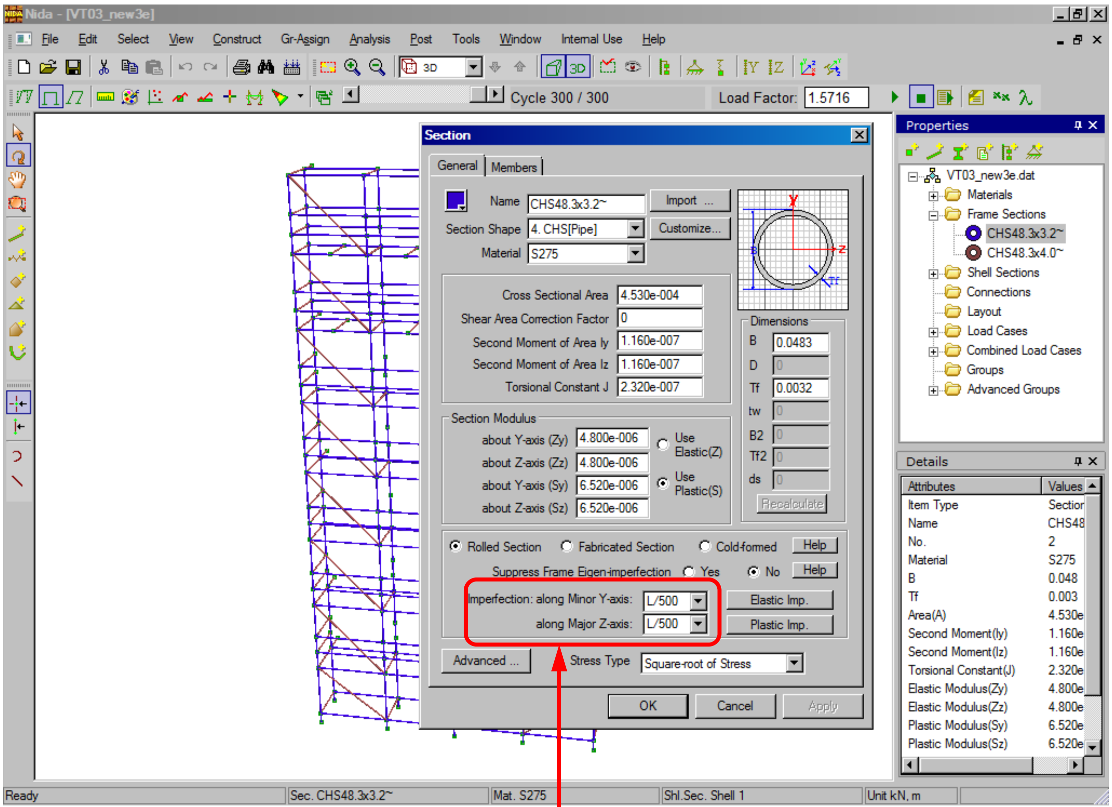
4.2.2 Member Imperfection by Curved Element

For practical members, initial bow and residual stress are unavoidable and must be considered in the buckling strength determination. Table 5.1 in the Eurocode-3 (2005) is the equivalent imperfection for these two sources of imperfections and they are the equivalent imperfection. The value of these imperfections cannot be measured from the initial bow or crookedness of the member but it can be determined by a curve-fitting procedure against the buckling strength vs. slenderness curve. In other words, we can try different values of imperfections to obtain a curve giving a 5% lower bound curve to the experimental curve. We can calculate the imperfection using the available Perry Robertson constants. For a compression member, the equivalent initial bow imperfection specified in Table 5.1 of Eurocode-3 (2005) below may be used in a second order direct analysis. If software uses straight element, it cannot model member imperfection as Table 5.1 and dividing a member to many elements has difficulty in assigning imperfection direction to follow the buckling mode.

Table 5.1: Design values of initial local bow imperfection e_0 / L

Buckling curve acc. to Table 6.1	elastic analysis	plastic analysis
	e_0 / L	e_0 / L
a_0	1 / 350	1 / 300
a	1 / 300	1 / 250
b	1 / 250	1 / 200
c	1 / 200	1 / 150
d	1 / 150	1 / 100

In computer program, we could input the imperfections as follows.



Input of member initial imperfections

Figure 9 Member imperfection in NIDA

The effects of imperfections could be considered approximately (or inaccurately) in member design when using the effective length method and the moment amplification method. We have different buckling curves in Eurocode-3 (2005) for effective length and moment amplification method and these buckling curves correspond to different imperfections for different sections indicated in Table 5.1 in Eurocode-3 (2005).

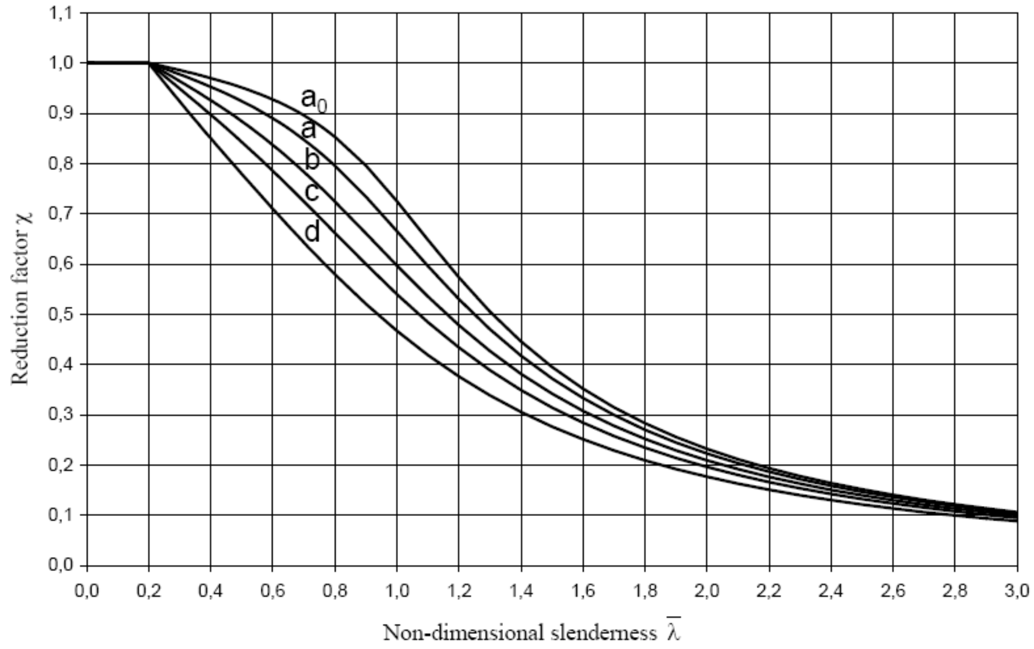


Figure 10 Buckling curves for sections with different imperfections ($\bar{\lambda} = \sqrt{\frac{Ap_y}{P_{cr}}}$)

4.3 Section Capacity Check (ϕ factor)

When the second-order direct analysis with full consideration P-Δ-δ effects and imperfections is used, we need not consider individual stability check nor effective length at all. According to clause 5.2.2(7) of Eurocode-3(2005) below, cross section capacity check in Equation (4) is sufficient to ensure the safety of the structure.

> (7) In accordance with (3) the stability of individual members should be checked according to the following:
>
> a) If second order effects in individual members and relevant member imperfections (see 5.3.4) are totally accounted for in the global analysis of the structure, no individual stability check for the members according to 6.3 is necessary.

And the strength and stability of members and frames can be checked symbolically on the cross section of every member as,

$$\frac{N_{Ed}}{Af_y} + \frac{(M_{y,Ed} + P\Delta_y + P\delta_y)}{M_{y,Rd}} + \frac{(M_{z,Ed} + P\Delta_z + P\delta_z)}{M_{z,Rd}} = \phi \leq 1 \qquad (4)$$

Where,

$M_{y.Ed}$ and $M_{z.Ed}$ the (action) design bending moment about the y and z axis and without consideration of second-order effects,

$M_{y.Rd}$ and $M_{z,Rd}$ are the design capacity to bending moment about the y- and z-axis. They can be considered as moment capacity about principal Y- and Z-axes (i.e. =fy S or =fy Z where S, Z=plastic or elastic modulus) and if the sections are also under the influence of lateral-torsional beam buckling, the moment resistance should be replaced as, $M_{y,Rd}$ = f_b Sy or f_b Zy).(For lateral-torsional buckling of beams, the use of M_b is a design approach and the rigorous method could include additional M-φ moment due to moment about the principal axis and the twist). A = cross sectional area, fy = design strength, ϕ = section capacity factor. If $\phi > 1$, member fails in section capacity check.

Δ = nodal displacement due to out-of-plumbness frame imperfections plus sway induced by loads in the frame

δ = displacement due to member curvature / bowing due to initial imperfection plus load at ends and along member length of a member. This is calculated using a curved stability function proposed by Chan and Gu (2000) and the element is not the cubic element which assumes the moment varies linearly along a member commonly adopted for linear analysis.

In software NIDA, different values of ϕ is indicated by different colours for easy identification and it could also be viewed in an Excel file for all load cases for easy identification.

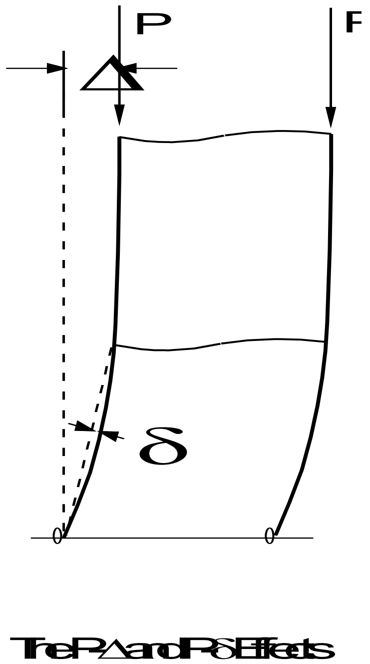
Take note that moments $M_{y.Rd}$ and $M_{y,Rd}$ and P-Δ and P-δ moments in Equation (4) are not evaluated separately and they are included conceptually in the moment expressions.

4.4 Second-Order Direct Elastic and Plastic Analysis

For ***second-order direct elastic analysis***, any member with $\phi>1$ indicates the reaching of design load of the complete structure.

For ***second-order direct plastic analysis***, members with $\phi>1$ will be inserted a plastic hinge until a collapse mechanism is formed and the ultimate design load needs to be found from the load vs. deflection curve.

As seen below, when we consider P-Δ and P-δ effects with their imperfections appropriately using curved element with curvature and buckling mode as imperfection mode, we need not worry hand-checking for flexural buckling, sway and non-sway frames, moment amplifications, change of stiffness in members when loaded, yielding, eccentric moment effect on member buckling, joint stiffness effects on column and frame buckling …



P-Δ and P-δ effects with imperfections

If we consider both P-δ and P-Δ effects & imperfections, we need not worry about the effective length and the design is more efficient and accurate.

Paradox : Why we do not simply or directly include the buckling effects by calculating the P-Δ and P-δ moments in analysis so we need not reduce the buckling strength for these P-delta effects ? (i.e. the effective length method which is affected by sway or non-sway nature of a frame and also it does not consider change of member stiffness required by codes)

Figure 11 P-δ and P-Δ effects in replacement of effective length and moment amplification method

4.5 Local and Lateral-Torsional Buckling

Local and lateral-torsional buckling needs to be considered in the design. As they are a type of local behavior and not related or sensitive to sway or non-sway characteristics of a frame, direct use of code formulae is adequate and has been considered in NIDA. For example, we need not classify a frame a sway or non-sway in design for local plate buckling nor lateral-torsional beam buckling and we could directly use formulae in codes for their design check. These local buckling effects can be considered by a method like the one by Trahair and Chan (2002).

5. Examples

These examples should be studied by educational version of NIDA which can be obtained by contacting the authors of this document at ceslchan@polyu.edu.hk .

Also, the simulation movie of examples 1 and 3 can be seen by typing "thenidachan" in "Youtube©". In these simulation movies, the colour of the members representing the values of section capacity factor ϕ in Equation (4) of this note are changing with increasing loads.

5.1 Tutorial 1 – Simple benchmark example for testing of software : A strut under axial force

The column of CHS 88.9x3.2, grade S275 steel and length 5m has a boundary condition as one end pin and one end fixed. Determine the axial load resistance and buckling load of the column by second order analysis. Do not assume effective length for the column as it is unknown for most compression members in real frames.

Area, $A = 8.6200 \times 10^{-4}\ m^2$

Second moment of area, $I = 7.9200 \times 10^{-7}\ m^4$

Elastic modulus, $Z = 1.7800 \times 10^{-5} m^3$

Plastic modulus, $S = 2.1360 \times 10^{-5} m^3$

Design strength or yield stress, $p_y = 275$ *MPa*

Software unable to do the first example should never be used to do a second-order analysis because of the following reasons.

If software cannot tell the design resistance or bucking load $P_{cr} = \pi^2 EI/(0.7L)^2$ as the load when the load vs. deflection curve is flat for the above column (see Figure 12), it cannot tell the design and buckling resistance of a frame.

Dividing a member to two equal-length elements is unable to check the critical section at the location with maximum curvature which is not at the mid-length.

Input of imperfections is too inconvenient when we use 2 elements per member since we need to follow the buckling mode shape otherwise the imperfection does good instead of harm to the design resistance of a frame which is NOT what we want.

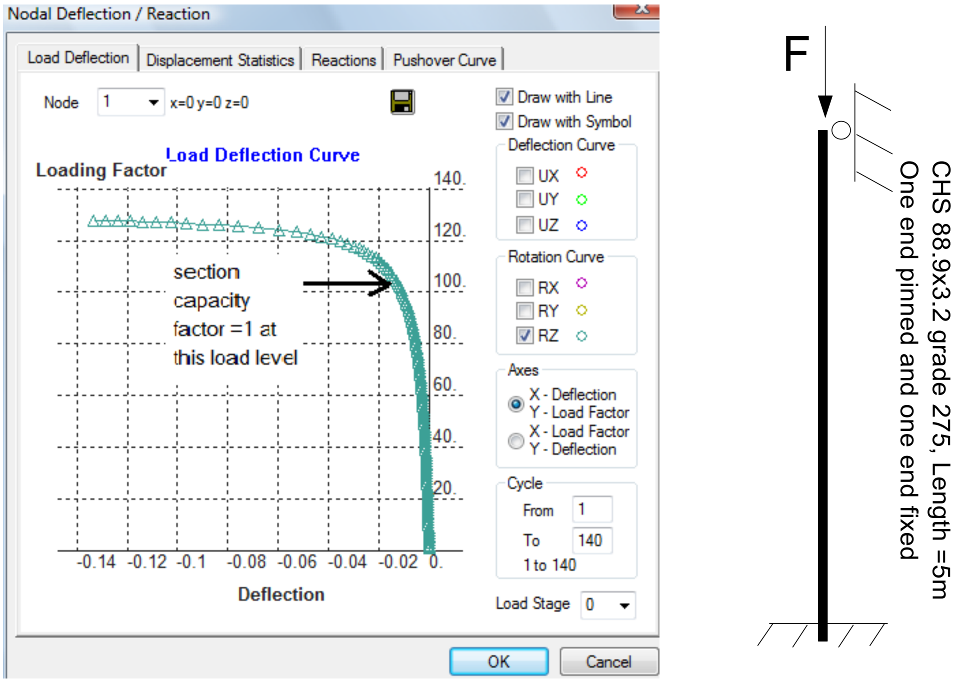


Figure 12 A strut under axial force

This results in the answer being over-estimated and the member is over-designed.

Method	Buckling resistance P_c (kN)	Error
1st order linear with L_e/L=0.7	108.9	N.A.
2nd order with imperfection L/500 to code	102.2	-6%
2nd order with imperfection L/1000 (not to code)	113.4	+4%
No imperfection	p_yA = 234	+118%

Using the effective length factor of L_e/L =0.7, the 1st order linear analysis from code is 108.9 kN

If we assume the imperfection as L/500 which is recommended in Hong Kong Steel Code (2011), the computed resistance is 102.2 kN (-6%) which is conservative.

If we assume the imperfection as L/1000 (smaller than Hong Kong Steel Code 2011), the computed resistance is 113.4 kN (+4%). This shows imperfection is important in determining the resistance. If one ignores imperfection, the resistance becomes pyA = 237 kN (+118% !) since no load vs deflection path could be plotted for a perfectly straight column which does not know where to deflect when under load.

5.2 Tutorial 2 – Snap-through buckling analysis of hexagonal frame

Please use NIDA to obtain the result for the hexagonal frame shown in Figure 13. Just type in the numbers and NIDA contains no imperial units. Assume diameter =0.793 in.

Case 1 Assume all supports are hinged

Case 2 Assume one support hinge and the opposite support restrained in one direction to prevent torsional mechanism and all other supports rollers.

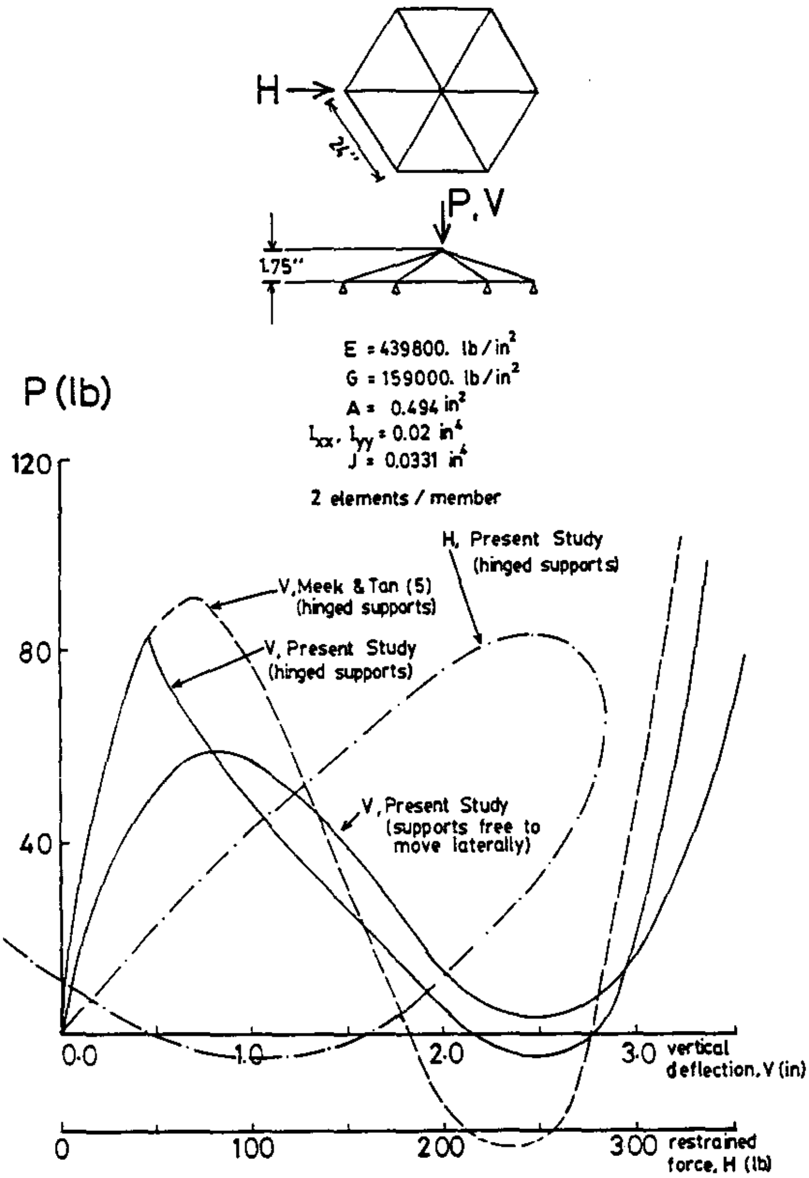


Figure 13 Snap-through buckling analysis of hexagonal frame (Chan, 1988)

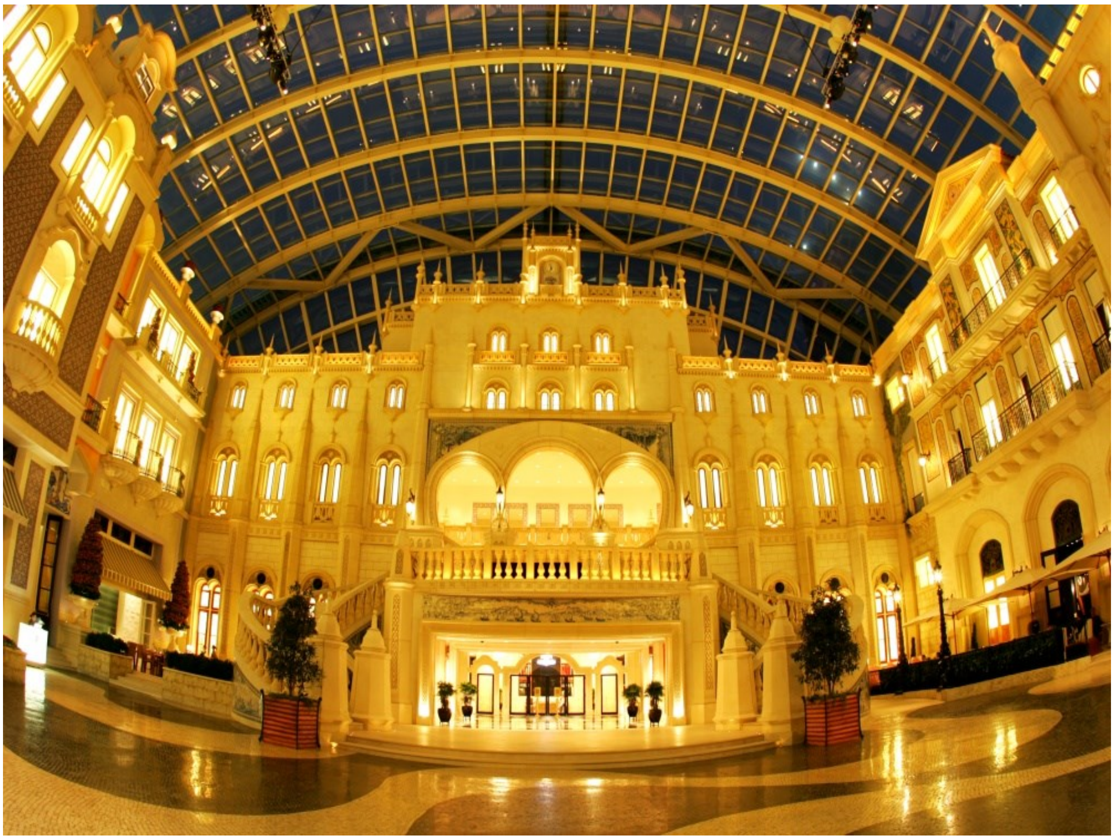
Nodal coordinates:

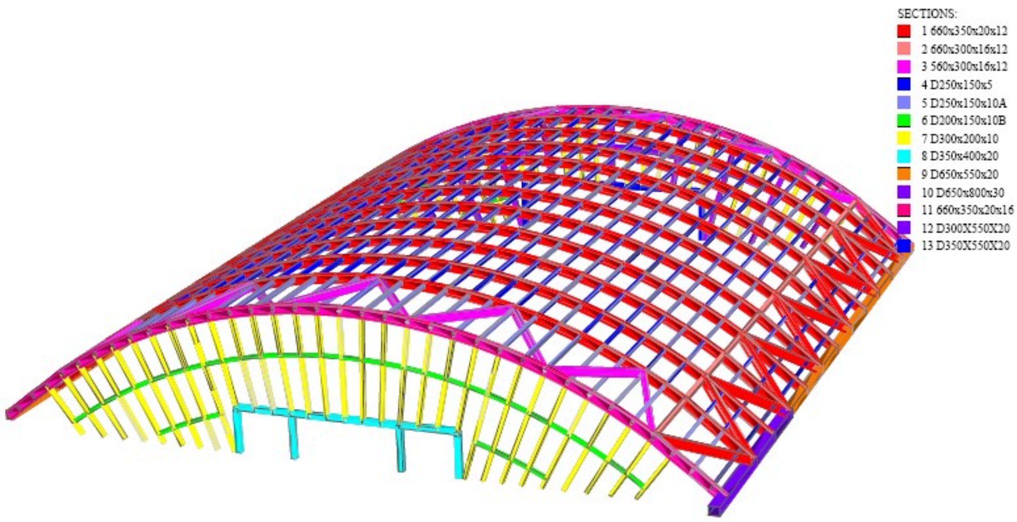
Node	X-coordinate	Y-coordinate	Z-coordinate
1	0	0	20.78
2	12	0	0
3	36	0	0
4	48	0	20.78
5	36	0	41.57
6	12	0	41.57
7	24	1.75	20.78

The simulation movie can be seen by typing "thenidachan" in "Youtube©".

5.3 Tutorial 3 – Second-order analysis for design of the skylight

This example checks the practicality of second-order direct analysis via a practical steel roof.




SECTIONS:
1 660x350x20x12
2 660x300x16x12
3 560x300x16x12
4 D250x150x5
5 D250x150x10A
6 D200x150x10B
7 D300x200x10
8 D350x400x20
9 D650x550x20
10 D650x800x30
11 660x350x20x16
12 D300X550X20
13 D350X550X20

6. Second-Order Direct Analysis vs. Effective Length

The differences between the new Second-order Direct Analysis and old Linear Analysis (LA) are summarised as follows.

Second-order direct analysis	**Linear analysis**
Design is combined with analysis – a system–based approach	Design is needed after analysis – a member-based approach
Design is by "section capacity check" and no more member design see note 1	Design is member-by-member
No need to assume effective length (L_e) which is replaced by P-Δ and P-δ effects computed in software	Uncertain effective length (L_e) is required to assume
Frame classification is not needed	Classification of frame is required but may not be possible for some types of frames like domes
More reliability as buckling is checked by rigorous non-linear theory	Less reliable leading to over-designing because of conservative assumption of buckling effect and effective length factor (Le/L)
Can be used in codes and it is recommended in codes as seen below	Can only be used in codes $\lambda_{cr} \geq 3$ for sway mode or structure dominated by sway buckling, but it is non-preferred in EC3 and LFRD
Design speed is fast with high efficiency as effective length is not needed for assessment	Tedious design procedure for assessment of effective length and thus less efficient
Safer as critical members will not be under-designed by wrong assumption of effective length	Less safe as some critical members are under-designed.
Lighter structural weight as redundant members are not over-designed	Heavy structural weight as redundant members are always over-designed
Wider application as seismic time-history is also based on second-order plastic analysis	More restrictive uses as it cannot be used in time-history seismic design
Complex analysis like structural fire engineering, progressive collapse analysis	Limited to simple problems and cannot be used in complex analysis like progressive collapse

can be carried out	analysis where effective length changes continuously
Simple to use as imperfections are automatically computed in software for all load cases without need of separated assessment.	Tedious to use as effective length varies with in different load cases and their computation cannot be automatic
Change of stiffness for compression and tension members are automatic and saves efforts in using reduction factor in LFRD code (2010) and also more accurate.	Analysis results are doubtful and the bending moment cannot be directly used. For e.g., a compression brace takes much smaller load than tension brace and this effect cannot be noted by linear analysis

Note 1: Clause 5.2.2(7) of Eurocode-3 states the individual member check is not needed in Second-order Direct Analysis method of design.

(7) In accordance with (3) the stability of individual members should be checked according to the following:

a) If second order effects in individual members and relevant member imperfections (see 5.3.4) are totally accounted for in the global analysis of the structure, no individual stability check for the members according to 6.3 is necessary.

7. Conclusions

Second-order direct analysis is a very useful tool for design but its use must be done very carefully otherwise under and over-design will occur. This brief lecture note gives a basic concept of the method and indicates the need for carefulness of engineers to use the method with suitable software. The linear, second-order indirect and second-order direct analysis are discussed and the worked examples are solved by the last method reliably and effectively.

See you again in www.nidacse.com

Thank you !

8. References

AISC360, 2010. "Specification for Structural Steel Buildings", American Institute of Steel Construction Chicago.

Chan, S.L. ,Geometric and Material Nonlinear Analysis of Beam-Columns and Frames using the Minimum Residual Displacement Method, *International Journal for Numerical Methods in Engineering*, vol. 26, 1988, pp.2657-2669.

Chan, S.L. and Gu, J.X., "Exact tangent stiffness for imperfect beam-column members", *Journal of Structural Engineering*, ASCE, 2000, vol.126, no.9, September, 2000, pp.1094-1101.

Chan, S.L. and Chui, P.P.T.,"Non-linear Static and Cyclic analysis of semi-rigid steel frames", Elsevier Science, 2000, pp.336.

Eurocode 3, Design of steel structures, BS EN 1993-1-1:2005.

Hong Kong Steel Code, Code of Practice for Structural Use of Steel 2011, Buildings Department, Hong Kong SAR Government.

Liew, R. J. Y., Chen, H., Shanmugam, N. E. and Chen, W. F., Improved nonlinear plastic hinge analysis of space frames. Engineering Structures, Elsevier, UK, 2000, 22, 1324-1338

Liu, Y.P., Chan, S.L. and Lam, D., Section V. Case Study for semi-rigid design, Section V, in Semi-rigid Connections Handbook, edited by Wai-Fah Chen, Norimitsu Kishi, and Masato Komuro–January 2011.

Trahair, N.S. and Chan, S.L., "Out-of-Plane Advanced Analysis of Steel Structures", Department of Civil Engineering, The University of Sydney, Research report No. R823, 2002.