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ICASS '09

edited by
S. L. Chan
Department of Civil and Structural Engineering, The Hong Kong Polytechnic University

Volume I
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Progress in Structural Stability and Dynamics

ICASS '09 / IJSSD

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Progress in Structural Stability and Dynamics

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These three volumes of proceedings contain 16 invited keynote papers and 146 contributed papers presented in the 6th international conference series on Advances in Steel Structures, with the first, second and third of the conference series held in Hong Kong, fourth in Shanghai and fifth in Singapore. As with the five previous conferences, this conference is intended to provide a forum for discussion and dissemination by researchers and designers of recent advances in analysis, behaviour, design and construction of steel, aluminium and composite steel-concrete structures.

An international conference of this magnitude would not have been possible without the supports and contributions of many individuals and organizations. The strong supports from The Hong Kong Polytechnic University, The Joint HKIE/ IStructE Structural Division, The Hong Kong Institute of Steel Construction and the sponsors from the academics and industry are greatly appreciated. The editorial contribution by Professors CM Wang, YB Yang, JN Reddy and R. R. asheed in organizing the Conference is greatly appreciated.

Last but not the least, we would like to thank all those involved in the conference who including staff in the General Office of the Department of Civil and Structural Engineering of the Hong Kong Polytechnic University, especially to Miss Miya Lau who assisted to organize the conference in the past year.
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RESEARCH AND DEVELOPMENT TOWARDS SUSTAINABLE STEEL CONSTRUCTION IN THE UNITED STATES

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Steel, sustainability, industry efforts, efficiency, economical, energy, recycling, economics, advanced applications, research

ABSTRACT
Sustainability is a major issue in today's business and construction climates, and governmental and private sector organizations all over the world are pursuing significant efficiencies for all areas of society. Construction plays a key role in these efforts, not the least because of its major impact on the gross domestic product (GDP), which is approximately 13 percent in the US. As a result, major research entities in the form of university, industry, and government researchers and other professionals are pursuing advanced studies that will demonstrate all-around efficient applications of construction materials and systems. The key word is sustainability, and through that the reduced emission of greenhouse gases in all forms.

The steel industry has taken a lead role in a large number of sustainability efforts in the United States, very much because steel is completely recyclable and therefore a fully sustainable material. Focusing on all aspects of construction and the efficient use of steel, combined with the changes that have taken place in steelmaking processes over the past 10 to 15 years, the industry has assembled an aggressive program for energy-efficient and economical construction. Most of the steel mills are now based on electric arc furnaces, using scrap steel and continuous casting of shapes, plates, and sheet. This has led to increased production efficiency and reduced energy consumption of waste materials, along with a very much improved environmental performance. Structural research and development play a central role in all of these activities, including the standardization efforts that are so important for designers and fabricators.
INTRODUCTION

Construction as a whole accounts for approximately 13 percent of the gross domestic product (GDP) of the United States. Recognizing the recyclability and other characteristics of steel, the industry has taken a lead role in a large number of sustainability efforts in the US. The efficiency of steel as a primary construction material recognizes the changes that have taken place in steelmaking processes over the past 15 to 20 years. Thus, the industry has now formulated an aggressive program for energy-efficient and economical construction. Most of the mills utilize electric arc furnaces which are fed steel scrap in lieu of iron ore and coke, and continuous casting of shapes, plates and sheet is used instead of the processing of large ingots into blooms and shapes and other products. This has led to significantly increased production efficiency and economies, with approximately 0.5 man-hours now being used to produce one ton of steel. At the same time the consumption of energy has been reduced by more than 60 percent, and the level of environmental pollutants and waste materials has been cut by nearly 70 percent.

The industry is working very closely with the United States Green Building Council (USGBC), which has evolved over the past dozen years into a major force for sustainable (“green”) construction. Activities such as the Leadership in Energy and Environmental Design (LEED) program have become major factors for architects and engineers, and a large number of individuals have become LEED Accredited Professionals through rigorous examinations. Although USGBC and the LEED program focus on buildings only, in all construction materials, the principles and methods of evaluation are now being applied to any number of different types of projects. The key words are energy, efficiency, economy and environmental performance.

THE STRUCTURAL STEEL SUPPLY CHAIN

In order to provide a realistic and detailed assessment of the sustainability capacities of steel and steel construction, it is important to be familiar with the key segments of the industry. A representative description of the various elements of the industry focuses on the following types of companies:

- Producers (steel mills)
- Steel service centers (stockists)
- Fabricators
- Erectors

For classification purposes in this paper, producers include mills that make hot-rolled steel shapes and plates and various forms of hot-rolled bars, and also sheet and strip products. Also in this group are companies that make cold-formed steel shapes from sheet, including roll-forming and press brake operations. Pre-engineered buildings (“metal buildings” in the United States) for storage warehouses, residential construction and rack structures are counted among fabricators.

For many years in the United States there was an antagonistic relationship between the steel mills and the other components of the steel supply chain. The steel mills were members of the American Iron and Steel Institute (AISI) – the fabricators and some special suppliers to the fabricating industry were members or associates of the American Institute of
Steel Construction (AISC). Companies such as steel erectors were members of neither organization. The overall steel construction market was very active and large projects were undertaken for many years, but the intra-industry relations were contentious and difficult under the best of circumstances. In many ways the business climate suffered badly, and although the construction economy in general was good, operations stagnated and too much time and effort was spent on entirely unproductive pursuits.

The industry changed drastically in the early 1980-s, when the construction market and the steel industry suffered enormous losses and many companies closed their doors. Integrated steel companies such as US Steel and Bethlehem Steel downsized and shuttered a number of their plants. Cities like Pittsburgh and Bethlehem, Pennsylvania; Cleveland, Ohio and Chicago, Illinois entered a period of depression-like conditions. But this was also the time the so-called mini-mills had become productive and major contributors to the market – steel mills that utilized electric arc furnaces with steel scrap and continuous casting in lieu of the basic oxygen furnaces with iron ore and coke and ingots of the integrated mills. The mini-mills have since become the center of activity of the American steel industry, with highly efficient production and competitive pricing for all types of products, including all of the production of hot-rolled shapes. In a word, they are no longer “mini”-mills by any stretch of the imagination. The American integrated mills are now much fewer and smaller; they produce only steel plate, sheet and strip, and provide but a relatively small part of the steel for construction, and then primarily for the welded built-up girders of industrial structures and bridges.

In a near-revolutionary development, the rolled shape mills joined AISC as full, active members in the late 1990-s. This opened up communications among the various companies and removed much of the former antagonism. Most importantly, it brought stability to the complex inter-relationships between all of the segments of the steel supply chain, and the steel construction industry has since become a model of efficiency and outlook for further developments of processes and products. As a result, the sustainability issues that are at the forefront today are now being addressed aggressively by all of the companies as a whole.

Effectively, the steel mills and their products fundamentally establish the sustainability characteristics of the material. The service centers, the fabricators and the erectors contribute to the use of a highly recycled material, but steel in itself is a function of the work of the mills. This is further emphasized by the Steel Recycling Institute, which was founded by the industry in the 1990-s. Taken together, these efforts attest to the importance of the entire sustainability program. It also illustrates why it is critical for the industry to address the subjects jointly – as well as the fact that the government and the various building code groups have the legislative power to ensure that sustainability in all forms is properly covered.

Finally, in the United States the design codes are prepared by private industry trade associations and professional organization groups, along with very significant input from researchers in academia. The code requirements are of course an integral part of the sustainability issue, primarily because efficient and economical usage of the materials is a fundamental criterion for state-of-the-art codes. As a result, major contributions are provided by all interest groups, and the outcome reflects extensive use of steel along with thoughtful consideration of all aspects of sustainability.
 SOME CURRENT INDUSTRY INITIATIVES AND RESULTS

The institutes representing the various components of the American steel production and steel construction industry are currently focusing on a number of aggressive undertakings. The fundamental principles are summarized by the Four “R”s program, focusing on (1) Reduce, (2) Reuse, (3) Recycle and (4) Restore. The program details and primary directions are given in the following.

“Reduce” Efforts

These efforts aim at reduced energy consumption in the production of steel, as well as the reduction of so-called greenhouse gas emissions (primarily carbon dioxide (CO₂)). The results to date show that energy consumption has been reduced by 60 percent since 1960; the reduction was 29 percent per ton during the 1990s alone. The energy usage reduction has largely been achieved through the change from basic oxygen to electric arc furnace operations of the mills. Data on greenhouse gases are more recent, specifically, to the effect that CO₂ emissions were reduced by 47 percent between 1990 and 2005. In the same period the overall industry emissions were reduced by nearly 70 percent. This outcome can be compared to the suggested measures of the Kyoto Protocol, which would have required reductions for all US industries of only 5.2% by 2012.

“Reuse” Efforts

The reuse activities focus on waste reduction, and have traditionally been more important for the operations of the traditional integrated mills. The concepts certainly have worldwide applicability, since approximately two thirds of the world’s steel mills are still based on iron ore and coke. For example, blast furnace slag is reprocessed for use in road building, glass making and other commercial applications. Similarly, the large quantities of gas that are produced in the making of coke are reprocessed and used for fuel in various applications.

“Recycle” Efforts

Steel is recognized as the most recycled material in the world. This applies very much to today’s electric arc furnace (EAF) steels, which are primarily based on the use of steel scrap. For example, the structural shapes that are now produced in the US average 88 percent recycled content (the remainder of the content is made up of materials such as general ferrous scrap, iron carbide (93 percent iron) and so-called pig iron, which is almost pure iron), and the shapes themselves are recycled at a rate of 98 percent. Hollow structural sections (rectangular or round) (referred to as HSS in the United States) are preferred by some architects and engineers for certain projects; the recycled content of such products is 88 percent when produced by an EAF mill; it is 29 percent when it is produced by a basic oxygen furnace (BOF) mill. Many of the sheet and plate products that are used to make HSS sections come from BOF mills; wide-flange shapes in the US come only from EAF mills. This actually means that HSS tubes are less “green” than wide-flange shapes, a fact that is generally not known. Of course, the HSS sections will eventually be recycled.

“Restore” Efforts

1 In this context it is interesting to note that the United States is not a signatory to the Kyoto Protocol. The attitude of the American steel industry reflects significant attention to societal and world needs,
The restoration activities of the American steel industry are much focused on the redevelopment of old steel industry properties, including land. The land is sometimes referred to as “brownfields”; many such properties have problems in the form of ground contamination of various forms. New land uses typically focus on light commercial activity. Old mill buildings may be torn down and the steel recycled; in some cases the buildings have been remade into offices or shopping malls, if suitable. As a showcase example, some of the Bethlehem Steel buildings in Bethlehem, Pennsylvania are being turned into a historical museum dedicated to the American steel industry.

Some “4R” Programs-Related Efforts

Quality certification programs have become very common in a number of industries, and the American Institute of Steel Construction took the lead when it established its program 20 years ago. It is similar to an ISO certification effort, although the AISC program focuses strictly on the needs of the fabricated steel industry. Such programs have now been expanded to steel erector companies and pre-engineered building companies. All of these efforts aim at achieving higher quality fabrication and construction that is consistent with the requirements of the codes, ensuring that sustainability issues are properly accounted for. It is anticipated that the AISC Specification, the steel design standard for the United States, will incorporate a chapter on Quality Assurance and Quality Control (QA/QC) in the next, the 2010 edition. This will earn additional “points” for steel construction under the LEED program of the USGBC, which is discussed in the following section of this paper...

STEEL CONSTRUCTION AND GREEN BUILDING

The United States Green Building Council (USGBC) has become a major force for sustainable or “green” construction of buildings. The well-known LEED program has evolved into an effort that provides professional certification for accredited individuals. The membership of the council of 13,500 individuals includes a broad variety of architects, engineers, contractors, representatives of material suppliers and building operation companies that supply HVAC equipment, for example, as well as different types of companies, some universities, etc. In brief, the activities of the USGBC and its LEED AP-s are now recognized as authoritative efforts for all sustainability aspects of building construction.

At this time relatively little attention is paid to the issues of sustainability for bridges and similar structures in the US, although the impact of the use of steel for elevated highways and transit structures is likely to have measurable effects. The difference from building construction is primarily the magnitude of the volume of materials use in such construction. However, research and development that is directed to the use of so-called high performance steels (HPS) has accelerated over the past five years, and it is recognized that the benefits are not only related to the use of steel itself, but also to the removal of certain forms of construction that offer a negative impact on sustainability. These efforts will likely increase significantly, although it is difficult at this time to assess how a LEED-type program could be developed for this area of construction.
LEED Standards and Their Applicability for Buildings

The LEED standards apply to buildings in any construction material, and the various award levels are based on a point system that recognizes all aspects of sustainability. Depending on the points, a project is awarded according to the following designations:

- Certified
- Silver
- Gold
- Platinum

The LEED standards have evolved over the past 15 years, and all proposed criteria are voted on by the members of USGBC. The standards are now very detailed in their assessment of the following features:

- New construction
- Existing buildings
- Commercial interiors
- Building core and shell
- Building functions

In assessing points to the individual projects, the standards are further refined to recognize the following categories, and the points that are awarded are then used to classify the project according to the USGBC award levels, as follows:

1. Sustainable construction sites
2. Efficiency of water supply
3. Energy and atmosphere
4. Materials and resources
5. Indoor environment quality
6. Innovation and design process

Obviously the sustainability issue goes well beyond the individual construction material and how and where it is produced, which is why the LEED APs are required to have a broad background and understanding of all aspects of the built environment.

All of the above subjects are important in the context of sustainability, but for construction materials and structural systems item numbers 3, 4 and 6 are especially applicable. For steel, the current materials and particularly the products that are made through the electric arc furnace and continuous casting processes do extremely well, as a result of the very large content of recycled materials (scrap, etc.) and the near-100 percent of steel that is recycled. The waste that is contributed by steel is effectively zero, since essentially all is recycled.

One issue that is difficult to assess is the regional materials question, as it is being addressed in the US today. Specifically, materials that can be provided from a local or near-local plant or supplier will get a higher LEED score. This is because the environmental impact of the transportation of the goods from the plant to the construction site will be less than if the goods have to be shipped from a far-away location. This is one of the advantages of the largest US steel producer today, the Nucor Corporation, since it has steel mills in many areas of the country, and transportation on distances are therefore somewhat limited. But this also
makes the assessment procedure much more complex, since each project has to be evaluated on an individual basis, and local versus larger region supply of steel scrap, for example, can be difficult to determine. Currently the American LEED criteria award regional material points on the basis of the percentage of the total mill scrap that is supplied from within a circle of 800 km (500 miles) from the project.

**Current Green Building Construction in the United States**

Based on information from the McGraw-Hill Construction Analysis group’s “SmartMarket Trends Report 2008” (February 2008), the current value of green building construction starts is larger than $12 billion. The volume is expected to increase to $60 billion by 2010. In the same vein, LEED projects in 2002 accounted for approximately 8 million square meters; for 2006 it was 64 million square meters. Considering the percentage of the non-residential and residential construction market that is attributable to steel, these numbers are impressive. It is also worth noting that the number and magnitudes of LEED projects are increasing exponentially.

**Design of Structures for Deconstruction**

With careful attention to constructability and de-constructability during the design and planning process, the entire life span of a structure can be considered and assessed very efficiently insofar as sustainability is concerned. In this context, it is recognized that 25 to 30 percent of the total waste produced each year in the US and UK can be attributed to material waste from new construction, renovation and demolition. 92 percent of the waste comes from renovation and de-molition – 8 percent comes from new construction. Much of the waste ends up in landfills, despite the fact that significant amounts can be recycled or at least salvaged, reducing the impact on landfills.

Structural steel in all forms should be recycled – and usually is, but that depends to some extent on the locale where the projects are undertaken. The impacts on society through sustainability are serious, and a concerted education is currently underway. It is led by the USGBC, but for steel the various arms of the industry are playing a lead role. Attention is especially paid by the design-oriented groups within the industry as well as the parts of organizations like the AISC and AISI where design standards are developed. Due to the close relationship between constructability and de-constructability, designs satisfying both needs simultaneously are becoming the norm – and this is at it should be.

Constructability is aided by simple systems and details, and the principles of building in steel are critical. Thus, the analyses focus on prefabrication and the development of practical modules, and especially the types of connections and fastening systems that are utilized for steel facilitate simpler assembly as well as disassembly. In this respect the standardization of connections is particularly useful, and such programs have been pursued vigorously by AISC and related organizations for many years. The design process therefore can be arranged to allow for considerations of reuse, which is one of the primary areas of the sustainability efforts of the US steel industry. The structure should be simple to assemble and build; much of the work will be done off-site, providing higher and more reliable quality; the structure will be completed sooner – and the owner will be able to derive revenue at a much earlier stage. Sustainability is therefore achieved at all levels of construction.
Reversing the construction process, joints that were easy to assemble will be easier to take apart; the tools that are used are simple; the energy consumed by the deconstruction process will be much lower – and the waste materials are reduced to very small amounts. None of the waste will be structural steel, which is fully recycled.

The benefits of a total design thus will include the entire life cycle of the structure, which means that realistic, life-cycle construction economies can be used to assess the true construction costs. These are among the most important of the benefits of carefully planned and built steel structures.

SUMMARY

An examination of the current sustainability efforts of the steel and construction industries of the United States has been provided. The use of steel has a long and successful history in the US, starting with the first tall building (1883) and bridge structure (1874) using what is now referred to as structural or constructional steel. The industry has gone through a major reshaping process over the past 15 to 20 years, from the mills with basic oxygen furnaces to today’s highly efficient mills with electric arc furnaces and continuous casting of shapes, plates and sheets. The quality has been significantly improved, assuring users that the structures will perform as intended.

The focus of sustainability has come to the fore over the past 15 years, and the steel industry is enjoying a reputation as the one with the most sustainable construction material of all. Research, design, planning and construction processes incorporate life-cycle considerations, and waste and related products are being minimized.

The industry focus continues to be on all aspects of energy, efficiency, economy and environmental performance. It is enjoying success in all of these efforts.

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JOINTS IN STEEL FRAMES SUBJECTED TO A COMPARTMENT FIRE: A T-STUB DESIGN MODEL

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KEYWORDS

Bolted, compartment fire, component method, composite frames, joints, steel frames, T-stubs.

ABSTRACT

This paper describes an analytical mechanics-based model to investigate the behaviour of bolted steel T-stub assemblies in joints in steel frames when subjected to elevated temperatures that are caused by a compartment fire. The modelling is unique, insofar as it considers the physical interaction of the end plate and column flange. The theoretical approach is based on the premise that the possible failure modes can be derived from simple beam theory, and the analytical representation is validated against tests published elsewhere. It is shown that the model can predict accurately the behaviour of the T-stub portion of the connection, and it has potential for including in prescriptive codes which adopt the component method of joint design.

INTRODUCTION

It is commonly accepted that the so-called component method [1] affords a rational procedure for joint or connection design in steel frames, and it has been embraced in detail in the Eurocode 3 [2]. A useful part of the component approach is the concept of a T-stub, that appears to have been first introduced by Zoetemeijer [3] in 1974, and it can model specific components such as a column flange in bending bolted to a beam end-plate in bending as shown in Figure 1.

Predictions of the force-displacement response of bolted steel T-stubs at ambient temperature have been developed by several researchers, including Agerskov [4] who reported an analytical model for the tension zone of a connection which he assumed reached its yield capacity when the end plate (the T-stub flange) developed its yield moment at the toe of the fillet weld; this model assuming the end plate to be attached to an infinitely rigid stratum comprising of the column flange. This model was extended by Shi et al. [5] to consider a flexible stratum. Other contributions at ambient temperature have been provided by Piluso et al. [6,7] Swanson and Leon [8,9] and others.

The behaviour of T-stub assemblies at elevated temperatures, as caused by a compartment fire in a steel framed building, has been studied by relatively few researchers. Spyrou and Davison [10] and Spyrou et al. [11,12] reported both analytical modelling and test results for bolted T-stubs. Their work assumed the end plate to be founded on a rigid medium (i.e. the column flange was rigid), but in reality the bolt forces are influenced
significantly by the flexible column flange, and some failure modes associated with a flexible column flange [5] are potentially excluded under the rigid-stratum assumption.

![Diagram of T-stub assembly and Free body diagram](image)

(a) T-stub assembly  
(b) Free body diagram

Figure 1: T-stub model

The intention of the present paper is to extend the Spyrou work undertaken at The University of Sheffield by incorporating a more thorough interaction of the end plate and the column flange. By incorporating empirical formulae for the degradation of the mechanical properties of the components of the T-stub assembly with temperature, it is possible to elucidate the 'ductile', 'semi-ductile' or 'brittle' nature of a T-stub. It is shown that increased temperatures can render an assembly that is ductile at ambient temperature as being brittle at elevated temperatures.

**MECHANICAL MODEL**

**General**

The mechanical modelling herein is founded on simple beam theory that incorporates both the flexural stiffness of the end plate and of the column flange, as well as the axial stiffness of the bolts. Theoretical predictions of the response of T-stub assemblies at elevated temperature are clearly complex, and so it is assumed herein that:

- 3-D effects including geometric nonlinearity can be disregarded.
- Bolt bending is neglected, so that only axial bolt deformations are considered.
- Prying action is incorporated by assuming the actions are localised to the edges of the end plate and column flange assembly.
- No prying force develops at plastic hinges at the line of the bolts or when the bolts have yielded.
- The end plate and column flange thicknesses are significantly small to preclude through-depth partial plasticisation.

By invoking classical beam theory [13], if the tension force acting on the T-stub assembly is $2F$ (Figure 1), then compatibility between the deflection of the T-stub at the bolt line and the elongation of the bolt at temperature $T$ requires that

$$\left[ \frac{1}{K_{ET}} + \frac{1}{K_{CT}} \right] F \left( \frac{\ell}{8} - \frac{\ell^3}{6} - B \left( \frac{\ell^2}{2} - \frac{2 \ell^3}{3} \right) \right) = \frac{B}{n_b K_{ST}}, \quad (1)$$

where $\ell = e/L$, $n_b$ is the number of bolts at each bolt line and $K_{ET}$, $K_{CT}$ and $K_{ST}$ are the flexural stiffness of the column flange and end plate and axial stiffness of the bolt at the elevated temperature $T$, given by
\[
K_{ct} = \frac{E_{st}}{L^3} \cdot I_s ; \quad K_{st} = \frac{E_{st}}{L^3} \cdot I_s \quad \text{and} \quad K_{bt} = \frac{A_{bt} E_{bt}}{\ell_b} .
\] (2)

In these equations, \( \ell_b \) is the length of the bolt, and \( E_{st} \) and \( E_{bt} \) are the elastic moduli of the structural steel and bolt material at temperature \( T \) respectively, which are related to their counterpart ambient temperature values \( E_{s0} \) and \( E_{b0} \) by

\[
E_{st} = \eta_{bs} \cdot E_{s0} \quad \text{and} \quad E_{bt} = \eta_{bs} \cdot E_{b0} .
\] (3)

Using these, Eqn. 1 can be written as

\[
\frac{1}{\eta_{bs} K_0} \left[ \alpha F - \beta B \right] = \frac{B}{n_0 \eta_{bs} K_{s0}} ,
\] (4)

where

\[
\alpha = \ell / 8 - \ell^2 / 6 ; \quad \beta = \ell^2 / 2 - 2\ell^3 / 3 ; \quad \text{and} \quad 1/K_0 = 1/K_{s0} + 1/K_{c0} .
\] (5)

Prying forces [14] develop in order to maintain equilibrium when the relative stiffnesses of the bolt and end plate and column flange cause changes in the curvature along the free edges of the tension region, and these forces can be obtained from Eqn. 4 as

\[
B = \alpha \left( \frac{\eta_{bs} \bar{K}_0}{n_0 \eta_{bs} K_{s0}} + \beta \right)^{-1} F \quad \text{and} \quad Q = B - F .
\] (6)

For prying action to develop, \( Q > 0 \) and so the relative stiffnesses of the bolt and the column flange and end plate assembly must be limited by

\[
\bar{K}_0 / K_{s0} \leq n_0 (\alpha - \beta) \eta_{bs} / \eta_{bs} .
\] (7)

If prying forces are present, the bending moment developed at the fillet weld on the end plate and the bolt line in either the end plate or column flanges are

\[
M_f = Bm - Q(e + m) \quad \text{and} \quad M_b = Qe ,
\] (8)

and if \( Q = 0 \) then \( M_f = Bm \) and \( M_b = 0 \). A plastic hinge develops at the fillet weld location, or at the bolt line, when

\[
M_f \text{ or } M_b = \min \left( M_{opt}, M_{opt} \right) ,
\] (9)

where

\[
M_{opt} = \eta_{by} f_{y0} S_e \quad \text{and} \quad M_{opt} = \eta_{by} f_{y0} S_e
\] (10)

are the plastic moments of resistance of the end plate and column flange respectively at temperature \( T \), \( f_{y0} \) and \( f_{y0} \) are the respective yield stresses, and

\[
\eta_{by} = f_{yst} / f_{y0} \]
(11)

describes the temperature-dependent degradation of the yield strength of structural steel.

**Formulation of First Yield Point**

First yield is characterised by the force \( F \) reaching the value

\[
1 F_0 = \begin{cases} 
\min \left( F_f, F_b, F_b \right) & Q > 0 \\
\min \left( F_f, F_b \right) & Q = 0 .
\end{cases}
\] (12)
where $F_1$ and $F_{bl}$ are defined as the values of $F$ applied to the T-stub to produce the first plastic hinge at the weld location in the end plate and the bolt line in the end plate (or in the column flange) and $F_b$ is the value of $F$ causing tensile yielding of the bolts. In the presence of prying forces ($Q > 0$), these forces are given by

$$F_1 = \eta_y \left( \frac{\eta_{Es} + \beta \frac{K_{b_0}}{K_0}}{n_y \eta_{Es}} \right) \min\left( \frac{M_{q0}}{M_{p0}}, \frac{M_{q0}}{m} \right)$$

(13)

$$F_{bl} = \frac{\eta_y \left( \frac{\eta_{Es} + \beta \frac{K_{b_0}}{K_0}}{n_y \eta_{Es}} \right) \min\left( \frac{M_{q0}}{M_{p0}}, \frac{M_{q0}}{m} \right)}{e(\alpha - \beta) \frac{K_{b_0}}{K_0} + m \beta \frac{K_{b_0}}{K_0}}$$

(14)

$$F_b = \eta_y \left( \frac{1}{\alpha} \frac{\eta_{Es}}{n_y \eta_{Es}} + \frac{n_y \beta}{\alpha} \right) B_{q0}$$

(15)

while in the absence of prying forces ($Q = 0$)

$$F_1 = \eta_y \cdot \min\left( \frac{M_{q0}}{m}, \frac{M_{q0}}{m} \right) \text{ and } F_b = n_y \eta_{q0} B_{q0}$$

(16)

where $\eta_{q0} = f_{y0}/f_{p0}$, with $f_{y0}$ and $f_{p0}$ being the yield strength of the bolt material at elevated and ambient temperatures respectively. The total central deflection of the T-stub at first yield (Figure 1) can be obtained from

$$\delta_y = \begin{cases} \frac{1}{24 \eta_{Es} K_0} \left( 1 - \frac{24 \alpha^2}{\eta_{Es} K_0 + \beta} \right) & Q > 0 \\ \frac{1}{n_y \eta_{Es} K_{b_0}} + \frac{m^2}{3L^2 \eta_{Es} K_0} & Q = 0. \end{cases}$$

(17)

(a) Case when $M_{q0} = M_{p0}$
Formation of Subsequent Yield Points

As the temperature increases and the force $F$ increases above its first yield point value $\gamma F_y$, other yielding locations develop that depend on the location of the first hinge.

For case (1) which is defined as that for which the first hinge occurs at the fillet weld on the end plate (or in the column flange), the bending moment at this location equals $M_{\text{opT}}$ (or $M_{\text{opT}}$). A second yield point can form at the bolt line of the T-stub assembly, or in the bolts themselves, characterised by the force $F$ reaching the value $2F_y$, where

$$2F_y = \gamma F_y + \gamma \Delta F$$

with

$$\gamma \Delta F = \min(\Delta F_{bl}, \Delta F_b)$$

in which

$$\Delta F_{bl} = \left[ \min(M_{\text{opT}}, M_{\text{opT}}) - \gamma F_y \right]/m$$

$$\Delta F_b = e\left(n_y \eta_y B_y - \gamma B_y \right)/(m + e)$$

with $\gamma Q_y$ and $\gamma B_y$ being determined by substituting $\gamma F_y$ from Eqn. 12 into Eqns. 6. Applying the condition $M = M_{\text{opT}}$ (or $M = M_{\text{opT}}$) in the model based on classical beam theory results in the central deflection of the T-stub assembly being given by

$$2\delta_y = \gamma \delta_y + \gamma \Delta F \cdot \left[ \frac{2m^2(e + m)}{3L^2 \eta_y K_0} + \left(1 + \frac{m^2}{e}\right) \frac{1}{n_y \eta_y K_b} \right]$$

It should be noted that when $\gamma \Delta F = \Delta F_{bl}$, the second and third plastic hinges form simultaneously owing to symmetry when $M_{\text{opT}} = M_{\text{opT}}$, and a mechanism will develop; the mechanism being identified as Mode 1 in Figure 2(a) and with ultimate values $F_u = 2F_y$ and $\delta_u = 2\delta_y$. However, if $M_{\text{opT}} < M_{\text{opT}}$ when $\gamma \Delta F = \Delta F_{bl}$, the force can be increased until the bolts yield, and then fracture, as identified by Mode VII in Figure 2(b). For this, yielding of the bolts occurs when
\[ ^2 \Delta F = n_y \eta_{yb} B_{y0} - ^2 B_y \]  

and

\[ ^3 \delta_y = ^2 \delta_y + ^2 \Delta F \left( \frac{1}{n_y \eta_{ey} K_{t0}} + \frac{m^3}{3L \eta_{Ey} K_{t0}^2} \right) \]  

where

\[ ^2 B_y = ^1 B_y + \Delta F \left( \frac{m}{e} + 1 \right) \text{ and } \frac{1}{K_{t0}} = \frac{L^2}{E_{t0}} \left( \frac{1}{I_c} + \frac{\eta_{Ey} E_{t0}}{\eta_{Ey} E_{y0}} \right) \]  

and \( \eta_{Ey} = \frac{E_{y0}}{E_{t0}} \) defines the degradation of the tangent modulus of structural steel. Finally, to reach bolt fracture in this case,

\[ ^3 \Delta F = n_y \left( \eta_{yb} B_{y0} - \eta_{yb} B_{y0} \right) \]  

and so the ultimate values are

\[ F_u = ^1 F_y + ^1 \Delta F + ^2 \Delta F + ^3 \Delta F \]  

\[ \delta_u = ^3 \delta_y + ^3 \Delta F \left( \frac{1}{n_y \eta_{ey} K_{t0}^2} + \frac{m^3}{3L \eta_{Ey} K_{t0}^2} \right) \]  

where

\[ K_{t0} = \frac{E_{t0} A_y}{t_b} \]  

and \( \eta_{yb} = \frac{f_{yb}}{f_{y0}} \) defines the degradation of the ultimate tensile strength of the bolt material and \( \eta_{Ey} = \frac{E_{y0}}{E_{t0}} \) defines the degradation of its tangent modulus at elevated temperature.

If \( ^1 \Delta F = \Delta F_f \) in Eqn. 19, the bolts yield and the applied load and deflections can be increased until they fracture (failure modes II of Figure 2(a) and VIII of Figure 2(b)) and the corresponding increment load \( ^2 \Delta F \) can be given by the right hand side of Eqn. 26; however the ultimate values of deflection and load are then obtained from

\[ F_u = ^1 F_y + ^1 \Delta F + ^2 \Delta F \]  

\[ \delta_u = ^2 \delta_y + ^2 \Delta F \left( \frac{1}{n_y \eta_{ey} K_{t0}^2} + \frac{m^3}{3L \eta_{Ey} K_{t0}^2} \right) . \]  

For case (II) which is that when first yielding occurs at the bolt line, the prying force cannot be increased further [10-12] and Eqn. 12 can be rewritten as

\[ ^1 \Delta F = \min \left( \Delta F_f, \Delta F_b \right), \]  

where

\[ \Delta F_f = \min \left( \frac{M_{of}}{m}, \frac{M_{of}}{m} \right) \text{, } \Delta F_b = n_y \eta_{yb} B_{y0} - ^1 B_y, \]  

and the central separation of the T-stub becomes
\[ \delta_y = \delta_y + \frac{1}{n_y \eta_{th} K_{th}} \left( \frac{1}{m^3 \eta_{es} K_0} + \frac{m^3}{3 L^3 \eta_{es} K_0} \right) \] (34)

Following a similar procedure to the previous case, when \( \Delta F = \Delta F_1 \) and \( M_{pt} = M_{pt} \), the T-stub assembly develops a mechanism (Figure 2(a), mode III) and the ultimate values are \( F_u = F_\gamma + \Delta F \) and \( \delta_u = \delta_\gamma \). When \( M_{pt} < M_{pt} \), the force can be increased further until the bolts yield and then fracture (failure mode IX of Figure 2(b)), and the corresponding load and deflection increments can be determined using Eqns. 23 and 24, and Eqns. 26 to 28, in which

\[
\delta_y = \delta_y + \Delta F_\gamma.
\] (35)

On the other hand, if \( \Delta F = \Delta F_b \) in Eqn. 32, a third plastic hinge may form at the fillet weld, or the bolts may fracture before the formation of this hinge at the fillet. In the former case, when \( M_{pt} = M_{pt} \) the T-stub develops the final mechanism shown in Figure 2(a) (failure mode IV), whilst when \( M_{pt} < M_{pt} \) the load and deflections can be increased (Eqns. 26 to 28) until the bolts fracture (failure mode X of Figure 2(b)). In the latter case where bolt fracture occurs first (Figure 2(a), mode V and Figure 2(b), mode XI), the second load increment is given by

\[
\Delta F = \Delta F_b,
\] (36)

in which \( \Delta F_b \) is given by the right hand side of Eqn. 26. The corresponding ultimate values of \( F_u \) and \( \delta_u \) are then obtained from Eqns. 30 and 31.

For case (III) for which the bolts yield first, the T-stub is not subjected to prying forces and any increase of load is carried by the bolts. In this case, the load may increase until the bolts fracture (failure modes VI of Figure 2(a) and XII of Figure 2(b)). Eqn. 19 may therefore be written as

\[
\Delta F = \Delta F_b,
\] (37)

where \( \Delta F_b \) is the same as the right hand side of Eqn. 26. The ultimate separation and tensile resistance of the T-stub assembly are then

\[
\delta_y = \delta_y + \Delta F \left( \frac{1}{n_y \eta_{th} K_{th}} + \frac{m^3}{3 L^3 \eta_{es} K_0} \right)
\] (38)

and

\[
F_u = F_\gamma + \Delta F.
\] (39)

**MODEL VALIDATION**

Spyrou *et al.* [11, 12] reported an experimental investigation of the behaviour of T-stub assemblies at Sheffield, in which separate results were given for the end plate and column flange separately. This is consistent with their modeling of the column flange or plate as a rigid stratum, whilst the present modeling accounts for both components being flexible. Because of this, the values of the appropriate stiffnesses in Eqns. 2 were chosen such that \( K_{et} \) or \( K_{et} \rightarrow \infty \).

Tables 1 and 2 show the geometrical and mechanical properties of typical specimens from Spyrou's experimental results used herein. The mechanical properties of the bolt material was taken from Theodorou [15] which was assumed to exhibit a bilinear stress-strain relationship, and the temperature-dependent properties of structural steel were taken from the Australian AS4100 Steel Code [16]. For all tests, the Young's modulus of the bolts at ambient temperature was taken as 215 kN/mm², and the yield and ultimate tensile strengths of the bolts at ambient temperature were 811 N/mm² and 886 N/mm².
TABLE 1
GEOMETRIC PROPERTIES OF SPECIMENS USED (MM)

<table>
<thead>
<tr>
<th>Test</th>
<th>e</th>
<th>m</th>
<th>t_e</th>
<th>t_f</th>
<th>d_b</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48.55</td>
<td>32.03</td>
<td>20.10</td>
<td>∞</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>83.22</td>
<td>28.03</td>
<td>∞</td>
<td>21.12</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>82.46</td>
<td>28.51</td>
<td>∞</td>
<td>21.14</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>31.85</td>
<td>35.895</td>
<td>∞</td>
<td>9.20</td>
<td>20</td>
</tr>
</tbody>
</table>

TABLE 2
MECHANICAL PROPERTIES OF TEST SPECIMENS

<table>
<thead>
<tr>
<th>Test</th>
<th>T_{T=stub} (°C)</th>
<th>T_{b} (°C)</th>
<th>f_{Y,T=stub} (N/mm²)</th>
<th>E_T_{stub} (kN/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>505</td>
<td>505</td>
<td>284</td>
<td>192</td>
</tr>
<tr>
<td>2</td>
<td>615</td>
<td>618</td>
<td>288</td>
<td>189</td>
</tr>
<tr>
<td>3</td>
<td>700</td>
<td>703</td>
<td>288</td>
<td>189</td>
</tr>
<tr>
<td>4</td>
<td>705</td>
<td>705</td>
<td>285</td>
<td>198</td>
</tr>
</tbody>
</table>

Figures 3 to 6 show a comparison of the test data of Spyrou with the predictions of the current model. It can be seen that the agreement is generally very good, with any differences being believed to be attributable to strain hardening and the post yield strength which were not accounted for in the present model. Nevertheless, any discrepancy is considered acceptable for providing practical design guidance, for which further work is currently being undertaken.

Figure 3: Theory – versus – Experiments (Test 1)

CONCLUSIONS

This paper has considered the behaviour of the beam-to-column connection in a steel frame when subjected to elevated temperatures. The study has been undertaken in the context of the component method of joint design that is used widely in the Eurocode and elsewhere. Unlike previous research, the present application has incorporated the flexibility of the end plate and the column flange, as well as the temperature degradation of the bolt material and structural steel. The analytical results have been compared with tests undertaken at University of Sheffield that are reported in the literature, where acceptable correlation has been seen to be achievable. The analytical model herein is simple, and the good correlation shows that the component method is appropriate for
elevated temperatures, and that acceptable design procedures can be formulated. It forms a useful foundation for steel joint design in a structural fire engineering approach.

Figure 4: Theory – versus – Experiments (Test 2)

Figure 5: Theory – versus – Experiments (Test 3)

Figure 6: Theory – versus – Experiments (Test 4)
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REFERENCES

LOCAL/DISTORTIONAL/GLOBAL INTERACTION IN LIPPED CHANNEL COLUMNS: BEHAVIOUR AND STRENGTH

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KEYWORDS
Lipped channel columns, Local/distortional/global mode interaction, Post-buckling behaviour, Ultimate strength, Shell finite element analysis, Experimental analysis.

ABSTRACT
This paper reports the available results of an ongoing numerical and experimental investigation on the post-buckling behaviour and strength of fixed-ended cold-formed steel lipped channel columns affected by local/distortional/global (flexural-torsional) mode interaction. Initially, ABAQUS shell finite element results concerning columns exhibiting identical local, distortional and global buckling loads (their geometries are identified through preliminary buckling analyses) are presented and discussed – otherwise identical columns containing critical-mode initial geometrical imperfections with different configurations are analysed. These numerical results include (i) elastic and elastic-plastic post-buckling equilibrium paths, (ii) curves and figures providing the evolution, along a given path, of the column deformed configuration and (iii) the characterisation of the column collapse mechanism. Then, the paper addresses the experimental investigation currently under way at COPPE (Federal University of Rio de Janeiro) and to be fully reported in the near future – experimental and numerical results concerning one test are presented and discussed.

INTRODUCTION
Most cold-formed steel members display very slender thin-walled open cross-sections, a feature making them highly susceptible to several instability phenomena, namely local, distortional and global (flexural or flexural-torsional) buckling (see Figs. 1(a)-(d)) – depending on the member length and cross-section shape/dimensions, any of these buckling modes may be critical. Since commonly used cold-formed steel member geometries may lead to similar local, distortional and global buckling stresses, the associated post-buckling behaviour, ultimate strength and collapse mechanism are likely to be strongly affected by interaction phenomena involving those buckling modes.
It is well known that thin-walled members exhibit stable local and global post-buckling behaviours with high and low post-critical strength reserve. On the other hand, recent studies (e.g., [1, 2]) showed that the distortional post-buckling behaviour fits in between the previous two and exhibits a non-negligible asymmetry with respect to the flange-lip motion (outward or inward).

Amongst the mode interaction phenomena affecting the column post-buckling behaviour, those stemming from the nearly simultaneous occurrence of local and global buckling are, by far, the better understood — indeed, virtually all current hot-rolled and cold-formed steel design codes account for them, either through the “plate effective width” concept (e.g., [3]) or by means of the increasingly popular “Direct Strength Method” (e.g., [4]). As for the local/distortional interactive buckling of lipped channel columns, this has attracted the attention of several researchers in the recent past (both numerical [5-7] and experimental [8, 9] investigations have been reported and some of this work already led to the proposal of novel Direct Strength Method approaches to account for this phenomenon [8-11]) and continues to foster fresh research activity, some of which is presented at this conference [12-15]. However, there are very few studies dealing with coupling phenomena involving both distortional and global buckling modes, namely distortional/global and/or local/distortional/global interaction — indeed, the authors are only aware of four very recent publications, all dealing with cold-formed lipped channel columns: numerical analysis of simply supported carbon steel columns [16-18] and experimental study on fixed-ended stainless steel columns [19]. The aim of the authors’ current research effort, which is partially reported in this paper, is to investigate, both numerically and experimentally, the post-buckling behaviour and ultimate strength of fixed-ended carbon steel lipped channel columns affected by local/distortional/global interaction.

The first part of the paper concerns the presentation and discussion of numerical results concerning the (i) elastic and elastic-plastic post-buckling behaviour, (ii) ultimate strength and (iii) failure mode nature of cold-formed steel (E=210 GPa, ν=0.3) fixed-ended lipped channel columns with cross-section dimensions and lengths leading to virtually coincident local (L), distortional (D) and global (G) buckling loads — the identification of these column geometries requires the performance of preliminary buckling analyses. One considers a fairly large number of columns differing only in the initial geometrical imperfection shape — the various configurations dealt with are linear combinations of the competing L, D and G buckling modes, normalised to exhibit amplitudes equal to 10% of the wall thickness t (local and distortional modes) and 0.1% of the column length L (global mode). All the numerical results presented were obtained through finite element analyses carried out in the code ABAQUS [20] that (i) adopt member discretisations into fine 4-node isoparametric shell element meshes (length-to-width ratio roughly equal to 1) and (ii) model the fixed-ended support conditions by attaching rigid plates to the column end cross-sections, which are then only allowed to exhibit longitudinal rigid-body motions — a detailed account of all the modelling issues can be found in [7, 21, 22]. The second part of the paper addresses the experimental investigation currently under way at COPPE (Federal University of Rio de Janeiro) and which will be fully reported in the near future — after providing a brief description on of the test program, set-up
and procedure, one compares the experimental results and numerical simulations concerning just one of specimens tested.

**BUCKLING BEHAVIOUR – COLUMN GEOMETRY SELECTION**

A sequence of buckling analyses (trial-and-error procedure) made it possible to identify a lipped channel column geometry (cross-section dimensions and length) ensuring almost coincident local, distortional and global buckling loads – these dimensions are: \( b_w = 75 \text{ mm} \) (web height), \( b_f = 65 \text{ mm} \) (flange width), \( b_l = 11 \text{ mm} \) (lip width), \( t = 1.1 \text{ mm} \) (wall thickness) and \( L = 235 \text{ cm} \) (length). The curve depicted in Figure 2(a) concerns columns with these cross-section dimensions and provides the variation of the critical buckling load \( P_{cr} \) with the length \( L \) (logarithmic scale). As for Figure 2(b), it shows the shapes of the (coincident) \( L=235 \text{ cm} \) column local, distortional and flexural-torsional buckling modes. The observation of these ABAQUS buckling results prompts the following remarks:

(i) The \( P_{cr} \) vs. \( L \) curve exhibits three distinct zones, corresponding to (i1) local buckling (\( L<120 \text{ cm} \)), (i2) local/distortional “mixed” buckling modes (almost horizontal plateau associated with extremely close buckling loads linked to local/distortional modes with different half-wave numbers) (\( 120<L<235 \text{ cm} \)), and (i3) global (flexural-torsional) buckling (\( L>235 \text{ cm} \)).

(ii) Figure 2(a) clearly shows that the \( L=235 \text{ cm} \) column has very close L, D and G critical loads, which indicates that the post-buckling behaviour and ultimate strength of such column will be highly affected by local/distortional/global interaction (\( L=L_{L/D/G} \)). The critical buckling loads are \( P_{cr,D4}=47.6 \text{ kN} \) (4 distortional half-waves), \( P_{cr,D5}=48.3 \text{ kN} \) (5 distortional half-waves), \( P_{cr,G}=48.6 \text{ kN} \) (single flexural-torsional half-wave) and \( P_{cr,L}=48.8 \text{ kN} \) (33 web-triggered local half-waves) – the corresponding buckling mode shapes are depicted in Figure 2(b).

![Figure 2: (a) Column \( P_{cr} \) vs. \( L \) curve (\( b_w = 75 \text{ mm} \); \( b_f = 65 \text{ mm} \); \( b_l = 11 \text{ mm} \); \( t = 1.1 \text{ mm} \)) and (b) \( L_{L/D/G}=235 \text{ cm} \) column local, distortional and global (flexural-torsional) buckling mode shapes](image)

**POST-BUCKLING BEHAVIOUR UNDER L/D/G MODE INTERACTION**

One now investigates the elastic and elastic-plastic post-buckling behaviour of fixed-ended columns with the geometry identified in the previous section – recall that its critical buckling load is equal to \( P_{cr}=47.6 \text{ kN} \) (\( \sigma_{cr}=190.4 \text{ MPa} \)) and it exhibits almost coincident local (33 half-waves), distortional (4-5 half-waves) and global (single half-wave) buckling modes.

**Initial Geometrical Imperfections**

\(^{1}\) In fixed-ended lipped channel columns, the distortional buckling loads associated with buckling modes exhibiting consecutive half-wave numbers are always very close – this is why two distortional buckling modes must be considered in this work.
Because in mode interaction studies the commonly used approach of considering critical-mode initial geometrical imperfections ceases to be well defined, due to the presence of more than one competing buckling modes that may be combined arbitrarily, an important issue consists of assessing how the initial geometrical imperfection shape influences the post-buckling behaviour and strength of the structural system under scrutiny – in particular, it is crucial to identify the most detrimental imperfection shape. In order to achieve this goal, one determines column equilibrium paths covering the whole critical-mode imperfection shape range – to ensure that such paths can be meaningfully compared, the following approach is adopted:

(i) Determine the “pure” critical buckling mode shapes, normalised so that the most relevant displacement has a unit value. Such displacement is (i) the mid-span mid-web flexural displacement ($w_L=1\,\text{mm}$ – local), (ii) the quarter-span flange-lip corner vertical displacement ($v_D=1\,\text{mm}$ – distortional) or (iii) the mid-span flange-lip corner vertical displacement ($v_G=1\,\text{mm}$ – global).

(ii) Scale down the above “pure” modes, leading to local, distortional and global imperfection magnitudes equal to $w_{L,0}=0.1\,t$, $v_{D,0}=0.1\,t$ and $v_{G,0}=L/1000$.

(iii) A given imperfection shape is obtained as a linear combination of the scaled competing buckling modes shapes – its coefficients $C_{L,0}$, $C_{D,0}$ and $C_{G,0}$ satisfy the condition $(C_{L,0})^2 + (C_{D,0})^2 + (C_{G,0})^2 = 1$. A better “feel” and visualisation of the initial imperfection shapes may be obtained by looking at a unit radius sphere drawn in the $C_{L,0}$–$C_{D,0}$–$C_{G,0}$ space, as shown in Figure 3(a): each “acceptable” shape lies on this sphere and can be defined by two angles ($\alpha$ and $\theta$). Figures 3(b) display the pure $D_4$ ($\alpha=0^\circ + \theta=0^\circ$ or $180^\circ$ – inward or outward flange-lip motions at quarter-span), $G$ ($\alpha=0^\circ + \theta=90^\circ$ or $270^\circ$ – clockwise or counterclockwise cross-section rotations) and $L$ ($\theta=0^\circ + \alpha=90^\circ$ or $270^\circ$ – inward or outward web bending at mid-span) initial imperfection shapes.

Figure 3: (a) Initial imperfection representation in the $C_{L,0}$–$C_{D,0}$–$C_{G,0}$ space and (b) pure distortional ($\alpha=0^\circ + \theta=0^\circ, 180^\circ$), global ($\alpha=0^\circ + \theta=90^\circ, 270^\circ$) and local ($\theta=0^\circ + \alpha=90^\circ, 270^\circ$) initial imperfection shapes.

---

2 No five half-wave distortional initial geometrical imperfections are dealt with here (only four half-wave ones). There are four and five half-wave distortional buckling modes associated with $P_{cr,D4}=55.9\,\text{kN}$ (the lower value) and $P_{cr,D5}=56.8\,\text{kN}$.

3 This vertical displacement is due to the cross-section rigid-body motion: rotation and minor-axis translation – since both cause upward or downward vertical displacements, it suffices to define the cross-section rotation sense (clock or counterclockwise).
Elastic Post-Buckling Behaviour

In order to investigate the elastic post-buckling behaviour of the $L_{DAG}$ column, a large number of otherwise identical columns with distinct initial imperfection shapes were analysed. However, due to space limitations, only some of the corresponding numerical results are presented here, adopting the following strategy: (i) presentation of the results concerning columns with the 6 pure critical initial imperfections shown in Figure 3(b), simply designated as $\theta=0^\circ$, $180^\circ$ (pure D, distortional), $\theta=90^\circ$, $270^\circ$ (pure global) and $\alpha =90^\circ$, $270^\circ$ (pure local) columns; (ii) after observing the previous post-buckling results, identification of the two most detrimental imperfection shape types (i.e., those leading to larger erosions of the column elastic post-buckling strength reserve), which define the “most detrimental imperfection plane”; (iii) presentation of results concerning 24 columns with different initial imperfection shapes lying on the above plane (with $15^\circ$ angle intervals).

Figures 4(a)-(c) show the upper parts of the equilibrium paths (i) $P/P_{cr}$ vs. $v/t$ ($v$ is half the sum of the two flange-lip corner vertical displacements), (ii) $P/P_{cr}$ vs. $w/t$ ($w$ is the mid-web flexural displacement, measured with respect to the web chord and after deducting the component stemming from the distortional deformation) and (iii) $P/P_{cr}$ vs. $\beta$ ($\beta$ is the web chord rigid-body rotation). It should be noted that $v$, $w$ and $\beta$ (i) are measured at the quarter-span section indicated in Figure 4(d) (S-S’ cross-section), where maximum distortional deformations occurs, and (ii) are well suited to quantify the evolution of the column distortional, local and global deformed configuration components along any given equilibrium path. As for Figures 4(d), they show the deformed configurations of the $\theta=0^\circ$, $180^\circ$ and $\alpha=90^\circ$ columns at the advanced post-buckling stage. From the observation of these post-buckling results, the following conclusions are drawn:

(i) All columns exhibit a stable post-buckling behaviour with a small-to-moderate post-critical strength reserve (see the various $P/P_{cr}$ vs. $v/t$ and $P/P_{cr}$ vs. $w/t$ curves displayed). This is somewhat surprising, since a previous study [17] on simply supported columns experiencing strong L/D/G mode interaction unveiled the existence of well defined limit points occurring quite below $P/P_{cr}=1.0$.

(ii) Generalised Beam Theory (GBT – e.g., [23]) provides the explanation for the distinct post-buckling behaviour nature exhibited by the simply supported and the fixed-ended lipped channel columns. The buckling results depicted in Figure 5(a) provide the variation of the critical buckling load $P_{cr}$ with the column length $L$ (for $L \geq 200\,$cm); (ii) the curve yielded by ABAQUS analyses and already shown in Figure 2(a), and (ii) the values obtained from GBT analyses carried out in GBTUL [24, 25] and including 18 deformation modes (4 global + 2 distortional + 12 local). As for Figure 5(b), it shows the GBT-based modal participation diagram, showing the contributions of each GBT deformation mode to the column buckling modes. Finally,
figures 5(c) display the GBT-based buckling mode shapes of the \( L = 300, 500, 700 \) cm columns and also the in-plane configurations of the 3 deformation modes that contribute to them. These GBT-based buckling results lead to the following comments:

(ii.1) The buckling curve descending branch corresponds to two distinct single half-wave buckling modes: (ii1) distortional-flexural-torsional \((2+4+6)\), for \(235 < L \leq 500 \) cm, and (ii2) flexural-torsional \((2+4)\), for \(L > 500 \) cm – the participation of mode 6 (antisymmetric distortional) gradually fades as the column length increases, until it practically vanishes for \( L = 500 \) cm.

(ii.2) This means that the \( L = 235 \) cm column single half-wave critical buckling mode, termed “global” until now, is indeed a “mixed” flexural-torsional-distortional mode, which combines about 35% of mode 2, 40% of mode 4 and 25% of mode 6. The same happened in the simply supported column affected by L/D/G interaction analysed in [17], where it was shown that the presence of mode 6 was responsible for the occurrence of (elastic) limit points. But why are there no such limit points in the fixed-ended column equilibrium paths?

(ii.3) Although a fully rational answer to the above question requires further investigation, which is currently under way and will be reported in the near future, it seems fair to anticipate that the explanation for the absence of elastic equilibrium path limit points has to do with both (ii1) the smaller contribution of mode 6 (it reached 90% in the simply supported columns analysed in [17]) and (ii2) the absence of a destabilising effective centroid shift associated with this anti-symmetric distortional deformation mode (e.g., [26]).

(iii) The columns with “global” (this designation will be retained for simplicity) initial imperfections \((\theta = 90^\circ \text{ or } 270^\circ)\) exhibit deformed configurations characterised by quarter-span (iii1) small outward flange-lip motions, (iii2) moderate inward or outward web bending and (iii3) significant clockwise or counterclockwise rigid-body rotations – the presence of non-null distortional and local (mostly) components indicates L/D/G interaction.

(iv) The small \( v \) values exhibited by some equilibrium paths do not correspond to minute distortional deformations. Indeed, they are due to the simultaneous presence of four \((D_4)\) and five \((D_5)\) half-wave distortional components, whose interaction leads to small \( v \) values at the quarter-span \((S-S')\) cross-section – therefore, this cross-section is not the most convenient to characterise the L/D/G interaction.
(v) The columns with distortional \( \theta=0^\circ, 180^\circ \) or local \( \alpha=90^\circ, 270^\circ \) initial imperfections have deformed configurations with no rigid-body rotations, i.e., only L/D interaction occurs (see Fig. 4(d)).

(vi) Because the local \( \alpha=90^\circ, 270^\circ \) initial imperfections always lead to higher column elastic post-buckling strengths than their global and distortional counterparts, one readily concludes that the most adverse L/D/G interaction effects occur for initial imperfections lying in the CD-CG plane. Note that this conclusion is in line with the findings reported earlier for simply supported columns [17]: the initial imperfection local component plays a lesser role, as far as the post-buckling behaviour of columns experiencing L/D/G interaction is concerned.

In view of the above facts, attention is now focused on the post-buckling behaviour of columns with 13 initial imperfections located in the CD-CG plane and corresponding to \( 0^\circ < \theta < 180^\circ \) (15° intervals – to ensure \( C_G \neq 0 \), the \( \theta=0^\circ \) and \( 180^\circ \) columns are replaced by \( \theta=1^\circ \) and \( 179^\circ \) ones)\(^4\).

Figures 6(a)-(c) show the upper parts of the \( P/P_{cr} \) vs. \( v/t \), \( P/P_{cr} \) vs. \( w/t \) and \( P/P_{cr} \) vs. \( \beta \) equilibrium paths, which now concern mid-span cross-section displacements – due to the relevant presence of D5 distortional deformations (they always come into play, even if only D4 initial imperfections are included in the analysis), this cross-section is more adequate to characterise the L/D/G interaction. Figure 6(d) shows the deformed configurations of the \( \theta=90^\circ \) column (pure global imperfections) at an advanced post-buckling stage. In order to illustrate some behavioural aspects detected in the above equilibrium paths and not visible in Figures 6(a)-(b), additional results are presented in Figures 7(a)-(c): more detailed representations of the \( \theta=90^\circ \) curves displayed in Figures 6(a)-(b), together with several column mid-span cross-section deformed configurations. The joint observation of all these post-buckling results prompts the following remarks:

(i) All equilibrium paths shown in figures 6(a)-(c) merge into single common curves, associated with mid-span (i1) clockwise web chord rotations, (i2) outward flange-lip motions and (i3) outward web bending (i.e., opposing the web bending caused by the distortional component) – this provides clear evidence of strong L/D/G interaction. Moreover, note that, since the column deformed configuration D5 distortional component has always the same sign (outward flange-lip motions at mid-span), the column post-buckling behaviour is symmetric with respect to \( \theta=90^\circ \).

(ii) The progressive emergence of significant distortional deformations at the column mid-span cross-section means that the L/D/G interaction involves also (or mostly) the non-critical 5 half-wave distortional buckling mode (D5). In spite of its slightly higher buckling load, this distortional mode is geometrically more akin to the global and local ones (maximum displacements at mid-span) and plays a key role in the column post-buckling behaviour, as can be seen in Figure 6(d) – note also the visibility of the column deformed configuration local component (barely perceptible in the simply supported columns analysed in [17]).

(iii) The equilibrium paths of the columns with a very small \( C_{G,0} \) value \( \theta=1^\circ, 179^\circ \) lie clearly above the remaining ones and reach a limit point prior to merging into the common curve – this is because of the very close proximity of the singular post-buckling behaviour of the \( \theta=0^\circ, 180^\circ \) columns which involves no cross-section rigid-body motions [17]. As for the other equilibrium paths, all of them are “smooth” (particularly those concerning columns with small \( C_{G,0} \) values) and evolve in a monotonic fashion.

\(^4\) Since the column post-buckling behaviour was found to be symmetric with respect to the deformed configuration global component sign (see Fig. 4(c)), there is no need to present post-buckling results concerning the \( 180^\circ < \theta < 360^\circ \) columns.
(iv) The most detrimental initial imperfections, in the sense that they lead to the lowest column post-buckling strength, are the pure “global” ones (θ=90°) – indeed, the corresponding equilibrium paths shown in Figures 6(a)-(c) lie below all the remaining ones.

![Figure 6: (a) P/P_cr vs. v/t, (b) P/P_cr vs. w/t, and (c) P/P_cr vs. β paths for C_D-C_G plane imperfections; (d) deformed configuration of the θ=90° column at the advanced post-buckling stage](image)

(v) In the advanced post-buckling stage (P/P_cr>0.93 and v/t>5), a gradual change occurs in the column common deformed configuration – this change is clearly visible in the θ=90° column equilibrium paths depicted in Figures 7(a)-(b). First of all, the rate of increase of the outward web bending displacements (w), measured with respect to the web chord and deducting the distortional component, progressively diminishes, until a “reversal” takes place (i.e., the web deformed configuration local component corresponds now to inward bending) – note also the emergence of a visible web double-curvature bending, stemming from the growth of the deformed configuration “global” component. Secondly, the rate of increase of the flange-lip distortional displacements (v) decreases, until another “reversal” takes place (i.e., the flange-lip assemblies start to move inward) – this means that while the mid-span cross-section rigid-body rotation and associated anti-symmetric distortion keep growing (see Fig. 7(c)), its symmetric distortion slowly starts to decrease.

**Elastic-Plastic Post-Buckling Behaviour**

Numerical results concerning the elastic-plastic post-buckling behaviour and collapse mechanism of the L/D/G column are now presented and discussed. Again due to space limitations, only the most relevant results are dealt with: those that correspond to columns with (i) initial imperfection shapes lying in the C_D-C_G plane (1°<θ<179° – 15° intervals), and (ii) yield stresses f_y=235, 355, 520 MPa, i.e., yield-to-critical stress ratios equal to f_y/σ_cr≈1.2, 1.9, 2.7. For comparative purposes,
some elastic results addressed earlier are presented again – they may be viewed as corresponding to $f_y = f_y / \sigma_{cr} = \infty$.

Figure 8(a) shows the upper portions ($P/P_{cr} > 0.6$) of four equilibrium paths $P/P_{cr}$ vs. $v/t$, concerning $\theta = 90^\circ$ columns (most detrimental initial imperfections) with different yield stresses. Figure 8(b) concerns the column with $f_y / \sigma_{cr} \approx 2.7$ and displays four plastic strain diagrams, corresponding to equilibrium states located along its post-buckling path (as indicated in figure 8(a)) and including the column collapse mechanism. Finally, Figure 9 (and its table) provides the column ultimate load ratios ($P_u / P_{cr}$) for all the $\theta - f_y$ combinations considered in this work. After observing these $1 \leq \theta \leq 179^\circ$ column post-buckling results, one is able to draw the following conclusions:

(i) The characteristics of the column elastic-plastic behaviour and collapse depend noticeably on the $f_y / \sigma_{cr}$ value. Indeed, for values close to 1.0 (e.g., $f_y / \sigma_{cr} \approx 1.2$), first yielding occurs when the column normal stress distribution is still “fairly uniform” and, therefore, precipitates a rather “abrupt” collapse. For higher $f_y / \sigma_{cr}$ values, the beginning of yielding does not precipitate the column collapse – indeed, a mild “snap-through” phenomenon takes place, followed by a strength increase up to a limit point. It is worth noting that, as the $f_y / \sigma_{cr}$ ratio increases, the “snap-through” phenomenon becomes less pronounced and the subsequent strength increase is larger.
In the column with \( \frac{f_y}{\sigma_{cr}} \approx 2.7 \) (the higher value considered), yielding begins at the bottom lip free end zone located in the vicinity of the column mid-span, as shown by the plastic strain distribution corresponding to the equilibrium state I in Figure 8(b). Then, the plastic strain distribution associated with the equilibrium point II indicates that yielding also begins at the top lip free end in the eighth-span cross-section zones. The column collapse, which corresponds to the equilibrium path (second) limit point, only takes place after the full yielding of the mid-span cross-section web-flange corners, thus forming a “distortional plastic hinge” – see diagram III in figure 8(b). Finally, diagram IV corresponds to a post-collapse equilibrium state and shows that yielding spreads along the upper half of the mid-span cross-section region, while all other column zones remain elastic.

Figure 9 shows that the \( \theta = 90^\circ \) column always exhibits the lowest ultimate load, which means that pure “global” initial imperfections are the most detrimental ones. Concerning the strength erosion due to the L/D/G interaction, the \( \theta = 90^\circ \) column ultimate load is equal to 74\% (\( f_y = 235 \) MPa), 81\% (\( f_y = 355 \) MPa) and 89\% (\( f_y = 520 \) MPa) of its critical buckling load.

Figure 9 (and its table) also shows that the variation of \( \frac{P_u}{P_{cr}} \) with \( \theta \) is less pronounced as \( \frac{f_y}{\sigma_{cr}} \) increases – this is due to the fact that the elastic equilibrium paths concerning the various \( \theta \) values are closer to each other when yielding begins (recall that they eventually merge into a common curve). Note that the interval centred at \( \theta = 90^\circ \) for which \( P_u \) is “almost uniform” (maximum and minimum values no more than 0.5\% apart) grows from \( 75^\circ \leq \theta \leq 105^\circ \) (\( \frac{f_y}{\sigma_{cr}} \approx 2.7 \)) to \( 15^\circ \leq \theta \leq 165^\circ \) (\( \frac{f_y}{\sigma_{cr}} \approx 1.2 \)).

**EXPERIMENTAL INVESTIGATION**

This section addresses the experimental investigation currently underway at COPPE (Federal University of Rio de Janeiro) and will be fully reported in the near future – after providing a brief description of the test program, set-up and procedure, the paper compares the experimental results and numerical simulations concerning just one of the specimens tested.

**Test Program, Set-Up and Procedure**

In order to ensure the occurrence of strong local/distortional/global mode interaction, four fixed-ended column geometries with close local, distortional and global buckling loads were first selected – the experimental investigation involves 12 tests (3 columns with each geometry) and the specimen average dimensions are given in Table 1, together with the corresponding buckling loads, calculated for \( E = 210 \) GPa and \( v = 0.3 \) (no thermal buckling load limitations, the wall thickness is always close to 1.1 mm). The specimens were cold-formed by press braking and the average of the yield stress values obtained from the coupon tests is equal to 343 MPa (4 tests and standard deviation of 2.9 MPa), which corresponds to the critical-to-yield stress ratios also given in Table 1 (\( P_r = A f_y \) is the column squash load) – moreover, the coupon tests carried out showed that the steel stress-strain curve exhibiting no well-defined yield plateau.

The initial geometrical imperfections of the lipped channel specimens were carefully measured – they were recorded for very low compressive loads, never exceeding 3\% of the experimental ultimate load (the initial loading was needed for appropriate positioning of the specimen in the test frame during deformation measurements). No residual stresses were measured and 6.3 mm thick steel plates were welded to the specimen end sections, thus ensuring fixed-ended support conditions – the positioning of these “rigid” end plates on the testing machine templates was carefully monitored, in order to minimise/eliminate the eccentricity of the applied
compressive load. The tests were performed in a self-equilibrated rigid frame under displacement control (stroke control of the hydraulic jacks) with a 100 kN capacity and a specially designed moving device (see Fig. 10(b)) was used to obtain the longitudinal profiles (variation along the length) of (i) the specimen initial imperfections (prior to the test) and (ii) two displacements during the test.

**TABLE 1**

MEASURED GEOMETRIES AND BUCKLING LOADS OF THE COLUMN SPECIMENS TESTED

<table>
<thead>
<tr>
<th>Specimen</th>
<th>b_w</th>
<th>b_l</th>
<th>b_l t</th>
<th>l</th>
<th>P_{cr,L}</th>
<th>P_{cr,D}</th>
<th>P_{cr,FT}</th>
<th>P_{cr,SH/P_y}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8</td>
<td>54.1</td>
<td>11.2</td>
<td>1.08</td>
<td>1650</td>
<td>60.1</td>
<td>58.8</td>
<td>62.2</td>
</tr>
<tr>
<td>2</td>
<td>2.4</td>
<td>59.3</td>
<td>11.0</td>
<td>1.07</td>
<td>2100</td>
<td>51.9</td>
<td>51.8</td>
<td>52.9</td>
</tr>
<tr>
<td>3</td>
<td>6.3</td>
<td>64.3</td>
<td>10.5</td>
<td>1.07</td>
<td>2352</td>
<td>48.8</td>
<td>47.5</td>
<td>49.6</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>77.5</td>
<td>11.9</td>
<td>1.07</td>
<td>2850</td>
<td>43.8</td>
<td>42.9</td>
<td>45.4</td>
</tr>
</tbody>
</table>

Figure 10(a) shows the location of the 7 displacement transducers employed to measure the column specimen initial geometrical imperfections and displacements during the test – Figure 10(b) shows a view of the specimen mid-span cross-section surrounded by transducers T3 to T6. In order to measure the specimen initial displacement longitudinal profiles, only transducers T1 to T6 were attached to the moving device, which was subsequently made to travel along the whole column length. Concerning the displacement measurements made during the test, the following methodology was employed:

(i) Transducers T2 to T5 and T7 were kept permanently at the mid-height level, in contact with the specimen by means long steel wires – this arrangement aims at ensuring contact with the specimen even when the cross-section rigid-body motion ceases to be small.

(ii) Transducers T1 and T6 were attached to the moving device, which was kept at the mid-height level during the application of the various load increments. At pre-defined applied load values, loading was stopped and the moving device travelled along the specimen length, thus providing the longitudinal profiles of the flange-lip corner vertical displacements for that load value5.

(iii) During the application of the load increments, the measurements of the 7 transducers (all located at the mid-height level) were continuously recorded and fed into a data acquisition system.

![Figure 10](image-url)

(a) Displacement transducer cross-section locations and (b) view of the moving device used to obtain the longitudinal profiles of some measured displacements

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5 These loading stops provide the explanation for the small load drops visible in the experimental equilibrium paths shown in Figure 12(a) for fairly high load values – they correspond to the specimen “relaxation” following a loading stop.
Comparison between Numerical and Experimental Results

The comparison between numerical and experimental results concerns just one test carried out at COPPE – the specimen (average) dimensions are $b_w=76.3\,\text{mm}$, $b_f=64.3\,\text{mm}$, $b_l=10.5\,\text{mm}$, $t=1.07\,\text{mm}$ and $L=235.2\,\text{cm}$, i.e., very similar to those considered in the numerical investigation reported earlier. The ABAQUS shell finite element numerical simulations performed exhibit the following characteristics:

(i) The steel material behaviour described by a linear-elastic/perfectly-plastic stress-strain curve defined by $E=210\,\text{GPa}$, $\nu=0.3$ and $f_y=343\,\text{MPa}$. However, a smaller yield stress value ($f_y=235\,\text{MPa}$) was also considered, when addressing some discrepancies observed between the numerical and experimental column post-buckling results.

(ii) The column end sections are fixed: with the sole exception of the rigid-body longitudinal displacement of the loaded end section, which is free, all the displacements and rotations are fully prevented.

(iii) Both the residual stresses (not measured in the tested specimens) and corner effects are neglected.

(iv) The initial geometrical imperfections model those measured in the tested specimen – they are obtained by means of the procedure illustrated in Figure 11 and described next. For each of the 6 cross-section displacements measured along the specimen length (transducers T1 to T6 – see Fig. 10(a)), the Fourier transform approach is used to obtain a linear combination of trigonometric functions that approximates the experimental values – Figure 11 shows a comparison between the experimental and numerical initial values of the horizontal displacements $d_i$ occurring along the bottom web-flange longitudinal edge (measured by transducer T 4). The 6 initial displacement approximation functions are then incorporated into the shell finite element mesh – whenever necessary, a linear displacement variation along the cross-section wall mid-line is assumed between the equally spaced web, flange or lip nodes.

![Figure 11: Experimental and numerical $d_4$ displacement values along the bottom web-flange edge](image)

Figure 12(a) show the comparison between the equilibrium paths $P$ vs. $d_1$ and $P$ vs. $d_6$, where $d_1$ and $d_6$ are the mid-span vertical displacements of the flange-lip corners (positive upward) (see Fig. 10(a)) (i) obtained during the test carried out at COPPE and (ii) provided by numerical simulations corresponding to yield stresses equal to $f_y=343\,\text{MPa}$ (coupon test average value) and $f_y=235\,\text{MPa}$ (value selected for comparative purposes) – the table below provides the experimental ($P_{u,E}$) and numerical ($P_{u,N}$) ultimate load values. As for Figure 12(b), it shows numerical ($f_y=343\,\text{MPa}$) and experimental representations of the column failure mode. The observation of these results prompts the following remarks:

(i) The three pairs of post-buckling equilibrium paths follow the same general trend and provide clear evidence of distortional/global interaction – indeed, they concern (i1) counterclockwise rotations (both $d_1$ and $d_6$ are positive, i.e., upward) and (i2) outward motions of flange-lip assemblies ($d_1$ is larger than $d_6$) at mid-span.
(ii) However, there are differences between the experimental and numerical \((f_y=343\,\text{MPa})\) curves. Qualitatively speaking, the numerical curve exhibits a feature absent from its experimental counterpart: a minor snap-through phenomenon followed by a further small load-carrying capacity increase up to the column collapse (also shown in Fig. 8(a)). Moreover, the numerical curve is associated with higher (ii1) ductility (displacements at collapse about twice those measured in the test) and (ii2) ultimate load \(P_{u,N}\) (8.8% above \(P_{u,E}\)).

(iii) Further investigation is required in order to provide rational explanations for the discrepancies described in the previous item. However, it seems logical to anticipate that the excessively simplistic steel material behaviour modelling is one of the reasons – indeed, the linear-elastic/perfectly-plastic constitutive law adopted overestimates the slightly non-linear curve obtained from the coupon tests, which were carried out in virgin sheet material and, therefore, do not account for the yield stress variation within the column cross-section (due to the press braking procedure). In order to illustrate this point, notice that the ABAQUS equilibrium paths determined for \(f_y=235\,\text{MPa}\) underestimate the experimental ultimate load by about 4.4% – i.e., a more realistic steel material modelling, which will most likely be “softer”, should bring the numerical and experimental ultimate loads closer together.

(iv) As shown in Fig. 12(b), there is a quite satisfactory match between the ABAQUS failure mode and the collapse mechanism observed during the test: both are almost symmetric and provide evidence of distortional/global interaction. Note, however, that local deformations are not perceptible in either of them (recall that they were clearly visible in the elastic post-buckling advanced stages – see Fig. 6(d)).
CONCLUSION

This paper reported the available results of an ongoing investigation on the post-buckling behaviour and strength of fixed-ended cold-formed steel lipped channel columns experiencing local/distortional/global mode interaction, whose geometries were identified through “trial-and-error” buckling analyses. Initially, shell finite element elastic (mostly) and elastic-plastic post-buckling results concerning otherwise identical columns with critical-mode initial imperfections exhibiting different configurations were presented and discussed in detail – it was found that (i) the global initial imperfections are the most detrimental ones and (ii) the L/D/G interaction causes a noticeable column strength erosion, whose quantification will be dealt with further ahead in this investigation. Then, the experimental investigation currently under way was very partially addressed (it will be fully reported soon) – a brief description of the test program, set-up and procedure, followed by the comparison between the experimental and numerical results concerning just one column specimen (a reasonable agreement was found, even if the numerical simulations need some refinement and a few discrepancies detected remain to be fully explained). Besides additional numerical simulations and experimental tests, future work will also aim at developing a DSM (Direct Strength Method) approach to design cold-formed steel lipped channel columns against L/D/G interaction.

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INTERACTION OF LOCAL AND DISTORTIONAL MODES IN THIN-WALLED SECTIONS

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KEYWORDS

Cold-formed steel; Buckling, Stiffening; Cross sections; Design, Local and distortional modes

ABSTRACT

The paper firstly reviews recent research in the USA, Portugal and Korea on design proposals for interaction between local and distortional buckling in thin-walled cold-formed lipped channel section columns. The paper then compares recent testing at the University of Sydney by Yang and Hancock, and Yap and Hancock with the Direct Strength Method of design of cold-formed column sections. Four different proposals to account for the interaction of local and distortional buckling are compared with the University of Sydney test data, and conclusions are reached.

INTRODUCTION

Thin-walled cold-formed steel sections, especially those made from high strength steel, are prone to buckling in the primary modes of local, distortional and/or flexural/flexural-torsional buckling. Recent design developments in standards and specifications, especially those of North America (AISI [1]) and Australia/New Zealand (Standards Australia [2]), account accurately for these three basic modes. However, the interaction between the modes is less clear and the accurate quantification is still under investigation. In particular, compression experiments on stiffened web channels (SWC) (Yap and Hancock [3]), channels with intermediate stiffeners in the web and flanges (Yang and Hancock [4]) and cross-shaped steel sections (SCR) (Yap and Hancock [5]) all in thin high strength steel (550 MPa yield) have demonstrated significant reductions in strength due to interaction between the local and distortional modes. Recent theoretical research by Silvestre, Camotim and Dinis ([6], [7], [8]) in Portugal, using the Finite Element Method, has demonstrated that lipped channel sections in compression with both simply supported and fixed ends, can undergo significant interaction between local and distortional buckling especially for sections with a high distortional slenderness. Experimental work in Korea by Kwon and Kim (Kwon, Kim and
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Hancock [9]) on both high strength and mild steel lipped channel sections with intermediate stiffeners in the web and separately the flanges has demonstrated significant interaction of local and distortional buckling.

The main purpose of this paper is to review the research to date, both theoretical and experimental, and to propose design methods based on the Direct Strength Method (DSM) incorporated in Refs [1] and [2].

DIRECT STRENGTH METHOD OF DESIGN

The distortional strength (N_{cd}) is defined by a simple equation based on the distortional buckling load (N_{od}) of the whole section in compression. The distortional buckling load can be calculated from finite strip buckling analysis computer programs such as THINWALL [10] or CUFSM [11]. The signature curve derived from these programs is as shown in Fig. 1 and the distortional buckling load is simply the product of the distortional buckling stress (f_{od}) shown at Point B in Fig. 1 with the gross area (A).

![Figure 1: Buckling modes and stresses of a simple lipped channel in compression](image)

For \( \lambda_d \leq 0.561 \)

\[
N_{cd} = N_y
\]  

(1)

For \( \lambda_d > 0.561 \)

\[
N_{cd} = 1 - 0.25 \left( \frac{N_{od}}{N_y} \right)^{0.6} \left( \frac{N_{od}}{N_y} \right)^{0.6} N_y
\]  

(2)

where the distortional slenderness \( \lambda_d \) is given by Equation 3.

\[
\lambda_d = \frac{N_y}{N_{od}}
\]  

(3)

and \( N_y \) is the squash load of the gross section given by the product of the yield stress (f_{y}) and the gross area (A).

To predict the local buckling strength (N_{cl}) , the DSM uses a variation of Equations 1, 2 and 3 as follows:
For \( \lambda_l \leq 0.776 \)

\[ N_{cl} = N_{ce} \quad (4) \]

For \( \lambda_l > 0.776 \)

\[ N_{cl} = \left[ 1 - 0.15 \left( \frac{N_{ol}}{N_{ce}} \right)^{0.4} \right] \left( \frac{N_{ol}}{N_{ce}} \right)^{0.4} N_{ce} \quad (5) \]

where the local buckling slenderness \( \lambda_l \) is given by Equation 6.

\[ \lambda_l = \sqrt{\frac{N_y}{N_{ol}}} \quad (6) \]

\( N_{ol} \) is the local buckling load computed by multiplying the local buckling stress \( (f_{ol}) \) at Point A in Fig. 1 by the gross area. \( N_{ce} \) is the column strength in the flexural or flexural-torsional mode determined by multiplying the column strength \( f_n \) in Equations 7 and 8 by the gross area A.

For \( \lambda_c \leq 1.5 \)

\[ f_n = (0.658 \lambda_c^2) f_y \quad (7) \]

For \( \lambda_c > 1.5 \)

\[ f_n = (0.877 \lambda_c^2) f_y \quad (8) \]

where the non-dimensional column overall slenderness is given by:

\[ \lambda_c = \sqrt{\frac{N_y}{N_{oc}}} \quad (9) \]

\( N_{oc} \) is the flexural or flexural/torsional buckling load of the gross section. The use of \( N_{ce} \) in Equations 4 and 5 allows for the interaction of local and overall buckling. When interaction of local and overall buckling is not considered, \( N_{ce} \) in Equations 4 and 5 is simply replaced by \( N_y \) to give:

For \( \lambda_l \leq 0.776 \)

\[ N_{cl} = N_y \quad (10) \]

For \( \lambda_l > 0.776 \)

\[ N_{cl} = \left[ 1 - 0.15 \left( \frac{N_{ol}}{N_y} \right)^{0.4} \right] \left( \frac{N_{ol}}{N_y} \right)^{0.4} N_y \quad (11) \]

**REVIEW OF RECENT RESEARCH**

*Work of Schafer at John Hopkins University, USA*

Ben Schafer at Johns Hopkins University produced a definitive paper entitled Local, Distortional and Euler Buckling of Columns [12]. In the paper, different predictions of the elastic Local (L) Distortional (D) and Euler (E) buckling loads are compared. For distortional buckling, they include those of Schafer [13] included in the 2007 AISI Specification [1], Lau and Hancock [14] included in the 2005 Australian/New Zealand Standard [2], and those of Desmond, Pekoz and Winter [15]. The predictions of both Lau and Hancock, and Schafer are shown to reproduce reasonable estimates for practical sections.

The paper also investigates predictions of distortional failure with an extensive range of test data mainly on lipped C's and Z's as well as C's with intermediate web stiffeners. Both effective width methods (EWM) and Direct Strength Methods (DSM) are considered. Interaction between L+E, D+E, and L+D modes are considered. The main conclusion in relation to L+D interaction is stated "In the majority of cases local plus distortional
interaction is identified as the controlling limit state, but predicted strengths are overly conservative. Interaction may still exist between these two modes, but in the available test data, local and distortional interaction does not appear significant”.

Work of Camotim, Dinis and Silvestre at Technical University of Lisbon, Portugal

Dinar Camotim, Pedro Dinis and Nuno Silvestre at the Technical University of Lisbon in Portugal have carried out extensive nonlinear finite element studies [6],[7],[8]. Ref. [6] published in 2006 gives a study of the interaction between local and distortional buckling in beams and columns, where the columns studied had simply supported ends. An approach is proposed (called the "NLD approach") where the \( N_{cd} \) from Equations 1 and 2 is substituted for \( N_y \) in Equations 10 and 11. The philosophy behind this approach assumes that the local/distortional interaction is similar to a local/overall interaction given by Equations 4 and 5. The conclusions are that the method is safe for beams and columns. It is similar to that of Yang and Hancock [4], and Schafer [12]. More recently, Silvestre, Camotim and Dinis [7],[8] have studied fixed ended channel columns. In these papers, an alternative approach (called the "NDL approach") is proposed where the \( N_{cl} \) from Equations 10 and 11 is substituted for \( N_y \) in Equations 1 and 2. The philosophy behind this approach assumes that the local/distortional interaction can be predicted by using the distortional strength with a limit at the local strength predicted by the DSM. Reference [8] shows that the NDL approach is satisfactory for slender (\( \lambda_d \geq 1.5 \)) simply supported sections but conservative for stocky (\( \lambda_d \leq 1.5 \)) simply supported sections, whereas the method is conservative for both slender and stocky fixed-ended sections. A "novel" approach based on the ratio of the distortional and local buckling half wavelengths is proposed.

Tests of Kwon and Kim at Yeungnam University, Korea

Recent testing at Yeungnam University in Korea [9] on lipped channel sections including plain and with intermediate stiffeners in the flanges, and alternatively the web, has shown significant interaction between local and distortional buckling. Most of the tests had local buckling occurring before distortional buckling. However, as local buckling has a strong postbuckling reserve, local and distortional buckling was found to interact in the post-local buckle range. The paper clearly demonstrates local/distortional interaction for many test sections and shows that the use of Equations 1 to 6 is unconservative. However, it also computes using the NLD approach and shows this to be generally conservative except for some mild steel sections tested by Kwon, Kim and Kim [16] which are accurately predicted.

DSM DESIGN CURVES COMPARED WITH UNIVERSITY OF SYDNEY TESTS

The main purpose of this paper is to compare the test results of the Stiffened Web Sections (SWC) [3], Web and Flange Stiffened Lipped Channels of Yang and Hancock [4] and Stiffened Cross Sections (SCR) [5] with design proposals based on the Direct Strength Method (DSM). The test results discussed in the paper are only for those of intermediate length and undergoing interaction of local and distortional buckling. The short length sections undergoing mainly local buckling are not discussed here. None of the SWC, Yang and Hancock nor SCR sections demonstrated overall flexural or flexural-torsional buckling.

The test results of both the SWC and SCR specimens that failed in the distortional buckling mode have been non-dimensionalised by dividing the ultimate failure load (\( N_{\text{test}} \)) with the
yield load \( N_y \) and plotted against the non-dimensional distortional slenderness
\[ \lambda_d = \sqrt{N_y / N_{ndl}} \]
defined in Equation 3. The non-dimensionalised SWC and SCR test results are plotted as the square (□) and triangle (Δ) points respectively, as shown in Fig. 2. Tests conducted by Yang and Hancock that failed in the distortional buckling mode are also non-dimensionalised and are plotted as diamond (◇) points as shown in Fig. 2. Tests conducted by Kwon and Hancock [17] on a section shape similar to the SWC specimens and failing in the distortional buckling mode are also non-dimensionalised and are plotted as asterisks (★) points as also shown in Fig. 2. The test results are compared against the non-dimensional slenderness of the DSM distortional buckling strength curve, as shown in Fig. 2 as the solid line with starred (★) points. The non-dimensional slenderness of the DSM distortional buckling strength curve was calculated based on the Spline Finite Strip Method (SFSM) [18] values of the elastic distortional buckling stresses for all the test specimens taking account of the fixed ends.

The SWC, SCR and Yang and Hancock test results, when plotted against the non-dimensionalised slenderness, can be seen to be decreasing over a very narrow slenderness range. This is due to the elastic local and distortional buckling stresses being relatively similar for a range of lengths of the test shape. There was a spread in the Kwon and Hancock results across a range of non-dimensional distortional slenderness because of the change in the lip sizes, allowing for an increase in the elastic buckling stresses. This spread of results allowed the calibration of the original DSM distortional buckling strength curve in design standards.

For the specimens failing in the distortional mode, the average non-dimensionalised SWC test results are predicted unconservatively by the DSM distortional buckling strength curve with higher values between 10.7% and 29%. The average SCR test results are also predicted unconservatively by the DSM distortional buckling strength curve with higher values of between 1.8% and 23.9%. The average Yang and Hancock test results are predicted unconservatively by the DSM distortional buckling strength curve with higher values between 7.4% and 21.7%. The average Kwon and Hancock test results are predicted by the DSM distortional buckling strength curve to have values that were both conservative and unconservative. The predicted values ranged from approximately 18.9% below the test results to 10.1% higher than the test results. This was a consequence of the test results spread across a range of non-dimensional distortional slenderness as previously mentioned. Since most of the chosen test specimens failed with the interaction of local and distortional buckling modes, it can be seen that the DSM distortional buckling strength curve is inadequate to account for specimens that exhibit such interaction of local and distortional buckling modes.
DESIGN PROPOSAL COMPARISONS

Four different design proposals have been presented by Yap [19] and summarized in [4]. They are:

Method 1: Multiple Distortional Buckling Modes
Method 2: Distortional Buckling Interacting with Overall Buckling
Method 3: Interaction of Local and Distortional Buckling
Method 4: Interaction of Local, Distortional and Overall Buckling

Method 1 - Multiple Distortional Buckling Modes

This method is intended to apply to sections that display multiple distortional buckling modes, such as the stiffened cross-shape channel column (SCR). In the SCR section [5], it was determined that two distortional buckling modes existed at different buckle half-wavelengths, namely the short half-wavelength distortional and long half-wavelength distortional buckling modes. As discussed in [5], if the distortional buckling stress is based on the SFSM value of buckling stress for the long half-wavelength, and Equations 1 to 3 for distortional buckling strength, then the proposed Method 1 distortional buckling strength curve for the 1200 mm, 1600 mm and 2000 mm test results is unconservative. This occurs because the elastic distortional buckling stress of the long half-wavelength distortional mode approaches the short half-wavelength distortional buckling stress, hence raising the proposed strength curve and predicting unconservatively

Method 2 – Distortional Buckling Interacting with Overall Buckling

The method is based on Equations 1 to 3 with $N_y$ replaced by $N_{ce}$ from Equations 7 to 9. The proposed nominal axial capacity for the interaction of distortional buckling and overall buckling ($N_{c,de}$):
For $\lambda_{d2} \leq 0.561$, 
\[ N_{c,de} = N_{ce} \]  
(12)

For $\lambda_{d2} > 0.561$, 
\[ N_{c,de} = \left[ 1 - 0.25 \left( \frac{N_{ad}}{N_{ce}} \right)^{0.6} \right] \left( \frac{N_{ad}}{N_{ce}} \right)^{0.6} N_{ce} \]  
(13)

where,
\[ \lambda_{d2} = \text{non-dimensional slenderness used to determine } N_{c,de} \]
\[ = \sqrt{\frac{N_{ce}}{N_{ad}}} \]

When the SWC test results are non-dimensionalised by dividing the results by the overall buckling strength ($N_{ce}$), the results are plotted against the proposed distortional buckling strength curve (Method 2) as the square (□) points shown in Fig. 3. Note that the horizontal axis is now $\lambda_{d2}$. The average non-dimensionalised SWC test results are predicted unconservatively by the proposed DSM distortional buckling strength curve with higher values between 8.1% and 20.7%.

When the SCR test results are non-dimensionalised by dividing the results by the overall buckling strength ($N_{ce}$), the results are plotted against the proposed distortional buckling strength curve (Method 2) as the triangle (Δ) points shown in Fig. 3. The average non-dimensionalised SCR test results are predicted slightly conservatively by the proposed DSM distortional buckling strength curve for the 600 mm specimens with a lower value of approximately 0.3 % and unconservatively for the longer length specimens with higher values between 3.8 % and 6.2 %.

Figure 3 : SWC, SCR and Yang and Hancock test results compared with Method 2 design curves

The Yang and Hancock test results for specimens failing in the distortional buckling mode are non-dimensionalised and plotted against the proposed distortional strength curve as the
diamond (◇) points shown in Fig. 3. The proposed strength curve predicts the average Yang and Hancock results unconservatively with higher values up to approximately 14.7%.

The average ratio of test results to design for this proposed distortional buckling strength curve is 0.92 compared to 0.9 of the original DSM distortional strength curve. This indicates that the proposed strength curve is slightly more accurate than the original DSM distortional buckling strength curve, and the scatter is much improved, 0.067 compared to 0.139. Therefore when compared to both test programs and Yang and Hancock results, the proposed distortional buckling strength curve is shown to be relatively accurate and reliable in predicting the design strength of compression members that exhibit interaction of buckling modes.

Adjusted Method 2 – Distortional buckling interacting with overall buckling

It has been shown that proposed design Method 2 is relatively accurate and reliable, and the accuracy can be improved by increasing the value of the exponent in Equation 6.4 from 0.6 to 0.65. By adjusting the exponent value, the slenderness limit has to be adjusted to remove discontinuity at the limit point.

The proposed nominal axial capacity for the interaction of distortional buckling and overall buckling ($N_{c,de}$):

For $\lambda_d^2 \leq 0.587$, 
$$N_{c,de} = N_{ce}$$  \hspace{1cm} (14)

For $\lambda_d^2 > 0.587$, 
$$N_{c,de} = 1 - 0.25 \left( \frac{N_{od}}{N_{ce}} \right)^{0.65} \left( \frac{N_{od}}{N_{ce}} \right)^{0.65} N_{ce}$$  \hspace{1cm} (15)

where,
$$\lambda_d^2 = \text{non-dimensional slenderness used to determine } N_{c,de}$$
$$\quad = \sqrt{\frac{N_{ce}}{N_{od}}}$$
$$N_{od} = \text{elastic distortional compression member buckling load}$$
$$\quad = Af_{od}$$

When the SWC test results are non-dimensionalised by dividing the results by the overall buckling strength ($N_{ce}$), the results are plotted against the proposed distortional buckling strength curve (Adjusted Method 2) as the square (□) points shown in Fig. 4. The average non-dimensionalised SWC test results are predicted unconservatively by the proposed DSM distortional buckling strength curve with higher values between 3.9% and 17.2%.
When the SCR test results are non-dimensionalised by dividing the results by the overall buckling strength \( (N_{ce}) \), the results are plotted against the proposed distortional buckling strength curve (Adjusted Method 2) as the triangle (Δ) points shown in Fig. 4. The average non-dimensionalised SCR test results are predicted conservatively by the proposed DSM distortional buckling strength curve for the 600 mm and 1200 specimens with a lower value of approximately 6.9% and 2.0% and slightly unconservatively for the 1600 mm and 2000 mm specimens with higher values between 1.2% and 0.2%.

The Yang and Hancock test results for specimens failing in the distortional buckling mode are non-dimensionalised and plotted against the proposed distortional strength curve as the diamond (◇) points shown in Fig. 4. The average non-dimensionalised Yang and Hancock test results are predicted conservatively by the proposed DSM distortional buckling strength curve for the 800 mm and 2000 mm specimens with lower values by approximately 5.8% and 9.3%, respectively and unconservatively for the 1300 mm specimens with a higher value of 6.3%.

The average ratio of test results to design for this proposed distortional strength curve is 0.979 compared to 0.9 of the original DSM distortional buckling strength curve. This indicates that the proposed strength curve (Adjusted Method 2) is much more accurate than the original DSM distortional buckling strength curve, and the scatter is also lower, 0.08 compared to 0.139. Therefore when compared to SWC and SCR test programs and Yang and Hancock results, the proposed distortional buckling strength curve (Adjusted Method 2) is shown to be highly accurate and reliable in predicting the design strength of compression members that exhibit interaction of local, distortional and overall buckling modes.

**Method 3 – Interaction of Local and Distortional Buckling Modes**

The proposed nominal axial capacity for the interaction of local and distortional buckling modes \( (N_{c,ld}) \) is simply the NLD method discussed earlier and given by:
For \( \lambda_{i3} \leq 0.776 \),
\[
N_{e_{ld}} = N_{cd}
\]  
(16)

For \( \lambda_{i3} > 0.776 \),
\[
N_{e_{ld}} = \left[ 1 - 0.15 \left( \frac{N_{ol}}{N_{cd}} \right)^{0.4} \right] \left( \frac{N_{ol}}{N_{cd}} \right)^{0.4} N_{cd}
\]  
(17)

where,
\[
\lambda_{i3} = \text{non-dimensional slenderness used to determine } N_{e_{ld}}
\]
\[
= \sqrt{\frac{N_{cd}}{N_{ol}}}
\]

When the SWC test results are non-dimensionalised by dividing the results by the DSM distortional buckling strength \( N_{cd} \) from Equations 1 and 2, the results are plotted against the proposed distortional buckling strength curve (Method 3) as the square \((□)\) points shown in Fig. 5. The average non-dimensionalised SWC test results are predicted conservatively by the proposed DSM distortional buckling strength curve for the 700 mm specimens with a lower value of approximately 16.1 % and unconservatively for the longer length specimens with higher values between 2.4 % and 13.0 %.

When the SCR test results are non-dimensionalised by dividing the results by the DSM distortional buckling strength \( N_{cd} \) from Equations 1 and 2, the results are plotted against the proposed distortional buckling strength curve (Method 3) as the triangle \((Δ)\) points shown in Fig. 5. The average non-dimensionalised SCR test results are predicted conservatively for all column lengths by the proposed DSM distortional buckling strength curve with lower values between 4.4 % and 35.6 %.

In addition to the SWC and SCR test results, Yang and Hancock’s test results for the specimens that failed in the distortional buckling mode are also non-dimensionalised and plotted as the diamond \((◊)\) points as shown in Fig. 5. The average non-dimensionalised Yang and Hancock test results are predicted conservatively for all column lengths by the proposed DSM distortional buckling strength curve with lower values between 22.6 % and 44.7 %.

The average ratio of test results to design for this proposed distortional strength curve is 1.147 compared to 0.9 of the original DSM distortional strength curve. This indicates that the proposed strength curve is much more conservative than the original DSM distortional buckling strength curve, however the scatter is wider, 0.174 compared to 0.139. This proposed strength curve, when taken with the proposed local/overall buckling strength curve, generally produces safe designs.
Method 4 – Interaction of Local, Distortional and Overall Buckling Modes

A strength curve that considers the interaction of all local, distortional and overall buckling modes may be necessary to safely account for the stiffened-web channel (SWC) specimens at longer lengths. To account for the interaction of local, distortional and overall buckling modes, the proposed strength curve for the interaction of distortional and overall modes, $N_{c,de}$ from Equations 14 and 15 is adopted as the overall buckling strength, $N_{ce}$.

The proposed nominal axial capacity for the interaction of local, distortional and overall buckling ($N_{c,lde0.5}$):

For $\lambda_{l4} \leq 0.673$, 

$$N_{c,lde0.5} = N_{c,de}$$

For $\lambda_{l4} > 0.673$, 

$$N_{c,lde0.5} = \left[ 1 - 0.22 \left( \frac{N_{ol}}{N_{c,de}} \right)^{0.5} \right] \left( \frac{N_{ol}}{N_{c,de}} \right)^{0.5} N_{c,de}$$

where, 

$$\lambda_{l4} = \text{non-dimensional slenderness used to determine } N_{c,lde0.5} = \sqrt{\frac{N_{c,de}}{N_{ol}}}$$

When the SWC test results are non-dimensionalised by dividing the results by the strength proposed in Method 2 ($N_{c,de}$), the results are plotted against the proposed distortional buckling strength curve (Method 4) as the square ($\Box$) points shown in Fig. 6. The average non-dimensionalised SWC test results are now predicted conservatively by the proposed DSM distortional buckling strength curve with lower values between 9.6 % and 30.1 %. As a
comparison, the SCR and Yang and Hancock test results are also non-dimensionalised and plotted as the triangle (Δ) and diamond (◊) points in Fig. 6. The average non-dimensionalised SCR test results are predicted very conservatively by the proposed DSM distortional buckling strength curve with lower values between 33.3 % and 52.6 %. The average non-dimensionalised Yang and Hancock test results are also predicted very conservatively by the proposed DSM distortional buckling strength curve with lower values between 46.4 % and 67.3 %. Hence, this proposed method can produce very conservative results at longer lengths.

The average ratio of test results to design for this proposed distortional strength curve is 1.362 compared to 0.9 of the original DSM distortional buckling strength curve. This indicates that the proposed strength curve always produces safe design values, though the design capacities are very much lower. The scatter of the results is much higher, 0.194 compared to 0.139, when compared to the original DSM distortional buckling strength curve.

![Figure 6: SWC, SCR and Yang and Hancock test results compared with Method 4 design curves](image)

**CONCLUSIONS**

Recent international research on the interaction of local and distortional buckling has been reviewed. It has been found that the interaction of local and distortional buckling may occur for some sections and may produce unsafe designs according to existing design methods. Recent testing on high strength cold-formed sections in compression at the University of Sydney has been compared with the Direct Strength Method which is shown to produce unsafe designs in some sections. Four different design proposals have been made to account for the interaction of local and distortional buckling for the tested sections.

The NLD method proposed by Schafer, and Yang and Hancock, normally produces safe designs but may be conservative in some cases as suggested by Schafer, and confirmed by Camotim, Silvestre and Dinis, and the test results in this paper. The proposals outlined by Camotim, Silvestre and Dinis, and also Method 2 in this paper reduce this conservatism.
REFERENCES


MODELLING THE CONFINEMENT EFFECT OF COMPOSITE CONCRETE-FILLED ELLIPTICAL STEEL COLUMNS

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KEYWORDS
Composite column, concrete-filled, elliptical hollow section, confined concrete, CFT, confinement, stress-strain relationship, finite element method, nonlinear analysis.

ABSTRACT
This paper describes the nonlinear finite element model developed through ABAQUS/Standard solver to investigate the confinement effect of the concrete-filled elliptical steel tube columns under axial compression. Based on the basic conception of stress-stain model of confined concrete, a modified stress-stain relationship of confined concrete for the elliptical steel section was proposed. The accuracy of the finite element modelling method and the modified stress-strain model were verified via a series of tests, which covered three concrete grades C30, C60 and C100, three tube wall thicknesses of 4mm, 5mm and 6.3mm. The compressive behaviour including ultimate load capacity, load vs. end-shortening relationships and failure modes obtained from the numerical modelling of concrete-filled elliptical steel tube columns were compared against the experimental results and good agreements were observed. The comparison and analysis presented in this paper indicate that the proposed model can be used to predict the compressive characteristics of short concrete-filled elliptical steel tube columns.

INTRODUCTION
Concrete-filled steel tube columns are being used increasingly in modern construction practice. It has been recognized that the axial load bearing capacity of a concrete-filled steel tube column may be higher than the sum of axial load bearing capacities of the concrete core and the corresponding hollow steel section. This increases in load bearing capacity is owing to the supporting effect of concrete core to the steel tube which delay or eliminate the inwards local buckling of the steel tube, while the confinement effect of steel tube to the concrete core enhanced the concrete strength, thus concrete-filled tube columns provide excellent structural
properties such as high load bearing capacity and high ductility. However, the structural behaviour of concrete-filled tube columns may be affected by many factors, such as geometry of steel section, column slenderness and member material properties. Although the use of concrete-filled steel tube columns may be traced back to 1960s, the main research on concrete filled tube columns started from late of 1990s. The state of art review on steel-concrete composite columns by Shanmugam and Lakshmi \cite{1} highlighted the significant research in this field. Previous research on the concrete-filled tube columns included experimental investigations and numerical modelling analyses of bearing capacity and design method for engineering practice, however most of them focused on concrete-filled columns with circular and square tube sections. In recent years, elliptical hollow sections have attracted significant attention from engineers and architects owing to their aesthetic appearance and structural efficiency \cite{2}. However, the lack of design guidance for elliptical hollow section and concrete composite columns impaired the more broad use in construction. Despite the increasing use of elliptical steel hollow sections in recent years, only a few literatures on the research of concrete-filled elliptical steel tube columns can be found \cite{3-7}. To enable further investigation of the structural behaviour of concrete-filled elliptical steel tube columns, a finite element model with modified stress-strain model for confined concrete were developed using ABAQUS/standard solver to predict the ultimate capacity and failure modes of concrete-filled elliptical steel stub columns under axial compression. Following the increasing use of concrete-filled steel tube columns in modern building, marked development in numerical models in prediction of the structural behaviour of concrete-filled tube columns under various loads has been made. Some dominant characteristics of concrete-filled tube columns with circular and square tube sections, such as the stress and strain relationship of confined concrete, interaction between steel tube and concrete core have been established and verified by researchers worldwide \cite{8-16}, however very little literature on modelling concrete-filled elliptical stub columns can be found. The aim of this paper is to develop a numerical model that can simulate and predict the compressive behaviour of concrete-filled elliptical steel tube columns.

**EXPERIMENTAL INVESTIGATION**

To understand the basis of the numerical modelling method and assess the accuracy of the numerical simulations, it is necessary to provide a brief description of the physical tests and the main experimental observations. Detailed results of the experimental studies have been presented through other publication \cite{4}.

**Stub Column Tests**

A series of concrete-filled elliptical hollow section (long outer diameter 150mm and short diameter 75mm) steel stub column specimens, each with the length of 300mm were tested to investigate the compressive behaviour of concrete-filled steel tube columns with different steel section wall thicknesses (4mm, 5mm and 6.3mm) and different concrete grades (C30, C60 and C100). All specimens were compositely loaded with concrete core and elliptical hollow section steel tube fully contacted. Table 1 shows the measured geometry of elliptical hollow sections and the steel material properties.
TABLE 1
MEASURED DIMENSIONS OF ELLIPTICAL TUBE SECTION AND MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>Specimen ref.</th>
<th>Long diameter (mm)</th>
<th>Short diameter (mm)</th>
<th>Tube wall thickness (mm)</th>
<th>Steel elastic modulus (N/mm²)</th>
<th>Steel yield strength (N/mm²)</th>
<th>Steel ultimate strength (N/mm²)</th>
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<tr>
<td>150x75x4</td>
<td>150.40</td>
<td>75.60</td>
<td>4.18</td>
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<td>150x75x4</td>
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<td>4.18</td>
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<td>5.12</td>
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<tr>
<td>150x75x5</td>
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<td>75.74</td>
<td>5.08</td>
<td></td>
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<td>369.0</td>
</tr>
<tr>
<td>150x75x5</td>
<td>150.28</td>
<td>75.67</td>
<td>5.09</td>
<td></td>
<td></td>
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</tr>
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<td>150x75x6.3</td>
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<td>75.45</td>
<td>6.32</td>
<td></td>
<td></td>
<td>216500</td>
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<td>150x75x6.3</td>
<td>148.92</td>
<td>75.56</td>
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<td>75.35</td>
<td>6.25</td>
<td></td>
<td></td>
<td>512.0</td>
</tr>
</tbody>
</table>

The measured compressive cube strength of concrete C30, C60 and C100 were 36.9Mpa, 59.8Mpa and 98.4Mpa respectively. For each specimen, horizontal and vertical strain gauges were installed to measure the hoop and axial strains respectively. The axial shortening of the stub column was recorded using linear variable displacement transducers (LVDTs) which were placed between the two loading plates contacted to both ends of the specimen. The test set up is shown in Figure 1.

![Figure 1: Test set up](image)

Main Experimental Findings

- The tests revealed typical failure modes. Outward local buckling of tube wall occurred but inward buckling was prevented by concrete core. An inclined shear failure of the concrete was clearly observed for columns with thinner tube wall (4mm and 5mm), while this was not evident for specimens with thicker tube wall (6.3mm) due to the greater confinement to the concrete core. Figure 2 gives typical failure modes.
- The compressive load bearing capacity of the concrete-filled elliptical steel stub column increased with the increase of the steel tube wall thickness owing to firstly the steel tube wall thickness being thicker than 4mm and 5mm, which provided more effective confinement to the concrete core.
- However, once the concrete core failed, it would appear that the grade of concrete had

51
little effect to the composite stub column capacity although the cracked concrete core might still contribute to the delay of the onset of local buckling of the steel tube.

Figure 2: Typical modes of failure

FINITE ELEMENT (FE) MODEL

A FE model was developed through ABAQUS/Standard solver to simulate the concrete-filled elliptical steel stub columns under axial compression. Figure 3 shows a typical FE model in which three-dimensional solid elements (C3D8) were used to model the main column components: elliptical steel tube and confined concrete core. Two rigid plates were used to simulate the loading plates contacting the column ends. The FE models were built closely to the experimental tests mentioned in the previous section. The results of the sensitivity study suggested that the appropriate mesh size would be 5~10mm for the steel tube and 10~15mm for the concrete core. While small element size would consume high amount of computer time, coarse mesh would cause numerical convergence problem and would not be able to reveal the important member deformation characteristics. In addition, in order to avoid premature steel tube wall buckling, there should be at least two layers of elements in the thickness direction of the tube wall.

Figure 3: Typical FE model mesh
It can be seen from Figure 1, direct contact existed between the loading plates of the hydraulic jack and the ends of the column; therefore contact function available in ABAQUS/Standard solver was used to simulate the interaction between rigid plate and column end surface. Contacts were defined as surface-to-surface contact with a finite sliding option. ‘Hard contact’ was assumed for the normal contact behaviour and rough friction was used in the tangential direction of the contact pairs to restrain the slip between rigid plate and column end. Due to the different material property, during loading process, a contact surface pairs would be formed between the inner surface of steel tube and concrete core, so surface-to-surface contact and ‘hard contact’ behaviour in the normal direction were also assumed, however a friction coefficient of 0.2 was used for the tangential direction to consider their interaction.

The boundary conditions of the FE model were according to the testing procedures. The lower rigid plate (supporting the specimen) contacting to the bottom of the column was fixed in all six directions and the upper rigid (loading) plate contacting to the top of the column was pinned in two horizontal directions but movement along the column axis was allowed. The load was applied as static uniform displacement at the upper rigid plate through the reference node, which is identical to the experimental loading procedure.

MATERIAL MODELS

Concrete Stress-Strain Relationship

It is well known that concrete material has good compressive strength but is brittle. The compressive strengths of confined and unconfined concrete are different. Previous research\cite{16} have revealed that circular steel tube with a lower value of diameter over wall thickness (D/t) ratio provided higher confinement effect to the concrete core due to the restraining action, but the confinement effect changes depending on the local buckling of the steel tube with different D/t ratios. Due to different section geometry, the confinement effect of the elliptical steel section might be different from that of the circular steel section, this suggested that different concrete stress-strain model should be adopted when simulate the structural behaviour of concrete-filled elliptical steel tube columns. Based on the basic conception of confined concrete material model, this section tried to develop a modified stress-strain relationship of confined concrete for concrete-filled elliptical steel tube columns.

Confinement Effects of Elliptical Hollow Sections

In order to demonstrate the different confining effects of circular section and the elliptical section, two idealised composite short columns with equal concrete core cross section areas and equal tube wall thicknesses subjected to axial compressive loads were modelled. Figure 4 shows the deformation modes. The contact forces between concrete core and steel tube at typical sections (C1, C2, E1, and E2) were abstracted to compare the confining effect. It was found that the contact forces between steel tube and concrete core in both section C2 and E2 were zero due to the contact opening. However, for typical tube-concrete core contacting sections (C1 and E1) as shown in Figure 4, although the relationships of load to end shortening for both columns were very similar, the contact forces between steel tube and concrete core were very different. This indicated the confinement effects were different in each case. In circular tube section C1, the confinement force changed very little along the section perimeter due to the symmetry (here only shows one monitored point). In elliptical
section E1, the confinement force changed evidently along the section perimeter with the highest confinement force occurred at the location very close to the sharp corner as shown by EHS-No.2 while the least contact force occurred at the flat corner of the elliptical section (EHS-No.4). However, the average confinement force in elliptical section was much smaller than that observed in the circular section by comparing the EHS-Ave and CHS-No.1 as shown in Figure 5. The comparison indicates that the circular steel tube section provided stronger confinement effect to the concrete core when the concrete was crushed.

Figure 4: Comparison of concrete-filled tube column with circular and elliptical sections

Figure 5: Comparison of confinement forces in circular and elliptical steel tubes

**Stress-Strain Model for Confined Concrete in Circular Steel Tube Columns**

The basic pattern of stress-strain relationship for unconfined concrete has shape as shown in Figure 6, where $f_{ck}$ is the unconfined compressive cylinder strength of concrete ($f_{ck} = 0.8 f_{ck,cub}$ and $f_{ck,cub}$ is the unconfined compressive cube strength of concrete). The strain $\varepsilon_{ck}$ corresponding to $f_{ck}$ for unconfined concrete may be taken as 0.003 according to ACI 318-95\textsuperscript{[17]}
Figure 6: Basic stress-strain curves of unconfined and confined concrete

The basic stress-strain relationship for general confined concrete is divided into three parts whose forms may be deduced from the featured values of unconfined concrete depending on the confining boundary conditions. For confined concrete in circular steel section, the maximum strength \( f'_{cc} \) and corresponding strain \( \varepsilon'_{cc} \) have been proposed by Mander et al.\(^{[10]}\) and validated by Ellobody et al.\(^{[14]}\) in predicting compressive behaviour of concrete-filled circular steel tube columns. The formulae can be described as:

\[
\begin{align*}
    f'_{cc} &= f_{ck} + k_1f'_i \\
    \varepsilon'_{cc} &= \varepsilon_{ck}(1 + k_2 \frac{f'_i}{f_{ck}})
\end{align*}
\]

Whose \( f'_{cc} \) and \( \varepsilon'_{cc} \) are defined as the ultimate strength and strain of the concrete-filled steel tube columns, respectively, \( f_{ck} \) and \( \varepsilon_{ck} \) are the strength and strain of the unconfined concrete, respectively, \( f'_i \) is the local compression stress at the inner surface. The study of Richart et al.\(^{[8]}\) suggests that \( k_1 = 4.1 \) and \( k_2 = 20.5 \) respectively. For concrete-filled circular steel stub column, Hu et al.\(^{[11]}\) proposed the value of parameter \( f'_i \) to be:

\[
\begin{align*}
    \frac{f'_i}{f_y} &= 0.043646 - 0.000832(D/t) \quad \text{For } 21.7 \leq D/t \leq 47 \\
    \frac{f'_i}{f_y} &= 0.006241 - 0.0000357(D/t) \quad \text{For } 47 \leq D/t \leq 150
\end{align*}
\]

where,

\[
\begin{align*}
    D &= \text{ the outer diameter of the tube section;} \\
    t &= \text{ the tube wall thickness;} \\
    f_y &= \text{ the yield strength of the tube section.}
\end{align*}
\]
The first part of the stress-strain curve defines the linear property of the confined concrete and the proportional limit stress can be assumed to be \(0.5f'_{cc}\) \[^{11, 14}\]. The initial Young’s modulus of the confined concrete followed the empirical formulation provided in ACI 318-95 \[^{17}\] and described as \(E'_{cc} = 4700\sqrt{f'_{cc}}\) MPA. The Poisson ratio of the confined concrete may be taken as 0.2.

The second part of the stress-strain model described the nonlinear portion starting from the proportional limit stress \(0.5f'_{cc}\) to the maximum confined concrete strength \(f'_{cc}\). This part was proposed by Saenz \[^{9}\] and described as following, which has been validated by Ellobody et al \[^{14}\] in numerical modelling of concrete-filled circular steel tube columns.

\[
f' = \frac{E'_{cc}E'}{1 + (R + R_E - 2)(\frac{E'}{E_{cc}}) - (2R - 1)(\frac{E'}{E_{cc}})^2 + R(\frac{E'}{E_{cc}})^3}
\]

where,

\[
R_E = \frac{E'_{cc}E'}{f'_{cc}}, \quad R = \frac{R_\sigma (R_\sigma - 1)}{(R_z - 1)^2} - \frac{1}{R_z}, \quad R_\sigma = R_z = 4.
\]

The third part of the curve starts from the maximum confined concrete strength, \(f'_{cc}\) and ends at \(f_u = r_k f'_{cc}\) with the corresponding \(\varepsilon'_u = 11\varepsilon'_{cc}\). For concrete-filled circular steel tube columns, Hu et al \[^{11}\] proposed the values of parameter \(k_3\) as following:

\[
k_3 = \begin{cases} 1 & \text{For } 21.7 \leq (D)/t \leq 40 \\ 0.0000339((D)/t)^2 - 0.0100085((D)/t) + 1.3491 & \text{For } 40 \leq (D)/t \leq 150 \end{cases}
\]

Based on the experimental studies carried out by Giakoumelis and Lam \[^{16}\], Ellobody et al \[^{14}\] suggested that the parameter \(r\) could be taken as 1.0 for concrete with cube strength of 30Mpa and 0.5 for concrete with cube strength of 100Mpa. Linear interpolation may be used for concrete with cube strength lies between 30 to 100Mpa.

**Stress-Strain Model for Confined Concrete in Elliptical Steel Tube Columns**

The comparative analysis of contact forces on the contact surface between the concrete core and the steel tube of elliptical concrete-filled tube section column and circular concrete-filled tube section column aforementioned has indicated that the confining effects in both cases were different. For concrete-filled elliptical tube columns subjected to axial compression, the maximum tube confinement force applied to the concrete core was at the long diameter corner. This force decreased along the perimeter to the minimum value at the short diameter corner. This force was compared with that of a concrete-filled circular steel tube column, in which, the areas of the concrete core and steel tube section were identical to those of elliptical concrete-filled steel tube column. It was found that the confinement force in the circular steel tube was between the maximum and minimum confinement forces in the elliptical steel tube section, but the average confinement force in the elliptical steel tube was smaller than that of the circular steel tube. This suggests that the total confinement effect of the elliptical tube...
section is smaller than that of a circular steel tube section, so for confined concrete in elliptical steel section column, the authors suggest a modified four-part stress-strain model to be adopted. This model included a “quick softening” section after the concrete is crushed and the affect of section deformation. The basic form of stress-strain relationship for confined concrete in concrete-filled elliptical steel tube column is proposed as in Figure 7.

The maximum strength $f'_{cc}$ and corresponding strain $\varepsilon'_{cc}$ of confined concrete can be calculated using Equations (1), however due to the confinement effect of elliptical tube section is different from circular tube section, for concrete-filled elliptical steel tube columns, modified parameters for Equations (1) should be adopted. According to the research by Yang H et al [4] and Hu et al [12], the authors suggested that the following parameter for concrete-filled elliptical steel tube columns:

\begin{align}
\begin{alignat*}{2}
    k_1 &= 6.770 - 2.645(a/b) \\
    k_2 &= 20.5 \\
    f'_{cc} &= f_y \left\{ \nu_1 - \nu_2 \left[ (a + b)/t \right] \right\} 
\end{alignat*}
\end{align}

where,

- $a$ = the long radius of the elliptical hollow steel section;
- $b$ = the short radius of the elliptical hollow steel section;
- $t$ = the wall thickness;
- $f_y$ = the yield strength of the elliptical hollow steel section.
- $\nu_1 = 0.043646, \nu_2 = 0.000832$ for $15 \leq (a + b)/t \leq 30$ and $a/b = 2$.

The first part and second part of the confined concrete stress strain curve are determined using as the same method as that for concrete-filled circular tube column. The first part of the curve is linear up to the proportional limit stress of $0.5f'_{cc}$, the initial Young’s modulus is
taken as \( E'_{cc} = 4700\sqrt{f'_{cc}} \) MPa; the second part is nonlinear and starting from the proportional limit stress of \( 0.5f'_{cc} \) and ending at the maximum confined concrete strength \( f'_{cc} \); the stress-strain relationship is given in Equation (3).

The third part of the stress-strain curve begins from the maximum confined concrete strength \( f'_{cc} \) and terminates at \( f'_e = \nu_3 (f'_{cc} - f'_u) + f'_u \) with parameter \( \nu_3 \) dependent on the steel section geometry. For \( 15 \leq (a + b)/t \leq 30 \) and \( a/b = 2 \), \( \nu_3 = 0.3 \). The corresponding strain is taken to be \( \varepsilon'_e = 10\varepsilon_{ck} \). The fourth part of the curve starts from \( f'_e \) and terminates when \( f'_u = \nu_4 f'_{cc} \) with the corresponding strain \( \varepsilon'_u = 30\varepsilon_{ck} \). At this stage, local buckling of the steel tube might occur; the value of parameter \( \nu_4 \) will depend on the strength of concrete. \( \nu_4 = 0.7 \) for C30 concrete and \( \nu_4 = 0.3 \) for C100 concrete. Linear interpolation may be used for concrete between C30 and C100, but \( f'_e \) should not be less than 30Mpa.

**Stress-Strain Model for Steel**

Coupons were cut from the elliptical hollow sections and tested to EN10002-1\(^{[18]} \) to determine the tensile strength. The coupons were cut from the region of maximum radius of curvature (i.e. the flattest portion of the section) and milled to specification. Some flattening of the ends occurred while gripping the specimen but this was well away from the ‘neck’ of the sample. The results from the coupon tests are summarized in Table 1. Figure 8 shows the stress-strain relationships for elliptical steel sections with different wall thickness.

![Figure 8: Stress-strain relationship for steel materials](image)

**COMPARISON BETWEEN NUMERICAL AND EXPERIMENTAL RESULTS**

Based on the method described in the previous section, nine stress-strain curves, as shown in Figure 9, were developed for the confined concrete of concrete-filled elliptical steel tube columns adopted in this research. It must be noted, formation of these stress-strain curves depended on the measured unconfined concrete strength, elliptical tube section geometry and material property adopted in the research presented in this paper.
Comparisons between the numerical modelling results and experimental results were carried out to verify the accuracy of the finite element model developed through ABAQUS/Standard solver and stress-strain model of confined concrete. The maximum compressive loads, load versus column end shortening relationships as well as the column deformation were investigated. Table 2 shows the comparison between the maximum loads ($P_t$) obtained from the experimental study and maximum loads ($P_n$) obtained from the FE models, it can be seen that very good agreements have been observed between both results. As shown in Table 2, the maximum difference observed between experimental results and numerical results was 4.9%.

The axial compressive loads vs. end shortening predicted by numerical models were compared with experimental results and very good agreements were achieved. Figure 10 shows the load vs. end-shortening curves for concrete-filled elliptical steel tube columns with different concrete grades (C30, C60 and C100) and wall thickness (4mm, 5mm and 6.3mm). It can been seen that the finite element models developed through ABAQUS/Standard solver, combining with the special confined concrete stress-strain models developed for concrete-filled elliptical steel tube columns successfully predicted the maximum axial load of columns and the axial load vs. end-shortening behaviour. In particular, the numerical models were able to capture the “big drop” of the load bearing capacity after the column experienced the maximum load. This is especially evident for columns filled with high strength concrete.

The deformation of the columns predicted by the numerical models was compared with the deformed shapes of the columns tested. The finite element models revealed important failure modes, such as outwards buckling, and concrete core shearing which were observed from the tests. However due to the FE models were established without considering geometry and material imperfections, the deformed shape revealed by numerical models were basic outwards buckling close to the column ends, as shown in Figure 13 (a) (b) (c). Nevertheless, with imperfection introduced to the FE model, asymmetric deformation can be captured as shown in Figure 13 (d).
### TABLE 2
**COMPARISON OF MAXIMUM LOADS (KN) OBTAINED BY EXPERIMENTS AND FE MODELS**

<table>
<thead>
<tr>
<th>Specimen ref.</th>
<th>Maximum loads by tests ($P_t$)</th>
<th>Maximum loads by FE models ($P_n$)</th>
<th>Difference (%) $(P_t-P_n)/P_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150x75x4-C30</td>
<td>839</td>
<td>843</td>
<td>-0.477</td>
</tr>
<tr>
<td>150x75x5-C30</td>
<td>981</td>
<td>959</td>
<td>2.242</td>
</tr>
<tr>
<td>150x75x6.3-C30</td>
<td>1192</td>
<td>1170</td>
<td>1.845</td>
</tr>
<tr>
<td>150x75x4-C60</td>
<td>974</td>
<td>958</td>
<td>1.643</td>
</tr>
<tr>
<td>150x75x5-C60</td>
<td>1084</td>
<td>1076</td>
<td>0.738</td>
</tr>
<tr>
<td>150x75x6.3-C60</td>
<td>1280</td>
<td>1217</td>
<td>4.922</td>
</tr>
<tr>
<td>150x75x4-C100</td>
<td>1265</td>
<td>1221</td>
<td>3.478</td>
</tr>
<tr>
<td>150x75x5-C100</td>
<td>1296</td>
<td>1329</td>
<td>-2.546</td>
</tr>
<tr>
<td>150x75x6.3-C100</td>
<td>1483</td>
<td>1449</td>
<td>2.293</td>
</tr>
</tbody>
</table>

Figure 10: Comparison of the numerical and experimental results for concrete-filled elliptical steel tube columns
CONCLUSIONS

This paper describes the methodology of the nonlinear finite element models developed through ABAQUS/Standard solver to investigate the structural behaviour of concrete-filled elliptical steel tube columns under axial compression. Based on the basic conception of stress-stain relationship of confined concrete, a modified stress-stain model of confined concrete for prescribed concrete–filled elliptical steel tube columns was developed. According to the comparisons and analyses between experimental results and predicted numerical results, the following conclusions can be made: (1) The FE model developed via ABAQUS/Standard solver may be used to predict the structural behaviour of concrete-filled elliptical steel tube columns. (2) The modified confined concrete stress-strain model validated via a series of concrete-filled elliptical steel tube columns successfully captured the stress-strain relationship of confined concrete in prescribed elliptical steel tube columns. (3) Numerical modelling may be used to reveal the failure modes of concrete-filled elliptical steel tube columns under axial compression.

ACKNOWLEDGEMENT

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ADVANCES IN AMERICAN STEEL DESIGN:
THE PROPOSED AISC 2010 SPECIFICATIONS

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KEYWORDS
Codes, design codes, steel buildings, composite construction, steel connections, frame
stability, local buckling, ultimate strength, allowable strength.

ABSTRACT

In 2005, the American Institute of Steel Construction issued its most recent and vastly
changed Specification for Structural Steel Buildings (ANSI/AISC 360-05), the governing
design document for metal structures in most legal jurisdictions in the USA. Work is almost
completed on the revisions to that edition, scheduled to appear in 2010. This paper describes
some of the major improvements in steel and composite design provisions that have come
about as a result of these documents, with emphasis on the changes for 2010. The changes for
2005 included the unification of both allowable strength design (ASD) and load and
resistance factor design methods (LRFD), and the incorporation into the main specification
of the design provisions for hollow structural sections and their connections, as well as for
single angles. Among the other major changes for 2010 are a complete revamping of the
methodologies for assessing stability of framed structures that include the use of the nominal
load approach; new provisions for composite columns and shear studs; updated material
requirements; and design provisions for fire. This paper will selectively describe some of
these changes and highlight those of practical significance.

INTRODUCTION

From its initial edition in 1923 until 1986, the American Institute of Steel Construction
(AISC) had used an allowable stress design (ASD) approach in its main design specification.
In 1986, it issued its first specification in ultimate strength (or Load and Resistance Factor
Design, LRFD) format as an alternative, but maintained the two different specifications
(ASD and LRFD) until its 2005 edition (AISC, 2005a; henceforth Specification). In the 2005 cycle of its main specification, AISC decided that it could no longer afford to maintain two separate specifications, and successfully implemented a set of unified provisions that merged both approaches. Beginning with the 2005 Specification, a single expression is given for the nominal resistance of a member or component \( R_n \), and that resistance is reduced by a resistance factor \( \phi \) for LRFD design or divided by a safety factor \( \Omega \) for the new allowable strength design (ASD).

The general format of the equations is:  
\[
R_u < \phi R_n \quad (LRFD) \quad R_d < \frac{R_n}{\Omega} \quad (ASD)
\]

Note that the nomenclature maintains the use of the terms resistance and safety factors. While the origin of the former is clear and rooted in reliability theory, the latter is based mostly on experience and cannot be given a consistent meaning across all forms of loading. While for simple loading conditions such as tension, the new Allowable Strength Design and the older Allowable Stress Design are identical design procedures, this is not a statement that can be generalized to the rest of the Specification.

With the 2005 cycle of document development, AISC also consolidated all its design provisions into four broad documents: basic requirements for steel buildings (AISC 2005a/ANSI 360-05), seismic design (AISC Seismic 2005b/ANSI 341-05), nuclear design (AISC 2006/N690) and contractual provisions (Code of Standard Practice for Structural Steel Buildings, AISC 2005c). These publications are available for free download at www.aisc.org. Through close coordination with other regulatory bodies, such as the International Code Council (IBC 2006) and ASCE (ASCE 7-05, 2005), AISC has been able to develop a set of consistent and rational design rules that can be applied to almost all metal and composite structures built in the USA.

As with any new code, the 2005 edition had gaps and unclear language. The 2010 edition is being used to address these editorial issues as well as to introduce new significant technical provisions. Examples of what can be considered editorial changes for 2010 include clarifying in the Scope section that (1) the Specification applies not only to steel but also composite construction, (2) the design provisions are tied exclusively to the load combinations in ASCE 7, and (3) the exception for seismic systems with a force reduction (R) factor of 3 also applies to composite systems. In the next sections, significant technical changes for 2010 are highlighted. To understand them properly, discussions on selected topics related to the 2005 edition are also included.

**DESIGN BASIS**

As noted above, a very significant change was introduced on the basis of design (Chapter B of the Specification) in the 2005 edition with the introduction of unified provisions for both ASD and LRFD design. In the 2010 edition, the changes are less obvious but nonetheless significant, including:

- Explicit discussion that for calculations related to structural integrity, (1) the nominal properties should be used and (2) that limit states related to yielding and deformation need not be met for connections. It is interesting to note that this is the first time that terminology “structural integrity” is included in the basis for design in the Specification.
- A requirement that at points of support, beams, girders and trusses need to be restrained.
against rotation about their longitudinal axis unless demonstrated by analysis that such restraint is not necessary.

- Inclusion in this section and clarification of the provisions for 10% moment redistribution from elastic analysis values for compact continuous beams. This provision had been moved to an Appendix in 2005 and thus seemed to have been eliminated to many readers of the Specification.
- Inclusion of a section on diaphragm and collectors. Although the Specification does not give detailed provisions in this regard, this section and accompanying discussion in the commentary have been added to highlight the importance of these elements in the overall strength, stiffness and deformation capacity of a structure.
- Differentiation between anchorage in composite members, to be designed by the provisions for composite members given in Chapter I, and anchorages at steel column bases, to be designed as connections per the provisions given in Chapter J.
- Table B4.1 which contains limits for local buckling has been separated into two parts: one for design of members subjected to compression and one for members subjected to flexure. A single limit is now given in these tables for slender and non-slender members under axial loads. The only technical change in these Tables is in the requirements for stems of tees in compression when subject to flexure.

**STABILITY**

The 2010 edition of the Specification presents a momentous change to the American provisions for stability. The Direct Analysis Method (DAM), the American adaptation of the nominal load approach which had been introduced in the 2005 edition in an Appendix, has now been moved to the front of the chapter on stability (Chapter C) indicating its preferred use. The DAM is the culmination of many years of development, beginning with an American-Australian joint effort in the mid1990’s (ASCE, 1997). The development of the methodology is chronicled in detail by White, et al. (2007) and brings the American specification in line to those in Europe and many Commonwealth countries that adopted similar approaches beginning a decade or so ago. The other methods, while still valid and available, have been moved to Appendices. The approach by the DAM is contrasted with the equivalent length method (ELM), the most commonly used method in the USA for many decades, in Table 1.

The DAM is the preferred method because the Specification now requires that “the effect of all of the following on the stability of the structure and its elements shall be considered: (1) flexural, shear, and axial member deformations, and all other deformations that contribute to displacements of the structure; (2) second-order effects (both P-Δ and P-δ effects); (3) geometric imperfections; (4) stiffness reductions due to inelasticity; and (5) uncertainty in stiffness and strength.” Thus, the Specification carries upfront a concise and complete list of the factors to be taken into account for stability design. In particular, the requirement that consideration be given to the uncertainty in stiffness and strength is one that most older stability approaches cannot fulfill. The requirement that P-d effects be considered for frame stability is waived if the ratio of maximum second-order to first-order drift is below 1.7 and no more than one third of the gravity load is carried by columns that are part of the lateral-load resisting system.
# TABLE 1
THE DIRECT DESIGN VS. THE EQUIVALENT LENGTH APPROACH

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Effective Length</th>
<th>Direct Analysis Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis Type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second-order elastic(1)</td>
<td>Second-order elastic(1)</td>
<td></td>
</tr>
<tr>
<td>Nominal out-of-plumbness (or Notional Load)</td>
<td>None(2)</td>
<td>$\Delta_o/L = 0.002$ (or 0.002$Y_i$)</td>
</tr>
<tr>
<td>Effective Stiffness</td>
<td>Nominal</td>
<td>0.8 * Nominal, except $EI_{eff} = 0.8\tau_b EI$ if $\alpha P_r &gt; 0.5P_y$</td>
</tr>
<tr>
<td>Axial Strength $P_n$</td>
<td>$P_n$ based on KL(3)</td>
<td>$P_n$ based on L (no K) (P_n = QP_y in some cases)</td>
</tr>
</tbody>
</table>

(1) Includes first-order elastic analysis with amplifiers  
(2) Minimum of $D_o/L = 0.002$ (or 0.002$Y_i$) required for gravity-only cases  
(3) $K = 1$ allowed when sidesway amplification $\leq 1.1$

---

(a) Results for leeward column

(b) Example frame

Figure 1 : Comparison of results between DAM and ELM
The new Chapter C is considerably more clear and direct than previous versions, and clearly puts a premium on the stability of the structural system as a whole rather than that of individual elements. It requires a 0.8 reduction on all member stiffnesses used in the analyses (Table 1), allows for the direct inclusion of imperfection to model P-D effects if the DAM is not chosen, and allows the use of all the member design approaches used in the rest of the Specification. Chapter C permits stability design by plastic methods (Appendix 1), effective length methods (Appendix 7.2) or other first-order methods (Appendix 7.3). A large number of trial designs for structures that represent most common structural design conditions have been carried out and extensively verified against advanced analyses (Figure 1(a)). Figure 1(b) shows an example of the superior results from the DAM when compared with an advanced plastic zone analysis and the ELM. It is hoped that design software will immediately adopt the DAM, providing a advanced tool for designers.

TENSION AND COMPRESSION ELEMENTS

Chapters D (Tension) and E (Compression) do not have any major technical changes in the 2010 edition, except for (1) revised expressions for the equivalent slenderness for built-up compression members with either welded or pretensioned bolts, and (2) revised inequalities to determine the critical stress to be used in the design of members with slender elements.

FLEXURAL MEMBERS AND INTERACTION EQUATIONS

Several apparently small but significant changes have been made to Chapter F (Flexure):

- The term Rm and the upper limit of 3.0 have been removed from the equation (F1-1) for the modification factor (C_b) for lateral torsional buckling, leaving the simple expression:

\[ C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C} \]

where \( M_{\text{max}} \) is the maximum moment and \( M_A, M_B \) and \( M_C \) are the absolute value of the moments at \( L/4, L/2 \), and \( 3L/4 \) of the unbraced segment.

- The expression for the limit (\( L_r \), Eq. 4-8) for the elastic lateral torsional limit for non-symmetric I-shaped members with compact and non-compact webs has been modified and clarifications added for the case when the compression flange is a small portion of the beam (Eq. 4-9b).

- Extensive rewriting of section F9 on tees and double angles loaded in the plane of symmetry was undertaken to change the provisions from a format using a critical stress to one using a nominal moment.

- The limits for web slenderness limits (h/t_w) in proportioning I-beams have been changed (Eqs. F13-3 and F13-4).

There are no significant changes in Chapter H (Combined Forces).
SHEAR

The only changes to Chapter G (Shear) regard how to determine the moment of inertia of transverse stiffeners and the design of transverse stiffeners for tension field action; the impact of these provisions in design will be minor.

COMPOSITE CONSTRUCTION

Composite provisions were substantially updated in the 2005 Specification. This was particularly true for composite column design, which had remained unchanged since the late 1970’s (SSRC 1979). Until 2005, the design of composite columns in the USA could be carried out under either the steel or concrete design specifications. In both cases, many of the benefits of composite action were not properly recognized as the design procedures were meant to mimic those of steel and concrete elements. To address this shortcoming, the 2005 edition of the Specification presented a completely new approach for the design of composite columns within the context of both load and resistance factor design (LRFD) provisions and allowable strength design (ASD) methodology (Leon, et al. 2007). The new provisions require that the strength of composite sections shall be computed based on first principles of mechanics and robust constitutive models for materials. Two approaches are given to satisfy this requirement. The first is the strain compatibility approach, which provides a general method. The second is the plastic stress distribution approach, which is a subset of the strain compatibility one. The main changes for 2010 in Chapter I (Composite Construction) include:

- Extensive editorial rearrangement of materials, including separating design into sections dealing with axial loads, flexure shear and force transfer. This rearrangement is accompanied by substantial clarification and extension of the provisions on the 2005 Specification.
- Where possible, ACI 318 provisions are used for all detailing requirements related to concrete. Of particular importance is the attempt to coordinate all requirements for concrete strength of steel headed steel stud anchors to ACI 318 Appendix D.
- The Specification sets the minimum strength of a composite column as equal to that of a bare steel column using the same steel section as the composite member, correcting an inadvertent mistake in the previous edition which would have given strength values lower than those for the bare steel in the case of extremely slender members because of differences in resistance factors.
- New material is added and revisions made to the load transfer provisions in a dedicated section.
- The resistance factor and safety factor for encased and filled composite beams were increased to 0.9 based upon assessment of new data.
- New provisions have been included for local buckling in composite members (Tables 2 and 3). The design of axially loaded members is now divided between slender and non-slender sections, while that of flexural members is divided into compact, non-compact and slender sections. This follows the same scheme as for steel sections given in Section B4, and extends the design of flexural members well beyond that contemplated in the 2005 Specification.
Following the creation of Chapter G to include all shear provisions in one location for steel sections, a new Section I4 includes all shear design provisions for composite members.

The provisions for checking beam-columns have been clarified. Three simple methods are allowed:

- Use of the conventional interaction equations of Chapter H. This approach basically validates the older approach of treating composite members as equivalent steel ones. It is recognized that for many practical situations this is a conservative approach.
- Use of simplified rigid-plastic approaches, similar to those in the Eurocodes. This is based on developing interaction surfaces for combined axial compression and flexure at the nominal strength level using the plastic stress distribution method (Leon et al. 2007).
- Use of the Tables in AISC Design Guide 6 (Griffis, 1992). This guide was developed following the ACI approach in the mid-1990s, using more conservative resistance factors. The value of this method is the availability of a large number of design tables, which considerably speeds up the iterations needed in the design process.

The issue of load transfer now merits a whole new section, including a subsection on load allocation between steel and concrete, force transfer mechanisms and detailing.

A new section has been added containing performance-based requirements for the design and detailing of composite diaphragms and collector beams. Supplemental information is provided in the commentary as guidance to designers.

A major new development is the approach of using the ACI terminology of steel anchors to refer to all connectors, including shear studs and channels. New steel anchor provisions for shear, tension, and interaction of shear and tension are provided for all composite construction. To simplify design, anchor design can be based on the provisions of Table 4.

Extensive revisions have been made to the commentary to justify the changes and provide additional guidance. The commentary, for example, contains a detailed discussion of bond transfer in composite members. It is expected that this material will result in new design provisions that will be incorporated into the main Specification in the 2015 editions.

CONNECTIONS

Chapter J does not have substantial technical revisions in the 2010 Specification, but a large number of editorial changes and references to new or updated material specifications. The main change is that bolts are now classified as either Class A (ASTM A325 and similar) or B (ASTM A490 and similar), with accompanying changes to reflect installation procedures and recommended uses.
TABLE 2
LIMITS FOR LOCAL BUCKLING IN COMPOSITE MEMBERS UNDER AXIAL COMPRESSION

<table>
<thead>
<tr>
<th>Description of Element</th>
<th>Width-Thickness Ratio</th>
<th>$\lambda_p$ Compact/Noncompact</th>
<th>$\lambda_r$ Noncompact/Slender</th>
<th>Max. Permitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sides of rectangular box and hollow structural sections of uniform thickness</td>
<td>$b/t$</td>
<td>$2.26 \sqrt{\frac{E}{F_y}}$</td>
<td>$3.00 \sqrt{\frac{E}{F_y}}$</td>
<td>$5.00 \sqrt{\frac{E}{F_y}}$</td>
</tr>
<tr>
<td>Round filled sections</td>
<td>$D/t$</td>
<td>$0.15 \frac{E}{F_y}$</td>
<td>$0.19 \frac{E}{F_y}$</td>
<td>$0.35 \frac{E}{F_y}$</td>
</tr>
</tbody>
</table>

TABLE 3
LIMITS FOR LOCAL BUCKLING IN COMPOSITE MEMBERS UNDER FLEXURE

<table>
<thead>
<tr>
<th>Description of Element</th>
<th>Width-Thickness Ratio</th>
<th>$\lambda_p$ Compact/Noncompact</th>
<th>$\lambda_r$ Noncompact/Slender</th>
<th>Max. Permitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flanges of rectangular box and hollow structural sections of uniform thickness</td>
<td>$b/t$</td>
<td>$2.26 \sqrt{\frac{E}{F_y}}$</td>
<td>$3.00 \sqrt{\frac{E}{F_y}}$</td>
<td>$5.00 \sqrt{\frac{E}{F_y}}$</td>
</tr>
<tr>
<td>Webs of rectangular box and hollow structural sections of uniform thickness</td>
<td>$h/t$</td>
<td>$3.00 \sqrt{\frac{E}{F_y}}$</td>
<td>$5.00 \sqrt{\frac{E}{F_y}}$</td>
<td>$5.00 \sqrt{\frac{E}{F_y}}$</td>
</tr>
<tr>
<td>Round filled sections</td>
<td>$D/t$</td>
<td>$0.09 \frac{E}{F_y}$</td>
<td>$0.35 \frac{E}{F_y}$</td>
<td>$0.35 \frac{E}{F_y}$</td>
</tr>
</tbody>
</table>

TABLE 4
LIMITS FOR ANCHORS

<table>
<thead>
<tr>
<th>Loading Condition</th>
<th>Normal Weight Concrete</th>
<th>Lightweight Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear</td>
<td>h/d ≥ 5</td>
<td>h/d ≥ 7</td>
</tr>
<tr>
<td>Tension</td>
<td>h/d ≥ 8</td>
<td>h/d ≥ 10</td>
</tr>
<tr>
<td>Shear and Tension</td>
<td>h/d ≥ 8</td>
<td>N/A +</td>
</tr>
</tbody>
</table>

$h/d =$ ratio of steel headed stud anchor shank length (not including head) to shank diameter.

+ Refer to ACI 318 Appendix D for the calculation of interaction effects of anchors embedded in lightweight concrete.
TUBULAR STRUCTURES

In the 2005 edition, design provisions for hollow structural tubes (HSS) had been moved from a stand-alone document into the main specification. This led to a very large number of pages and equations being added to the document, not always in a format consistent with the rest of the specification. In the 2010 edition, the first three sections of Chapter K (Tubular Structures) have been reorganized, with many of the provisions now consolidated in a clear tabular format.

OTHERS

The older Chapter M (Fabrication, Erection, and Quality Control) has been split into new Chapters M (Fabrication and Erection) and N (Quality Control). This serves to emphasize the importance of quality control, for which explicit requirements and specifications exist, as opposed to fabrication and erection issues which tend to be more generic. Former Appendices 1 and 7 dealing with Stability have been morphed into Appendices 1, 6, 7 and 8 to accommodate the move of the DAM to the main Specification. Appendices 2 (Ponding), 3 (Fatigue), 4 (Fire) and 5 (Load Testing) remain unchanged.

CONCLUSIONS

This paper has given a brief overview of changes to the 2010 AISC Specification. The changes in this document are less significant than those in the 2005 edition, with the exception of the provisions for stability and composite elements. This Specification, along with its companion documents, provide state-of-the-art guidance for the design of metal structures.

ACKNOWLEDGEMENTS

The author would like to thank his colleague Dr. Don White for providing the data in Figure 1.

REFERENCES

[1] ACI. Building Code Requirements for Structural Concrete, ACI 318-05, American Concrete Institute, Farmington Hills, Michigan, 2005.


STUDY ON A NOVEL STEEL-CONCRETE COMPOSITE BEAM

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KEYWORDS
Steel-concrete web-embedded composite beam, Push-out test, Pull-out test, Local compression test, Flexural capacity, Construction stage, Service stage

ABSTRACT
A novel steel-concrete composite beam, web-embedded composite beam, is proposed. This composite beam is made up of a concrete slab and a steel beam with reverse T section. The isosceles trapezoid notch is cut at the top of the web of the steel beam, and the notched web is embedded in the concrete slab to act as shear connectors. To solve the problem that the steel beam with reverse T section is likely to buckle in lateral direction, cushion blocks laid on the bottom flange of the steel beam is proposed to support the weight of concrete for the slab during construction of such a composite beam. The experimental and theoretical studies on the capacities of the steel beam with reverse T section with loading at the bottom flange are conducted. Through the test on 10 push-out specimens, the bearing capacity of the notched web connectors embedded in the composite beam is investigated. Test results suggest that the bearing capacity of the shear connector increases with the improvement of the concrete grade and the thickness of the steel plate. Based on the test results, the formulas to calculate the shear bearing capacity of the connectors is obtained and the configuration requirements on the shear connector in web-embedded composite beams are proposed. To study the character to resist up-lift of the shear connector in web-embedded composite beams, pull-out tests on 6 specimens were conducted. Test results suggest that the notched web-embedded shear connection has sufficient ability to resist up-lift. To study the local compression performance of the shear connector in web-embedded composite beams, local compression tests on 4 specimens were conducted. The damage mechanism is observed and analyzed, and the formula to calculate the local compression bearing capacity of the shear connector in embedded composite beams is concluded concerning main influence factors. To study the performance of the composite beam in service stage, 4 specimens were designed to be tested,
the influence factors and the damage mechanism were researched, and test results suggest that the bending capacity of the web-embedded composite beam can be calculated by the plastic method.

INTRODUCTION

Steel-concrete composite beams employ shear connectors to combine the concrete slab and the steel beam to work cooperatively, which take advantage of the high tensile strength of the steel and the high compressive strength of the concrete. Composite beams have higher stiffness comparing with steel beams, and better seismic performance comparing with concrete beam [1]. Since 1920s, many researches on composite beams have been conducted. Viest I.M [2] collected the test results of 185 steel-concrete composite beams and 249 push-out specimens, and conducted comparison on the different methods to calculate the bearing capacity and deflection of composite beams in 1960. In 1975 Johnson R.P. [3] conducted experimental investigation on composite beams with partial shear connection and complete shear connection and found that the slip and lift effect between steel beam and concrete slab is also small even for the composite beam with partial shear connection if the steel beam is in elastic stage. In 1990, Crisinel [4] conducted tests on 3 specimens of simply supported steel-concrete composite beams using steel angle as shear connectors and concluded that behavior of this kind of shear connectors is as good as that of shear studs. In 1997, Richard [5] tested 44 simply supported steel-concrete composite beams under static and fatigue loads, resulting that the crack produced in concrete slab first under fatigue load, and the failure of the specimens was caused by the expansion of the cracks.

![Figure 1: Conventional composite beam and web-embedded composite beam](image)

Conventional composite beams are made up of steel beams with H-shaped section and concrete slabs. Usually headed stud shear connectors are used to combine the steel beam and the concrete slab to work as an integrated composite beam, as shown in Figure 1a. In a conventional composite beam, the top flange of the steel beam is located at the vicinity of the neutral axis of the composite section and the material of the top flange of the steel beam is not effectively used. For saving steel, a novel composite beam is proposed with the top flange of the steel beam removed and the isosceles trapezoid notch at the top of the web of the steel beam to work as shear connectors embedded in the concrete slab. This novel composite beam is named as web-embedded composite beam, as shown in Figure 1b.

Comparing with a conventional composite beam the novel web-embedded composite beam has the following advantages as
1) It does not need the top flange of the steel beam, which can save about 20% of steel comparing with the conventional composite beams;
2) It does not need the additional shear connectors, which can save material and the relevant exppanse;
3) The notched web connector not only has higher shear bearing capacity to develop complete shear connection, but also can decrease the reduction of the stiffness of the composite beam caused by slipping effect;
4) The fireproofing cost can be reduced because the area of the steel beam exposed to fire is reduced.

However, for practical application of this novel composite beam, the problems listed below need to be studied:
1) The performance the steel beam with reverse T section for the web-embedded composite beam during construction stage;
2) The capacity to resist shear and lift of the notched shear connector;
3) The local compressive character of the web-embedded composite beam; and
4) The flexural capacity of the web-embedded composite beam during service stage.

THE PERFORMANCE OF THE WEB-EMBEDDED COMPOSITE BEAM DURING CONSTRUCTION STAGE

The Construction Method

The design of a composite beam during construction stage is different from that during service stage. At the construction stage, the concrete is not cast or does not develop full strength, so the construction load is undertaken by the steel beam. Because the section of the steel beam is a reverse T, its lateral stiffness and capacity to resist lateral-torsional instability is low, which is not favorable for the steel beam to support the load during construction. Elastic stability theory indicates that the stability of the flexural members is related to the loading position, and the lower the loads related to the shear center of the cross-section of the members, the larger the capacity of the stability of the members [6]. Considering the special reverse T section of the steel beam, cushion blocks are proposed to be placed on the bottom flange to support the weight of the concrete for the slab through the concrete form and purlins, as shown in Figure 2. Using this construction method, the loading position is at the bottom flange of the steel beams, and then the stability of the steel beams with reverse T sections can be improved to satisfy the safety demands during construction stage.

![Figure 2: Illustration for construction of web-embedded composite beam](image-url)
Theoretical Analysis of the Flexural-torsional Buckling of the Steel Beam with Reverse T Section

Loaded on two sides of the web at the bottom flange

Because of randomness of construction, construction load can be applied at two sides or one side of the web at the bottom flange of the steel beams. When the steel beam with reverse T section is loaded at two sides of the web, the critical maximum moment in the beam can be obtained by the elastic theory as [6]:

\[
M_{cr} = \beta_1 \frac{\pi^2 EI_y}{l^2} \left[ \beta_2 \beta_y + \left( \beta_2 \beta_y \right)^2 + \frac{G I_y^2}{\pi^2 EI_y} \right]
\]

where \( \beta_1 \) is the modified coefficient of the critical moment determined by different types of loads; \( \beta_2 \) is the modified coefficient to monosymmetry section at different types of loads; \( a \) is the distance from the load position to the shear center of the steel beam section.

Loaded on one side of the web at the bottom flange

When the load is on one side of the web, the steel beam with reverse T section will be under bending and torsion. Under this circumstance, the load can be decomposed into symmetric and asymmetric ones (see Figure 3).

\[
P = \frac{P}{2} + \frac{P}{2}
\]

Figure 3: Decomposition of the load

The theory presented above can be used to solve the problem of the member under the symmetric load. When the member is undertaken the asymmetric load, the member is under torsion. Because the warp moment inertia of the reverse T section is nearly 0, the section of the member can warp freely. So there is only shear stress on the section produced by the free torsion. According to the yield criterion of the shear stress, the bearing capacity of the steel beam under torsion can be determined as

\[
P_z = \frac{4 I_t f_v}{n b_y t_{max}}
\]

where \( I_t \) is the free torsion moment inertia; \( t_{max} \) is the thickness of the thickest plate for the steel beam; \( b_y \) is the width of the bottom flange; \( n \) is the number of the loads on the steel beam; and \( f_v \) is the shearing strength of the steel.

When the bottom flange of the steel beam with reverse T section is loaded on single side of the web during construction stage, the bearing capacity of the steel beam can be determined by Rankine method as [7]:
\[
\frac{1}{P_{bz}} = \frac{1}{P_b} + \frac{1}{P_z}
\]

where \( P_{bz} \) is the bearing capacity of the steel beam under combined bending and torsion; \( P_b \) is the bearing capacity of the steel beam under bending, which can be obtained with Eqn. 1; and \( P_z \) is the bearing capacity of the steel beam under torsion.

**Experimental Study of the Steel beam with the Reverse T Section**

To verify the theory described herein above, tests on 4 specimens of the steel beam with reverse T section were conducted. The hydraulic jacks were used as the loaders. The wooden blocks were used as cushion blocks to support the jack. The specimens and the test setup are shown in Figure 4. The main parameters of the specimens are listed in Table 1. The yielding strength of the steel for the specimens is obtained from the material tests to be 300MPa, and the tensile strength is 380MPa. All of the specimens were simply supported. The vertical and lateral deflections of the specimens were measured at the location of the middle of the span.

**TABLE 1**

PARAMETERS OF SPECIMENS

<table>
<thead>
<tr>
<th>Specimens</th>
<th>TB-1</th>
<th>TB-2</th>
<th>TB-3</th>
<th>TB-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web(mm×mm)</td>
<td>215×8</td>
<td>265×10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flange(mm×mm)</td>
<td>100×10</td>
<td>150×12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam span(mm)</td>
<td>3360</td>
<td>3640</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear span(mm)</td>
<td>980</td>
<td>1070</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height of tooth(mm)</td>
<td>70</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance of tooth(mm)</td>
<td>240</td>
<td>300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Load mode | double side | single side | double side | single side |

(a) Arrangement of strain gauges and displacement meters

(b) Loaded at single side

(c) Loaded at double sides

Figure 4: Test set-up
The load-displacement curves were recorded, as shown in Figure 5. Test results suggest that the vertical deflection of the specimen is larger when the steel beam was loaded on two sides of the web and the lateral deflection is larger when the steel beam was loaded on one side of the web. The failure of the specimen is shown in Figure 6.

![Figure 5: Experimental load-displacement curves of the steel beams](image1)

![Figure 6: Failure mode of the steel beam with reverse T section](image2)

<table>
<thead>
<tr>
<th>No.</th>
<th>Span (mm)</th>
<th>Shear Span (mm)</th>
<th>Measured Load (kN)</th>
<th>Theoretical Load (kN)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB-1</td>
<td>3360</td>
<td>980</td>
<td>28.8</td>
<td>23.5</td>
<td>-11.3%</td>
</tr>
<tr>
<td>TB-2</td>
<td>3360</td>
<td>980</td>
<td>17.9</td>
<td>16.9</td>
<td>-5.6%</td>
</tr>
<tr>
<td>TB-3</td>
<td>3640</td>
<td>1070</td>
<td>53.5</td>
<td>48.6</td>
<td>-9.1%</td>
</tr>
<tr>
<td>TB-4</td>
<td>3640</td>
<td>1070</td>
<td>23.5</td>
<td>28.3</td>
<td>16.9%</td>
</tr>
</tbody>
</table>
The ultimate loads obtained from tests were listed in Table 2. And the results obtained by the theoretical method were also listed in Table 2. Though there are some differences between the measured values and the theoretical predictions, the errors are acceptable for the purpose of structural engineering.

THE SHEAR-RESISTANCE OF THE CONNECTORS IN WEB-EMBEDDED COMPOSITE BEAMS

Push-out Test

To study the shear-resistance performance of the connectors in web-embedded composite beams, push-out tests were employed. To simulate the mechanical status of the notched web in a web-embedded composite beam, a 50mm wide gap was retained between the concrete slab and the steel beam in push-out specimens. Concerning the concrete strength and the shape of the notch, 10 push-out specimens were designed for tests, as shown in Figure 7 and Table 3. In Table 3, \( f_{cu} \) is the measured value of concrete strength; \( h_i \) is the height of the concrete slab; \( t_w \) is the thickness of the notched steel plate; \( h_t \) is the height of the tooth of the notched web, \( w_b \) is the width of the tooth at bottom; \( w_t \) is the width of the tooth at top; \( \rho_h \) is the transverse reinforcement ratio; and \( l \) is the total length of the specimens.

Table 3: Parameters of push-out specimens

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Concrete grade</th>
<th>( f_{cu} ) (N/mm²)</th>
<th>( h_i ) (mm)</th>
<th>( t_w ) (mm)</th>
<th>( h_t ) (mm)</th>
<th>( w_b ) (mm)</th>
<th>( w_t ) (mm)</th>
<th>( \rho_h )</th>
<th>l (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH-1, SH-2</td>
<td>C20</td>
<td>27.5</td>
<td>120</td>
<td>10</td>
<td>80</td>
<td>100</td>
<td>200</td>
<td>0.56%</td>
<td>840</td>
</tr>
<tr>
<td>SH-3, SH-4</td>
<td>C40</td>
<td>42.5</td>
<td>120</td>
<td>10</td>
<td>80</td>
<td>100</td>
<td>200</td>
<td>0.56%</td>
<td>840</td>
</tr>
<tr>
<td>SH-5, SH-6</td>
<td>C30</td>
<td>34.2</td>
<td>120</td>
<td>10</td>
<td>80</td>
<td>100</td>
<td>200</td>
<td>0.56%</td>
<td>840</td>
</tr>
<tr>
<td>SH-7, SH-8</td>
<td>C30</td>
<td>34.2</td>
<td>120</td>
<td>10</td>
<td>80</td>
<td>100</td>
<td>200</td>
<td>0.70%</td>
<td>840</td>
</tr>
<tr>
<td>SH-9, SH-10</td>
<td>C30</td>
<td>34.2</td>
<td>100</td>
<td>8</td>
<td>60</td>
<td>80</td>
<td>160</td>
<td>0.84%</td>
<td>740</td>
</tr>
</tbody>
</table>
The yielding strength of the steel for the shear connectors of the specimens is obtained from the material tests to be 349MPa, and the tensile strength is 480MPa. HRB335 steel bar was used in push-out test specimens. The yielding strength of the steel bar is obtained to be 370MPa. The concrete strength was obtained through the standard strength test after 28 days since the concrete was poured. The diameter of the steel bar is 8mm and the thickness of the concrete cover is 15mm. Various thicknesses of steel plates, 8mm, 10mm and 12mm were used to for the notched shear connectors.

All tests were conducted with hydraulic testing machine YE-2000, the loading capacity of which is 2000kN. The push-out displacements were measured with the displacement meter YHD-30, the maximum capacity of which is 30mm.

**Test Phenomena and Results**

When the load is less than 0.6 times of the ultimate push-out force $V_u$, the specimens remain in elastic state. When the specimens are loaded at the level of 0.6-0.7$V_u$, the transverse concrete crack emerges, as shown in Figure 8a. When larger loads apply, the sound of the concrete cracking can be heard, and the longitudinal concrete crack emerges and extends along the longitude of the specimen, as shown in Figure 8b. The main test results are shown in Table 4.

![Horizontal cracking](image1)

![Vertical cracking](image2)

Figure 8: Horizontal and vertical cracks of push-out specimens

<table>
<thead>
<tr>
<th>No</th>
<th>Cracking load (kN)</th>
<th>Ultimate load (kN)</th>
<th>Cracking displacement (mm)</th>
<th>Ultimate displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH-1</td>
<td>98.50</td>
<td>135.20</td>
<td>0.31</td>
<td>1.85</td>
</tr>
<tr>
<td>SH-2</td>
<td>100.70</td>
<td>140.60</td>
<td>0.35</td>
<td>1.92</td>
</tr>
<tr>
<td>SH-3</td>
<td>110.50</td>
<td>165.20</td>
<td>0.40</td>
<td>1.50</td>
</tr>
<tr>
<td>SH-4</td>
<td>112.20</td>
<td>170.40</td>
<td>0.21</td>
<td>0.65</td>
</tr>
<tr>
<td>SH-5</td>
<td>110.10</td>
<td>140.20</td>
<td>0.40</td>
<td>1.12</td>
</tr>
<tr>
<td>SH-6</td>
<td>105.60</td>
<td>135.80</td>
<td>0.25</td>
<td>0.79</td>
</tr>
<tr>
<td>SH-7</td>
<td>120.10</td>
<td>185.10</td>
<td>0.15</td>
<td>0.75</td>
</tr>
<tr>
<td>SH-8</td>
<td>122.80</td>
<td>180.10</td>
<td>0.18</td>
<td>0.56</td>
</tr>
<tr>
<td>SH-9</td>
<td>90.10</td>
<td>105.60</td>
<td>0.11</td>
<td>1.90</td>
</tr>
<tr>
<td>SH-10</td>
<td>85.60</td>
<td>110.40</td>
<td>0.11</td>
<td>1.50</td>
</tr>
</tbody>
</table>
Capacity of Shear-resistance

From the observation of test results, the failure mode of the push-out specimens is longitudinal crack of the concrete slab. It is found that the shear-resistance capacity of the push-out specimens is related to the concrete grade and the thickness of the notched web. Formula to calculate the bearing capacity of a shear connector at the top of notched web can be obtained on the basis of the test results as:

\[ N_c = 0.85t_w \sqrt{E_c f_c} + 57.25 \text{(kN)} \]  

where \( E_c \) is the elastic modulus of concrete in MPa; \( f_c \) is concrete strength in MPa; and \( t_w \) is the thickness of the notched web in mm.

THE LIFT-RESISTANCE OF THE SHEAR CONNECTOR IN WEB-EMBEDDED COMPOSITE BEAMS

The Requirements of the Shear Connector to Resist Lift

In a composite beam the lift force is caused by the trend of separation between the steel beam and the concrete slab, as shown in Figure 9. Usually the separation displacement at the ends of the composite beam is the largest. To resist the separation, the shear connector in composite beams should have enough capacity to resist the lift. Currently, it believes that the capacity of the shear connector in a composite beam to resist the lift should reach 10% of the shear-resistance capacity of the connector [8].

Pull-out Tests

Tests were conducted on 6 specimens to investigate the lift-resistance capacity of the connector in web-embedded composite beams. The specimens and test set-up are shown in Figure 10. The hijacker was used as the loader, and an I25a steel beam was used as the reaction beam. To avoid flexural failure of the concrete slab, 2 I16a steel beams were used to guarantee the homogeneity of the lift force to pull the notched web connector out from the concrete slab. The main parameters of the specimens are shown in Table 5, in which \( h_c \) is the height of the concrete slab; \( t_w \) is the thickness of the notched steel plate; \( h_t \) is the height of the tooth of the notched steel plate; \( w_b, w_t \) are the width of the tooth at bottom and at top respectively; \( d \) is the diameter of the steel bar; \( \rho_h \) is the transverse reinforcement ratio; and \( l \) is the total length of the specimens.
Figure 10: Pull-out specimen and test setup

TABLE 5
THE MAIN PARAMETERS OF THE PULL-OUT SPECIMENS

<table>
<thead>
<tr>
<th>Group</th>
<th>No.</th>
<th>$h_i$ (mm)</th>
<th>$t_w$ (mm)</th>
<th>$h_o$ (mm)</th>
<th>$w_3$ (mm)</th>
<th>$w_4$ (mm)</th>
<th>$d$ (mm)</th>
<th>$\rho_s$</th>
<th>$l$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B-1</td>
<td>120</td>
<td>10</td>
<td>80</td>
<td>100</td>
<td>200</td>
<td>8</td>
<td>0.56%</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>B-2</td>
<td>120</td>
<td>10</td>
<td>80</td>
<td>100</td>
<td>200</td>
<td>8</td>
<td>0.56%</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>B-3</td>
<td>100</td>
<td>8</td>
<td>60</td>
<td>60</td>
<td>120</td>
<td>8</td>
<td>0.84%</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>B-4</td>
<td>100</td>
<td>8</td>
<td>60</td>
<td>60</td>
<td>120</td>
<td>8</td>
<td>0.84%</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>B-5</td>
<td>150</td>
<td>12</td>
<td>110</td>
<td>100</td>
<td>200</td>
<td>10</td>
<td>0.70%</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>B-6</td>
<td>150</td>
<td>12</td>
<td>110</td>
<td>100</td>
<td>200</td>
<td>10</td>
<td>0.70%</td>
<td>1000</td>
</tr>
</tbody>
</table>

Test Phenomenon and Results

It is found that the direction of the longitudinal cracks of specimens vary with the transverse reinforcement ratio. The direction of the cracks in specimen B-1 and B-2 which had lower transverse reinforcement ratio extended obliquely and the direction of the cracks in other specimens which had larger transverse reinforcement ratio extended straightly, as shown in Figure 11. The main test results are listed in Table 6.
TABLE 6
THE MAIN RESULTS OF PULL-OUT TESTS

<table>
<thead>
<tr>
<th>No</th>
<th>B-1</th>
<th>B-2</th>
<th>B-3</th>
<th>B-4</th>
<th>B-5</th>
<th>B-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift displacement at failure (mm)</td>
<td>1.2</td>
<td>1.5</td>
<td>2.2</td>
<td>1.3</td>
<td>3.1</td>
<td>2.8</td>
</tr>
<tr>
<td>Pull-out capacity (kN)</td>
<td>81.2</td>
<td>77.1</td>
<td>45.0</td>
<td>36.7</td>
<td>84.9</td>
<td>80.5</td>
</tr>
<tr>
<td>The direction of the crack</td>
<td>oblique</td>
<td>oblique</td>
<td>straight</td>
<td>straight</td>
<td>straight</td>
<td>straight</td>
</tr>
</tbody>
</table>

The ratio of the capacities of the notched web connectors to resist lift and to resist shear are listed in Table 7. The shear resistance capacity of the connector was obtained with Eqn. 4. It can be observed that the notched web connectors have good capacity to resist lift.

TABLE 7
THE RATIO OF THE PULL-OUT CAPACITY AND SHEAR-RESISTANCE CAPACITY

<table>
<thead>
<tr>
<th>No</th>
<th>B-1</th>
<th>B-2</th>
<th>B-3</th>
<th>B-4</th>
<th>B-5</th>
<th>B-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pull-out capacity (kN)</td>
<td>81.2</td>
<td>77.1</td>
<td>45.0</td>
<td>36.7</td>
<td>84.9</td>
<td>80.5</td>
</tr>
<tr>
<td>Shear-resistance capacity (kN)</td>
<td>113</td>
<td>113</td>
<td>92.9</td>
<td>92.9</td>
<td>137</td>
<td>137</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.72</td>
<td>0.68</td>
<td>0.48</td>
<td>0.40</td>
<td>0.62</td>
<td>0.59</td>
</tr>
</tbody>
</table>

LOCAL COMPRESSION CAPACITIES OF THE WEB-EMBEDDED COMPOSITE BEAMS

In a web-embedded composite beam, the steel beam with reverse T section is embedded in concrete slab. So when the composite beam is subjected local compression, the concrete is punched by the web of the steel beam with reverse T section, and the local compression strength failure may occur before the overall failure of the composite beam. To study the local compression capacity of the composite beam, local compression tests were conducted.

Local Compression Tests

Local compression tests on 4 specimens were conducted. An oil jack is used to apply local load through a cushion block on the top of the specimens. The cushion block is a 110mm square and 40mm thick of steel plate. The specimen and the test setup are shown in Figure 12.
The yielding strength of the steel for the specimens is obtained from the material test to be 300MPa, and the tensile strength is 380MPa. The yielding strength of the steel bar is obtained to be 314MPa. The concrete strength was obtained through the standard strength test. The compression strength of C20 concrete was obtained to be 24.4MPa, and that of C30 concrete be 30.7MPa. The main parameters of the 4 specimens are listed in Table 8. In Table 8, \( h_c \) is the height of the concrete slab; \( t_w \) is the thickness of the notched steel plate; \( h_t \) is the height of the tooth of the notched steel plate; \( w_t, w_r \) are the width of the tooth at bottom and at top respectively; \( d \) is the diameter of the steel bar; \( \rho_h \) is the transverse reinforcement ratio; and \( l \) is the total length of the specimens.

**TABLE 8**

<table>
<thead>
<tr>
<th>No</th>
<th>( h_c ) (mm)</th>
<th>Concrete grade</th>
<th>( h_t ) (mm)</th>
<th>( w_t ) (mm)</th>
<th>( w_r ) (mm)</th>
<th>( t_w ) (mm)</th>
<th>( d ) (mm)</th>
<th>( \rho_h ) (mm)</th>
<th>( l ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CQ-1</td>
<td>130</td>
<td>C30</td>
<td>80</td>
<td>100</td>
<td>200</td>
<td>10</td>
<td>8</td>
<td>0.81%</td>
<td>1000</td>
</tr>
<tr>
<td>CQ-2</td>
<td>130</td>
<td>C20</td>
<td>80</td>
<td>100</td>
<td>200</td>
<td>10</td>
<td>8</td>
<td>0.81%</td>
<td>1000</td>
</tr>
<tr>
<td>CQ-3</td>
<td>120</td>
<td>C30</td>
<td>70</td>
<td>80</td>
<td>160</td>
<td>8</td>
<td>8</td>
<td>0.70%</td>
<td>960</td>
</tr>
<tr>
<td>CQ-4</td>
<td>120</td>
<td>C20</td>
<td>70</td>
<td>80</td>
<td>160</td>
<td>8</td>
<td>8</td>
<td>0.70%</td>
<td>960</td>
</tr>
</tbody>
</table>

**Test Phenomenon and Results**

When the load was low, the relationship between the load and the local compressive deformation was linear. When the specimens were subjected to a load grater than 0.7\( Pu \) (\( Pu \) is the ultimate local compressive load), the longitude crack emerged at the middle of the lateral sides of the specimen, as shown in Figure 13a. With the load further increasing, the longitudinal concrete crack extended rapidly, as shown in Figure 13b. The sinkage emerged on the top of the specimen when it failed, as shown in Figure 13c.
The main results of the tests are listed in Table 9.

### TABLE 9
MAIN RESULTS OF LOCAL COMPRESSION TESTS

<table>
<thead>
<tr>
<th>No.</th>
<th>CQ-1</th>
<th>CQ-2</th>
<th>CQ-3</th>
<th>CQ-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local compressive deformation (mm)</td>
<td>3.93</td>
<td>4.63</td>
<td>3.86</td>
<td>4.02</td>
</tr>
<tr>
<td>Local compression capacity (kN)</td>
<td>670.4</td>
<td>610.9</td>
<td>592.6</td>
<td>433.8</td>
</tr>
<tr>
<td>Theoretical results (kN)</td>
<td>640.4</td>
<td>535.3</td>
<td>563.5</td>
<td>445.4</td>
</tr>
<tr>
<td>Relative difference</td>
<td>-4.5%</td>
<td>-12.4%</td>
<td>-4.9%</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

Note: theoretical results are obtained by Eqn. 5.

**Predication of Local Compression Capacity**

It can be observed from the test results that the local compression capacity is related to the concrete grade, the local compressive area and the distance from the top of the concrete slab to the top of the web notch of the steel beam with reverse T section. The local compression mechanical model of the web-embedded composite beam is shown in Figure 14. With the model, the formula to calculate the local compression capacity of the composite beam is obtained as:

\[
F_l = 1.5 \beta f_{c} (b + a)(l + a) \tag{5}
\]
where \( F_l \) is the local compression capacity; \( \beta_c \) is the concrete strength coefficient; \( f_{ck} \) is the characteristic value of concrete compressive strength; \( b_l \) and \( l \) are the width and length of the local compressive area respectively; and \( a \) is the distance from the top of the concrete slab to the top of the notch. The local compression capacities obtained with Eqn. 5 of the 4 local compressive specimens are listed in Table 9. It can be seen that the predication results have a good agreement with the experimental values.

LOAD-BEARING CAPACITIES OF THE WEB-EMBEDDED COMPOSITE BEAMS IN SERVICE STAGE

Analysis of the Web-embedded Composite Beam

Plastic theory can be employed to calculate the flexural capacity of web-embedded composite beams. According to whether the neutral axis is in the concrete slab or not, the composite beams can be divided into 2 types. If the neutral axis of the composite beam is in the concrete slab, it is classified into the 1st type, and the steel beam is in tension. If the neutral axis of the composite beam is in the steel beam, it is classified into the 2nd type and some part of the steel beam will be compressed [9].

First type of web-embedded composite beams

For the first type of the composite beam (Figure 15), \( A_f \leq b_l h_c f_c \) and

\[
M_{lp} = b_c x f_c y \tag{6}
\]

First type of web-embedded composite beams

Figure 15: Stress distribution of the first-type of composite beam

in Eqn. 6

\[
x = A_f / (b_c f_c) \tag{7}
\]

where \( M_{lp} \) is the flexural moment capacity of the composite beam; \( x \) is the depth of the compression zone of the concrete slab; \( y \) is the distance from the centroid of the tensile stress of the steel beam to the centroid of the compressive stress of the concrete slab; \( A \) is the cross-sectional area of the steel beam; \( f \) is the design value of the strength of steel; and \( f_c \) is the design value of the compressive strength of concrete.
Second type of web-embedded composite beams

For the second type of the composite beam (Figure 16), \(Af > b_h f_c\) and

\[
M_{bp} \leq b_h f_c y_1 + A_c f y_2
\]

Figure 16: Stress distribution of the second-type of composite beam

in Eqn. 8

\[
A_c = 0.5(A - b_h f_c f / f)
\]

where \(A_c\) is the compressive cross-sectional area of the steel beam; \(y_1\) is the distance from the centroid of the tensile stress of the steel beam to the centroid of the compressive stress of the concrete slab; and \(y_2\) is the distance from the centroid of the tensile stress of the steel beam to the centroid of the compressive stress of the steel beam.

Flexural Tests of the Web-embedded Composite Beams

Test on 4 specimens of web-embedded composite beams were conducted. The main parameters of the 4 specimens are shown in Figure 17 and Table 10. In Table 10, \(h\) is the height of the whole combined cross section; \(h_l\) is the height of the tooth of the notched steel plate; \(w_p\) and \(w_t\) are the widths of the tooth at bottom and at top respectively.

Figure 17: Specimens for flexural tests of web-embedded composite beams

TABLE 10
MAIN PARAMETERS OF THE SPECIMENS (mm)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Concrete grade</th>
<th>Span</th>
<th>(h)</th>
<th>(w_p)</th>
<th>(w_t)</th>
<th>(h_l)</th>
<th>Longitudinal bar</th>
<th>Transversal bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECB-1</td>
<td>C20</td>
<td>3360</td>
<td>320</td>
<td>160</td>
<td>80</td>
<td>70</td>
<td>Φ8@100</td>
<td>Φ10@100/200</td>
</tr>
<tr>
<td>ECB-2</td>
<td>C30</td>
<td>3360</td>
<td>320</td>
<td>160</td>
<td>80</td>
<td>70</td>
<td>Φ8@100</td>
<td>Φ10@100/200</td>
</tr>
<tr>
<td>ECB-3</td>
<td>C20</td>
<td>3540</td>
<td>380</td>
<td>200</td>
<td>100</td>
<td>80</td>
<td>Φ8@100</td>
<td>Φ8@80/160</td>
</tr>
<tr>
<td>ECB-4</td>
<td>C30</td>
<td>3540</td>
<td>380</td>
<td>200</td>
<td>100</td>
<td>80</td>
<td>Φ8@100</td>
<td>Φ8@80/160</td>
</tr>
</tbody>
</table>
**Test Phenomenon and Results**

Flexural failure occurred to the 4 specimens. When the load reached the ultimate value, the remarkable flexural deflection developed (Figure 18a), the concrete at the zone of the mid-span of the specimens was crushed (Figure 18b), and longitudinal cracks also emerged on the concrete slab at the location of the web at the steel beam (Figure 18c).

![Figure 18: Test phenomenon of flexural tests of the web-embedded composite beams](image)

The load-deflection curves of the 4 specimens are shown in Figure 19. It can be concluded that plastic design method can be used to determine the flexural capacity of the web-embedded composite beams as the plasticity of the steel beam and the concrete can be fully developed.

![Figure 19: Load-deflection curves of the specimens](image)
The load-bearing capacities of the specimens obtained through measurement and prediction are listed in Table 11. It can be found that the plastic method can be employed to predict the flexural capacity of the web-embedded composite beams satisfactorily.

<table>
<thead>
<tr>
<th></th>
<th>ECB-1</th>
<th>ECB-2</th>
<th>ECB-3</th>
<th>ECB-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical value/kN</td>
<td>133.5</td>
<td>136.2</td>
<td>238.4</td>
<td>244.9</td>
</tr>
<tr>
<td>Measured value/kN</td>
<td>142.2</td>
<td>150.6</td>
<td>228.1</td>
<td>235.4</td>
</tr>
<tr>
<td>Relative error</td>
<td>6.3%</td>
<td>9.4%</td>
<td>-4.6%</td>
<td>-4.2%</td>
</tr>
</tbody>
</table>

CONCLUSIONS

Through the experimental and theoretical studies of the web-embedded composite beams, the following conclusions can be drawn as follows:

(1) Web-embedded composite beam is a new type of the composite beam with the similar performance as the conventional composite beam and reduction of cost.

(2) Cushion blocks can be used to transform the load from the concrete for the slab to the bottom flange of the steel beam and to enhance the stability of the steel beam with reverse T section for the web-embedded composite beams during construction stage.

(3) The notched shear connector embedded in the concrete slab has wonderful shear-resistance capacity, lift-resistance capacity and local compression capacity.

(4) As conventional composite beams, the flexural capacity of the web-embedded composite beams can be determined with using plastic method.

REFERENCES


NOVEL DEPLOYABLE STRUCTURAL SYSTEMS

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KEY WORDS

Mobile, foldable, transformable, retractable, adaptive, robustness, renewable energy, protective structures

ABSTRACT

This paper deals with the development of deployable structures for civilian and military application. While various forms of deployable shelters are reviewed, there is no clear indication of protection ability of the existing shelters. Tests on armour materials by field blast and laboratory bullet shootings confirm the possibility of robustness enhancement for deployable structures. It is concluded that the marriage of deployable forms and protective materials allows emergence of new deployable protective shelters, which is a balance of low weight, deployability, and protection level. Another important direction of developing deployable form is in energy harvesting system for buildings. Deployable system in this design includes electrical devices to collect energy and its mobility allows high level of solar energy yield.

INTRODUCTION

This paper discusses two aspects of deployable structures, robustness and environmental interaction. These features are rarely mentioned in literature as making the structure deployable is already a challenging task and the past research mainly falls in this category.

Deployable structures have been developed for the last few decades with major focus on civilian and space applications. Various forms of such applications can be found in Kronenburg (1998), Ishii (2000), Gantes (2001), Ma et al. (2009), Vu et al. (2006, 2008), and Oungrinis, (2006).
Deployable systems are also commonly found in military operations due to their convenient in transportation and rapid transformation e.g. foldable bridges or deployable communication boom. Military deployable shelters contribute significantly to the growth of army technologies. However, most of them remain in tentage form with no clear evidence of capability to resist blast and fragments. This paper will discuss the possibility of utilizing armour materials towards enhancing robustness of deployable shelters. The concept of deployable protective shelters is emerged in such context.

Transformation of deployable structure is a potential source of energy creation and allows adaptation of the system to external environment change. This point will be illustrated further in section 3 of this paper showing the conceptual development of a mobile solar energy harvesting façade system. The development of such a system requires multi-disciplinary research effort.

MILITARY DEPLOYABLE SHELTERS

Existing shelters

The existing rapidly deployed shelters are classified as following, which is based on the mechanical feature of the systems

- Container-based
- Strut assembly (truss or frame)
- Pantograph-based
- Pneumatic
- Movable panels

Portable shelter requires no crane or special equipment for deployment. A group of 2 to 5 persons are able to handle the whole installation. It may span up to 9 m.

Large and heavier shelter requires crane or special lifting equipment to assist deployment due to relatively high weight. The range of span is from 9 m to 60 m.

Container-based shelters

Container-based design is among the most popular instant shelter design available today due to its simplicity in manufacturing and installation. Container-based shelters are used in many area including military, offshore living quarter, and mobile accommodation. The deployment of the shelter can be from stack form (Fig. 1) or from the ISO container form (Fig. 2).

Major components of expandable container shelters include the steel strut and steel sheet welded together to form container shape. Some other components are hinged to each other to allow deployment by rotation as shown in Fig. 1.

The advantages of container-based shelters are
- Very fast deployment (time/area)
• Well protect human or equipment from weather effects or shrapnel

Their disadvantages are
• Limited in height (and thus the flexibility in usage is not significant)
• Limited in shapes and sizes
• Heavy (high weight of material/area to be covered)
• Consume a lot of space during transportation

Figure 1: Deployment of container-based shelter from stack forms
(http://www.uniteamcontainer.com/shelter/quickrapidreaction.html)

Figure 2: Deployment of container-based shelter from ISO containers
http://www.euro-shelter.com/francais/index1.htm)

Figure 3: Rapid-built aircraft shelter.
Rapidly Deployable Aircraft Protective Shelter (RDAPS) series have been developed by the authors to provide sheltering for military aircraft in case of urgent deployment (Vu et al., 2009; Ma et al., 2009). The primary feature of such shelter is to allow very rapid deployment comparing with conventional aircraft shelters while offering reliable protection against weapon effects from near misses of munition explosion. The proposed shelter system utilizes readily available shipping (ISO) container to form the side enclosures and roof is made from deployable lightweight cable strut structure with suspended fabric for covering. The container-based walls can be filled with different protective materials to mitigate weapon effects from near misses of munition explosion. The lightweight fabric roofing system is designed to protect the aircraft from weathering effects. Though not designed to mitigate weapon effects, the roofing system is detailed to reduce the risk of damaging the protected asset in the event of localised failure (Vu et al, 2009).

**Strut assembly**

![Figure 4: Planar frame shelter](http://www.rubb.com/products_military_rapid_environmental_shelter.asp)

![Figure 5: Deployable shelter (Temmerman et al., 2006)]

![Figure 6: Strut-assembly shelter classified by US Jocotas, 2007]
Strut assembly is among the most common forms of shelters and thus its improved forms with instance feature are also easily adopted. The usage of strut assembly is flexible in terms of shapes and sizes. Because of its popularity, there are many suppliers in the world. Good speed in construction can be gained when the system is well modularized.

Strut assembly can be in the form of modular planar frame (Fig. 4). It can be connected by special hinges and deployed on site as shown in Fig. 5. This approach requires high volume during transportation. Strut assembly is also one of the rare chosen forms of military shelters by US Jocotas, 2007 as shown in Fig. 6.

The major advantages of strut assembly are structural efficiency and flexibility in shapes and sizes. However, preparation for deployment of strut assembly is time consuming and very difficult to be improved. Generally speaking, the preparation for deployment of strut assembly requires high human effort which is very close to the conventional construction techniques.

**Pantograph-based shelters**

![Figure 7: Krishnapillai deployable structure](image)

![Figure 8: Drash deployable shelter (www.milsys.co.uk)](image)

Pantographs or Scissor-Like Elements (SLEs) are common components in design of deployable structures. The reason is that the SLEs once connected will behave like a kinematic chain. The deployment is therefore well facilitated. Several ideas of using a set of SLEs to form a shelter in dome shape or barrel vault shape can be found in Figs. 7, 8.
The major advantage of SLE system is the compactness of folded form and the relatively quick deployment. However, the drawback is low structural stiffness and poor robustness. SLE systems are mainly used for small span applications (Vu et al., 2006a&b).

Pneumatic and Lightweight Membrane Shelters

Pneumatic technology is not new since it has long been applied in pneumatic tires for cars, motorbikes, and airp lanes. The main advantage of this technology is due to its lightness and compactness as well as its speed in deployment. A pneumatic air-beam technology has been developed for large span shelters as shown in Fig. 9.

The major drawback of this technology is maintenance and replacement as the membrane material is not readily available. Also, the system is very sensitive to tearing and the strength and stiffness can be quickly reduced to zero. This system has been recently adopted by US Natick (JOCCOTAS 2007).

Various forms of lightweight membrane shelters can be generated as shown in Fig. 10. The system is named as butterfly wing structure developed by Tran and Liew (2006; 2008). The structure has three main components: membrane, cable and arched frame. The ends of the arched frames are coincided circumferentially and are pinned so that the arches can rotate perpendicularly to the frame-planes. The membrane attached to the arches, therefore, will be tensioned by rotating these arches outwards. This opening process is done by tensioning the fans of cables which are radiate from anchor points outside of each arch to every joint of the arched frames. Strength and stiffness of the structure are achieved by stressing the cable against the anchor points. The final shapes of the structure after deploying look like the butterflies spreading their wings.

Versatility of Butterfly-shaped membrane structures is achieved by combining basic unit in many ways to suit the shape and the size of the applications. The number of wings of the “butterfly” ranges from two to many, depending on the requirement of covered area. Typically, the larger the area needs to be covered, the more wings it is needed. The arched frames themselves can have many shapes ranging from shallow to tall. The inclined angle of the arch with respect to the ground is also adjustable resulting in the different shape and height of the structure. The curvature of the membrane is also various depending on the level of prestress. The cable anchorage points may be close or distant and more than one points may be used each side of an.
arch. By combining different shapes in different configurations, an infinite of structural forms is achievable Tran and Liew (2006; 2008).

**Movable/foldable panels**

These systems consist of movable components which are made from panels. Special joints are developed to allow the deployment of the panels to form the whole system as shown in Fig. 11.

A customised medium-size personnel protective shelter (Fig. 12) has been developed to shield from various weapon effects. The shelter can accommodate up to 15 personnel and can either be mounted on trailer or equipped with wheels for mobility. The proposed design displays an optimised balance of high speed automated lightweight deployment and protective ability. The distinctive features of the shelter are high stacking ratio (5:1), high level of automation and mobility. Resistance to bullet penetration to US-NIJ standard and blast loadings will be discussed in Sections 2.2 and 2.3.
Bullet and fragment resistance of armour materials

The 200 mm x 200mm test plate with 4 nos. 9mm diameter hole was designed due to the limitation of test specimen support fixture. The location and arrangement for shots were followed by STANAG multiple hit testing (Fig. 13). According to STANAG multiple hit testing, the application of multi-hit conditions to armour component ballistic testing require a number of geometric parameters be defined in order to obtain a reproducible and fair evaluation. Ballistic performance of the tested materials is shown in table 1.
TABLE 1
BALLISTIC PERFORMANCE OF ARMOURED MATERIALS

<table>
<thead>
<tr>
<th>S/N</th>
<th>Test Materials</th>
<th>NIJ Standard 0108.01</th>
<th>STANAG (12 mm FSP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Level IIIA (9 mm FMJ)</td>
<td>Level 1</td>
</tr>
<tr>
<td>1</td>
<td>3 mm thick SECURE MS</td>
<td>Pass</td>
<td>Fail</td>
</tr>
<tr>
<td>2</td>
<td>6 mm thick XAR-450</td>
<td>Pass</td>
<td>Pass</td>
</tr>
<tr>
<td>3</td>
<td>6 mm thick OPTIM 700</td>
<td>-</td>
<td>Pass</td>
</tr>
<tr>
<td>4</td>
<td>6 mm thick ST 52-3</td>
<td>-</td>
<td>Pass</td>
</tr>
<tr>
<td>5</td>
<td>8 mm thick Mild Steel</td>
<td>Pass</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>26 plies Twaron CT 714 Fabric</td>
<td>Fail</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>58 plies Twaron CT 714 Fabric</td>
<td>Pass</td>
<td>-</td>
</tr>
</tbody>
</table>

The hardness of the 12.7 mm FSP projectile (286 HB) was 24% less than the hardness of the target plate (356 HB). The range of velocities used in the present investigation shows significant projectile erosion. The projectile was shortened and flattened at tip for the impact velocities of 305 m/s and 402.2 m/s. The substantial deformation, flattening and forming of mushroomed shape of the FSP projectile nose has occurred under the impact velocities of 560 m/s and 563 m/s (Fig. 14).

Figure 13: 6 mm test plate after 4 shots of FSP projectile with different velocities.

Figure 14: Deformation of projectiles by impact
TABLE 2
PROPERTIES OF 12.7MM FRAGMENT SIMULATED PROJECTILE (FSP)

<table>
<thead>
<tr>
<th>Type</th>
<th>Weight (g)</th>
<th>Hardness (HB)</th>
<th>Yield Strength (MPa)</th>
<th>Tensile Strength (MPa)</th>
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</thead>
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<tr>
<td>FSP-1 (AISI 4340)</td>
<td>13.4 ± 0.13 g</td>
<td>286</td>
<td>758</td>
<td>930</td>
</tr>
<tr>
<td>FSP-2 (KRUPP 1191)</td>
<td>13.4 ± 0.13 g</td>
<td>160-180</td>
<td>330</td>
<td>590</td>
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</table>

The test result clearly showed that the increased in hardness of the 12.7 mm FSP projectile can increase the depth of penetration on the target plate (Fig. 15). The range of velocities used in the present investigation shows significant projectile erosion. The projectile was shortened and flattened at its tip for the impact velocities of 310 m/s and 420 m/s. The substantial deformation, flattening and forming of mushroomed shape of the FSP projectile nose has occurred under the impact velocities of 560 m/s. The properties of FSP are shown in Table 2.

Blast resistance of armour steel and steel-concrete composite material

Blast was carried out to validate blast pressure resistance of armour steel and composite materials at distance of 15 m and 5 m as shown in Fig. 16 below.

It was found that 6 mm armour steel resists 100 kg TNT at 15 m well. No penetration was observed and deformation is limited to about 10 mm for plate size of 1 m x 1m. A good alternative is steel-concrete steel composite plate as it can resist penetration well though its material is much heavier and requires extra effort for fabrication.
Deployable Solar Panels for Energy Harvesting

Given the increases in both the environmental and economic costs of energy, there is a high demand to design and build more sustainable and low-energy-consuming building systems. Among all the renewable energy and/or clean energy solutions, solar is one of the affordable solutions that can be applied on an individual’s property that is self-sustaining and with low maintenance. The high initial cost and the low energy yield of the solar panels limit the development of photovoltaic market. In this research, a movable PV building façade system will be developed.
Considering that each building side receives solar energy only for several hours a day, this research plans to design a movable PV system to utilize as much available sunlight as possible with as small amount of PV array as possible. A PV system adaptive to manifold shapes of building envelop is developed to move over a long guide rail on the façade to follow the sun. The PV system will be moved only several times every day. Hence, the consumed energy by actuating mechanism is very little. Taking an example of a general facing-south building with quadrangle sections, it may be moved only twice a day. The concept is shown in Fig. 17. The east and south façades are covered to absorb the sunlight in the morning. In the afternoon the PV system will be driven to the south and west façades to receive the solar energy. After the sunset, it will go back to the original position for the operating in the next day.

The core of this idea is to minimize the cost of expensive PV system integrated in tall building facades (Fig. 17) while maximize the yield of solar energy.

(1) A movable PV building facades have the following advantages: (a) maximizing the working time of each PV panel while minimizing the used PV panels with lower cost; (b) suitable variety of building shapes not limited to circular ones; (c) meeting the special layout demands of local appearance by adjusting the horizontal spaces between PV panels, and (d) economizing more PV panels with the taller building and wider flank surfaces relative to the face one.

(2) Simplified solar tracking systems suitable for different locations: (a) maximizing solar gain of each PV panel at any working time while minimize the complexity of system; (b) following the sun from the east to the west while fixing PV panels in an optimal slope position by single axis design, and (c) changing the incidence angle in a defined interval of time using timing control.

(3) A self-powered control and drive systems are developed to (a) complete the operation of responding to the sun’s position autonomously only based on the solar power and (b) feed the produced energy to the electrical grid in the building.

Figure 17. Deployable solar façade for multi-storey buildings
CONCLUSIONS

This paper discusses the two new aspects of developing deployable structural systems: protective and robustness for military applications and mobility/adaptivity for clean energy harvesting. From the study of deployable shelter forms and armour material properties, it is indicative that if a balance of system weight, deployability, and protection level is achieved in a design, deployable protective shelter is a feasible concept. The later part of this paper addressed an important direction for development of deployable structure which is towards the design of high-rise building for clean energy harvesting. The deployable façade system is similar to retractable roof but the system is applicable to the façades of multi-storey buildings.

ACKNOWLEDGEMENT

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REFERENCES


ENHANCING THE ROBUSTNESS OF STEEL AND COMPOSITE BUILDINGS

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KEYWORDS

Alternative load path, Column removal, Composite structures, Nonlinear static response, Parametric studies, Progressive collapse, Robustness, Semi-continuous beams, Simplified model

ABSTRACT

Methods intended for use in the design of steel and composite building frames covering the provision of adequate robustness so as to guard against progressive collapse are in the process of moving from prescriptive to quantitative. An approach that recognises all the important complex physical phenomena, employs a realistic criterion of failure and is capable of being implemented at a variety of levels has been devised at Imperial College London. Recent further development of this method has streamlined the necessary analyses, thereby making it possible to conduct parametric studies that provide insights into the link between changes to the structure and quantitative measures of resistance to progressive collapse. This approach is used herein to examine a number of different arrangements, from which key features of behaviour are identified. It has therefore been possible to isolate those structural modifications with the greatest potential for improving robustness and, moreover, to associate quantitative measures of that improvement with each scheme. Results for a series of arrangements, including several proposals put forward by Industry as methods of improving robustness based on the simple notion of increased tying capacity, are examined herein and general principles for improvement identified.
INTRODUCTION

Progressive Collapse of building structures is a topic that has come much to the fore in recent years. It does, however, have a chequered history – with its generally agreed genesis being the Ronan Point Collapse of 1962. This was followed by a somewhat fallow period characterised by Bruce Ellingwood’s comment at the 1997 SEWC Conference in San Francisco:

“There is currently a virtual absence of research activity or interest in the United States in the topic”.

and a recent surge of interest stimulated most notably by the World Trade Centre collapses in New York. But it is also possible to identify aspects of the topic in studies of WW2 bomb damage to structures in London, in measures designed to combat the IRA bombing campaigns and in various forensic studies on public safety issues that typically followed any major bomb blast on a significant structure e.g. the Murragh Building in Oklahoma City. As interest in the more scientific treatment of the topic has increased so also has the realisation that knowledge and evidence from adjacent fields e.g. seismic resistant design, can inform the process of developing strategies and procedures to better address a lessening of both the likelihood of a progressive collapse failure and the consequences should one be initiated.

This paper briefly reviews the current state of the art relating to both the development of a better understanding of progressive collapse and its treatment in design and then proceeds to present new developments in the Imperial College London approach, concluding with illustrative results for a range of cases of both steel and composite frames. These highlight certain limitations in existing design approaches.

POSSIBLE DESIGN APPROACHES

The Ronan Point collapse demonstrated to the Structural Engineering Community in the UK the need for resistance to Progressive Collapse to become part of the routine design process for at least some types of structure. The absence of any basic theory meant that this could not follow the usual structural design approach of:

- Identifying a representative form of applied loading.
- Devising an analysis model from which to calculate structural response to this loading.
- Identifying key outputs from this analysis e.g. moments, stresses, deflections etc.
- Comparing the calculated values of these key outputs with suitable limits.

Instead, rather more indirect and essentially prescriptive approaches were devised. These fell into one of three types:

- Tying force.
- Alternate load path.
- Key elements.

Requiring that building frames be adequately tied together, including the specific requirement for steel frame structures that beam to column connections possessed a certain limited tying capacity (the ability to transmit an axial force from the beam into the column) should ensure greater robustness i.e. the ability that in the event of the structure suffering an incident for which it had not been specifically designed it would not suffer disproportionately. These
concepts of robustness (a property of the structure) and disproportionate collapse (the initiation of progressive collapse) thus entered the vocabulary.

Tying has the advantage of being simple to appreciate, easy to implement through simple design calculations and being readily achievable in practice providing the required tying forces are not too large. Its main disadvantage is that it is entirely prescriptive in nature i.e. it is of the form “providing the provisions are satisfied behaviour will be better than if they are not”, but there is no basis for comparing alternative arrangements nor for assessing the actual margin of safety against progressive collapse. Because of its simplicity, it has been widely, if not always correctly, used in the UK and has been introduced in other parts of the world as they too, developed design provisions against progressive collapse.

In the alternate load path approach a member is notionally removed and the ability of the damaged structure to resist some reduced level of applied load is then assessed. Most usual is column removal with the damaged floor(s) bridging over the increased span. The level of structural analysis used to examine the behaviour of the damaged structure varies from static applications using elastic theory and individual members to sophisticated numerical approaches including dynamic effects, large deformations and inelastic material behaviour. Clearly this approach entails more effort than does a simple application of the tying force approach; in return it provides information on the relative merits of different configurations and allows the designer to assess the effects of structural changes in a quantitative fashion.

In certain cases e.g. transfer girders, damage to a particular element would leave the structure with no alternate load path. Such members may be designed as “key elements”, with their basic design being conducted to a larger load factor so as to provide a (supposedly) greater margin of safety to the structure. One of the most obvious key elements is the single spindle supporting the wheel of the London Eye; it is of considerable interest to read how the engineers responsible took into consideration the many features with the potential for unwanted consequences and rationalised their approach so as to provide an acceptably safe structural component.

Clearly tying forces and key elements can only be regarded as prescriptive approaches yielding no information on the behaviour of the damaged structure and thus providing no insights into how a designer’s knowledge of structural principles might be utilised in the search for the best solution to a given set of circumstances. Alternate load path analysis does, however, offer more possibilities – particularly if it can be developed in a way that models the key features of progressive collapse whilst remaining tractable in terms of mathematical and computational complexity. It is, therefore, not surprising, that, particularly post WTC collapse, considerable effort has been focussed on developing and applying this approach, both by the research communities and by those responsible for deriving design rules.

Work conducted at Imperial College London during the past 5 years has been aimed at developing a complete design method – essentially based on the alternate load path concept of sudden column removal – that combines sufficient rigour that all essential physical features are correctly modelled with the simplicity of application necessary if it is to be attractive for use in practice. Its core is a quantitative assessment of the ability (or not) of the damaged structure to attain a new equilibrium position in its grossly deformed state. It incorporates dynamic effects (but without the need for dynamic analysis), allows for gross changes of geometry and inelastic material behaviour and recognises that the key governing property is the ability of the beam to column connections to deliver the necessary rotations.
Implementation may be at member, floor, substructure, complete frame or whole building level. Originally the analysis step was conducted using ADAPTIC [1] – although any suitable software package may be employed. The most recent advance is the possibility to use only “hand calculations” based on an extended slope-deflection approach suitable for the rapid examination of many alternative arrangements [2]. In parallel, work has also been conducted on modelling the behaviour of connections subject to the combined beam axial load plus moment forms of loading that arise during a progressive collapse [3].

IMPERIAL COLLEGE METHOD FOR PROGRESSIVE COLLAPSE ASSESSMENT

The Imperial College method [4, 5] provides a simplified framework for accurate prediction of the dynamic capacity of steel and composite structures subject to sudden column removal. The simplicity of the method is illustrated by the possibility of predicting the complex nonlinear dynamic response of multiple floor systems based on the nonlinear static responses of the individual beams.

Figure 1a depicts a typical steel-framed multi-storey structure subject to sudden removal of a peripheral column. Provided the remaining columns are able to resist the redistributed load, only the floors above the removed column are affected. The directly affected areas of these floors can be isolated and the interaction with the surrounding structure can be simulated with appropriate boundary conditions (Figure 1b). The Imperial College framework provides a simple multi-level assessment approach for prediction of the response of such multiple floor systems. According to this approach, the beam models at the lowest level of structural idealization (Figure 1d) can be assembled to obtain the corresponding floor system response (Figure 1c) and the floor system responses can be used to assemble the response of the multiple floor system. Provided the affected floors are identical in terms of structure and loading, the latter assembly can be omitted since in that case, the response of a single floor system predicts sufficiently the frame resistance against progressive collapse.

Figure 1: Levels of structural idealization for progressive collapse assessment
Regardless of the level of structural idealization, the Imperial College method uses three main assessment stages for estimation of the capacity against progressive collapse:

- Prediction of the nonlinear static response of the damaged structure under gravity loading (by using either analytical or numerical models).
- Estimation of the dynamic response by converting the nonlinear static to the corresponding pseudo-static response based on an energy-equivalence concept.
- Determination of the dynamic capacity by comparing the available and required connection ductility levels.

The framework has been applied to assess the resistance of a typical multi-storey building by considering both bare steel and composite frames and flexible bare steel connection arrangements [6]. The same bare steel frame with rather more substantial semi-continuous connections has also been examined [2]. The results from both studies have shown that adequate connection tying force capacity as defined by the British code may not ensure resistance to progressive collapse.

ANALYTICAL METHOD FOR PREDICTION OF THE BEAM STATIC RESPONSE

For the analysis of typical semi-continuous composite beams (Figure 2a) suffering progressive collapse due to column loss, the structural system depicted in Figure 2b is adopted. In the region of hogging bending moment, the stiffness \( (EI) \) of the composite beam is reduced to the cracked stiffness \( (EI') \). The axial restraint provided by the adjacent structure is simulated by a linear extensional spring \( (K_s) \). The nonlinear rotational springs \( (S) \) and \( (S') \) represent the connection mechanical models which should account for both the connection rotations and axial deformations. In the presence of beam axial load \( (N) \), the connection response is expressed by the following \( M-N-\Phi \) relationship [3]:

\[
\Phi = M\alpha_1 + Nz\beta_1 - \gamma_1 \\
\Phi' = M'\alpha_1' + Nz'\beta_1' - \gamma_1'
\]

where \( \alpha, \beta \) and \( \gamma \) are associated with geometrical and material properties of each connection, whereas \( z \) and \( z' \) correspond to the levels of application of \( N \).

The beam nonlinear static response is the relationship between the monotonically applied static uniformly distributed load \( (q) \) and the corresponding beam deflection \( (w) \). The beam deflection is associated with deformations of the various components (Figure 2c) such as beam bending, rotations of the support \( (\Phi') \) and mid-span \( (\Phi) \) connection and axial deformation of the adjacent structure \( (\Delta_s) \).
For the structural system of Figure 1b, the bending moment distribution can be estimated according to the stiffness method. The connection bending moments ($M$ and $M'$) and the bending moment at the theoretical point of inflexion ($M_a$) (Figure 2d) are associated with the sum of the corresponding equivalent nodal forces of the clamped structure (Figure 3a) and the nodal forces caused by the rotations/deformations of the released structure (Figure 3b). From the equilibrium equations of the two sections at the beam deflected state (Figure 2d), $\Phi_a$ and $u_a$ can be isolated and expressed in terms of the remaining parameters. Therefore, explicit equations linking the connection bending moments with the beam axial load, the beam deflection and the beam loading are obtained as follows:

$$ M = N\lambda + q\mu + \nu $$  \hspace{1cm} (2.a)

$$ M' = N\lambda' + q\mu' + \nu' $$  \hspace{1cm} (2.b)

where $\lambda$, $\mu$ and $\nu$ are associated with $w$, $L$, $EI$, $EI'$ and $a_1$, $\beta_1$, $\gamma_1$, $z$, $z'$. 
Figure 3: Bending moment distribution

From Figure 2d, the overall equilibrium equation of the system can be obtained:

\[ M' = \frac{qL^2}{2} - Nw - M \]  

The total axial deformation can be estimated quite accurately by a second order approximation:

\[ \frac{w^2}{2L} = u + u' + \frac{N}{K^a} \]  

- \( K^a \) is the equivalent axial stiffness of the beam \((EA/L)\) and the adjacent structure \((K_a)\)
- \( u \) and \( u' \) are the connection axial deformations which can be expressed in terms of the connection applied loading in a similar fashion to the connection \(M-N-\Phi\) relationship (Eqn. 1):

\[ u = M\alpha_2 + Nz\beta_2 - \gamma_2 \]  

\[ u' = M\alpha'_2 + Nz'\beta'_2 - \gamma'_2 \]  

By solving the set of Eqns. 1-5, \( M, M', N \) and \( w \) can be expressed in relation to the system geometrical and material properties and the beam loading \( q \).

VALIDATION OF THE SIMPLIFIED ANALYTICAL METHOD

The analytical method described in the previous section has been validated against the nonlinear analysis program ADAPTIC [1] via examination of various beam arrangements. Since the proposed method allows prediction of the nonlinear static response of bare steel and composite beams, both types have been examined. Figure 4 shows the connection type (endplate minor-axis beam-to-column) and the typical arrangement that has been adopted and it indicates the constant and variable parameters. The various beam arrangements are described in Table 1. By considering the bare steel beam in Case 1 as a starting point, the
influence of the highlighted structural parameters on the ability of the method to accurately follow behaviour is demonstrated via examination of these cases.

The variable parameters are the beam span ($L$) and depth ($D$), the plate thickness ($t_p$), the number of reinforcement bars and bolt rows and the beam axial restraint. When the latter parameter is present, it is approximately provided by the axial stiffness of the adjacent beam and connection which are considered identical to the corresponding components of the beam system under consideration. For each case, the beam span-to-depth (including the concrete slab depth) ratio ($L/D^t$), the rebar ratio ($p$) and the connection normalised stiffness ($R$) and strength ($m$) are calculated and presented in Table 1. The component characteristics are defined based on the provisions of EC3 [7] and EC4 [8]. Bi-linear characteristics with 1% strain hardening are adopted for the tensile and rigid-plastic for the compressive components. Full shear connection between the concrete slab and the steel beam is provided in all composite cases.

### TABLE 1

**BEAM ARRANGEMENTS USED FOR VALIDATION OF THE PROPOSED METHOD**

<table>
<thead>
<tr>
<th>Case no.</th>
<th>$L$ (m)</th>
<th>$D$ (mm)</th>
<th>$t_p$ (mm)</th>
<th>rebar rows</th>
<th>bolt restr.</th>
<th>$L/D^t$</th>
<th>$p$ (%)</th>
<th>$R$</th>
<th>$m$</th>
<th>critical comp.</th>
<th>$P_{d,Rd}$ (kN)</th>
<th>$q_{d,Rd}$ (kN/m)</th>
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<td>11.7</td>
<td>Br6</td>
<td>553</td>
<td>46.1</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>533</td>
<td>8</td>
<td>4</td>
<td>6</td>
<td>no</td>
<td>9.0</td>
<td>1.5</td>
<td>11.7</td>
<td>rebar</td>
<td>283</td>
<td>23.6</td>
</tr>
<tr>
<td>15</td>
<td>9</td>
<td>603</td>
<td>12</td>
<td>8</td>
<td>8</td>
<td>yes</td>
<td>12.3</td>
<td>2.0</td>
<td>37.7</td>
<td>Br8</td>
<td>501</td>
<td>27.8</td>
</tr>
<tr>
<td>16</td>
<td>9</td>
<td>603</td>
<td>12</td>
<td>8</td>
<td>8</td>
<td>no</td>
<td>12.3</td>
<td>2.0</td>
<td>37.7</td>
<td>Br8</td>
<td>501</td>
<td>27.8</td>
</tr>
</tbody>
</table>

The Imperial College method has been applied for conversion of the nonlinear static into the corresponding pseudo-static (P-s) responses. The dynamic capacities and the critical components obtained by the simplified method are presented in Table 1. Both the dynamic capacities ($P_{d,Rd}$) of double-span ($2L$) beams subjected to sudden removal of the intermediate column and the corresponding distributed loads ($q_{d,Rd}$) – to enable comparisons between beams with different spans – are given. These capacities are determined based on indicative average connection failure criteria. For the bare steel components the maximum deformation is taken as the minimum between 30mm, which is the average value based on experimental results, and the deformation associated with failure of the bolts in tension. The maximum deformation of the rebar at failure is obtained based on the model proposed by Anderson et al. [9]. The noted bare steel critical components correspond to the connection under hogging bending moment (support connection).
The resulting P-s responses obtained from both analyses are compared in Figures 5-7. It is confirmed that the proposed method is able to predict the beam nonlinear static response with excellent accuracy. Furthermore, the method provides accurate predictions for the system capacity and critical component for the adopted failure criteria. Besides comparisons between the P-s responses obtained from both analyses, each figure shows the influence of specific parameters on overall system behaviour.

Figure 4: Arrangement of flush endplate beam-to-column composite connection

Figure 5: Influence of connection tensile components on the beam P-s response
Figure 5 stresses the significant influence of the connection tensile components on the beam P-s response. The connection tensile capacity increases by either increase in the reinforcement ratio or in the endplate thickness and the beam P-s response increases accordingly as illustrated in Figures 5a and 5b respectively. The substantial difference between the P-s responses of bare steel and corresponding composite beams with any reinforcement ratio is explained mainly by the difference in beam \((EI/L)\) and connection \((S_J)\) stiffness. Variation of these parameters among composite beams with different reinforcement ratios is considerably less pronounced.

Figure 6 shows that the axial restraint provided by the surrounding structure may enhance performance in various ways depending on the balance between the properties of the system components. Comparison between cases 15 and 16 in Figure 6a establishes that the axial restraint becomes less effective as the beam span-to-depth ratio increases. In the same figure, comparison between cases 5 and 6 shows that in axially restrained beams, the compressive arching action is limited when the connection compressive capacity is significantly lower than the tensile resistance and the system response increases in the subsequent tensile catenary action. The exact opposite happens when the connection compressive capacity is
The response is enhanced by the relatively low beam span-to-depth ratio.

Figure 7: Influence of beam span-to-depth ratio on the P-s response

The effects of beam span-to-depth ratio on the response of axially restrained beams are examined further in Figure 7. Figure 7a compares the responses of identical beams with different spans. In Figure 7b, the beam span is kept constant whereas the beam section and connection arrangements change. It is confirmed that the response during the compressive stage increases significantly as the beam span-to-depth ratio decreases and the connection compressive resistance increases. In particular, the response during the compressive arching action may govern the system capacity as in case 13.
PARAMETRIC STUDIES ON TYPICAL FLOOR SYSTEMS

A number of different column removal scenarios depending on the position of the initial damage within the frame arrangement depicted in Figure 8 have been considered and the ability of the corresponding affected floor areas to sustain the redistributed loads have been assessed based on the Imperial College method. The beams have been designed as both bare steel and composite based on the column grid dimensions and unfactored loads shown in Figure 8, for the combination of actions 1.35$q_D$+1.5$q_I$ at the ULS and for the unfactored imposed loading at the SLS. The same beam sections and connection arrangements have been used in both directions of the frame, as shown in Table 2. The remaining dimensions and properties are defined in Figure 4. (The column web has been considered as rigid in the major-axis beam-to-column connections of the transverse beams.) The resulting beam span-to-depth ratios ($L/D^T$) and the ratios ($a$) between the design bending moments and beam plastic moment capacities in each direction are similar for both frame types (Table 2).

![Figure 8: Plan view of the floor area and depiction of the different column removal scenarios](image)

**Table 2**

<table>
<thead>
<tr>
<th>Frame type</th>
<th>Bare steel beam section</th>
<th>Endplate (mm)</th>
<th>bolt rows</th>
<th>rebar no.</th>
<th>Longitudinal beam</th>
<th>Transverse beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bare steel</td>
<td>UB457x152x52</td>
<td>450 10 90 90</td>
<td>4</td>
<td>---</td>
<td>17.8 0.68 18.4 0.20</td>
<td>11.1 0.85 11.5 0.20</td>
</tr>
<tr>
<td>Composite</td>
<td>UB305x102x33</td>
<td>312 10 90 70</td>
<td>3 4</td>
<td>18.1 0.67 35.6 0.58</td>
<td>11.3 0.88 22.3 0.58</td>
<td></td>
</tr>
</tbody>
</table>

The beams that are affected by edge or corner column removal when their direction is perpendicular to the corresponding edge are conservatively treated as cantilevers, ignoring the relatively low contribution of the connection in the region of sagging bending moment. On the other hand, the beams that are affected by removal of a penultimate column (I2, I4, E2 in the longitudinal and I3, I4, E4 in the transverse direction respectively) and are oriented perpendicularly to the corresponding edge are treated as axially unrestrained. The remaining beams are considered as axially restrained with the degree of axial restraint to be approximately defined as described in the previous section.
The beam nonlinear static responses have been obtained by the proposed hand-calculation method and they have been converted to pseudo-static according to the Imperial College framework. The floor pseudo-static response for each column removal scenario has been obtained from the corresponding responses of the affected beams based on the assembly procedure of the Imperial College method. The P-s responses of the beams and the resulting floor P-s responses for the different column removal cases are depicted in Figures 9 and 10 for bare steel and composite frames respectively. The level of the vertically applied loading (Demand) at the time of column removal for each case is defined based on the combination of actions $q_D + 0.25q_I$ recommended by the GSA [10]. The resulting capacity-demand ratios ($r$) are also presented in column (a) of Table 3.

An initial general observation regarding the overall behaviour is that both structures exhibit similar responses and capacities. However, the bare steel frame response increases compared to the composite as the degree of axial restraint increases (E1, E3, I3, I2 and especially I1). The exact opposite happens when the floor areas are axially unrestrained in both directions (E2, E4, I4) and especially when the floor is composed of cantilever beams only (C1). This is explained by the responses of the individual members shown in Figures 9a and 10a and by the comment regarding the beam axial restraint of the previous section (Figure 6). Comparison between the responses of axially restrained and unrestrained composite beams is similar with the relevant correlations shown in Figure 6a. Both the axially restrained and unrestrained beams (as well as floor areas of same arrangements: I1-4, E1-E2, E3-E4) initially behave almost identically due to the relatively high connection tensile capacity (including the rebar) compared to the connection compressive resistance (beam flange) – this is stressed in Figure 10d – which reduces the compressive arching effects. The response increases significantly for relatively large beam deflections but usually only after failure. On the other hand, the initial responses of the axially restrained bare steel beams are substantially higher than the responses of the corresponding unrestrained members due to the opposite balance between the connection tensile and compressive resistances. As this is more pronounced in the transverse beam due to the shorter length (Figure 7), there is a notable variation between the responses of the different bare steel floor systems.
Figure 9: Progressive collapse analysis of bare steel frame

b. Floor P-s responses - Internal column removal

c. Floor P-s responses - Edge column removal

d. Floor P-s responses - Corner column removal
Therefore, it is concluded that the response of the bare steel systems is limited by the connection tensile resistance, especially in the absence of axial restraint, whereas the high connection tensile capacity of the composite systems is not completely utilised due to premature failure of the compressive beam flanges in the initial compressive stage when an axial restraint is present. To overcome these limitations, the critical components are enhanced by increasing the connection endplate thickness and by stiffening the beam flanges. Each modification is implemented individually and the new capacity-demand ratios are presented respectively in columns (b) and (c) of Table 3.

By increasing the endplate thickness, an increase of around 20% in the capacity-demand ratio of the axially unrestrained bare steel floors is obtained as expected (column (b) in Table 3). However, the ratio of the floors with axially restrained transverse beams (I1, I2, E3) is also increased with approximately the same rate. As depicted in Figure 9, these systems exhibit a softening or plateau response after the compressive arching action. By enhancing the connection tensile components, the compressive arching action is substituted by a monotonically increased response. On the other hand, enhancing the connection tensile components of the composite frame has negligible or negative effects on system performance since the response is limited by the capacity of the beam compressive flange.

![Graph showing P-s responses of individual beams](image-url)
Figure 10: Progressive collapse analysis of composite frame
TABLE 3
FLOOR DYNAMIC CAPACITIES FOR VARIOUS COLUMN REMOVAL SCENARIOS

<table>
<thead>
<tr>
<th>Case No</th>
<th>Demand $P_o$ (kN)</th>
<th>Capacity-Demand ratio ($r = P/P_o$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bare steel</td>
<td>Composite</td>
</tr>
<tr>
<td>(a) $t_p=10$mm, w/o FS</td>
<td>(b) $t_p=12$mm, w/o FS</td>
<td>(c) $t_p=10$mm, w/ FS</td>
</tr>
<tr>
<td>11</td>
<td>872</td>
<td>1.13</td>
</tr>
<tr>
<td>12</td>
<td>872</td>
<td>1.00</td>
</tr>
<tr>
<td>13</td>
<td>872</td>
<td>1.00</td>
</tr>
<tr>
<td>14</td>
<td>872</td>
<td>0.85</td>
</tr>
<tr>
<td>E1</td>
<td>436</td>
<td>1.09</td>
</tr>
<tr>
<td>E2</td>
<td>436</td>
<td>0.87</td>
</tr>
<tr>
<td>E3</td>
<td>436</td>
<td>1.34</td>
</tr>
<tr>
<td>E4</td>
<td>436</td>
<td>1.08</td>
</tr>
<tr>
<td>C1</td>
<td>218</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Provided the response is governed by tensile connection components, stiffening of the beam flanges has no effect on the capacity as observed in most bare steel cases in Table 3, column (c). However, it affects considerably the response and capacity of the floors with axially restrained transverse beams which as explained above are governed by compressive arching effects (11, 12, E3 in Figure 9). On the other hand, the response of the axially restrained composite floors is enhanced significantly by flange stiffening as expected, whereas the effects on the axially unrestrained composite systems are either slightly negative or inconsiderable. The corner composite floor column removal demonstrates a rather stiffer initial response but the capacity remains approximately the same as in column (a) of Table 3 due to rebar fracture.

CONCLUSIONS

It has been argued that structural engineers require design approaches to guard against progressive collapse that are essentially similar to those used for the ultimate and serviceability limit states i.e. methods that permit quantitative comparisons to be made between alternative arrangements. Further developments to the Imperial College London approach, which aims to provide such a facility, have been briefly presented. Illustrative results for different column removal scenarios for a pair of steel and composite frames have been used to illustrate the effects of different parameters. The findings confirm the complex nature of the interplay between the various physical phenomena, especially the importance of the stage in the load-deformation responses of the individual members selected as the failure criterion. A particularly important feature is whether “failure” occurs in the tensile or the compressive parts of the beam to column connections and whether this develops during the compressive arching or tensile membrane stage of the beam’s response.
REFERENCES


IMPLEMENTATION OF THE EUROCODES
PROGRESS TOWARDS A VALUABLE OUTCOME

G.W. Owens
President of the Institution of Structural Engineers
Consultant to the Steel Construction Institute

KEYWORDS

ABSTRACT

This paper presents the progress that is being made on the implementation of the Eurocodes for steel and composite construction, namely Eurocodes 0, 1, 3 and 4. After a brief review of the timetables and processes for implementation, it describes some of the most significant European and UK initiatives that have been undertaken to assist the adoption by practising engineers. It summarises the wide range of paper and electronic design aids that are being produced. It draws attention to some of the real progress that has been made towards a harmonised approach to design, thereby in the longer term assisting cross-border trade in construction in Europe. It illustrates this progress by reference to: harmonisation within the original documents, the harmonised output from the Access-Steel project, beam to column connection design for braced multi-storey buildings and element design for portal frames proportioned by elastic/plastic analysis. It highlights some of the areas where further work is of greatest importance, in order to ensure the successful adoption of the Eurocodes. It offers some predictions for the future adoption of the Eurocodes, both in Europe and internationally, emphasising the importance of both education for undergraduates and conversion training for practising engineers.

INTRODUCTION

The Eurocode drafting process began in 1975, when the Commission of the European Community decided to implement article 95 of the European Treaty for construction. This article generally seeks ‘to eliminate technical obstacles to trade and the harmonisation of technical specifications’. In 1989, the activity of preparation and publication of the Eurocodes passed to CEN. There was something of a false dawn in the early 1990’s, when draft, ENV, versions of many of the specified documents were published and some expected the speedy finalisation of those documents and their application in practice.
The completion of the documents and their publication in the official languages of English, French and German took much longer. It was 2005 to 2006 before even the first tranche of publications was available. A few of the envisaged documents still remain unpublished. However, sufficient were available from 2005/6 onwards to cover the design of most practical structures.

Publication of the core publications was not the end of the story. In order for any Eurocode document to be used in any specific country, it needs to be accompanied by a National Annex which governs its national application. A National Annex has two functions. Firstly and most importantly, it defines the values of Nationally Determined Parameters (NDP) that are used for national application. Secondly, it provides an opportunity for selected national over-rides of specific clauses, where there are specific national concerns.

It is interesting and encouraging to note the evolution in National Annexes that has occurred in the past 15 years. When they were produced for the ENV versions, they were generally notable for their length and depth. At this early stage, they were seen as an opportunity to rewrite large portions of the core document, where national experts felt that insufficient notice had been taken of their views by the international drafting progress. However, when the National Annexes came to be written for the final (EN) documents, a remarkable transformation had occurred. There was almost an international competition to see which country could prepare the shortest National Annexes! In general, they accepted the recommended values of NDPs and imposed minimal modifications to other content. In effect, the emphasis had shifted from national protectiveness to a real commitment to international harmonisation.

ALTHOUGH GREAT PROGRESS HAS BEEN MADE IN THE PUBLICATION OF BOTH THE EURONORMS AND THE NATIONAL ANNEXES, A FEW REMAIN OUTSTANDING.

Table 1 summarises the situation for the general design of steel and composite structures.

<table>
<thead>
<tr>
<th>Eurocode</th>
<th>Status of National Annexes in Autumn 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>EN 1990 – Basis of structural</td>
<td>Main document published</td>
</tr>
<tr>
<td>design</td>
<td>NA for Amendment A1 and Annex A2 for bridges expected 2009</td>
</tr>
<tr>
<td>EN 1991 – Actions on Structure</td>
<td>Eight documents published</td>
</tr>
<tr>
<td></td>
<td>NA to EN 1991-3 Actions induced by cranes and machinery expected 2009</td>
</tr>
<tr>
<td>EN 1993 – Design of Steel</td>
<td>Nine documents published</td>
</tr>
<tr>
<td>Structures</td>
<td>The following are due in 2009:</td>
</tr>
<tr>
<td></td>
<td>NA to EN 1993-1-7:2007 Plated Structures subject to out of plane loading</td>
</tr>
<tr>
<td></td>
<td>NA to EN 1993-3-1:2007 Towers and Masts</td>
</tr>
<tr>
<td></td>
<td>NA to EN 1993-3-2: 2008 Chimneys</td>
</tr>
<tr>
<td></td>
<td>NA to EN 1993-5: 2007 Piling</td>
</tr>
<tr>
<td></td>
<td>NA to EN 1993-6: 2007 Crane supporting structures</td>
</tr>
<tr>
<td>EN 1994 – Design of composite</td>
<td>All four documents published</td>
</tr>
<tr>
<td>and concrete structure</td>
<td></td>
</tr>
</tbody>
</table>
STRUCTURE, PHILOSOPHY AND CONTENT OF THE EUROCODES

Existing national standards for the design of steel and composite structures adopt a range of approaches in their style and content. These differences reflect different national histories of design evolution. However, in most countries, the tradition was for the documents to:

- Be complete, so that designers did not generally need to refer to other documents and textbooks for background information.
- ‘Post process’ the basic design approaches, so that derivative material (for example the shear strength of slender webs or the strength of unrestrained beams) was directly incorporated, frequently in tabular form.
- Simplify approaches, so that design could be carried out by manual calculation.
- Order material so that, as far as is practical, it reflects the order of the design process.

The Eurocodes have several significant differences:

- While every design approach is unambiguously stated, the format and content requires other inputs. For example, where the elastic critical moment for an unrestrained beam (M_cr) is used, its value is not defined and has to be determined elsewhere.
- Only the basic design approach is stated, leaving the designer to look elsewhere, or use his own judgement, for important design parameters, for example, the effective lengths of columns and unrestrained beams.
- The documents implicitly recognise the role of computers in design and, in some instances, present design approaches that are complex and impractical for manual design.
- The material is ordered to reflect the structure of the applied science behind structural design rather than any design sequence.

These differences lead to the requirement for two additional types of information for practical implementation.

1. **Non-contradictory, complementary information (NCCI)**

   The Eurocodes recognise the incompleteness of their information by encouraging the development, by industry and designer bodies, of these explicit pieces of additional information. Since the underlying purpose of the Eurocodes is to harmonise the design process, there are clear benefits if NCCI can likewise be harmonised.

2. **Flowcharts and worked examples**

   One of the greatest concerns of any designer adopting a new design standard is to ensure that all the design steps have been executed and in the correct order. This concern is compounded when the sequence of the standard does not reflect the sequence of the design process. In these circumstances, there is great value in flow charts that explicitly state, in appropriate detail, all the steps that need to be undertaken for each design activity. There is further value in worked examples that demonstrate all relevant aspects of the new standard and design sequence.

EUROPEAN AND UK NATIONAL IMPLEMENTATION INITIATIVES

There were two key European implementation projects that had a major impact on the practical implementation of the Eurocodes for steel and composite construction.

**Access Steel**

This 4 million Euro project was undertaken from 2005 to 2007 with sponsorship from the eContent programme of the European Union and six major steel producers: Arcelor Mittal,
Corus, Peiner Trager Ruukki, Voestalpine and SSAB. It was undertaken under the leadership of the Steel Construction Institute by the leading technical institutes in Europe, CTICM, Labein, RWTH Aachen, SBI and the SCI, with additional input from CSC, Corus, Arcelor Mittal and eTeams.

The output site is maintained by the Steel Alliance, a joint venture between SCI and CTICM. Figure 1 shows the home page of this valuable resource. The home page clearly shows the multi-lingual nature of the resources which are available in English, French, German, Spanish and Czech. The primary targets for this material were the major usages of steel in Europe i.e. low-rise multi-storey buildings, single storey buildings and light gauge construction, where there are seen to be major opportunities for growth in usage.

Figure 1: Home page of Access Steel – a multi-lingual resource for the implementation of the Eurocodes

<table>
<thead>
<tr>
<th>Type of Resource</th>
<th>Number of Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case Studies</td>
<td>31</td>
</tr>
<tr>
<td>Client Guides</td>
<td>4</td>
</tr>
<tr>
<td>Data</td>
<td>10</td>
</tr>
<tr>
<td>Scheme Development</td>
<td>57</td>
</tr>
<tr>
<td>Flowcharts</td>
<td>45</td>
</tr>
<tr>
<td>NCCI</td>
<td>59</td>
</tr>
<tr>
<td>Worked Examples</td>
<td>71</td>
</tr>
</tbody>
</table>

Table 2 summarises the different types of content:
- **Case Studies** illustrate specific successful projects in steel.
- **Client Guides** put forward, in very succinct terms, the key advantages of using steel for particular applications.
- **Data** are detailed sets of information derived from the Eurocodes that are necessary for design in practice.
- **Flowcharts** address all the principal tasks in element design.
• **NCCI, Non Contradictory Complementary Information** provide the additional information needed for design in practice.

• **Scheme Development** provide the guidance necessary for design development. It helps the designer, inexperienced in steel, collaborate with the architect and client to define an initial structural solution that makes best use of steel.

• **Worked Examples** demonstrate the principal tasks in element design.

**Steel buildings in Europe**

This major project is being undertaken from 2008 to the present. The clients are Arcelor Mittal, Corus and Peiner Trager, with additional sponsorship from the Research Fund of the European Coal and Steel Community. It is being undertaken as a joint venture between the SCI and CTICM under the Steel Alliance. The project is developing design guidance that is based on best European practice. Input has been sought from Belgium, Germany, Spain and Italy from workshops organised by the relevant National Independent Promotion Organisation.

**TABLE 3**

**DESIGN GUIDES BEING PRODUCED BY THE STEEL BUILDINGS IN EUROPE PROJECT**

<table>
<thead>
<tr>
<th>Title</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSB01</td>
<td>An architect’s guide to multi-storey steel buildings</td>
</tr>
<tr>
<td>MSB02</td>
<td>Conceptual design of multi-storey steel buildings</td>
</tr>
<tr>
<td>MSB03</td>
<td>Actions on multi-storey buildings</td>
</tr>
<tr>
<td>MSB04</td>
<td>Detailed design of multi-storey steel buildings</td>
</tr>
<tr>
<td>MSB05</td>
<td>Design of pinned connections in multi-storey steel buildings</td>
</tr>
<tr>
<td>MSB06</td>
<td>Fire engineering of multi-storey steel buildings</td>
</tr>
<tr>
<td>MSB07</td>
<td>Model construction specification for multi-storey buildings</td>
</tr>
<tr>
<td>MSB08</td>
<td>Resistance of steel members</td>
</tr>
<tr>
<td>MSB09</td>
<td>Resistance of pinned connections</td>
</tr>
<tr>
<td>MSB10</td>
<td>Technical software specification for composite beams</td>
</tr>
<tr>
<td>SSB01</td>
<td>An architect’s guide to single storey steel buildings</td>
</tr>
<tr>
<td>SSB02</td>
<td>Conceptual design of single-storey steel buildings</td>
</tr>
<tr>
<td>SSB03</td>
<td>Actions on single storey buildings</td>
</tr>
<tr>
<td>SSB04</td>
<td>Detailed design of portal frames</td>
</tr>
<tr>
<td>SSB05</td>
<td>Design of trusses in single storey buildings</td>
</tr>
<tr>
<td>SSB06</td>
<td>Design of built-up members in single storey buildings</td>
</tr>
<tr>
<td>SSB07</td>
<td>Fire engineering of single storey buildings</td>
</tr>
<tr>
<td>SSB08</td>
<td>Steel building envelope</td>
</tr>
<tr>
<td>SSB09</td>
<td><em>Introduction to computer software</em></td>
</tr>
<tr>
<td>SSB10</td>
<td>Model construction specification for single buildings</td>
</tr>
<tr>
<td>SSB11</td>
<td>Moment connections</td>
</tr>
</tbody>
</table>

Table 3 summarises the design guidance that is being produced in Phase One. In Phase Two, it will be localised to accommodate local National Annexes and other relevant local conditions and will then be translated and disseminated in target national markets. In addition to these major European activities, individual European countries are developing their own, national, guidance. Table 4 summarises the publications programme in the UK.
Almost all the information discussed above is in the form of documents. These can and will be disseminated both on paper and electronically. However, there is also a growing number of electronic interactive resources that are being developed.

Figure 2 shows one of the interactive worked examples from the Access Steel project. The user may input his own parameters and, in effect, use the tool to carry out his own element design – in this case a column.

Figure 2: Interactive worked example for a pin ended column from the Access Steel Project
Figure 3 shows a typical example of a screen from the eBlue Book. This valuable aid provides an electronic version of all the data provided by the traditional SCI/BCSA ‘Blue Book’.

Further electronic aids are being developed as part of the ‘Steel Buildings in Europe’ project on member resistances and simple connections.

HARMONISATION IN PRACTICE

If the overall objective of the Eurocodes is to be achieved, namely the removal of barriers to cross border trade in construction, all aspects of the process have to be harmonised. More importantly to society and the quality of the Built Environment, harmonisation could, and should, lead to the sharing of best practice between national construction communities.

It can be argued that the steel construction sector has made more progress towards effective harmonisation than any other construction sector. What follows captures some of the highlights of what has often been a difficult and time consuming process.

Harmonisation within the Eurocodes

From the outset, the code drafting committee, CEN250/TC3, attached a high importance to debating technical issues through to a final conclusion wherever possible. This approach had its drawbacks; in many cases, drafting the Steel Eurocodes took longer than those of other material sectors. By contrast, other committees have developed Eurocodes where only the overall framework is common. In such cases, variations in the detailed application of many of these approaches are permitted, enabling national communities to continue to adopt their current practice.

While this approach undoubtedly simplifies the drafting process, it is difficult to see how it can ever lead to harmonisation, or to the exchange of best practice.
The Nation ally Determ ined Pa rameters prov ide a v ery u seful guide to the de gree of harmonisation that has been achieved. The figures below are revealing.

Number of NDPs in EN 1992-1-1: 2004 123
Number of NDPs in EN 1993-1-1: 2005 25
Number of NDPs in EN 1994-1-1: 2005 19

Harmonisation within ‘Access Steel’

Once again, the em phasis on harm onisation was dete rmined from the outset of this project. Preparation of first drafts of all the 250 resources was distributed throughout the project technical partners. Once available, this draft material was circulated to the other partners for comment. This commentary and review process was not taken lightly. Overall, the same effort was needed to comment on and then finalise the draft resources as was require for the initial drafting.

Some difficult topics required round table discussions at a series of special workshops. At the outset, several project partners were sceptical about the degree of harmonisation that could be achieved. In the event, the entire team was very encouraged by, and proud of, the high level of success that was achieved. Finally, out of all the topics originally proposed, only one had to be abandoned. It related to the use of friction to restrain beams where they are supporting precast planks on th eir top flanges. Some countries argued that, in appropriate circumstances, friction alone may be used for restraint. Other countries always insist on positive attachment between the floor and beam. No compromise was possible – you cannot half mobilise friction! So this single resource was not completed.

Harmonisation within the ‘Steel Building Project’

This project extended the drive for harmonisation into detailed design guidance. Two examples are presented below, at opposite ends of the spectrum of complexity. One addresses the detailed modelling of connections in simple braced frames. The other considers probably the most complex structure in conventional steelwork design, namely beam and column stability in plastically designed portal frames.

Analysis of braced frames for gravity loads

In technical literature, there are many different approaches to the design assumptions for beam to column connections in braced frames. Many of these connections have proportions that lead to semi-rigid behaviour. The Eurocodes devote considerable attention to the design of such connections. There may be worthwhile benefits in the consideration of semi-rigid behaviour in certain circumstances. However, the complexity of these methods and the difficult interaction between connection behaviour and overall analysis make them unsuitable for routine construction. Fortunately, the Eurocodes also permit simple approaches. EN 1993-1-1 Section 5.1.2(1) states that ‘The effects of the behaviour of the joints in the distribution of internal forces and moment within a structure, may generally be neglected, but where these effects are significant (such as in the case of semi-continuous joists) they should be taken into account, see EN 1993-1-8.'
EN 1993-1-8 Section 5.2.2.1(2) states:

‘A joint may be classified on the basis of experimental evidence, experience of previous satisfactory performance in similar cases or by calculations based on test evidence’.

In the UK, the treatment of such joints as ‘simple’ underpins much of our current market success. It leads to economical connections and minimises column sizes (beams are larger than would be achieved by semi-rigid methods but this disadvantage is more than offset by the other advantages.

AT THE NATIONAL WORKSHOPS ASSOCIATED WITH THIS PROJECT, THERE WERE SPECIFIC DISCUSSIONS ON THE TREATMENT OF THE CONNECTIONS SHOWN IN FIGURE 4. IT WAS MOST ENCOURAGING TO LEARN THAT ALL THE COUNTRIES INVOLVED ADOPTED SIMILAR APPROACHES IN PRACTICE, ALBEIT WITH SLIGHTLY DIFFERENT APPROACHES TO THE ECCENTRICITIES, Table 5 presents the details.

<table>
<thead>
<tr>
<th>Country</th>
<th>Major Axis Eccentricity</th>
<th>Minor Axis Eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>$h/2$</td>
<td>0</td>
</tr>
<tr>
<td>Netherlands</td>
<td>$h/2$</td>
<td>0</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>$h/2 + 100$</td>
<td>$t_{w}/2 + 100$</td>
</tr>
<tr>
<td>Germany</td>
<td>$h/2$</td>
<td>0</td>
</tr>
<tr>
<td>France</td>
<td>$h/2$</td>
<td>0</td>
</tr>
<tr>
<td>Spain</td>
<td>$h/2$</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 4: Design models for beam / column connections in simple construction
Design of Columns and Rafters in Portal Frames

In this complex situation, the nature of the harmonisation is different. The European drafting process and the associated background studies, had demonstrated a shortcoming in some established national practices, which had to be resolved.

The plastic design of portal frames had developed in the United Kingdom, in isolation from input from any other country. Because of the size of the market, this form of construction had been the subject of many design aids and the development of increasingly sophisticated software. Underpinning the whole process was an assumption that there was no adverse interaction between in-plane and out-of-plane instability of the rafters and columns.

A rafter or column is subject to vertical loading, axial load, $N$, and end moments $M_1$ and $M_2$. In plane, the rafter is restrained at its two ends, translationally by the column and the change in direction of the rafter at the apex and rotationally by the 'rigid' connections to the same elements. Out of plane, the rafter is restrained at its ends and is also restrained by the purlins attached to the top flange, possibly augmented by purlin stays that also stabilise the bottom flange and provides torsional restraints.

A background paper to ECCS/TC8 carried out detailed numerical studies on the behaviour of rafters (or columns) in this situation. Two of the most significant figures are reproduced as Figure 5 and Figure 6. The problem is most significant when the two slendernesses, $\lambda_y$ and $\lambda_z$ are similar. Figure 5 examines the case for a simple short rafter without any intermediate restraints. Figure 6 considers the behaviour of the same section six times greater in length, with intermediate lateral restraints, so that $\lambda_z$ is unchanged.

This report was prepared in 1999, where different approaches to the design equations were still being produced. Both graphs show the various design approaches being considered at that time. The FB equations are for flexural buckling only, because the member is fully torsionally restrained. LTB denotes behaviour where the torsional restraints are removed and lateral torsional buckling may occur.

As shown in Figure 5, where $\lambda_y << \lambda_z$, the interaction between $M_y/M_{pl,y}$ and $N/N_{pl}$ is essentially linear, simplifying design considerably. However, in Figure 6, the interaction is significantly concave requiring greater complexity in design approach, if economy is to be preserved.
After considerable extra work, Equations 6.61 and 6.63 in EN 1993-1-1 were finally agreed in committee to cover the design of such beam columns. In their original form, they cover both major and minor axis actions. Where \( M_z \) does not occur and the section is fully symmetric, as in portal frames, the equations simplify to:

\[
\frac{N_{Ed}}{X_y N_{Rk}} + k_y \frac{M_{y,Ed}}{X_{LT} M_{y,Rk}} \leq 1.0 \quad \text{ex Eq. 6.61(1)}
\]

\[
\frac{N_{Ed}}{X_z N_{Rk}} + k_z \frac{M_{y,Ed}}{X_{LT} M_{y,Rk}} \leq 1.0 \quad \text{ex Eq. 6.62(2)}
\]
It is the cross term, $k_{xy}$, that addresses the interaction between in-plane and out-of-plane buckling.

So much for the background research. The important harmonisation point was that major design communities in the UK and elsewhere had to consider an additional behavioural effect for an entire class of structures, if their designs were to be in accordance with the Eurocodes. The solution was to adopt a pragmatic effect to the system lengths to be used for the element checks and to recognise the beneficial effects of in-plane continuity on effective length factors.

At the time of writing, final values of effective length factors have still to be agreed but Figure 7 shows the current draft approach. This will be subject to further review and verification before it is finalised.

![Figure 7: Draft design approaches for in-plane behaviour of portal frames](image)

Thus, in this case, the harmonisation process has improved the rigour of the underlying design philosophy without imposing significant additional penalties on design outcomes.

**FUTURE PROGRESS ON EUROCODE IMPLEMENTATION**

As this paper has indicated, much has already been done to ensure that the Eurocode for steel and concrete construction will be successfully implemented. However, much remains to be done. The entire Eurocode system remains relatively little used, though this should change after March 2010, the moment at which the period of official coexistence with national standards begins. Commentators observe that small but worthwhile economies are achieved from design to the Eurocode. These primarily arise from reductions in load factors while the resistances are, overall, broadly unchanged from the basic national standards. Once the potential for even modest savings is realised, this will drive significant application for design and build projects, a substantial portion of the overall market place.

The implementation activities undertaken so far have largely been provided by the sector’s supporting infrastructure. During the next phase of the implementation, this will be
supplemented by enhanced course activity (both at universities and for practising engineers) and practical usage. In addition the first tranche of practical software will be completed and launched.

With all these activities under way, every detailed assumption of and practical application of the Eurocodes will be put to scrutiny. As issues arise, they will feed back into the supporting infrastructure, for example by questions to the SCI Advisory Service or Q and A sessions on courses. Once fed back, they will be resolved, hopefully in a harmonised way using existing international relationships. It is difficult to predict what issues arise but the following predictions are offered.

1. Simplification of wind loading.
   The wind loading section of EN 1991 is complex and difficult to use. It is hoped that simplifications can be found that are not unduly conservative.
2. General simplification of the load combinations.
   Experience will develop over which load combinations govern different classes of structure in practice. Simplified guidance will follow.
3. Design for serviceability.
   As with most of the national standards the Eurocodes will replace, most attention is currently devoted to the ultimate limit state. Serviceability is also important, especially for longer span structures. Harmonised criteria need to develop. In addition ‘design for stiffness’ may become more prevalent. This might finally encourage the uptake of semi-rigid design approaches.

ADOPTION OUTSIDE EUROPE
The driving force for the development of the Eurocodes was to break down trade barriers in Europe. What has emerged is a very advanced set of design and construction standards increasingly supported by the steel construction sector. It is already clear that many countries that have previously looked to individual countries in Europe for guidance on design standards are not looking to adopt the Eurocodes. They very nature of the dual core standard, National Annex approach lends itself to such adoption. For example, personal interactions by the author have confirmed the adoption of the Eurocodes in Malaysia, Singapore and South Africa. Even where a country is not adopting the Eurocodes wholly, they are using them as major source documents for the ongoing revision of their National Standards.

CONCLUSIONS

1. The Eurocodes and National Annexes now provide an advanced framework for the general design of steel and composite structures. Outstanding specialist standards will shortly be available.
2. The nature of the Eurocodes differs from many National Standards requiring substantially more supporting material.
3. A wide range of paper and electronic design aids for general construction is now, or will very shortly, be available.
4. The steel sector has achieved substantially more harmonisation than other sectors. This investment will pay substantial dividends in the future from economies in design aids and design software.
5. As practical application increases from March 2010 onwards, further harmonised guidance should be available to address issues as they arise.
6. The Eurocodes, as a comprehensive and advanced system of design standards, are finding increased usage outside Europe.

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SEISMIC DESIGN OF HIGH-RISE STEEL BUILDINGS IN SHANGHAI

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KEYWORDS
High-rise steel building, seismic design, design specification, ductility design, time history analysis, hysteretic model

ABSTRACT

Typical structural systems of high-rise steel buildings in Shanghai were introduced with examples of landmark buildings, including moment frame, braced frame, steel frame-bracing/steel plate shear wall, mega frame, frame with viscous dampers and steel-concrete hybrid structure. Development of design specifications for tall steel buildings in Shanghai was summarized as well as technical inspection of seismic fortification of tall buildings designed beyond the code limits. The newly upgraded Specification for Steel Structure Design of Tall Buildings (DG/TJ08-32-2008) was illustrated in detail, focusing on its seismic ductility design method, which featured ductility consistency between the structural system, beam-to-column connections and plastic members of a structure as well as requirement of non-linear time history analysis in favor of collapse prevention under rare earthquakes.

INTRODUCTION

Shanghai is located in seismic prone area. The earliest earthquake in Shanghai appearing in historical records dated back to 1475. Among the earthquake records, the most severe ones in Shanghai occurred in 1624 (M 4.8, epicenter in Shanghai), 1668 (M 8.5, epicenter in Shandong province), 1853 (M 6.75, epicenter in southern Yellow Sea) and 1927 (M 6.5, epicenter in southern Yellow Sea). All the above events reached Seismic Intensity 6 of China and all the recorded earthquakes in Shanghai were tectonic earthquakes. Although Shanghai is a region subject to earthquakes of low intensity and low frequency and the maximum recorded intensity was 6, most districts of Shanghai were assigned with Seismic Intensity 7 during seismic zoning, considering Shanghai is after all a megalopolis.
The earliest high-rise steel building in China – Park Hotel was born in Shanghai in 1934 and it kept the honor of “the tallest building in far east” for 48 years. After that, no high-rise steel buildings were built in Shanghai until 1980s, during which 5 buildings were erected. Up to now, many tall buildings have been constructed with steel structures and the height record has reached 492m (See Table 1). This paper will review the state-of-practice of seismic design of high-rise steel buildings in Shanghai.

**TABLE 1**

<table>
<thead>
<tr>
<th>No.</th>
<th>Project</th>
<th>Height (m)</th>
<th>No. of stories</th>
<th>Building Area (×10⁴ m²)</th>
<th>Steel consumption (×10⁴ ton)</th>
<th>Structural system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shanghai World Financial Center</td>
<td>492</td>
<td>101</td>
<td>38.2</td>
<td>6.7</td>
<td>Mega column-n-mega brace-SRC core tube structure</td>
</tr>
<tr>
<td>2</td>
<td>Jin Mao Tower</td>
<td>420.5</td>
<td>88</td>
<td>28.9</td>
<td>1.4</td>
<td>Composite frame-RC core tube</td>
</tr>
<tr>
<td>3</td>
<td>Shanghai Shimao International Plaza</td>
<td>333</td>
<td>60</td>
<td>13.6</td>
<td>0.95</td>
<td>Composite frame-RC core tube</td>
</tr>
<tr>
<td>4</td>
<td>Shanghai Donghai Plaza</td>
<td>245</td>
<td>3+52</td>
<td>10</td>
<td>0.34</td>
<td>Steel frame-RC core tube</td>
</tr>
<tr>
<td>5</td>
<td>Shanghai Pudong International Financial Mansion</td>
<td>230</td>
<td>3+53</td>
<td>12</td>
<td>1.1</td>
<td>Steel frame-RC core tube</td>
</tr>
<tr>
<td>6</td>
<td>Shanghai International Shipping Building</td>
<td>210</td>
<td>3+48</td>
<td>10</td>
<td>0.95</td>
<td>Steel frame-RC core tube</td>
</tr>
</tbody>
</table>

**TYPICAL STRUCTURAL SYSTEMS OF TALL STEEL BUILDINGS IN SHANGHAI**

*Moment Frame*

Shanghai Park Hotel (See Figure 1) is 83.3m high with 2-story underground and 22 aboveground. Steel moment frame was adopted as well as reinforced concrete slab.

![Figure 1: Shanghai Park Hotel](image)
Braced Frame

Shanghai Jin Sha Jiang Hotel (See Figure 2) is 42m high with 2-story underground and 12 story aboveground. Its steel consumption was about 1300 t in total and 68kg per unit floor area. It was the first tall steel building which was designed by Chinese engineers and utilized domestic H profile steel.

Steel Frame-Bracing/Steel Plate Shear Wall

Shanghai Jin Jiang Tower is 153 m tall with 1-story underground and 43-story aboveground. Outriggers were used at the 23rd floor and the top floor to reduce horizontal displacement. Braces and steel plate shear walls were installed in the core tube for the stories above and below the 23rd floor, respectively (See Figure 3). Its total steel consumption was 8500t.

Mega Frame

Shanghai Stock Exchange Building (See Figure 4) is 121m high with 3-story underground and 30-story aboveground. It is composed of a 31m high, 63m long overbridge and two 36m×21m towers, which provides a sound background for the application of steel mega frame. Its total steel consumption was 9000t.
Frame with Viscous Dampers

The Sci-tech Complex of Tongji University is 48.6m \times 48.6m in plan and 98 m tall with 1-story underground and 21-story aboveground. Its outline is a regular prism but its floors take an irregular configuration of “L” in plan, which rotates clockwise once for every three stories (See Figure 5). Outer braces with dampers were employed to control seismic effect of coupled lateral-torsional vibration.

Steel-concrete Hybrid Structure

Shanghai World Financial Center Tower is 492m tall with 3-story underground and 101-story aboveground, being presently the tallest building in China. It employs a lateral resistant structural system with the combination of a mega-frame structure, a reinforced concrete core and braced steel core connected by three 3-story outrigger trusses (See Figure 6). The mega-frame structure is composed of mega columns, mega braces and belt trusses.

DESIGN SPECIFICATIONS

Design Specifications for Tall Steel Buildings in Shanghai

In order to meet the need of modernization construction, Shanghai local government sponsored 8 research projects in 1985. The research topics covered damping ratio, seismic design spectrum, panel zone of beam-column joint, mixed type of beam-to-column connection, welded thick plate column of box section, hysteretic model and composite column of tall steel buildings as well as mutual interference of wind effect between tall
buildings. Based upon these projects and recent engineering practices, the first design specification for tall steel buildings in China – Standard DJB08-32-92 was published in 1993. After over 10 years of implementation, this specification was upgraded to the formal specification DG/TJ08-32-2008 in 2008, with significant adjustment and additional design provisions regarding ductility design of steel structures, transverse wind response, wind interference, semi-rigid beam-to-column connections and buckling restrained braces etc.

Meanwhile, Shanghai Code for Seismic Design of Buildings (DGJ08-9-2003) provided seismic design provisions for steel buildings not more than 12 stories or not higher than 40m.

A special type of steel-concrete hybrid structural system – steel frame-RC core structure was once suspected to be vulnerable to earthquake damage, as it lacks redundant lateral resisting system. However, over half tall steel buildings in earthquake-prone area of China employed this system. Based upon systematic research by Tongji University, the first design specification for the hybrid system in China – Standard DG/TJ08-015-2004 was put forward in Shanghai in 2004.

Technical Inspection of Seismic Fortification of Tall Buildings Designed beyond the Code Limits

Regarding structural systems of tall buildings can be too complex for design specifications to cover all the cases with accurate seismic design requirements, China Ministry of Construction issued Ministry of Construction Order No. 59 in 1997 and revised it as Ministry of Construction Order No. 111 in 2002. They were temporary and formal regulations on the requirement of technical inspection of seismic fortification of tall buildings designed beyond the code limits. Tall buildings beyond applicable height limit and those with non-applicable structural systems or especially irregular configuration are required to be subject to technical inspection by officially authorized committees. Later in 2006, the ministry issued a technical outline for such technical inspection (Ministry of Construction Order No. 220).

SEISMIC DESIGN METHODOLOGY

Seismic Design Criteria

According to China National Standard GB50011-2001, seismic design criteria are defined as the following three levels: (1) No damage under frequent earthquakes with exceeding probability of 63.2% in 50 years; (2) Repairable damage under design earthquakes with exceeding probability of 10% in 50 years; and (3) Collapse prevention under rare earthquakes exceeding probability of 2~3% in 50 years.

Definition of Seismic Action

In favor of response spectrum analysis, response spectra suggested by DGJ08-9-2003 is shown in Figure 7 with $\alpha$ being earthquake influence coefficient, $T_g$ characteristic period, $\gamma$ attenuation coefficient, $\eta_1$ and $\eta_2$ modification factors and $T_s$ structural natural vibration period. The values for earthquake maximum influence coefficient $\alpha_{max}$ are listed in Table 2. Regarding the equivalent base shear method, the base shear force $F_{EK}$ is equal to $\alpha_1 G_{eq}$, where $G_{eq}$ is the equivalent total mass and $\alpha_1$ is the earthquake influence coefficient corresponding to the fundamental period of the structure.
Regarding time history analysis, 4 accelerograms are suggested by DGJ08-9-2003 as shown in Figure 8 while accelerogram amplitude for each specific analysis can be determined with reference to Table 2.
Ductility Design

Ductility design method under design earthquake was adopted in Standard DG/TJ08-32-2008 in place of elastic design method under frequent earthquake in Standard DJB08-32-92. However, design formula have been scaled to the form of frequent earthquake so as to make use of seismic action as defined in Figure 7 and Table 2, which is compatible to the definition in China National Standard GB50011-2001. Process of ductility design method suggested by DG/TJ08-32-2008 is as follows:

Step 1: Select an appropriate structural system according to expected ductility capacity. As listed in Table 3, four ductility types are defined according to ductility capability of structural systems with beam-to-column connections and member cross sections of different ductility levels.

Step 2: According to features and ductility requirement of the target structural system, determine if a structural member is allowed to be plastified under design earthquakes. As can be seen in Table 3, there are three section types of plastic members, which are defined as follows: section type A can form a plastic hinge with the rotation capacity required for plastic hinges; section type B can develop its plastic moment resistance, but has limited rotation capacity; section type C can have its extreme compression fibre reach its yield strength, but local buckling is liable to prevent development of the plastic moment resistance. Cross section of a plastic member should be proportioned not exceeding the limiting width-thickness ratio as listed in Table 4. Meanwhile, there are three types of beam-to-column connections as listed in Table 3. Improved Traditional Type and Improved Type are required to have a rotational capacity no less than 0.02 and 0.03 rad, respectively, while no such requirement is suggested for Traditional Type. Prequalified beam-to-column moment joints for the three connection types are provided by Standard DG/TJ08-32-2008 as depicted in Figure 9.

Step 3a: Perform elastic structural analysis using the applicable one of equivalent base shear method and mode-superposition response spectrum method.

Step 3b: Do design checks for structural members and connections by means of Eqn. 1.

\[ S \leq R / \gamma_{RE} \]  

where \( R \) is design value of member capacity; \( \gamma_{RE} \) is seismic adjustment factor of member capacity, taking 0.75, 0.80, 0.85 and 0.90 for beam/column, brace, connection plate/bolt, and weld, respectively; \( S \) is design value of seismic effect of permanent load, horizontal and vertical seismic actions, and wind load, of which partial safety factors are \( \gamma_{GE}, \gamma_{Ehk}, \gamma_{Evk}, \gamma_{wk} \) respectively. \( \gamma_{RS} \) is ductility factor of target structural system, taking 1.0, 0.85, 0.70, 0.60 for Ductility Type I, II, III, IV of structural systems, respectively.
<table>
<thead>
<tr>
<th>Ductility type</th>
<th>Structural system</th>
<th>Type of beam-to-column connection</th>
<th>Section type of plastic members</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Ordinary frame</td>
<td>Traditional C</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ordinary CBF</td>
<td>Traditional C</td>
<td>Single</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ordinary frame-RC panel</td>
<td>Traditional C</td>
<td>Single</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ordinary frame/truss tube</td>
<td>Traditional C</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ductile frame</td>
<td>Improved Traditional</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ductile CBF</td>
<td>Improved Traditional</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ductile frame-RC panel</td>
<td>Improved Traditional</td>
<td>B</td>
<td>Dual</td>
</tr>
<tr>
<td></td>
<td>Ordinary frame-steel plate RC panel</td>
<td>Improved Traditional</td>
<td>B</td>
<td>Single/dual</td>
</tr>
<tr>
<td></td>
<td>Ductile frame/truss tube</td>
<td>Improved Traditional</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ordinary tube-in-tube, bundled tubes</td>
<td>Traditional C</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Highly ductile CBF</td>
<td>Improved Traditional</td>
<td>B</td>
<td>Dual</td>
</tr>
<tr>
<td></td>
<td>Ductile EBF</td>
<td>Improved Traditional</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BRBF</td>
<td>Improved Traditional / Improved Traditional</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Highly ductile frame-RC panel</td>
<td>Improved B</td>
<td>Dual</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ductile frame-steel plate RC panel</td>
<td>Improved B</td>
<td>Single/dual</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ductile frame-composite steel wall</td>
<td>Improved Traditional</td>
<td>B</td>
<td>Single/dual</td>
</tr>
<tr>
<td></td>
<td>Ductile frame-steel plate wall</td>
<td>Improved Traditional</td>
<td>B</td>
<td>Single</td>
</tr>
<tr>
<td></td>
<td>Highly ductile frame tube</td>
<td>Improved B</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ductile tube-in-tube/bundled tubes</td>
<td>Improved Traditional</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Highly ductile frame</td>
<td>Improved A</td>
<td></td>
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<tr>
<td></td>
<td>Highly ductile EBF</td>
<td>Improved A</td>
<td>Single/dual</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Highly ductile BRBF</td>
<td>Improved A</td>
<td>Single/dual</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Highly ductile frame-steel plate wall</td>
<td>Improved A</td>
<td>Dual</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Highly ductile tube-in-tube/bundled tubes</td>
<td>Improved A</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: When the ratio of design shear of moment frame to base shear of overall structure is no less than 25 %, a structure is recognized as a dual lateral system. Otherwise, it is a single lateral system.

Step 3c: Do design check for story drift of the structure under frequent earthquakes using Eqn. 2.
where \( \Delta/h \) is elastic story drift due to the same seismic load effect combination of Eqn. 1; \( \gamma_d \) is displacement amplification factor, taking 1.0, 1.18, 1.43, 1.67 for Ductility Type I, II, III, IV of structural systems, respectively. \[ \frac{\Delta}{h} \] is allowable story drift of structures under frequent earthquakes: (1) 1/400 for buildings having non-structural elements attached to the structure; (2) 1/300 for buildings having ductile non-structural elements; (3) 1/200 for buildings having non-structural elements fixed in a way as not to interfere with structural deformations.

Step 4: Do performance check for story drift of the structure under rare earthquakes. Allowable story drift of structures under rare earthquakes are prescribed as 1/50 for steel moment frames and 1/70 for all other structural systems. Non-linear time history analysis on the structure for at least 3 accelerograms of rare earthquakes is required by Standard DG/TJ08-32-2008 for structural systems of Type III and IV and for extremely irregular structural systems. Simplified methods are applicable to take the place of non-linear time history analysis for other structural systems. Hysteretic models for different structural members are suggested by the standard as shown in Figure 10.

![Diagram](a)

![Diagram](b)

![Diagram](c)

![Diagram](d)

![Diagram](e)

![Diagram](f)

![Diagram](g)
Figure 9: Beam-to-column moment joint suggested by DG/TJ08-32-2008:
(a) Traditional Type, (b)-(e) Improved Traditional Type, and (f)-(j) Improved Type

<table>
<thead>
<tr>
<th>TABLE 4</th>
<th>LIMITING WIDTH-THICKNESS RATIOS FOR BEAMS AND COLUMNS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
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<tr>
<td>Beam</td>
<td>free flange in I or box sections</td>
</tr>
<tr>
<td></td>
<td>stiffened flange in box sections</td>
</tr>
<tr>
<td></td>
<td>web plates in I or box sections</td>
</tr>
<tr>
<td>Column</td>
<td>free flange in I or box sections</td>
</tr>
<tr>
<td></td>
<td>stiffened flange in box sections</td>
</tr>
<tr>
<td></td>
<td>web plates in I or box sections</td>
</tr>
</tbody>
</table>

Note: $N_b$, $A$ and $f$ are axial force, cross sectional area and design value of steel strength of the column, respectively.
Figure 10: Hysteretic models suggested by Standard DG/TJ08-32-2008: (a) Multi-spring model for beams and columns, (b) Hysteretic model for steel considering damage accumulation, (c) Yield surface model for beams and columns, (d) Axial hysteretic model for common braces, (e) Axial hysteretic model for buckling restrained braces (BRB), (f) Shear hysteretic model for joint web panels

PERSPECTIVES

Challenges for seismic design of high-rise steel buildings in Shanghai will always be twofold. On one hand, more and more steel skyscrapers beyond the scope of design codes are to be built. An over 600m-tall building – Shanghai Center Tower has been under construction. On the other hand, there will be many important needs to improve the design specifications. Among others, research efforts have been devoted to accounting for risk of successive earthquakes in seismic design for over a decade. Last but not the least, it should be noted that modern tall steel buildings in Shanghai have not experienced severe earthquakes.

REFERENCES


BEHAVIOUR AND DESIGN OF HOLLOW AND CONCRETE FILLED STEEL COLUMNS SUBJECTED TO IMPACT LOADS

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KEYWORDS
Bridges, buildings, columns, composite construction, concrete filled steel columns, hollow sections, impact loading, structural design

ABSTRACT
A comprehensive test program has been carried out recently at the University of Western Sydney and University of Wollongong to investigate the performance of stainless steel concrete filled steel tube (CFST) columns subjected to impact loading. Concurrently, material tests using Split Hopkinson Pressure Bar (SHPB) apparatus were also conducted at Hunan University, China. These test results are reported in this paper, where the performance of stainless steel CFST columns is compared with that of carbon steel CFST columns. The main objective of this paper is to compare the performance of stainless steel CFST columns with carbon steel CFST columns. Moreover, the behaviour of in-filled tubes under impact loading is also compared with that of hollow sections. Generally, the stainless steel specimens showed improved energy-dissipating behaviour compared with their carbon steel counterparts, especially when concrete was used to fill the hollow tubes.

INTRODUCTION
Blast, earthquakes, fire and hurricanes are extreme events for buildings and infrastructure and warrant innovative structural engineering solutions. The subject of interest in this paper is to reduce the vulnerability of building and infrastructure systems to extreme events through cost-effective protective systems utilising high-performance steels. The specific extreme event that is considered is that of impact and blast loads on structural elements of critical infrastructure such as bridges, buildings and offshore structures. The Australian government defines critical infrastructure as those physical facilities that, if destroyed, degraded or...
rendered unavailable for an extended period, would significantly impact on the social or economic well-being of the nation. This would mainly include buildings, bridges and offshore structures. Figure 1 illustrates some examples of extreme events associated with impact loading that have caused significant damage to critical infrastructure. Figures 1 (a) and (b) represent examples of major catastrophes in Australia associated with impact which have resulted in major losses in life. The Granville train disaster of 1977 was caused by a train derailment and the impact of a central pier causing an overhead bridge to collapse, Figure 1 (a). The Tasman bridge collapse resulted from a ship impact of a central pier and the collapse of the main deck in 1975.

(a) Granville train disaster, 1977  
(b) Tasman bridge collapse, 1975

Figure 1: Examples of extreme events causing damage of critical infrastructure

Studies on blast loading (Cormie et al. [1]) have also highlighted the importance to the design structures against progressive collapse, (Starossek [2]) and thus structural robustness, (Knoll and Vogel [3]) is now an important limit state that needs to be considered in design. Concrete filled steel columns have been used extensively over recent decades in both multi-storey buildings and bridges. Significant experimental and theoretical studies have been carried out on thin-walled concrete filled steel columns, for application in regions where construction economy is the driving force in construction. This has been mainly for aseismic regions. Thick-walled steel tubes used in concrete filled steel columns have been the subject of greater application in seismic regions. The thick-walled nature of the columns has allowed confinement to be maximised resulting in increased ductility of these columns. The heightened interest throughout the world recently in protecting critical infrastructure from accidental or extreme loads has had structural engineers seeking systems or elements which performed well under seismic conditions to be considered for impact or blast style loading scenarios. This is the focal point of this paper, which will concentrate on the behaviour of concrete filled steel columns under transverse impact loads. The energy absorption capability of concrete filled columns has been the subject of investigation by Xiao et al. [4]. They carried out experiments on concrete filled columns under axial effects and found that there were significant increases in the capacity associated with the high strain rates. The most recently reported research by Kang and Liew [5] considered the dynamic increase in yield strength of steel and concrete in various loading situations and for members subjected to pure bending an approximate 25% increase in yield strength was observed.
EXPERIMENTS

This paper will present the results of three sets of experimental series, namely
1. Static tests;
2. Impacts tests; and

Static Tests

There were two sets of static tests performed which can be further categorised as mild steel static tests; and stainless steel static tests. Table 1 outlines the test specimens used. Note that the nominal cross-section dimensions are the same for all of the sections and the compactness \((b/t)\) was based on the measured values. The column members were tested as beams and had an overall length of 2850 mm and these were tested in three point bending as illustrated in Figures 2, 3 and 4. Figure 5 provides some representative load-deflection graphs which illustrates the effect of concrete infill and the use of stainless steel in improving ductility.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Nominal Dimensions (mm x mm x mm)</th>
<th>((b/t)) Compactness</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS1</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>MSH1</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>MS1a</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>MSH1a</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>MS2</td>
<td>150 x 150 x 5</td>
<td>35.5 Non-Compact</td>
</tr>
<tr>
<td>MSH2</td>
<td>150 x 150 x 5</td>
<td>35.5 Non-Compact</td>
</tr>
<tr>
<td>SS1</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>SSH1</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>SS1a</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>SSH1a</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>SS2</td>
<td>150 x 150 x 5</td>
<td>35.5 Non-Compact</td>
</tr>
<tr>
<td>SSH2</td>
<td>150 x 150 x 5</td>
<td>35.5 Non-Compact</td>
</tr>
</tbody>
</table>
Figure 2: Static test facility at University of Wollongong

Figure 3: Static test facility at University of Western Sydney

Figure 4: Schematic diagram of test set up
Impact Tests

There were two sets of impact tests performed which can be further categorised as mild steel impact tests; and stainless steel impact tests. Table 2 outlines test specimens that were tested. Note that the nominal cross-section dimensions are the same for all of the sections and the compactness \((b/t)\) was based on the measured values. The column members were tested as beams and had an overall length of 2850 mm and these were tested in three point bending as illustrated in Figure 6. Figure 7 and 8 provide typical responses showing the peak and residual loads from the impact tests.

**TABLE 2**

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Nominal Dimensions (mm x mm x mm)</th>
<th>((b/t)) Compactness</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS3</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>MSH3</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>MS3a</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>MSH3a</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>MS4</td>
<td>150 x 150 x 5</td>
<td>35.5 Non-Compact</td>
</tr>
<tr>
<td>MSH4</td>
<td>150 x 150 x 5</td>
<td>35.5 Non-Compact</td>
</tr>
<tr>
<td>SS3</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>SSH3</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>SS3a</td>
<td>100 x 100 x 5</td>
<td>23.7 Compact</td>
</tr>
<tr>
<td>SSH3a</td>
<td>100 x 100 x 5</td>
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<tr>
<td>SSH4</td>
<td>150 x 150 x 5</td>
<td>35.5 Non-Compact</td>
</tr>
</tbody>
</table>
Figure 6: Impact test facility at University of Wollongong

Figure 7: Dynamic load vs. deflection graphs for mild steel, hollow column
Split Hopkinson Pressure Bar Tests

Split Hopkinson Pressure Bar (SHPB) tests have been carried out at Hunan University to ascertain the stress-strain behaviour at high strain rates for both carbon and stainless steel. The apparatus and specimen used is illustrated in Figures 9 and 10 respectively.
Figure 9: The tension split Hopkinson bar apparatus at Hunan University, China

Figure 10: Specimen dimensions (Unit: mm)

Figure 11 shows the incident, the reflected and the transmitted pulses recorded in the experiment for a typical specimen. As can be seen, the shape of the incident pulse in the SHPB experiment was close to rectangular.

Figure 11: Typical forms of waves acquired in the tests

According to the one-dimensional elastic wave propagation theory [6-8], the stress (σ), strain (ε) and strain rate (ε̇) in a specimen can be determined using Eq. (1) to Eq. (3), respectively.
where $E$ is Young’s modulus of the split bars; $A_b$ and $A_s$ are the cross-sectional areas of the pressure bar and specimen, respectively; $C_0$ is the speed of an elastic wave in a bar; and $l_0$ is the effective length of a specimen.

Figure 12 (a) and (b) shows the measured stress-strain curves for the stainless steel and carbon steel materials, respectively, using the MTS material testing machine. As can be seen, the strength of the stainless steel is higher than that of the carbon steel. Although the stainless steel showed much higher ductility than the carbon steel, the elongation of the stainless steel under a strain rate of 0.05 s$^{-1}$ decreased significantly compared with that under a strain rate of 0.005 s$^{-1}$. No such phenomenon was observed for the carbon steel material, as shown in Figure 12(b).
To demonstrate the strain rate effect at the initial loading stage, initial stress-strain curves are shown in Figure 13. From this figure, obvious strain rate hardening effect can be observed for the stainless steel, but not for the carbon steel. Based on the results, it seems that higher strain rate sensitivity is expected for the stainless steel material.
Figure 13: Effect of strain rate on the initial stress-strain curves

THEORY

Theoretical calculations have been carried out to determine the flexural capacity of the composite column members for each of the test configurations, including static tests; and impact tests. The capacity of the columns in the static tests was calculated assuming a rigid plastic analysis for the cross-section at midspan. The material strengths for the steel and concrete were based on the mean material strengths based on the tested values, which included mean yield strength of the steel and mean compressive strength of the concrete as illustrated in Figure 14. The capacity of the columns in the static tests was calculated assuming a rigid plastic analysis for the cross-section at midspan and incorporating the increased material strengths proposed by Kang and Liew [5] to account for the dynamic increase at ultimate. The material strengths for the steel and concrete were based on the mean material strengths based on the tested values, which included mean yield strength of the steel and mean compressive strength of the concrete.

Figure 14: Rigid plastic analysis procedure for cross-section capacity of concrete filled steel columns
Comparisons of the experimental and theoretical results are presented here for both the static tests and impact tests. Table 3 summarises the comparisons of experimental and calculated static moment capacity. Note that for MSH2 and SSH2 the comparisons are on the unconservative side and this is due to premature local buckling which occurred in the tests. Table 4 illustrates the comparisons of the dynamic moment capacity to the static moment capacity for the mild and stainless steel specimens respectively. Apart from SSH3a and SSH4 which may have failed by local buckling in the impact tests, all the tests show an increased capacity associated with the dynamic tests and the increases varies from 2 to 64 %.

### TABLE 3
**COMPARISON OF EXPERIMENTAL AND CALCULATED STATIC MOMENT CAPACITY**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$M_{ue}$ (kNm)</th>
<th>$M_{uc}$ (kNm)</th>
<th>$M_{ue}/M_{uc}$</th>
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<tr>
<td>MSH1 27.6</td>
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<td>0.96</td>
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<td>MS1 33.6</td>
<td>30.6</td>
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<td></td>
</tr>
<tr>
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<td>29.1</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>MS1a 33.7</td>
<td>37.7</td>
<td>1.12</td>
<td></td>
</tr>
<tr>
<td>MSH2 57.7</td>
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<td></td>
</tr>
<tr>
<td>MS2 83.1</td>
<td>72.3</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
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<td>1.06</td>
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<tr>
<td>SS1 38.1</td>
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<td>1.32</td>
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<td>SS2 109.7</td>
<td>88.3</td>
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TABLE 4
COMPARISON OF STATIC AND DYNAMIC MOMENT CAPACITY

<table>
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<th>(M_{ude}) (kNm)</th>
<th>(M_{ude}/M_{uc})</th>
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<td>1.08</td>
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CONCLUSIONS AND FURTHER RESEARCH

This paper has presented the results of both static and impact tests of hollow and concrete filled carbon and stainless steel sections. The test results reveal that both the concrete infill and the use of stainless steel results in an increased ductility and strength over the conventional mild carbon steel sections. In addition this paper presented the preliminary results of stress-strain behaviour of both carbon and stainless steel specimens tested at high strain rates. These initial results show that there is a significant increase in the stiffness and capacity when stainless steel is used over conventional carbon steel.

Further research is required to ascertain material property behaviour for concrete at high strain rates as well as steel at appropriate rates which are akin to those observed in the impact tests of the beams in this report. Analytical models calibrated with test results could be used to assist with a further parametric study which would be useful to extend the range of parameters wider than that considered in the test program presented.
ACKNOWLEDGEMENTS

This project was supported by the Australian Research Council Discovery Grants Scheme under the program number DP0879733 and the International Research Initiatives Scheme provided by the University of Western Sydney. This support is gratefully acknowledged. The author would like to thank Professor Zhong Tao, Dr Mithra Fernando and Messrs Daniel Gallaty, Arben Osmani, David Talbot and Mohamad Yousuf of the School of Engineering at the University of Western Sydney for their assistance with experiments carried out in Kingswood, Sydney. Furthermore, the author would like to acknowledge the support of Dr Alex Remennikov and Messrs Ian Bridge, Peter Fairbrother, Alan Grant, Bob Rowland and Scott Waudby from the Faculty of Engineering at the University of Wollongong for their assistance with experiments carried out in Wollongong. Finally the author would like to thank Professor Yan Xiao and Dr Bo-Sheng Chen from the College of Civil Engineering, Hunan University, China for their assistance with carrying out the Split Hopkinson Pressure Bar Tests.

REFERENCES

HYDROELASTIC ANALYSIS OF THE LARGE FLOATING STEEL PLATFORM AT MARINA BAY IN SINGAPORE

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KEYWORDS

Hydroelastic analysis, floating steel platform, modeling, linear potential theory, wave action

ABSTRACT

This paper presents the hydroelastic analysis of the large floating steel platform that was constructed on the Marina Bay in Singapore. The challenge posed in the hydroelastic analysis lies in modeling an equivalent plate that captures the dynamic flexural behaviour of the floating platform. By using a plate model with plate strips along the connecting boundaries of the floating platform, their Young’s moduli and Poisson’s ratios are adjusted so that the free vibration frequencies and mode shapes of the equivalent plate model match the results of the rigorous finite element model of the floating platform. The analysis shows that the hydroelastic response of the floating structure is relatively mild due to the calm waters in the Marina Bay when compared to the static bending response from imposed loads.

INTRODUCTION

A considerable interest on pontoon-type, very large floating structures (VLFS) was generated when the Japanese naval architects and structural engineers constructed a 1-km floating airplane runway on Tokyo Bay in 1998 (see Figure 1). The floating steel structure was so large that it was referred to as the Mega-Float by the Japanese builders and it was recorded as the world’s largest man-made floating island in the Guinness Book of Records. Its construction provides valuable information into the behaviour of such an unprecedented large floating structure under the action of waves, currents, wind and imposed loads, its effect on the sea environment as well as the problems encountered in the construction of the floating
structure. For more information on the Mega-Float, one may refer to the papers by Suzuki [1, 2].

Figure 1: Mega-float in Tokyo Bay
<www.jasnaoe.or.jp and www.marinetalk.com>

The VLFS technology provides an alternative solution to the traditional land reclamation method for creating land from the sea. These floating structures have the following advantages: (1) they are especially cost effective when the water depth is large and the seabed is soft; (2) they have little effect on the water quality, marine habitat and current flow; (3) they have a shorter construction time so that investment may be monetized rapidly; (4) they can easily be expanded or contracted because of their modular construction and thus giving greater flexibility to the owners for different uses; (5) they are not affected by seismic shocks since they are inherently base isolated; (6) their position with respect to the water level is constant, thereby making them ideal for applications as floating piers, container berths and cruise terminals and (7) their interior space may be used for storage purposes, offices and car parks.

The Japanese have used the VLFS technology for floating emergency rescue bases (in Tokyo Bay, Ise Bay and Osaka Bay), floating oil storage facilities (in Shirashima and Kamigoto Islands), and floating ferry piers in Ujina, Hiroshima. The Americans have constructed large
pontoon-type floating bridges in Seattle, USA, and the United Arab Emirates has its own 300m long floating bridge across the Dubai Creek.

Singapore’s first application of VLFS technology is for a floating platform on the Marina Bay. The floating steel structure measures 120mx83mx1.2m and is the world’s largest floating performance stage (see Figure 2). The final design of the steel floating platform comprises 15 identical pontoons of dimensions 40mx16.6mx1.2m. These pontoons are connected to one another by side connectors and corner connectors which involve steel studs. The detail engineering design and implementation of the floating steel platform at the Marina Bay may be obtained from the conference paper by Koh and Lim [3]. Singapore is planning to build a mega floating fuel storage facility (FFSF) to cater for the increasing demand for oil storage capacity [4]. Such FFSF may double up as bunker cum mooring system for ships, thereby relieving traffic congestions in the Singapore harbour and decreasing the turnaround time for ships.

![Figure 2: Floating steel platform on Marina Bay, Singapore](image)

**CONSIDERATIONS FOR HYDROELASTIC ANALYSIS**

As the floating steel platform is relatively flexible due to its small depth in relation to the length dimensions, it exhibits elastic behaviour, similar to a thin plate, when subjected to wave actions. In other words, the response of the floating platform in the vertical direction cannot be assessed only by its rigid body motions. It is thus crucial to estimate the flexural response of the floating platform when designing it. However, for the design of the mooring system (that keeps the floating platform in position), the floating platform may be considered as a rigid-body since it has a large flexural rigidity in the horizontal directions. When the dynamic response of the floating platform due to waves is analysed, the pressure of the water acting on it must be evaluated at the same time. If the floating platform moves, the water surrounding the platform also moves. Thus the pressure surrounding the floating platform changes in order to satisfy the Bernoulli’s equation. On the other hand, if the pressure changes, the motion of the floating platform is affected. This mutual relationship is referred
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to as a fluid-structure interaction. Since the vertical motion of the floating platform comprises elastic deformations, the fluid-structure interaction is called hydroelasticity. Hence it is necessary to perform hydroelastic analyses of the platform under the action of waves found in the Marina Bay. A comprehensive literature survey on the various techniques for hydroelastic analysis may be obtained from the papers by Kashiwagi [5] and Watanabe et al. [6].

In this paper, we present the modeling and hydroelastic analyses of Singapore’s floating steel platform with the view to determine the deflections and stresses induced when the platform is subjected to the estimated worst wave loading in the Marina Bay. The challenge in this study is to figure out a simple plate model for the hydroelastic analysis and yet it is capable of capturing the dynamic flexural behaviour of the floating platform.

FINITE ELEMENT MODEL OF FLOATING PLATFORM

In the finite element model, the floating platform is discretised appropriately with a combination of plate and beam elements using the finite element software ABAQUS. The top and bottom plates are discretised with a sufficient number of 4-node quadrilateral and 3-node triangular plate elements. The stiffeners of the top plate and bottom plate of the pontoon are modeled as beam elements of equivalent flexural rigidity as the actual angle-section members. Likewise, the rest of the pontoon, that is, the side plates and stiffeners as well as the interior webs are discretised with a combination of plate and beam elements. A total of about 12,000 nodes are used to discretise one typical pontoon. Figure 3 shows the refined finite element model for a single pontoon that measures 40mx16.6mx1.2m and the interior design of the pontoon.

![Finite element model and interior design of a single pontoon](image.png)

The finite element model of the entire platform (see Figure 4) is obtained by joining the finite element models of the pontoon by using the side connectors (see Figure 5) and the corner connectors (see Figure 6). The 15 pontoons are laid out in a 5 by 3 configuration to obtain a large platform of dimension 120mx83mx1.2m. Each pontoon is connected to each other on the longer side at 5 locations and on the shorter side at 2 locations by the side connectors which are modeled using appropriate plate elements with the connecting steel studs modeled as rod beam elements. These rod elements connect the separate bodies and make them bend together. Besides the side connectors, each pontoon is also connected to the other pontoons at the 4 corners through corner connectors. The latter are again modeled using appropriate plate elements.
elements and the connecting studs modeled as rod beam elements. The rod beam elements connect the pontoon to the corner connector using the “tie” constraint in ABAQUS where its top and bottom segments are tied (or fused) to the corresponding top and bottom brackets inside the corner connector and the middle segment is tied to the bracket inside the male part of the pontoon. A total of about 200,000 nodes (or approximately 1.2 million degrees of freedom) are used to discretize the entire floating platform, i.e. the 15 pontoons as well as all the side and corner connectors. The described finite element model will be used to calibrate a simpler plate model for the hydroelastic analysis.

**EQUIVALENT SOLID PLATE MODEL FOR HYDROELASTIC ANALYSIS**

For the hydroelastic analysis, it is not possible to use the aforementioned detail finite element model due to the huge number of degrees of freedom. An equivalent solid plate model has to be found for the hydroelastic analysis. This equivalent plate will have the same length and overall depth dimensions as the actual floating platform, but its Young’s modulus and Poisson’s ratio have to be tweaked to furnish similar flexural and dynamic behaviour as the actual floating platform. In order to do this, we perform a free vibration analysis of the actual floating platform modeled by the detail finite element model with Winkler springs that simulate the buoyancy force. By adjusting the Young’s modulus and Poisson’s ratio so as to match the fundamental frequency of vibration of the equivalent solid plate model and the actual floating platform model, the equivalent Young’s modulus and Poisson’s ratio may be obtained for use in the hydroelastic analysis. Note that the selfweight of the floating platform is 2787.5 tonnes.

Figures 7-9 show the first 3 vibration modes of the floating platform. The natural frequencies are 0.99 Hz, 1.02 Hz and 1.04 Hz for the first, second and third vibration modes, respectively. By observing the vibration mode shapes of the floating platform, it is clear that a simple isotropic equivalent plate model will not suffice as the mode shapes of such a simple model
do not correspond to the ones observed. Moreover, the mode shapes show some rotations at the joining lines between the pontoons.

In view to obtain similar vibration mode shapes, we propose the equivalent model consisting of 15 solid isotropic plates that model the pontoons (1.2m in depth), 20 corner connectors (4m x 4m in length and in width and 1.2m in depth), and plate strips (1m in width and 1.2m in depth) along their boundaries to simulate the connection regimes (see Figure 10). The Young’s moduli and Poisson’s ratio for the various components are shown in Table 1. The adoption of a relatively low modulus for the corner connectors and the plate strips captures
the floating platform’s flexibility at the joining lines. The first three frequency values and mode shapes are given in Figs. 11-13 which appear to be rather similar to those for the actual floating platform (see Figs. 7-9). This equivalent plate model will be used for the hydroelastic analysis.

![Figure 10: Proposed equivalent plate model](image)

**TABLE 1**

<table>
<thead>
<tr>
<th>Materials</th>
<th>Pontoon</th>
<th>Strip</th>
<th>Corner connector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>10</td>
<td>0.05</td>
<td>0.3</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Mass Density (kg m⁻³)</td>
<td>233.22</td>
<td>233.22</td>
<td>233.22</td>
</tr>
</tbody>
</table>

![Figure 11: First vibration mode shape of equivalent plate (frequency = 0.995 Hz)](image)
HYDROELASTIC ANALYSIS AND RESULTS

The sea-water is assumed to be inviscid and incompressible fluid and its motion is irrotational. Accordingly, the wave is modeled by the linear potential theory where the velocity potential $\phi$ is solved using the boundary element method [7]. We consider a time-harmonic problem which implies that the time dependency is represented by $e^{i\omega t}$, where $i = \sqrt{-1}$, $\omega$ is the angular frequency of the wave, and $t$ is the time variable.

Figure 14 is the schematic diagram of the water-floating platform problem. The governing equation of the velocity potential is given by

$$\nabla^2 \phi = 0 \text{ for the water domain } \Omega$$

(1)

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator. The boundary conditions are given by

$$\frac{\partial \phi}{\partial z} = i\omega w \text{ under the floating platform}$$

(2)

$$\frac{\partial \phi}{\partial z} = \frac{\omega^2}{g} \phi \text{ on the free surface}$$

(3)
\[ \frac{\partial \phi}{\partial z} = 0 \quad \text{on the seabed} \quad (4) \]

where \( g \) is the gravitational acceleration, \( \omega \) the angular frequency of the wave, \( w(x) \) the surface displacement and \( x = (x, y) \).

At \( |x| \to \infty \), the velocity potential must satisfy the Sommerfeld radiation condition \([7]\), i.e.

\[ \lim_{x \to \infty} \sqrt{x} \left( \frac{\partial}{\partial x} - ik \right) (\phi - \phi_{inc}) = 0 \quad (5) \]

From Eqs. (1)-(4), we can derive (by using separation of variables) the incident wave which is given by \([7]\)

\[ \phi_{inc}(x) = i \frac{gA \cosh[k(z + h)]}{\omega \cosh(kh)} \exp[i(k \cos \theta + y \sin \theta)] \quad (6) \]

where \( A \) is the wave amplitude (which is half of the wave height \( H \)), \( h \) the water depth, \( \theta \) the incident angle, \( g \) the gravitational acceleration, and \( k \) the wave number which is related to the wavelength \( \lambda \) by

\[ k = \frac{2\pi}{\lambda} \quad (7) \]

Moreover, the wavelength is related to the wave period \( T \) by

\[ \left( \frac{2\pi}{T} \right)^2 = g \frac{2\pi}{\lambda} \tanh\left( \frac{2\pi h}{\lambda} \right) \quad (8) \]

The platform is modeled as a perfectly flat plate with free-edges. The more refined Mindlin plate theory \([8]\) is adopted as it takes into account the effects of rotary inertia and transverse shear deformation. Better prediction of the stress resultants are expected since they are expressed at most in terms of first derivatives of the deflection and rotations in the Mindlin plate theory. The motion of the floating platform is described by the following equations \([8]\)

\[ \kappa G I \left[ \nabla^2 w + \frac{\partial}{\partial x} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \right] + \rho_s t \omega^2 w = \rho g w - i \rho \omega \phi(x, y, -d) \quad (9a) \]

\[ D \left[ \frac{1-\nu}{2} \nabla^2 \psi_x + \frac{1+\nu}{2} \frac{\partial}{\partial x} \left( \frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y} \right) \right] \right)^2 = \kappa G I \left[ \psi_x + \frac{\partial w}{\partial x} \right] + \frac{\rho_s t}{12} \omega^2 \psi_x = 0 \quad (9b) \]

\[ D \left[ \frac{1-\nu}{2} \nabla^2 \psi_y + \frac{1+\nu}{2} \frac{\partial}{\partial y} \left( \frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y} \right) \right] \right)^2 = \kappa G I \left[ \psi_y + \frac{\partial w}{\partial y} \right] + \frac{\rho_s t}{12} \omega^2 \psi_y = 0 \quad (9c) \]
where \( w \) is the deflection of the plate, \( \psi_x \) and \( \psi_y \) the bending rotations of the normal of the cross-section about the \( y \) and \( x \) axes, respectively, the dynamic pressure \( p(x,y) \) relates to the velocity potential on the bottom surface of the floating platform, \( d \) is the draft, \( t \) the thickness of the plate, \( \rho \) the mass density of water, \( \rho_p \) the mass density of the floating platform, \( D \) the flexural rigidity of the plate, \( G = E/[2(1+\nu)] \) the shear modulus and \( \kappa \) the shear correction factor. The free-edge boundary conditions of the floating platform are given by

\[
M_{nn} = D \left( \frac{\partial \psi_n}{\partial n} + \nu \frac{\partial \psi_n}{\partial s} \right) = 0, \quad M_{ns} = D \frac{1-\nu}{2} \left( \frac{\partial \psi_n}{\partial s} + \frac{\partial \psi_s}{\partial n} \right) = 0, \quad \text{and} \quad Q_n = \kappa G t \left( \psi_n + \frac{\partial w}{\partial n} \right) = 0
\]

(10)

where \( M_{nn} \) is the bending moment, \( M_{ns} \) the twisting moment, \( Q_n \) the shear force, and \( n \) and \( s \) denote the normal direction and the tangential direction to the plate edge.

For the hydroelastic analysis using the modal expansion method, we have used 30 modal functions. The modal functions are calculated using MSC/NASTRAN, where CQUAD4 (4-node quadratic plate) elements are used. The first three elastic vibration modes are shown in Figs. 11-13. The generalized displacement field is expanded by a series of the products of the modal functions \( f_i(x,y) \) and the complex amplitudes \( \zeta_i \):

\[
w(x,y) = \sum_{i=1}^{30} \zeta_i^w f_i^w(x,y), \quad \psi_x(x,y) = \sum_{i=1}^{30} \zeta_i^\psi_x f_i^\psi_x(x,y) \quad \text{and} \quad \psi_y(x,y) = \sum_{i=1}^{30} \zeta_i^\psi_y f_i^\psi_y(x,y)
\]

(11)

The potential \( \phi(x,y,z) \) can be expressed by the sum of the diffraction and radiation potentials, \( \phi_D(x,y,z) \) and \( \phi_r(x,y,z) \) based on the linear theory, and the radiation potentials can be further decomposed as follows:

\[
\phi(x,y,z) = \phi_D(x,y,z) + \phi_r(x,y,z) = \phi_D(x,y,z) + i\omega \sum_{i=1}^{30} \zeta_i \phi_i(x,y,z)
\]

(12)

where \( \phi_i(x,y,z) \) is the radiation potential corresponding to the unit-amplitude motion of the \( l \)-th modal function. Note that the diffraction potential \( \phi_D(x,y,z) \) is composed of

\[
\phi_D(x,y,z) = \phi_{in}(x,y,z) + \phi_S(x,y,z)
\]

(13)

where \( \phi_S(x,y,z) \) is the scattering potential. By substituting Eqs. (11) and (12) into Eqs. (1)-(4), we have the governing equation and boundary conditions for each of the unit-amplitude radiation potentials and the diffraction potential. Then, the potentials are solved by using the Green’s function method. For details, see Ref. [9]. It is assumed that the floating platform has a constant draft of \( d = 0.273 \text{ m} \) at its static equilibrium state.
Once the complex amplitudes $\zeta_i$ are determined, the complex amplitudes for the bending and twisting moments can be determined by

$$
\overline{M}_{xx} (x, y) = \sum_{i=1}^{30} \zeta_i M_{xxf} (x, y), \quad \overline{M}_{yy} (x, y) = \sum_{i=1}^{30} \zeta_i M_{yyf} (x, y), \quad \overline{M}_{xy} (x, y) = \sum_{i=1}^{30} \zeta_i M_{xyf} (x, y)
$$

(14)

where $\overline{M}_{xx}$ and $\overline{M}_{yy}$ are the complex amplitudes for the bending moments in $x$ and $y$ directions, respectively; and $\overline{M}_{xy}$ for the twisting moment. The modal functions $M_{xx}$, $M_{yy}$ and $M_{xy}$ are calculated by the eigenvalue analysis using MSC/NASTRAN.

The normal and shear stresses induced in the bottom plate are calculated by

$$
\sigma_{xx} (x, y) = \frac{\overline{M}_{xx} (x, y)}{Z}, \quad \sigma_{yy} (x, y) = \frac{\overline{M}_{yy} (x, y)}{Z}, \quad \tau_{xy} (x, y) = \frac{\overline{M}_{xy} (x, y)}{Z}
$$

(15)

where $Z$ is the sectional modulus for the bottom plate. Since the thickness of the bottom plate is 8mm whereas the thickness of the deck plate is 12mm, the maximum von Mises stress occurs in the bottom plate. The von Mises stress is given by

$$
\sigma_v = \sqrt{\frac{1}{2} \left( (\sigma_{xx} - \sigma_{yy})^2 + \sigma_{xx}^2 + \sigma_{yy}^2 \right) + 3 \tau_{xy}^2}
$$

(16)

Note that the calculation of Eq. (16) is performed in the time-domain using the following equations:

$$
\sigma_{xx} = \text{Re}\left[ \sigma_{xx} e^{i \omega t} \right], \quad \sigma_{yy} = \text{Re}\left[ \sigma_{yy} e^{i \omega t} \right], \quad \tau_{xy} = \text{Re}\left[ \tau_{xy} e^{i \omega t} \right]
$$

(17)

For the given period $T = 2.4s$, we consider two cases of different water depths, $h = 3m$ and $h = 0.8m$ to investigate the effect of tidal variation. For the water depth $h = 3m$ and a wave period $T = 2.4s$, the wavelength corresponds to 8.75m whereas for the shallow water depth of $h = 0.8m$ and wave period $T = 2.4s$ the wave length corresponds to 6.09m. The considered wave angles are $\theta = 0^0$ (head wave) and $\theta = 90^0$ (beam wave). The wave height is assumed to be $H = 0.5m$. 

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The hydroelastic analysis of the plate was carried out by using an in-house developed hydroelastic software. The deflections and von Mises stress distributions are shown in Figs. 15 to 18 for unit-amplitude waves (i.e. $A = 1$ m or $H = 2$ m). Therefore, as an example, for $H = 0.5$ m (i.e. $A = \frac{1}{4}$ m), we obtain the maximum deflection as $0.344/4 = 0.086$ m (i.e. a draft of $0.273 \pm 0.086 = 0.187$m to $0.359$m) and the maximum von Mises stress as $103/4 = 26$MPa which are rather small when compared to those due to static loads. So the wave forces will not pose a problem. Note that the above maximum von Mises stress occurs at the position of corner connecter (Figure 18). Thus, the actual maximum von Mises stress for the bottom plate of the pontoons is $101/4 = 25$MPa, which is even smaller (Figure 15).

CONCLUDING REMARKS

The plate model used in the hydroelastci analysis of the floating platform on Marina Bay is rather complex with the allowance for corner connectors and plate strips with significantly lower moduli of rigidity as compared to that of the pontoon. These corner connectors and plate strips are necessary to simulate the connectors.

The hydroelastic analysis show that the deflection and von Mises stresses due to the wave load are rather small when compared to those due to static loads. This is primarily due to the benign sea condition on site. If the platform is used in a harsh sea environment, the deflections and von Mises stresses due to wave action will be relatively large due to its flexibility.
REFERENCES


Figure 15: Deflection and von Mises stress in the bottom plate of floating platform under head sea wave of period $T = 2.4s$ and water depth $h = 0.8m$. Maximum deflection is $w = 0.147m$ and maximum von Mises stress for pontoons is 101MPa for unit-amplitude waves
Figure 16: Deflection and von Mises stress in the bottom plate of floating platform under beam sea wave of period $T = 2.4s$ and water depth $h = 0.8m$. Maximum deflection is $w = 0.098m$ and maximum von Mises stress for pontoons is 30MPa for unit-amplitude waves.
Figure 17: Deflection and von Mises stress in the bottom plate of floating platform under head sea wave of period $T = 2.4\text{s}$ and water depth $h = 3\text{m}$. Maximum deflection is $w = 0.089\text{m}$ and maximum von Mises stress for pontoons is 63 MPa for unit-amplitude waves.
Figure 18: Deflection and von Mises stress in the bottom plate of floating platform under beam sea wave of period $T = 2.4s$ and water depth $h = 3m$. Maximum deflection is $w = 0.344m$ and maximum von Mises stress for pontoons is 103 MPa for unit-amplitude waves.
RIGID MECHANICS AND APPLICATIONS TO NONLINEAR STRUCTURAL ANALYSIS

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KEYWORDS

Beam, corector, geometric nonlinear analysis, plate, postbuckling, predictor, rigid body rule, rigid element, shell.

ABSTRACT

In the incremental-iterative analysis of elastic nonlinear structures, great saving in computation can be achieved if use is made of the different nature of the predictor and corrector phases. The predictor relates to solution of the structural displacements for given load increments, which affects only the number of iterations. In fact, the equations used in the predictor need not be exact, but should be accurate to the level not to mislead the direction of iterations. In this study, it is demonstrated that the use of the linear stiffness matrix \([k_e]\) and a rigid-body qualified geometric stiffness matrix \([k_g]\) in the predictor is sufficient for most purposes. The rigid-body qualified geometric stiffness matrix \([k_g]\) can be derived from the virtual work equation for a rigid displacement field. In contrast, the corrector is concerned with computation of the force increments for given deformations of each element, and the updating of initial element forces at the end of each increment. For a nonlinear analysis with practically small incremental steps, the element force increments can be computed using only the linear stiffness matrix \([k_e]\), and the initial nodal forces updated by the rigid body rule. For the present purposes, the two-dimensional beam element will be used as the vehicle of illustration of the ideas involved. The procedure will then be extended to treat the three-dimensional beam structures and plates and shells. The effectiveness of the ideas presented herein will be demonstrated in the solution of some nonlinear problems involving the postbuckling response.
INTRODUCTION

In the past half a century, great advances have been made in structural nonlinear mechanics. A partial review of the related previous works can be found in Ref. [1]. Based on the updated Lagrangian (UL) formulation, the purpose of this paper is to present a conceptually simple, but procedurally robust, approach for the nonlinear analysis of elastic structures, by taking advantage of the different properties of each phase of the incremental-iterative analysis. Such a procedure may appear to be non-conventional in the first stance. Nevertheless, its applicability has been thoroughly tested in the solution of a number of nonlinear problems [2,3].

For the incremental-iterative analysis of elastic nonlinear structures, two typical phases can be identified, i.e., the predictor and corrector phases, in addition to the error-checking phase. The predictor relates to solution of the structural displacement increments \{ \mathbf{U} \} for given load increments \{ \mathbf{P} \} based on the incremental structural equation, \[ \mathbf{K} \mathbf{U} = \mathbf{P} \] [4,5]. Concerning the mechanism of incremental-iterative analysis, let us refer to the schematic diagram shown in Figure 1. The slopes AB and CD represent the tangent stiffness used in the predictor or computing the displacement increments \{ \mathbf{U} \}, given the load increment \{ \mathbf{P} \} = \{ \mathbf{P}_2 \} - \{ \mathbf{P}_1 \}. Clearly, a convergent solution can always be obtained, regardless of whether the stiffness matrix (i.e., the slope) has been updated or not. In fact, this phase affects only the direction of iteration or the number of iterations, but not the accuracy of solution. As such, the stiffness matrix \[ \mathbf{K} \] used in the predictor need not be exact, but must be accurate enough not to misguide the direction of iteration [6]. Such a point allows us to be released from the burden of deriving highly nonlinear elements, as conventionally attempted. In this study, it will be demonstrated that using the linear stiffness matrix \[ \mathbf{k}_e \] and a rigid-body qualified geometric stiffness matrix \[ \mathbf{k}_g \] is all we need for the predictor. The geometric stiffness matrix \[ \mathbf{k}_g \] that is rigid-body qualified can be derived from the virtual work equation for a rigid displacement field, for which the effort required is considerably less than that of a conventional element.

The corrector is concerned with recovery of the force increments \{ \mathbf{f} \} for the element displacement increments \{ \mathbf{u} \} and the updating of element forces at the end of each increment. With reference to Figure 1, the corrector \mathbf{f} or the first iteration is represented by the segment CE. This segment, as well as other similar segments in Figure 1, should be made as accurate as possible. Otherwise, the unbalanced forces computed (as indicated by BC) will be inaccurate.
and the iterations following will be meaningless. As a matter of fact, the corrector determines the accuracy of solution. However, for the case where each incremental step is practically small, the element force increments \{\mathbf{f}\} can be computed with sufficient accuracy using only the linear stiffness matrix \([k_e]\), and the initial nodal forces \{\mathbf{f}_0\} can be updated using the rigid body rule. By so doing, the amount of computation involved in this phase is greatly reduced compared with the conventional approaches.

The error-checking phase relates to computation of the unbalanced forces as the difference of the total applied loads and total internal resistant forces for the updated geometry of the structure. Whenever the unbalanced forces are greater than preset tolerances, another iteration involving the predictor and corrector phases should be conducted.

**INCREMENTAL NONLINEAR ANALYSIS**

In an incremental nonlinear analysis, three states can be identified for a structure: the initial state \(C_0\), the last (calculated) state \(C_1\), and the current state \(C_2\) (Figure 2). The step characterizing the behavior of the structure from \(C_1\) to \(C_2\) is referred to as an incremental step. It is assumed that all the (initial) forces acting on the structure are in equilibrium at \(C_1\), as long as iterations have been performed to remove the unbalanced forces.

![Figure 2: History of deformation of a structure](image)

Based on the updated Lagrangian (UL) formulation, the virtual work equation for a finite element at \(C_2\) but with reference to \(C_1\) can be given in a linearized sense as [7]:

\[
\int_C e_{ij} \delta e_{ij} dV + \int_C \tau_y \delta \gamma_y dV = \delta R - \delta R
\]  

(1)

in which the first two terms denote the strain energy and potential energy, \(V\) = volume, \(C_{ijkl}\) = constitutive coefficients, and \(\tau_y\) = Cauchy stresses of the element at \(C_1\), \(\delta\) = variation of the quantity following, and \(\gamma_y\) = linear and nonlinear components of the strain increments,

\[
\gamma_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad \gamma_y = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)
\]  

(2a,b)

where \(u_i\) = displacement increments and \(x_i\) = coordinates of the element at \(C_1\). The external virtual works \(\delta R\) and \(\delta R\) at \(C_2\) and \(C_1\), respectively, can be given as

\[
\delta R = \int_S \delta t_i \delta u_i dS + \int_C \delta f_i \delta u_i dV, \quad \delta R = \int_S \delta t_i \delta u_i dS + \int_C \delta f_i \delta u_i dV
\]  

(3)

where \(S\) = surface area at \(C_1\), \(\delta t_i\) and \(\delta f_i\) = surface tractions at \(C_2\) and \(C_1\), respectively, and \(\delta t_i\) and \(\delta f_i\)
and \( f_i \) = body forces. For a 2D beam element, the virtual work equation in Eq. (1) can be transformed into an incremental stiffness equation for the element as [7]

\[
\left( [k_e] + [k_g] \right) \{u\} = \{ \delta f \} - \{ f \}
\]  

(4)

where \( \{u\} \) = the displacement increments. The linear stiffness matrix \([k_e]\), geometric stiffness matrix \([k_g]\) and nodal loads \(\{ f \}\) and \(\{ \delta f \}\) respectively acting on the element at \(C_2\) and \(C_1\) are

\[
\delta U = \int_V C_{ijkl} e_i^k e_j^l \delta e_i e_j dV = \{ \delta u \}^T [k_e] \{u\}, \quad \delta V = \int_V \tau_{ij} \delta \eta_i \eta_j dV = \{ \delta u \}^T [k_g] \{u\}
\]  

(5a,b)

\[
\delta R = \{ \delta u \}^T \{ \delta f \}, \quad R = \{ \delta u \}^T \{ f \}
\]  

(6)

Eq. (4) serves as the basis for incremental-iterative analysis. Here, we shall demonstrate that with a robust path-tracing scheme, the use of the linear stiffness matrix \([k_e]\), plus a rigid-body qualified geometric stiffness matrix \([k_g]\), is all that we need for nonlinear analysis.

**RIGID BODY RULE**

Consider a bar sitting on the surface of the earth and subjected to a load \(P\) in Figure 3(a) at \(C_1\). For equilibrium, a reaction of the same magnitude, but in opposite sense, is developed at the bottom of the bar. When the earth rotates by an angle \(\theta\) in \(C_2\), the load \(P\) acting on the bar will rotate following the rotation, but with the magnitude kept unchanged. An overall result is the preservation of equilibrium of the bar at \(C_2\), as in Figure 3(b). The fact that an initially stressed bar remains in equilibrium after a rigid rotation with the acting forces remaining unchanged in magnitude has been referred to as the rigid body rule [7,8].

Figure 3: Initially loaded bar: (a) before rigid rotation, (b) after rigid rotation

Figure 4: Initially stressed 2D beam: (a) before rigid rotation, (b) after rigid rotation

The rigid body rule is a general principle, which holds for any structure under loadings in
equilibrium at $C_1$. It can be inferred that if the structure is to be represented by any assembly of finite elements, then each of the composing elements, which are initially stressed and in equilibrium at $C_1$, should exhibit the same behavior over a rigid rotation. Naturally, all the initial forces acting on the element, including the axial forces, shear forces, and bending moments, should remain unchanged in magnitude, as illustrated in Figure 4. Such a rule should be obeyed in updating the initial nodal forces of each element of a structure undergoing rigid rotations.

NATURAL DEFORMATION AND RIGID DISPLACEMENTS

The displacements $\{u\}$ of each element of a structure in each increment can be divided into two parts as the rigid displacements $\{u\}_r$ and natural deformations $\{u\}_n$. In general, the rigid displacements constitute a great portion of the incremental displacements of each element that is initially stressed, and the natural deformations are relatively small for structures well represented by the finite elements. Such a characteristic can be appreciated from the buckling of the cantilever shown in Figure 5. For an element with initial nodal forces $\{f\}$ undergoing the displacements $\{u\}$ from $C_1$ to $C_2$, the effect of the rigid displacements $\{u\}_r$ is to rotate the initial nodal forces $\{f\}$ from $C_1$ to $C_2$ without altering their magnitudes, according to the rigid body rule. For an analysis with practically small increments, the elastic force increments $\{f\}$ can be computed by considering only the natural deformations $\{u\}_n$ in relation with the linear stiffness matrix $[k_e]$, that is,

$$\{f\} = [k_e] \{u\}_n = [k_e] (\{u\} - \{u\}_r) = [k_e] \{u\}_n$$  \hspace{3cm} (7)

where it is recognized that no actions will be induced by the rigid displacements $\{u\}_r$.

GEOMETRIC STIFFNESS MATRIX FOR 2D BEAM

The geometric stiffness matrices $[k_g]$ will be derived for a rigid element with the nodal degrees shown in Figure 4(a). The nodal forces $\{f\}$ and $\{f\}'$ acting at $C_1$ and $C_2$ are

$$\{f\} = \begin{bmatrix} F_a \\ F_b \\ M_a \\ M_b \end{bmatrix}, \quad \{f\}' = \begin{bmatrix} F_a' \\ F_b' \\ M_a' \\ M_b' \end{bmatrix}$$

\hspace{3cm} (8)

For the element to be in equilibrium at $C_1$, the following conditions hold:
\[ F_x = F_{sb} = -\frac{1}{L}, \quad F_y = F_{yb} = -\frac{1}{L}, \quad F_{z} = -\frac{1}{L}M_{sw} + \frac{1}{L}M_{zb} \]  \hspace{1cm} (9)

\[ M_z = -M_{sw}(1 - x/L) + M_{zb}(x/L) \]  \hspace{1cm} (10)

The element forces at each cross section can be defined as the stress resultants as

\[ F_x = \int_{d}^{} r_{xx} dA, \quad F_y = \int_{d}^{} r_{xy} dA, \quad M_z = -\int_{d}^{} r_{xz} y dA \]  \hspace{1cm} (11)

where \( r_{xx} \) and \( r_{xy} \) are axial and shear stresses. By the plane section hypothesis, the displacements \((u_x, u_y)\) at a point of the cross section can be related to those at the centroid \((u, v)\) as

\[ u_x = u - yv', \quad u_y = v \]  \hspace{1cm} (12)

For the rigid beam, the strain energy in Eq. (5a) simply vanishes, i.e., \( U = 0 \). The geometric stiffness matrix \([k_g]\) for the rigid beam can be derived by substituting the strains \( \eta_x \) and \( \eta_y \) in Eq. (2b) into Eq. (5b), together with the expressions in Eqs. (11) and (12), i.e.,

\[ \{\delta u\}^T [k_g] \{u\} = \int \left( \left[ r_{xx} \delta \eta_x + 2 r_{xy} \delta \eta_y \right] dV - \int \left[ F_x \delta (u'' + v'') \right] dx + \int \left[ F_y \delta (u'v') \right] dx \]  \hspace{1cm} (13)

For a rigid rotation, the axial and lateral displacements \((u, v)\) can be related to the nodal ones as

\[ u = (1 - x/L)u_a + (x/L)u_b, \quad v = (1 - x/L)v_a + (x/L)v_b \]  \hspace{1cm} (14)

subject to the condition \( u_a = u_b \). For the present constrained problem, the following must be considered in deriving each term of Eq. (13): (1) The two variational terms are equal to zero: \( \delta u'' = 0, \quad \delta (u''v') = 0 \), for the rigid displacement field. (2) The variation of a quantity that is zero in the rigid displacement field need not be zero. For instance, \( u' = (u_a - u_b)/L = 0 \) as implied by the constraint, but \( \delta u' = (\delta u_a - \delta u_b)/L \) is unequal to 0 since \( \delta u_a \neq \delta u_b \). (3) The variational term \( u'' \delta v' \) is zero, but is retained in the derivation for the sake of symmetry of the matrix. By substituting the rigid displacement field in Eq. (14) into Eq. (13), and using the conditions in Eqs. (9)-(10), one can derive the geometric stiffness matrix \([k_g]\) for the 2D rigid beam as

\[ [k_g] = \frac{1}{L} \begin{bmatrix} 0 & -F_{xb} & 0 & 0 & F_{yb} & 0 \\ -F_{yb} & F_{xb} & 0 & F_{yb} & -F_{xb} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ F_{yb} & -F_{xb} & 0 & -F_{yb} & F_{xb} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]  \hspace{1cm} (15)

To demonstrate that the geometric stiffness matrix derived above is rigid-body qualified, let us consider a rigid displacement field \( \{u\}_r \):

\[ \{u\}_r = \{0, 0, \theta, 0, L\theta, \theta\}^T \]  \hspace{1cm} (16)

By substituting Eqs. (15) and (16) into the finite element equation in Eq. (4), one can obtain the element forces \( \{\hat{f}\} \) at \( C_2 \), but with respect to \( C_1 \), as

\[ \{\hat{f}\} = [k_g] \{u\}_r + \{\hat{f}\} \]  \hspace{1cm} (17)

which are equal in magnitude to the element forces \( \{\hat{f}\} \) at \( C_2 \) with reference to \( C_2 \), as predicted by the rigid body rule in Figure 4(b). Thus, the stiffness matrix \([k_g]\) is rigid-body qualified.

PREDICTOR AND CORRECTOR FOR NONLINEAR ANALYSIS
By looping over all the elements of a structure, the element stiffness equation, Eq. (4), can be assembled to yield the structural equation for the incremental step from $C_1$ to $C_2$:

$$
[K][U] = \{\dot{\gamma}P\} - \{\dot{\gamma}P\}
$$

(18)

where $[K]$ = stiffness matrix, $\{U\}$ = displacement increments from $C_1$ to $C_2$, and $\{\dot{\gamma}P\}$ and $\{\dot{\gamma}P\}$ = applied loads of the structure at $C_1$ and $C_2$, respectively. In a step-by-step analysis, all the information of the structure under the applied loads $\{\dot{\gamma}P\}$ at $C_1$ is known a priori. By increasing the external loads from $\{\dot{\gamma}P\}$ to $\{\dot{\gamma}P\}$ by a small amount, we are interested in the displacement increments $\{U\}$ of the structure generated from $C_1$ to $C_2$, which has to be obtained in an iterative manner, due to the nonlinear nature of the problem considered.

In an incremental-iterative nonlinear analysis, distinction should be made between the predictor and corrector phases. The predictor is concerned with solution of the structural displacement increments $\{U\}$ from the equation in Eq. (18), while the corrector relates to computation of the element force increments $\{f\}$ from the displacement increments $\{u\}$ made available for each element from the structural displacements $\{U\}$. As was stated previously, the predictor affects only the speed of convergence or the number of iterations for each incremental step. Thus, the stiffness matrix contained in the structural matrix $[K]$ need not be exact. In the numerical studies, it will be demonstrated that using the linear stiffness matrix $[k_e]$ and the rigid-body qualified geometric stiffness matrix $[k_g]$ in the predictor enables us to solve a wide range of nonlinear problems involving the postbuckling response. On the other hand, the corrector should be made as accurate as possible, since it determines the accuracy of solution. Based on the UL formulation, it will be demonstrated that so long as the rigid body rule is exactly followed in updating the initial nodal forces $\{\dot{\gamma}f\}$ from $C_1$ to $C_2$, the element force increments $\{\dot{\gamma}f\}$ at each iterative step can be computed using merely the linear stiffness matrix $[k_e]$, that is,

$$
\{\dot{\gamma}f\} = \{\dot{\gamma}f\} + \{f\} = \{f\} + [k_e][u]
$$

(19)

using Eq. (7), where the initial forces $\{\dot{\gamma}f\}$ acting at $C_1$ with reference to $C_1$ are directly regarded as those acting at $C_2$ with reference to $C_2$ according to the rigid body rule.

The strategy as presented above for dealing with the predictor and corrector enables us to circumvent the complicated procedure of formulation for highly nonlinear elements. The advantage is more obvious when dealing with the 3D beams, plates, and shells [3], for which the geometric stiffness matrix cannot be derived in a straightforward manner.

**EXTENSION TO 3D RIGID BEAM**

The procedure presented for deriving the geometric stiffness matrices $[k_e]$ for the 2D rigid beam can be easily extended to that for the 3D rigid beam, with 6 DOFs at each node shown in Figure 6. For the 3D beam with uniform solid sections, we should consider two planes of bending and the twisting action, in addition to the axial elongation. As for the rigid displacement field, the axial and twisting displacements are assumed to be linear, subject to the constraint that the axial and twisting displacements at the two ends of the beams are the same. Also, the lateral and rotational displacements are linear, with the same restraint for the rotations at the two ends. Following the same procedure as the one for the 2D beam, the geometric stiffness matrix can be derived for the 3D beam as
Here, $0$ is a $3 \times 3$ null matrix containing all zero entries, and

$$
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 \\
-1 & 0 & 0 \\
0 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

The submatrices $[h_a]$ and $[i_a]$ associated with end $a$ can be obtained by replacing the moment terms ($M_{ab}$, $M_{ab}$, $M_{ab}$) in the submatrices $[h_b]$ and $[i_b]$ by the terms ($M_{aa}$, $M_{aa}$, $M_{aa}$), respectively. The matrix as presented in Eq. (20) is asymmetric, due to the asymmetry of the submatrices $[i_a]$ and $[i_b]$ originating from the boundary terms [3,7].

EXTENSION TO 3D RIGID PLATE

Consider the triangular plate element (TPE) with three translational and three rotational degrees of freedom (DOFs) at each node in Figure 7, which is chosen for its compatibility with the 12-DOF beam element derived above. As far as the rigid body behavior of an element is concerned, only the initial forces acting on the element and the external shape of the element need to be considered. The elastic properties that are essential to the deformation of the element, such as Young’s modulus, areas of cross sections and moments of inertia, can be totally ignored. As such, the rigid behavior of the TPE can be simulated as if it were composed of three rigid beams lying along the three sides of the element, as shown in Figure 8. This enables us to derive the geometric stiffness matrix for the rigid TPE in a heuristic, easy way [3].
Figure 8 shows the nodal forces acting on each of the three beam elements, named as beam 12, beam 23 and beam 31. In this figure, $F_{ij}^k$ and $M_{ij}^k$ denote the nodal force and nodal moment, respectively, acting on beam $ij$ at $C_1$, with the right subscript $k$ denotes the direction and the nodal point at which the force or moment is acting. One feature with the rigid element is that there are more unknowns than the equations available. For the present case, each beam has 12 unknowns and we have a total of 36 unknowns. In contrast, we have 6 equations of equilibrium for each node, 6 equations of equilibrium for each beam, and 6 equations of equilibrium for the entire TPE. Thus, the total number of equations available is $42 (3 \times 6 + 3 \times 6 + 6)$. However, there are 12 dependent equations, due to the fact that the rigid behavior of the TPE can actually be represented by two rigid beams. Thus, the total number of equations available reduces to 30, but the total number of unknowns we have is 36. This can be resolved by arbitrarily setting 6 constants solved from the 36 equations to zero [3].

**JOINT EQUILIBRIUM CONDITIONS IN THE DEFORMED POSITION**

Both the geometric stiffness matrix $[k_{ij}]^{beam}$ for the rigid beam, as given in Eq. (20), and the $[k_{ij}]^{TPE}$ matrix for the TPE are asymmetric, which relate to the behavior of nodal moments undergoing 3D rotations. Such a property of asymmetry is restricted to the element level, but not the structure level. It has been demonstrated that the anti-symmetric parts of the geometric
stiffness matrices of all beam elements meeting at the same joint will cancel out, once the conditions of equilibrium are enforced for the joint at $C_2$. As a result, the stiffness matrix assembled for the structure is symmetric. The same is also true for the TPE element.

**GENERALIZED DISPLACEMENT CONTROL METHOD**

The Generalized Displacement Control Method [7, 10] is adopted as the path-tracing scheme for the following reasons: (1) Numerical stability in passing the limit points can always be assured, in that the applied loads are not kept constant in iterations. (2) The variation of the structural stiffness is reflected in determining the load increments via the generalized stiffness parameter (GSP). (3) The direction of loading is automatically reversed once a change in the sign of GSP is detected. In fact, this method has been demonstrated to be a reliable method for solving the postbuckling responses of structures with multi winding loops [2, 3].

**NUMERICAL STUDIES**

To verify the points of view concerning the predictor and corrector in the analyses, both the elastic stiffness matrix $[k_e]$ and the rigid-body qualified geometric stiffness matrix $[k_g]$ will be adopted in the predictor for assembly of the structural stiffness matrix $[K]$. In contrast, only the elastic stiffness matrix $[k_e]$ will be included in corrector for computation of the element force increments $\{f\}$, that is, $\{f\} = [k_e] \{u\}$. The total element forces at $C_2$ is simply computed as $\{f\}_{total} = \{f\}_{rigid} + \{f\}_{elastic}$ according to the rigid body rule, where $\{f\}_{rigid}$ denotes the initial nodal forces existing on the element. Such a combination of the predictor and corrector will be referred to as the scheme P1C1 in the numerical studies to follow.

**Example 1. Shallow dome** Figure 9 shows a hinged shallow arch which is subjected to a vertical load $P$. The following data are assumed: $L = 100$ in, $h = 5$ in, moments inertia $I = 1$ in$^4$, Young’s modulus $E = 2000$N/mm$^2$ and section area $A = 1$ in$^2$.

In the finite element modeling, the arch was first represented by 25 straight beam elements. The portion of the arch covered by the central element was then divided into two elements, in order to provide a nodal point at the apex of the arch for application of the vertical load. Here are two cases were considered: the first is the load $P$ applied at the central node which is known to be the symmetrical. In the second case, the central load was dislocated and applied at the node nearest to the center of the arch to account for the effect of imperfection in loading. The results obtained for both cases were shown in Figure 10, which compares quite well with those existing in the literature [11].

**Example 2. Hinged cylindrical shell** A cylindrical shell subjected to a central load $P$ on the top surface is shown in Figure 11 [12]. The longitudinal boundaries of the shell are hinged and immovable, whereas the curved edges are completely free. The following data are adopted: $E = 3.10275$ kN/mm$^2$, $R = 2540$ mm, $L = 254$ mm, $h = 12.7$ mm, $\theta = 0.1$ rad, $\nu = 0.3$. Because of symmetry, only one quarter of the shell is considered and modeled by an $8 \times 8$ mesh. For the shell with thickness $h = 12.7$ and 6.35 mm, the results obtained for the central deflection of the shell have been plotted in Figs. 12 and 13, respectively. All the results shown here, indicated as the P1C1 curves, agree well with those existing in the literature.
**Figure 9:** Shallow arch

**Figure 10:** Load-deflection curves for shallow arch: (a) perfect, (b) imperfect

**Figure 11:** Hinged cylindrical shell

**Figure 12 Shell with** $h = 12.7$ mm

**Figure 13 Shell with** $h = 6.35$ mm
CONCLUSIONS

An incremental-iterative nonlinear analysis generally involves three phases, i.e., the predictor, corrector, and error-checking phases. The predictor relates to solution of the structural equations \( [K] \{U\} = \{P\} \) for the displacement increments \( \{U\} \) given the load increments \( \{P\} \). The corrector is concerned with recovery of the force increments \( \{f\} \) from the displacement increments \( \{u\} \) for each element of the structure, which is made available through the structural displacements \( \{U\} \). In this paper, as well as in our previous papers [2,3], advantage was taken of the different features of the predictor and corrector for nonlinear analysis. It has been demonstrated that a linear stiffness matrix \( [k_e] \) plus a rigid-body qualified geometric stiffness matrix \( [k_g] \) is all that we need for constructing the structural stiffness matrix \( [K] \) for use in the predictor. Further, the element force increments \( \{f\} \) at the end of each incremental step can be computed using only the linear stiffness matrix \( [k_e] \) for given element displacement increments \( \{u\} \), while the initial nodal forces \( \{f\}^{0} \) should be updated according to the rigid body rule. The advantage of the proposed approach is not only on the efficiency of computation, as demonstrated in Refs. [2] and [3], but on the simplicity of formulation.

ACKNOWLEDGEMENTS

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REFERENCES

EVALUATION OF STRUCTURAL BEHAVIOR OF STEEL MEMBER AFFECTED BY THE PRESENCE OF GUSSET-PLATE

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KEYWORDS

Steel structure, Passive-control structure, Gusset-plate, Beam-column connection, Effective length of beam, Cyclic loading test.

ABSTRACT

Buckling restrained braces (BRBs) show that the casing restricts global buckling which serves to balance the tensile and compressive forces in the brace and improve the energy dissipation and plastic deformation capacities of the brace. Nevertheless, the majority of these studies have tested isolated braces or simple subassemblies which neglect the influence of the framing components and the gusset-plate on the system performance. In this paper, frame subassemblies with the gusset-plate were subjected to cyclic lateral loading to investigate structural behaviors of beam-column frame subassemblies affected by the presence of the gusset-plate. Two pairs of specimens were tested. Each pair consisted of the conventional moment-resisting beam and the beam with the gusset-plate attachment to the beam and column flanges. In the first pair, a rectangular hollow section (RHS) was chosen for the column. On the other hand, a wide flange H-shaped section was chosen for the column in the second pair. It was found that effective length of the beam shortened by the presence of the gusset-plate connections. It was indicating that critical section of the beam was moved to the toe of the gusset-plate. On the other hand, effective length of the column was hardly affected by the gusset-plate when RHS was used for the column.

INTRODUCTION

In the Northridge and Kobe earthquakes, some buildings lost their structural functions, although many buildings avoided collapse as to save human life. The loss caused the termination of social and industrial activities, and severe economic loss. At the design stage of seismic design in urban areas, it is important to consider restoring structures immediately...
after an earthquake. Most of high-rise buildings are designed according to Damage-Controlled Design (Wada et al. [1]) seen in Figure 1.

This system consists of a primary frame and dampers. The primary frame only supports gravity and is able to remain in the elastic range during an earthquake, because dampers absorb the input energy of the earthquake. Therefore, the buildings designed as Damage-Controlled Design can be used continuously by repairing or exchanging dampers after an earthquake. However, the majority of studies on dampers have tested isolated dampers or simple subassemblies which neglect the influence of framing components and gusset-plate on the system performance. Recently, some design-level and beyond design-level cyclic loading tests of frame subassemblies with buckling-restrained braces were carried out. These tests showed good behavior of the braces, and the results indicated a number of important considerations for the design of buckling-restrained braced frames and also of braced frames in general (Mahin et al. [2]). In this paper, four frame subassemblies with the gusset-plate connections were subjected to cyclic loading. The objectives of the tests were to verify structural behavior of the beam affected by the presence of the gusset-plate, removing influences of a brace forces.

![Figure 1: (a) Concept of Damage-Controlled Structures; (b) dampers; (c) primary frame](image)

**TEST PLAN**

A constant comparison was used for all specimens to investigate structural behaviors of beam-column frame subassemblies affected by the presence of the gusset-plate. Capacity limitations of the testing equipment, as well as constraints on the overall size of the test specimen, dictated that the specimens were approximately half of the actual building bay width and story height. The specimen was cut out from the frame with a bay width of 3.0 m and a story height of 2.2 m. The tests were cantilever beam, cyclic-load tests with a stiff, strong column as seen in Figure 2. A lateral support was applied to the beam during cyclic loading.

Two pairs of specimen were tested. Each pair consisted of the conventional moment-resisting beam (B) and the beam with the gusset-plate attachment to the beam and column flanges (G). In the first pair, a rectangular hollow section (RHS) was chosen for the column. On the other hand, a wide flange H-shaped section was chosen for the column in the second pair. Overall details are complied in Table 1.

Beam was made of section (depth × flange width × web thickness) of 300×150×6.5×9. Steel grades JIS S S400 were chosen for flange and web of the beam. As shown in TABLE 2, mechanical properties were obtained from tensile coupon tests according to JIS-1A. Cold-formed RHS columns with 250 mm depth, 12 mm thickness, and BCR295
steel grade were used for specimen BOX_G and BOX_B. Columns for specimen H_G and H_B were made of section of 250×250×9×14 with JIS SS400 steel grade.

**TABLE 1**

<table>
<thead>
<tr>
<th>Specimen Shape of Columns</th>
<th>Gusset-plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box_G RHS-Roll</td>
<td>Attached</td>
</tr>
<tr>
<td>Box_B RHS-Roll</td>
<td>Nothing (bare)</td>
</tr>
<tr>
<td>H_G Wide Flange</td>
<td>Attached</td>
</tr>
<tr>
<td>H_B Wide Flange</td>
<td>Nothing (bare)</td>
</tr>
</tbody>
</table>

**TABLE 2**

MECHANICAL PROPERTIES OF STEEL PLATE USED FOR SPECIMENS

<table>
<thead>
<tr>
<th>Sample plate</th>
<th>Grade</th>
<th>$\sigma _y$ [MPa]</th>
<th>$\sigma _u$ [MPa]</th>
<th>$\varepsilon _u$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam-flange SS400</td>
<td></td>
<td>342</td>
<td>461</td>
<td>29</td>
</tr>
<tr>
<td>Beam-web SS400</td>
<td></td>
<td>406</td>
<td>490</td>
<td>22</td>
</tr>
<tr>
<td>Gusset-plate SM</td>
<td></td>
<td>490</td>
<td>536</td>
<td>27</td>
</tr>
</tbody>
</table>

Figure 2: Test set-up and details of beam-column-gusset plate connection:
(a) test set-up; (b) specimen Box_G; (c) specimen H_G (unit : mm)

Figure 3: Loading program

A detail of the gusset-plate in stalled at the beam-column connections is shown in Figure 2. The gusset-plate was attached to the beam and column flange by using shop-welding of fillet welds. Gusset-plates are fabricated in many different configurations. The most common
configuration in Japan, rectangular non-compact type, was used for the specimen BOX_G and H_G. Steel grades JIS SM 490, stronger than the beam and column, were chosen for the gusset-plate.

Quasi-static loading was carried out following a simple loading program shown in Figure 3. The loading programs were based on rotation angles of the specimen, which were 1/200, 1/100, 1/50, 1/33, 1/25, and 1/20 radian. The cantilever beams had a length L from the face of the column to the center of the load. The total tip deflection was due to elastic and plastic flexural deformation of the beam and connection. A rotation angle of the specimen can be found out by dividing the total tip deflection by L. Note that length L from the face of the column to the center of the load was used regardless of the presence of the gusset-plate.

TEST RESULTS AND CONSIDERATION

Specimen Performance

A shear force versus a rotation angle of the beam is plotted for all specimens in Figure 4. As observed in Figure 4, all specimens exhibited stable hysteretic behavior during 1/50 radian rotation angle cycles. Locally buckled beam flange led to degrading hysteretic characteristics over 1/50 radian rotation angle cycles. Locally buckled beam flanges grew up shown in Figure 5. As compared in place of the locally buckled beam flanges, those of test specimen BOX_G and H_G were observed at the toe of the gusset-plate.

“Skelton curves”, which are cut out from overall behaviors of the beam, are plotted for all specimens in Figure 6. The gusset-plate attached led to a roughly 35% increase in the yield strength. It was indicating that the critical section of the beam was moved to the toe of the gusset-plate. And a roughly 25% increase in initial elastic stiffness was caused by the gusset-plate in specimen H_G. On the other hand, initial elastic stiffness in specimen BOX_G was increased by only 5%, indicating that RHS column is hardly affected by the presence of the gusset-plate when a rectangular hollow section (RHS) was used for the column.

![Figure 4: Overall behaviors of the beam with gusset-plate connections: (a) Box_B; (b) Box_G; (c) H_B; (d) H_G](image-url)
Figure 5: Photographs of locally buckled beam flange of test specimen
(a) Box_B; (b) Box_G; (c) H_B; (d) H_G

Distribution of the Principal Stress at the Gusset-Plate

Distribution of the principal stress at the gusset-plate at the cycle of ±1/200 radian amplitude is shown in Figure 6. It was found that shear forces were transferred between the beam, the column, and the gusset-plate. In particular, the principal stress, shear-transfer, was concentrated on the toe of the gusset-plate. As compared in distribution of the principal stress at the toe of the gusset-plate, the shear-transfer between the toe of the gusset-plate and RHS column flange was less than that of test specimen H_G.

Figure 6: Distribution of the principal stress at the gusset-plate:
(a) Box_G; (b) H_G
Figure 7: Distribution of bending moment at the beam and column: (a) specimen Box_G; (c) specimen H_G (unit:kN-m)

**Distribution of Bending Moments at the Beam and Columns**

Distribution of bending moments at the beam and columns at the cycle of ±1/200 radian amplitude is shown in Figure 7. Bending moments were decreased between the toe of the gusset-plate and beam-end by the presence of the gusset-plate. It was indicating that shear forces were transferred between the beam, the column, and the gusset-plate. On the other hand, bending moments were hardly affected by the presence of the gusset-plate when a rectangular hollow section (RHS) was used for the column.

**CONCLUSIONS**

This paper presents an experimental study of beam-column subassemblies to verify structural behavior of the beam affected by the presence of the gusset-plate, removing influences of brace forces. It was found that effective length of the beam shortened by the presence of the gusset-plate connections. It was indicating that the critical section of the beam was moved to the toe of the gusset-plate. On the other hand, effective length of the column was hardly affected by the gusset-plate when a rectangular hollow section (RHS) was used for the column.

**REFERENCES**


GLOBAL AND LOCAL ELASTIC BUCKLING OF THIN-WALLED BEAMS WITH OPEN ELLIPTIC CROSS SECTIONS

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KEYWORDS
Elliptical cylindrical shells, Thin-walled beams, Elastic buckling.

ABSTRACT
Subject of the paper are cold-formed thin-walled beams with open elliptic cross sections. The problems of lateral buckling and local buckling of these beams in pure bending states are considered. The system of differential equations of the equilibrium is derived on the basis of the theorem of minimum total potential energy. The system of equations is reduced to a generalized eigenvalue problem with the use of the Galerkin method. The critical states for these beams are determined. Moreover, the Finite Strip Method (FSM) and the Finite Element Method (FEM) investigations are carried out, and the calculation results are compared with the analytical solution.

INTRODUCTION
Typical shapes of open cross sections of thin-walled beams are C-section and I-section. Strength and buckling problems of these beams are described in many monographs and papers of the 20th century, for example in chronological order by Vlasov [1], Bleich [2], Timoshenko and Gere [3], Murray [4], Bažant and Cedolin [5], Weiss and Giżejowski [6], and Trahair [7]. Rasmussen [8] presented a general bifurcation analysis of thin-walled beams. Shapes of flanges or webs of cold-formed thin-walled channel beams are rather complicated. Taking into account the papers of Davies [9] and Magnucki and Paczos [10] these shapes can be graphically illustrated, as for example in Figure 1.

Figure 1: Channel beams with various shapes of flanges or webs
Hancock [11-14] formulated the direction of further development of fundamental research in the field of strength and buckling of cold-formed thin-walled beams with the use of numerical methods and experiments. Camotim and Silvestre [15] collected and described the results of this research. These problems are intensively studied, for example by: Chu et al. [16], Silvestre and Camotim [17-18], Magnucki and Ostwald [19-20], Cheng and Schafer [21], Bambach [22], Schafer [23], Dinis and Camotim [24], Pastor and Roure [25-26] and Magnucka-Blandzi [27]. Strength and stability problems of cylindrical shells with non-circular cross section or thin-walled beams with closed elliptic cross sections presented Soldatos [28], Chan and Gardner [29], Ruiz-Teran and Gardner [30]. Joniak et al. [31] presented results of numerical and experimental investigations of elastic buckling of thin-walled beams with open circular cross sections in pure bending state.

The subject of the paper are thin-walled beams with open elliptic cross sections in pure bending state. These beams of length $L$, depth $H$, and wall thickness $t$ are simply supported and carry two equal moments $M_0$ (Figure 2).

**GLOBAL BUCKLING OF THE BEAM**

*Analytical Solution of Lateral-Torsional Buckling of the Beam*

Scheme of the elliptic cross section with principal axes $yz$ with the origin $O(0,0)$ is shown in Figure 3.

Geometric properties of the cross section are defined by the following dimensionless parameters

$$x_1 = \frac{b}{a}, \quad x_2 = \beta, \quad x_3 = \frac{t}{b},$$

(1)

where:

- $a$ and $b$ - dimension of the cross section,
- $\beta$ - angle ($0 \leq \beta < \pi/2$),
- $t$ - thickness of the wall.

Depth of the beam

$$H = 2a + t = a(2 + x_1 x_3).$$

(2)

The ellipse in the polar coordinate
\[ r(\phi) = \frac{b}{\sqrt{1 - (1 - x_i^2)\sin^2 \phi}}, \quad \text{for} \quad 0 \leq \phi \leq \frac{\pi}{2} + \beta \]  

(3)

The total area \( A \) and the geometric stiffness \( J_{sv} \) for Saint-Venant torsion of the cross section

\[ A = 2btf_0(x_i), \quad J_{sv} = \frac{2}{3} b^3 f_0(x_i) \]

(4)

where

\[ f_0(x_i) = \int_0^{\frac{\pi}{2} + \beta} \frac{f_e(x_i)}{\left[1 - (1 - x_i^2)\sin^2 \phi\right]^{\frac{3}{2}}} \, d\phi, \]

\[ f_e(x_i) = \sqrt{\left[1 - (1 - x_i^2)\sin^2 \phi\right]^3 + \left(\frac{1-x_i^2}{2}\right)^2 \sin^2 (2\phi)}. \]

The location of the centroid of the elliptic cross section – the principal axis \( y \) (Figure 3a).

\[ e_0 = \frac{f_1(x_i)}{f_0(x_i)} b, \quad \text{where} \quad f_1(x_i) = \int_0^{\frac{\pi}{2} + \beta} \frac{f_e(x_i)\cos \phi}{\left[1 - (1 - x_i^2)\sin^2 \phi\right]^{\frac{3}{2}}} \, d\phi. \]

(5)

Moments of inertia of the area of the elliptic cross section with respect to the principal axes \( y z \)

\[ J_y = 2b^3 f_2'(x_i), \quad J_z = 2b^3 f_3'(x_i) \]

(6)

where

\[ f_2(x_i) = \int_0^{\frac{\pi}{2} + \beta} \frac{f_e(x_i)\cos^2 \phi}{\left[1 - (1 - x_i^2)\sin^2 \phi\right]^{\frac{3}{2}}} \, d\phi - \frac{f_1^2(x_i)}{f_0(x_i)}, \]

\[ f_3(x_i) = \int_0^{\frac{\pi}{2} + \beta} \frac{f_e(x_i)\sin^2 \phi}{\left[1 - (1 - x_i^2)\sin^2 \phi\right]^{\frac{3}{2}}} \, d\phi. \]

The location of the shear centre – the point \( C \) (Figure 3)

\[ z_c - z_B = \frac{1}{J_z} \int_A \omega_B y \, dA = a \frac{f_4(x_i)}{f_3(x_i)}, \]

(7)

where:

\[ \omega_B(\phi) = ab \tilde{\omega}_B(\phi) \] - the warping function with respect to the auxiliary point \( B \) (Figure 3b),

\[ \tilde{\omega}_B(\phi) = \phi - \arctan\left[ \frac{(1-x_i^2)\sin(2\phi)}{(1+x_i^2) + (1-x_i^2)\cos(2\phi)} \right], \quad f_4(x_i) = \int_0^{\frac{\pi}{2} + \beta} \frac{f_e(x_i)\tilde{\omega}_B(\phi)\sin \phi}{\left[1 - (1 - x_i^2)\sin^2 \phi\right]^{\frac{3}{2}}} \, d\phi. \]

The warping moment of inertia of the elliptic cross section

\[ J_\omega = \int_A \omega^2 \, dA = 2a^5 tx_i^3 f_5(x_i), \]

(8)

where:
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\[ \omega(\varphi) = ab \tilde{\omega}(\varphi) \]  
the warping function with respect to the the shear centre – the point C (Figure 3b),

\[ \tilde{\omega}(\varphi) = \tilde{\omega}_b(\varphi) - \frac{\sin \varphi}{\sqrt{1 - (1 - x_i^2)\sin^2 \varphi}} f_4(x_i), \quad f_5(x_i) = \int_0^{\pi} f_4(x_i) \tilde{\omega}(\varphi) d\varphi. \]

The critical moment and the global critical stress of lateral-torsional buckling of beams in pure bending state [5, 6, 7]

\[ M_{cr}^{(glob)} = \frac{\pi E}{L} \left[ \frac{J_S V_{cr}}{2(1+\nu)} \right] \left[ 1 + 2(1+\nu) \frac{\pi^2 J_{as}}{L^2 J_{SV}} \right], \quad \sigma_{cr}^{(glob)} = \frac{M_{cr}^{(glob)}}{J_z} a. \]  

**Numerical Calculations of Critical Stresses of Lateral-Torsional Buckling of Beams**

The family of thin-walled beams with constant area \( A \) of the cross section, and constant depth \( H \) of the beam is numerically studied with the use of the Analytical Solution (AS), Finite Strip Method (FSM) and Finite Element Method (FEM). The following data are assumed for the beams: Young’s modulus \( E = 2.05 \cdot 10^5 \text{ MPa} \), Poisson ratio \( \nu = 0.3 \), the area of the cross section \( A = 623.6 \text{ mm}^2 \), the depth \( H = 200 \text{ mm} \), and two constraints \( 2\pi/3 \leq \gamma_e = 3\pi/4 \), and \( 0.4 \leq x_1 \leq 0.9 \). The first constraint refers to the relative arc length \( \gamma_e \) of the middle lines of open elliptic cross sections \( \gamma_e = \gamma_e = A/(2\pi) = x_1 f_5(x_1) \) (Eqn. 4). The results of the numerical studies showed occurrence of an effective proportion of semi-axes of the elliptic cross section \( x_{1,ef} = 0.6 \) of the beam with respect to the maximal global critical stress (Eqn. 9). The values of this critical stress as functions of relative length \( \lambda \) of the effective beam are shown in Figure 4a, and as function of relative arc length \( \gamma_e \) of middle lines of the open elliptic cross section are shown in Figure 4b. The critical stresses of lateral-torsional buckling \( \sigma_{cr}^{(glob)} \) decrease with the growth of the relative length \( \lambda \) of the beam, and increase with the growth of the relative arc length \( \gamma_e \) of the middle line of the open elliptic cross section.

**LOCAL BUCKLING OF THE BEAM**

**Analytical Solution of Local Buckling of the Beam**
The middle surface of the thin-walled beam (Figure 2) as an elliptic cylindrical shell is described in the orthogonal coordinates \( x \) and \( s \) (Figure 5).

Figure 5: Scheme of the elliptic cylindrical shell

The general theory of shells with assumption of the Kirchhoff-Love hypotheses is described for example by Ventsel and Krauthammer [32]. The elastic strain energy of the shell, and the work of the load are determined as follows:

\[
U_e = \frac{E}{2(1-\nu^2)} \iint_{\omega} \left( \varepsilon_{xx}^2 + 2\nu \varepsilon_{xx} \varepsilon_s + \varepsilon_s^2 + \frac{1-\nu}{2} \gamma_{ss}^2 \right) dx ds dz, \quad W = \frac{1}{2} \iint_{\omega} \left( \frac{\partial w}{\partial x} \right)^2 dx ds, \tag{10}
\]

where \( \varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_s = \frac{\partial u}{\partial s} - \frac{w}{R}, \quad \gamma_{ss} = \frac{\partial u}{\partial s} + \frac{\partial v}{\partial x} \) - linear strain-displacement relations, \( w(x,s) \) - the deflection,

\[
u(x,s,z) = u_0(x,s) - z \frac{\partial w}{\partial x}, \quad v(x,s,z) = v_0(x,s) - z \frac{\partial w}{\partial x}, \quad \text{and} \quad u_0(x,s), v_0(x,s) \] - displacements in the directions of the \( x \) and \( s \) coordinate lines for the middle surface of the shell, moreover, \( 0 \leq x \leq L, -s_e \leq s \leq s_e, -\frac{t}{2} \leq z \leq \frac{t}{2}, \) and \( R \) - the principal radius of curvature, \( ds = Rd\theta \) - the linear element.

Taking into account the principle of minimum of the total potential energy \( \delta(U_e - W) = 0 \), integrating over the thickness \((-t/2 \leq z \leq t/2)\), and integrating by parts over the middle surface of the shell, and defining the force function, one obtains a system of two partial differential equations in the following form:

\[
DV^4 w + \frac{1}{R} \frac{\partial^2 \Phi}{\partial x^2} + N_s^0 \frac{\partial^2 w}{\partial x^2} = 0, \quad V^4 \Phi - \frac{Et}{R} \frac{\partial^2 w}{\partial x^2} = 0, \tag{11}
\]

where \( D = \frac{Et^2}{12(1-\nu^2)} \) - the flexural rigidity, \( V^4 = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial s^2} + \frac{\partial^4}{\partial s^4} \) - the biharmonic operator, \( \Phi(x,s) \) - the force function, and \( N_s = -\partial^2 \Phi/\partial s^2, \ N_x = -\partial^2 \Phi/\partial x^2, \ N_{ss} = \partial^2 \Phi/\partial x \partial s \).

Two longitudinal edges \( (s = \pm s_e) \) of the shell are free. Thus, values of the following quantities are zero: the bending and twisting moments \( M_s \big|_{s=s_e} = M_x \big|_{s=s_e} = 0 \), the transverse effective shear force \( V_s \big|_{s=s_e} = 0 \), and the in-plane normal and shear forces \( N_s \big|_{s=s_e} = N_x \big|_{s=s_e} = 0 \). The detailed description of these boundary conditions for the circular cylindrical shell presented Magnucki and Mackiewicz [33].

The system of stability Eqs. 11 is approximately solved, assuming two unknown functions.
\[ w(x,s) = tw_a \left( \sin \frac{3\pi s}{2s_e} + \alpha_3 \sin \frac{\pi s}{2s_e} \right) \sin \frac{m\pi s}{L}, \quad \Phi(x,s) = -Et^2af\left( \sin \frac{\pi s}{s_e} + \frac{1}{2} \sin \frac{2\pi s}{s_e} \right) \sin \frac{m\pi s}{L}, \]  

where: \( w_a, f, \alpha_3 = \left( 9 + 4\sqrt{k^2} \right) \left( 1 + 4\sqrt{k^2} \right), \) \( k = ms_e/L \) - dimensionless parameters of functions.  

The intensity of normal force-load is assumed as follows \( N_s^0 = N_0 \sin(\pi s/s_0), \) where \( s_0 = s(\pi). \)  

These functions for \( s = \pm s_e \) accurately satisfy the boundary conditions of the statement problem, however, for \( s = 0 \) gives the bending moment \( M_s = 0. \) From the Galerkin orthogonal conditions of the Eqns.11 with respect to the functions (Eqns.12) the lower estimation of the critical stress of local buckling of the elliptic cylindrical shell in pure bending is obtained in the following form

\[
\sigma_{CR}^{(Local)} = \min_k \left\{ \sigma_0 \right\}
\]

where:

\[
\sigma_0 = N_0 = \frac{1}{320k^2C_N} \left[ \frac{\pi^2}{12(1-\nu^2)} \left( \frac{t}{s_e} \right)^2 \left( 9 + 4k^2 \right)^2 + \alpha_3^2 \left( 1 + 4k^2 \right)^2 \right] + \left( \frac{16}{\pi} \right)^2 \left( \frac{s_e}{a} \right)^2 \frac{k^4C_w^2}{20 + 16k^2 + 5k^4}
\]

and, for example \( C_w = 0.5264 + 0.5532\alpha_3, \) \( C_N = 0.3603 + 0.0211\alpha_3 + 0.4198\alpha_3^2, \) for \( \tilde{s}_e = 2\pi/3, \)

and \( C_w = 0.6231 + 0.5804\alpha_3, \) \( C_N = 0.3404 + 0.1262\alpha_3 + 0.3687\alpha_3^2, \) for \( \tilde{s}_e = 3\pi/4. \)

**Numerical Calculations of Critical Stresses of Local Buckling of Beams**

The family of thin-walled beams with constant area of the cross section \( A = 623.6 m^2, \) and constant depth of the beam \( H = 200 mm \) is numerically studied with the use of the Analytical Solution (AS), Finite Strip Method (FSM) and Finite Element Method (FEM). The calculation results are values of critical stresses and shapes of local buckling. Example values of these critical stresses of thin-walled beams with relative arc length \( \tilde{s}_e = 2\pi/3 \) of the middle line are the following: \( \sigma_{CR}^{(Loc-AS)} = 432.1 MPa, \) \( \sigma_{CR}^{(Loc-FSM)} = 492.5 MPa, \) \( \sigma_{CR}^{(Loc-FEM)} = 500.2 MPa. \) Example shapes of local buckling of these beams are shown in Figure 6.

The difference of critical stresses of FEM and FSM is smaller \( \Delta\sigma_{\sigma_{CR}}^{(FEM-FSM)} = 1.5\%, \) however, the difference of critical stresses of FEM and AS is greater \( \Delta\sigma_{\sigma_{CR}}^{(FEM-AS)} = 13.6\%. \) Values of critical stresses \( \sigma_{CR}^{(Loc)} [MPa] \) of local buckling of the family of thin-walled beams with elliptic cross sections, as function of relative arc length \( \tilde{s}_e \) of the middle line, are shown in Figure 7.

**CONCLUSIONS**

Theoretical studies of global and local buckling of thin-walled beams with constant area \( A = const \) of open elliptic cross sections under pure bending showed:

- Occurrence of the effective proportion of semi-axes of the elliptic cross section \( x_{1,ef} = 0.6 \) with respect to the maximal critical stresses of lateral-torsional buckling.
- Increase of the critical stress \( \sigma_{CR}^{(Glob)} \) of lateral-torsional buckling with the growth of the relative arc length \( \tilde{s}_e \) of the middle line of the open elliptic cross section.
The difference of critical stresses of local buckling of FSM or FEM and the analytical lower estimation (AS). The difference is below fourteen per cent.

a) The FSM result (CUFSM – B. Schafer)  
b) The FEM result (ABAQUS)

Figure 6: Example shapes of local buckling of thin-walled beams (\( \bar{\chi}_e = 2\pi/3 \))

Figure 7: The influence of relative arc length \( \bar{\chi}_e \) of middle line of the open elliptic cross section on the critical stress \( \sigma_{CR}^{(Loc)} [\text{MPa}] \) of local buckling

REFERENCES

LATERAL BRACING FORCE OF IPE-240 BEAMS AT ULTIMATE LOAD

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Lateral torsional buckling, Lateral restraint, Restraint stiffness, Restraint location, Steel beam.

ABSTRACT
Almost traditionally the design load in an intermediate lateral restraint against lateral torsional buckling of beams has been taken as a percentage of the maximum value of the factored force in the compression flange. National design codes have suggested values of around 2%. This has mainly been backed up by theoretical elastic analyses. Inelastic analysis on braced columns have yielded higher bracing forces. To investigate the influence of inelastic behaviour of beams subject to lateral torsional buckling on the forces in a single lateral brace, geometrical and material non-linear analyses including imperfections (GMNIA) were performed on IPE240 beams with different lengths, support conditions and types of loading. The location of the non-rigid lateral brace is varied along the length of the beam and along the height of the section. The stiffness of the lateral restraints was determined in such a way that the elastic critical moment is 95% of the elastic critical moment with rigid lateral restraints. This will keep the reduction in buckling load to a maximum of 5%. It was found that the forces in the restraints can be larger than design forces suggested by building codes.

INTRODUCTION
The design of a single intermediate lateral support for a beam against lateral torsional buckling follows a simple suggestion for strength as published in 1947: “our general office practice is to design bracing members for a force of 2% of the compression flange force” (Throop [1]). In a 1966 publication (Lay & Galambos [2]) it states that in 1925 a more conservative design value of 2.5% was given in “The General Specifications for Steel Railway Bridges (U.S.)”. A stiffness requirement for a mid span restraint attached above the shear center of the section such that the beam would buckle in the second mode of instability was first suggested in 1951 (Flint [3]) to be of the order of 10 to 15 times the lateral bending stiffness of the beam. It was also shown experimentally and theoretically that the location of the brace up the height of the section has an influence on the buckling load of the beam.
An expression for the force in a mid-span restraint of a beam subject to equal end moments and an elastic point support against lateral movement at the shear center was first obtained from an elastic analysis including a maximum crookedness of beam length $L/1000$ as allowed by AISC specifications (Zuk [4]). Further study of the behaviour of lateral torsional buckling of beams with elastic restraints has mostly restricted to elastic analysis of members with equal end moments or central point loads (Winter [5], Massey [6], Lay & Galm bos [7], Schmidt [8]). Such investigations resulted in relationships between beam and brace stiffness in graphical form (Mutton & Trahair [9]) and closed form solutions (Tong & Chen [10]).

In a 1964 publication on lateral supports of inelastic columns (Pincus [11]) it was shown that the required stiffness of a lateral brace is larger for an inelastic than an elastic column. The first published study on bracing requirements for inelastic beams (Lay & Galm bos [6]) suggests procedures for obtaining the section area and section modulus for the brace but concludes that the use of 2% of the compression flange area still holds true. Recently it was suggested (Lui & Khanse [12]) that for the inelastic design of braced columns, the maximum brace force be taken as 4% of the column strength.

By the nineteen seventies it became possible to numerically establish relationships between increased stability of beams and restraint stiffness (Nethercot & Rockey [13], Net hercot [14]). In these parametric studies initial imperfections were not taken into account. In a further development (Johnson & Cafolla [15]) an out-of-straightness of $L/667$ was used in a second order elastic finite element analysis on bridge girders where discrete lateral restraints were replaced by an equivalent smeared lateral stiffness over the length of the beam in order to establish conditions for “stiff” restraints.

In this paper results are presented of a numerical parameter study on lateral torsional buckling of IPE240 beams with three different lengths. All members were elastically restrained by a single lateral brace. The location of the restraint was varied over the length of the beam and over the height of the section. The stiffness of the lateral support is determined in such a way that the elastic critical moment is 95% of the elastic critical moment of the same beam with a rigid lateral restraint. It is taken that these “near rigid restraints” will have little influence on the lateral buckling load of the structure but allow a more realistic study of the force in the brace at beam failure. Three different load cases were considered including mid-span point loads and a uniformly distributed load. The geometric and material non-linear analyses of the structure include imperfections for bracing systems as defined by Eurocode 3 [16].

**FINITE ELEMENT SIMULATIONS**

**FE-Model and Loading**

Simulations with the general finite element program ANSYS 10.0 using GMNIA (Geometric and Material Non-linear Inelastic Analysis) for restrained beams yield ultimate lateral torsional buckling loads. The FE model of the beams as shown in Figure 1 consists mainly of 4-node shell elements based on Mindlin-Reissner shell theory. Using shell elements only would cause the root radii between flange and web to be ignored. To compensate for this, elastic-plastic rectangular hollow section beam elements are used at the flange-web intersections (Bruins [17]). Two different types of support with torsion restraints yielding fork conditions are modelled: a simple support as shown in Figure 1a and a fixed support, Figure 1b, where warping is not restrained. The fixed support is used in a propped cantilever. The stiff (beam) elements are employed to allow warping but to prevent distortion of the cross-section.

The lateral restraint is modelled by a single link element as shown in Figure 2. Its position varies along the longitudinal beam axis ($x$) and the section height ($y$). In case the lateral restraint is attached to the web, stiffeners have been introduced to prevent local distortion of the cross-section.
The concentrated load is applied at center span to a single node requiring the introduction of stiffeners.

![Stiff elements](image1)

(a) Simple support  (b) Fixed support

Figure 1. Support conditions

**Material and Imperfections**

The steel grade employed in all analyses was S235 with yield strength 235 N/mm². A bilinear stress-strain curve without hardening was used with a modulus of elasticity of 210 kN/mm².

The imperfection pattern used is according to Eurocode 3 Clause 5.3.3(1) [16]. For the analysis of bracing systems which are required to provide lateral stability within the length of beams the imperfections should be included by means of an equivalent imperfection of the beam to be restrained, in the form of an initial bow imperfection of $L/500$.

**PARAMETER STUDY**

The parameter study is performed on restrained IPE240 beams for three load cases as shown in Figure 3. In the first case, a concentrated load is applied at mid-span on a simply supported beam. The load is applied at the intersection of the web and top flange. Three span lengths are considered: 7.2m, 5.4m and 3.6m. The second case is a uniformly distributed load on a simply supported beam applied at the web-top flange intersection. The third load case is a statically indeterminate system. It resembles the first load case where one of the supports is fixed such that it is free to warp but not to twist. For the last two load cases only one span length is considered: 7.2m. The location of the lateral braces is varied: six locations in longitudinal direction from support to mid span at regular spacing and 3 locations up the height of the section from shear center to the top flange at regular intervals.

![Lateral restraint](image2)

Figure 2. Lateral restraint
Reduced Elastic Critical Moment

Replacing rigid lateral restraints by elastic beam bracing will result in reduced elastic critical moments. As it is not feasible to make braces with infinite stiffness it is suggested to use a design criterium based on a specific reduction in the elastic critical moment of the restrained beam with a rigid lateral restraint, e.g. 95% $M_{cr;100\%}$. This allows axial strains in the brace to take place. It will also limit the reduction in ultimate load to a maximum of 5%. This proposal requires the brace to be given a specific stiffness for selected locations along the length of the beam and up the height of the steel section. It is therefore necessary to first obtain the elastic critical moment of the restrained beam with a rigid brace before an adjusted stiffness of an elastic restraint can be determined. The non-rigid restraint allows a GMNI A of the restrained beam and to obtain the force in the lateral brace in addition to the lateral displacement of the beam at the location of the restraint.

Figure 4 shows the development of the elastic critical moment with increasing brace stiffness for a 7.2 m long beam subject to a central point load with a mid span brace attached to the top flange. Analyses showed that at a brace stiffness of 0.480 kN/mm the beam buckled in the first mode. At a restraint stiffness of 0.500 kN/mm the beam displayed a second buckling shape.

It is clearly shown that employment of 95% of the elastic critical moment will cause first mode buckling of the beam. The reduced brace stiffness will yield slightly smaller ultimate loads for the beam. It should be noted here that a perfect first mode buckling failure over the length of the beam is only obtained for unrestrained beams. The presence of a brace will distort this shape.
Figure 5. Force-displacement curves for a beam with flexible central top flange restraint

Figure 6. Load case 1 for 7.2m beam

**Bracing Force at Ultimate Load of Beam**

In the GMNA of a 7.2 m long laterally braced beam, the force in the elastic restraint is obtained at the ultimate mid span point load as shown in Figure 5 for a beam with a single restraint attached to the top flange at center span. For this example the 7.52 kN force in the lateral restraint is 2.19% of the ultimate compressive force in the top flange.

**Influence of Elastic Critical Moment on Bracing Force**

The axial forces in the brace and its stiffness are given in Figure 6 for a 7.2m long simply supported beam which is subjected to a concentrated load on the top flange at mid span, i.e. load case 1. It shows the axial forces in the brace for the situation with a rigid lateral restraint, giving 100% $M_{cr}$, which are compared to the forces in a non-rigid brace where the elastic critical moment has been reduced to 95% $M_{cr}$. It can be seen that the differences in axial bracing forces for the two situations are small at most locations and that their variations along the beam and up the section are similar. The diagram indicates clearly that rigid braces do not always attract larger forces than non-rigid restraints. It also shows that bracing forces are in most cases larger than 2% of the maximum compressive force in the top flange. For a restraint near the supports this can be as much as 5%.

Figure 7 also displays the required axial restraint stiffnesses in order to obtain 95% of the full elastic critical moment of the beam, i.e. a critical moment due to rigid bracing. It can be observed that restraints closer to the support require a larger axial stiffness. It should be noted here that a midspan brace located at shear center demands a much larger stiffness than when placed at the top flange.
Influence of Location of Non-Rigid Restraint on Bracing Force

Load case 1

Figures 6, 7 and 8 show restraint forces and the required restraint stiffnesses for three lengths of simply supported beams subject to a point load at midspan. All diagrams show identical trends for the restraint forces. At center span, the force in the brace attached to the top flange is at or below 2% of the compressive force in that flange. For braces that are connected below the top flange, the forces are at or below the 3% level. Towards the beam supports, the restraining forces first increase marginally, over the quarter span nearest the supports the brace forces increase rapidly and obtain values of up to 5% of the compressive flange force of the 7.2m and 5.4m long beams, and 9% for the 3.6m beam.

The stiffness required for the restraints in order to obtain 95% of the full elastic critical moment in general increases over the length of the beams towards the supports. It is also observed that the bracing stiffnesses for the short 3.6m long beams are about 10 times larger than for the longer 7.2m beams.

Load case 2

Figure 9 shows the restraint forces and required brace stiffness for a 7.2m long beam subjected to a uniformly distributed load. The forces in the braced and the required brace stiffnesses are quite similar to the results for the 7.2m beam with a point load at center span. In nearly all cases the brace forces are above the 2% level.
Load case 3

The forces in the restraints and the brace stiffnesses for a propped cantilever beam in Figure 10 show completely different development patterns along the length of the beam when compared to the results for the simply supported beams as given in Figures 6 and 9. The forces in the braces are quite small and in many cases below the 2% level. The required axial stiffnesses for the restraints are also very small.

CONCLUSIONS

A numerical investigation has been done on the behaviour of single braces attached to beams against lateral torsional buckling. The restraints have been given finite stiffnesses in order to allow lateral displacement of the beam at brace location. The brace stiffnesses used in this study were selected such that the elastic critical moment was 95% of the elastic critical moment with rigid braces. It was found that the forces in rigid braces are not always larger than in non-rigid braces. The non-rigidity of the restraints did yield slightly decreased ultimate loads for the beams.

From the study of the IPE-240 beams with different length and support conditions subject to point loads and uniformly distributed loads it can be concluded that the bracing forces are for the majority of analysed cases larger than 2% of the maximum compression force in the top flange.
REFERENCES


FINITE ELEMENT INVESTIGATION OF PERFORATED STEEL BEAMS WITH DIFFERENT WEB OPENING CONFIGURATIONS

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KEYWORDS

Perforated sections, shear-moment interaction curves, Vierendeel mechanism, non-linear Finite Element Analysis, coupled shear utilisation ratios, plastic hinges, various standard and non-standard web openings.

ABSTRACT

The objective of this work is to investigate and compare, through an analytical study, the behaviour of perforated steel beams with different shape configurations and sizes of web openings. In this investigation the ‘Vierendeel’ failure mechanisms of steel beams with web openings are examined through a Finite Element study. The shear and flexural failures of standard perforated sections are controlled mainly by the size (i.e. depth) of the web openings, whilst the ‘Vierendeel’ mechanism is primarily controlled by the critical length of the web openings. Three main categories of web opening shape configurations and sizes are considered in this work. Standard, non-standard and elongated web opening configurations are examined, each with three different opening sizes. Four Advanced UB beams are used in the investigation in order to cover a range of sections and demonstrate the main differences in behaviour. The results of this comprehensive FE study are presented and include the position of plastic hinges, the critical opening length of perforated steel sections and the ‘Vierendeel’ parameters. The yield patterns and the failure modes do not differ dramatically. The results of this study are considered as relevant for practical applications as: (i) the reduction of the moment capacities of the tee-sections due to combination of axial and shear forces is smaller compared to the previous conservative linear interaction formula, and (ii) the formation of the initial plastic hinges at the low moment side (LMS) of the top tee-sections of the web openings does not usually cause failure, meaning that the beams can continue to carry additional load until all four plastic hinges are formed in the vicinity of the web openings and a ‘Vierendeel’ mechanism is fully established.
INTRODUCTION

The main objective of the work presented here is to investigate the moment-shear (M/V) interaction behaviour of perforated steel beams having different shapes and sizes of web openings. The Finite Element method is used to investigate the behaviour of full scale steel beams with two large isolated openings in the web, symmetrical about the centre-line. To compare the efficiency of the various shapes of web openings, eleven forms are considered, six of these being standard configurations, including elongated web openings, whilst the other five are elliptical (Figure 1). Web openings C, D, E and F have identical web opening areas, but the last three are rotated by an angle of 45° and hence have a decreased web opening depth. Web opening G is rotated by the same angle but the web opening area is increased and its depth is equal to 0.8h where h is the depth of the beam. It is well known that the width of the web opening influences the load carrying capacity of the perforated sections and that the critical opening length ‘c’ is the main dimensional parameter. The associated values of ‘c’ are listed above the web opening configurations in Figure 1 and in Table 1. Opening depths, d_o, equal to 0.8h, 0.65h and 0.5h are investigated.

Figure 1: Geometric configurations of web openings (left) and rotated elliptical web openings (right)

Non-linear moment-shear interaction curves are used to present the results of this investigation. A FE model using both geometrical and material non-linearity is used to allow for load redistribution across the web opening following the formation of the first plastic hinge. According to Chung et al. [1] four typical middle-range steel beams commonly used in practice can be investigated in order to obtain useful results. From this work by Chung et al. [1], and from a comprehensive FE analyses conducted by Tsavdaridis [2] on beams UB457x152x52, UB457x152x82, UB610x229x101 and UB610x229x140, beam size UB457x152x52 was selected to represent this study as it produced the most conservative results.

PARAMETRIC STUDY

FEA Model

In order to simulate the structural behaviour of the perforated sections and investigate the ‘Vierendeel’ mechanism, a finite element model was established that included both material and geometric non-linearity. The ANSYS four-noded quadrilateral plastic shell element, SHELL181, was used to model the web, flanges and stiffeners. Fine mapped mesh configuration was incorporated in order to avoid discontinuities in stress contours across element boundaries, mainly in the vicinity of the web openings. For the material modelling of steel, a bi-linear stress-strain curve with an elastic modulus, E, of 200kN/mm² and a tangent
modulus, $E_T$, of 1000kN/mm$^2$ was adopted, together with a kinematic hardening rule (BKin) and the Von-Mises yield criteria. The analyses were performed on simply supported beams of 5m span under a uniformly distributed load. Ten different positions of that web opening along the half length of the beams were considered. The FE model was calibrated against test data, published by Tsavdaridis et al. [3], on a perforated steel I-section with two circular web openings.

**Plastic Hinge Positions**

Von-Mises stresses are used to reveal the plastic hinges in the vicinity of the web openings which are formed at both ends of the tee-sections. The positions of these plastic hinges are influenced by the magnitude of the global shear force and bending moment. The shear forces produce additional moments (‘Vierendeel’ action), as first mentioned by Bower [4] and Redwood [5]. By understanding the movement and the critical positions of these plastic hinges, the actual critical opening length can be obtained. Figure 2 shows these hinge positions for different web opening shapes for section UB457x152x52, with web openings of depth equal to 0.8h and at a distance, $x$, of 0.284m from support.

![Figure 2: Von-Mises stresses of beams subjected to high shear forces](image)

This FE study found that in the case of non-standard web openings, the structural performance of the perforated sections is strongly affected by not only by the opening depth and critical length, but also the web opening shape. In perforated sections with elliptical web opening configurations, the yield zones overlapped significantly when the web opening positions were changed. Generally, it can be concluded that in terms of stress distribution, perforated sections with vertical and rotated elliptical web openings have a better performance compared to circular and hexagonal web openings. This was expected, especially for vertical elliptical web openings, as the web opening width is narrower.

**Moment /Shear (M/V) Interaction Curves**

A practical design method proposed by Chung et al. [6], is related to ‘coupled’ shear capacities and allows for the ‘Vierendeel’ mechanism. The behaviour of perforated sections is characterized by three actions: global bending action, global shear action and local ‘Vierendeel’ action. The moment shear interaction curves obtained from the finite element investigation are presented in Figure 3, where the vertical axis is the ‘coupled’ shear ratio and the horizontal axis is the moment ratio. At failure, the global shear force, $V_{o,Sd}$ (Eqn. 1) and
Perforated sections with elliptical web openings generally behave similarly to the other web opening configurations. However, from the FE analysis, it was found that as the web opening size increased the utilisation ratios of the perforated sections also increased. Most affected are the perforated sections with web openings of $d_o = 0.5h$, where both the $x$- and $y$-intercepts are significantly reduced and an uneven stress distribution is observed. The rotation of the elliptical web openings is a key parameter. By rotating all sizes of web openings $45^\circ$ to the vertical centre-line of the web opening, a reduction of depth of around $11.5\%$ is obtained. However, this dissimilarly affects the design capacity values, $V_{o,Rd}$ and $M_{o,Rd}$. Consequently, regarding the standard web openings it can be concluded that the bigger the web opening size the higher the reduction of the utilisation ratio is.

Investigating the perforated sections with inclined web openings G with $d_o = 0.8h$, $0.65h$ and $0.5h$, it can be seen that the utilisation ratios for all three web opening sizes are close to each other and the overall behaviour is similar to the other sections with elliptical web openings. Even though web opening G has only a $3.7\%$ smaller web opening area than web opening A, the $M/V$ curves are quite different. This is due to shift of the stress concentration points, where the web opening shape and opening length controls their behaviour. Overall, it can be concluded that when perforated sections with elliptical web openings are used, the opening length of the web openings is the main parameter.
Figure 3: M/V interaction curves for various web opening configurations obtained from FEA

<table>
<thead>
<tr>
<th>Opening Types</th>
<th>Opening Shapes</th>
<th>Opening Length, c</th>
<th>0.44h</th>
<th>0.5h</th>
<th>0.57h</th>
<th>0.65h</th>
<th>0.7h</th>
<th>0.8h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>A</td>
<td>0.23</td>
<td>N.A.</td>
<td>0.95</td>
<td>N.A.</td>
<td>0.86</td>
<td>N.A.</td>
<td>0.75</td>
</tr>
<tr>
<td>Typical</td>
<td>B</td>
<td>0.43</td>
<td>N.A.</td>
<td>0.92</td>
<td>N.A.</td>
<td>0.82</td>
<td>N.A.</td>
<td>0.65</td>
</tr>
<tr>
<td>Non-Standard</td>
<td>C</td>
<td>0.14</td>
<td>N.A.</td>
<td>0.71</td>
<td>N.A.</td>
<td>0.88</td>
<td>N.A.</td>
<td>0.92</td>
</tr>
<tr>
<td>Elliptical</td>
<td>D</td>
<td>0.25</td>
<td>0.60</td>
<td>N.A.</td>
<td>0.79</td>
<td>N.A.</td>
<td>0.88</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>0.25</td>
<td>0.59</td>
<td>N.A.</td>
<td>0.78</td>
<td>N.A.</td>
<td>0.87</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>0.25</td>
<td>0.59</td>
<td>N.A.</td>
<td>0.72</td>
<td>N.A.</td>
<td>0.87</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>0.21</td>
<td>N.A.</td>
<td>0.71</td>
<td>N.A.</td>
<td>0.79</td>
<td>N.A.</td>
<td>0.74</td>
</tr>
<tr>
<td>Standard</td>
<td>H</td>
<td>1.00</td>
<td>N.A.</td>
<td>0.65</td>
<td>N.A.</td>
<td>0.48</td>
<td>N.A.</td>
<td>0.26</td>
</tr>
<tr>
<td>Elongated</td>
<td>I</td>
<td>1.16</td>
<td>N.A.</td>
<td>0.56</td>
<td>N.A.</td>
<td>0.35</td>
<td>N.A.</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>J</td>
<td>2.00</td>
<td>N.A.</td>
<td>0.46</td>
<td>N.A.</td>
<td>0.23</td>
<td>N.A.</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>2.23</td>
<td>N.A.</td>
<td>0.37</td>
<td>N.A.</td>
<td>0.20</td>
<td>N.A.</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 1 summarizes the values of the ‘coupled’ shear ratios, for perforated sections obtained from the comprehensive finite element investigation of the current study. ‘Coupled’ capacities are found to depend not only on the shapes and sizes of web openings, but also on the applied global shear forces and moments. Hence, the position of the web opening as well as the loading arrangement affects the ‘coupled’ shear capacities.
Moment/Shear Ratios for Various Section Sizes

From the FE analysis, it is found that in an I-section with large web openings the shear area of the web is significantly reduced, and hence the shear areas of the flanges are taken into consideration in assessing the shear capacity of the perforated section [6]. Hence, in the aforementioned FE analyses the following assumption, presented in Eqn. 5, is utilised. Table 2 presents the differences by using the latter assumption and the ones based on simple plastic section analysis, where the shear area of an I-section is taken either as \( h \times t_w \) for practical reasons, or as given in Eqn. 6 below, from in ENV (1993-1-3) EC3 [7].

\[
A_v = h t_w + Z \left(0.75 t_f^2\right) \\
A_v = A - 2b t_f + (t_w + 2\tau) t_f
\]

For beam sections with thick flanges, the increase in shear capacities can exceed 30%. This percentage is decreased for perforated sections with smaller web opening sizes. Also, the \( t_w/h \) ratio affects the bending capacity of the perforated sections to a lesser extent.

Table 2 presents the ‘coupled’ shear ratios from the present FE study for one web opening of each category according to the aforementioned assumptions, together with the percentage differences between them. The Eqn. 5 shows a greater decrease of the utilisation factor when UB 457x152x52 sections are investigated. It is seen that both the \( t_f \) and \( t_w \) parameters are considered, although \( t_f \) is generally domain.

‘Vierendeel’ Parameter

The ‘Vierendeel’ moment across the opening is resisted by the plastic moment capacities of the sections, which may or may not be stiffened. For beams with circular openings Redwood [8] proposed an effective size of rectangular opening. A ‘Vierendeel’ parameter, \( \nu_v \), is used for this purpose and is defined in Eqn. 7 [6].

\[
\nu_v = \frac{V_{o,Rd,Vi}}{4M_{T,Rd,c}}
\]

Where \( M_{T,Rd} \) is the basic shear capacity of the tee-sections under zero axial and shear forces, \( V_{o,Rd,Vi} \) is the global ‘coupled’ shear capacity of perforated sections as obtained from FEA, and \( c \) is the critical opening length.

Figure 4 shows typical values of the ‘Vierendeel’ mechanism for perforated sections under zero global moment. The results show that the ‘Vierendeel’ parameter increases as the critical opening length is increased and tends towards unity, thus illustrating the importance of the ‘Vierendeel’ mechanism.
TABLE 2
COUPLED SHEAR RATIOS OF TEE-SECTIONS

<table>
<thead>
<tr>
<th>Section Sizes</th>
<th>Opening Shape</th>
<th>Opening Size d_o/h</th>
<th>t_f/t_w</th>
<th>t_w/h</th>
<th>h_x/t_w ratio</th>
<th>EC3 Eqn. 6 ratio</th>
<th>[5] Eqn. 5 ratio</th>
<th>h_x/t_w Vs. EC3 (%)</th>
<th>h_x/t_w Vs. [5] (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UB 610x229x140</td>
<td>A</td>
<td>0.8</td>
<td>1.69</td>
<td>0.0212</td>
<td>1.11</td>
<td>0.90</td>
<td>0.77</td>
<td>18.9</td>
<td>30.6</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1.47</td>
<td>1.17</td>
<td>1.02</td>
<td>0.76</td>
<td>0.62</td>
<td>0.53</td>
<td>18.4</td>
<td>30.3</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.96</td>
<td>0.76</td>
<td>0.78</td>
<td>1.27</td>
<td>0.98</td>
<td>1.01</td>
<td>22.8</td>
<td>20.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.47</td>
<td>0.39</td>
<td>0.39</td>
<td>0.77</td>
<td>0.60</td>
<td>0.44</td>
<td>17.0</td>
<td>17.0</td>
</tr>
<tr>
<td>UB 610x229x101</td>
<td>A</td>
<td>0.8</td>
<td>1.41</td>
<td>0.0174</td>
<td>1.17</td>
<td>0.93</td>
<td>0.77</td>
<td>20.5</td>
<td>34.2</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1.27</td>
<td>0.98</td>
<td>1.01</td>
<td>0.77</td>
<td>0.60</td>
<td>0.44</td>
<td>22.1</td>
<td>42.8</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.95</td>
<td>0.72</td>
<td>0.75</td>
<td>0.77</td>
<td>0.60</td>
<td>0.44</td>
<td>22.1</td>
<td>42.8</td>
</tr>
<tr>
<td>UB 457x152x82</td>
<td>A</td>
<td>0.8</td>
<td>1.80</td>
<td>0.0225</td>
<td>1.16</td>
<td>0.87</td>
<td>0.92</td>
<td>25.0</td>
<td>20.7</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.95</td>
<td>0.72</td>
<td>0.75</td>
<td>0.77</td>
<td>0.60</td>
<td>0.44</td>
<td>22.1</td>
<td>42.8</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0.27</td>
<td>0.20</td>
<td>0.22</td>
<td>0.77</td>
<td>0.60</td>
<td>0.44</td>
<td>22.1</td>
<td>42.8</td>
</tr>
</tbody>
</table>

**Position of First Plastic Hinge**

Non-linear FE analyses have shown that failure is dominated by web post for low axial force in the tee-section, and ‘Vierendeel’ for high axial force in the tee-section. Curved beam theories by Olander [9] and Sahmel [10] were used in order to assess the results. The overall view is that as the angle $\phi_p$ of the plastic hinge is increased, the ratio of axial force to shear force also increases.

The angle $\phi_p$ of the first plastic hinge always occurs at the LMS of the top tee-section. After the formation of the first plastic hinge, there is load redistribution across the web opening and the plastic hinges are formed in a slightly different way. Typically, the larger the opening length, c, the higher the $\phi_p$ value is. However, as it shown from Figure 5 the shape of the web opening also significantly affects the position of the plastic hinges. Furthermore, it can be concluded that in perforated sections with polygonal web openings, the stress usually tends to concentrate at the sharp corners of the polygons. Also, it should be clearly stated that perforated sections with non-standard elliptical web openings behave similarly to the standard non-polygon web openings in terms of the sequence of the plastic hinges formation, independent of the web opening shape.
CONCLUSIONS

Finite element analyses of perforated sections with different standard and non-standard web opening configurations show how the ‘Vierendeel’ mechanism is affected by the shapes and sizes of the web openings. The investigation on novel elliptical web openings presents some new results and others that update current knowledge. The FE analysis provides a good prediction of the ‘Vierendeel’ loads but the results are more applicable if calibrated against other FEA results found in the literature and/or against experimental work with similar test configurations. The effects of the flange and web thicknesses, as well as the critical web opening length and depth are presented herein through a comprehensive parametric FE investigation.

REFERENCES


BUCKLING STRENGTH OF THIN WALLED MEMBERS WITH PROFILED SECTIONS

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KEYWORDS
Thin-walled structure, profiled plate, Transfer matrix, Point matrix, Local buckling.

ABSTRACT
In recent year the thin-walled members with profiled sections, such as the trapezoidal and shelled forms are used. It is present that this profiled section is used to increase the strength of deck plate and to improve the bond of concrete. Accordingly there is a little on the application of compressive thin-walled member.

Usually the stiffener is used for compressive thin-walled member. In this paper, the buckling strength of the box-section with profiled section and stiffener are compared, and the behavior of box-section with profiled section is investigated, in order to verify the possibility of application to compressive member.

Generally, the buckling behavior of the thin-walled box-section with these complicated forms is only obtained as approximate solution by the finite-element method.

In order to obtain the exact solution of these complicated structures, a transfer matrix method for elastic buckling problems of thin-walled box-section with profiled section is presented.

The analytical local and overall elastic buckling loads of thin-walled box-section with profiled section can be obtained simultaneously. Furthermore, a technique to estimate the buckling mode shapes of these structures is also shown.

The results are given as two local buckling behaviors for box-section with open stiffener. The first local buckling behavior is in the local buckling shape to become a joint at the position of the each stiffener, and the second local buckling behavior is the local buckling shape of individual plate to compose box-section.

For box-section with profiled sections the second local buckling behavior is only generated.
INRODUCTION

In recent years, a lot of thin-walled members are using deck plate of bridge, section steel and etc. in broad range of structural applications. And there are numerous forms for the stiffened plate with open and closed stiffeners and the profiled sections worked thin plate itself. These profiled sections grow up the stiffness by working the triangular, square and shellforms for the part of thin plate.

In this paper, it is clear that the strength as the compressive members to estimate the buckling strength and the buckling mode shapes of the thin-walled members with profiled sections.

Furthermore, in order to clarify the buckling strength of the thin-walled box section with profiled sections, it is researched for the propriety in comparison with the open stiffeners and profiled sections. The analytical local and overall elastic buckling load of box-section with profiled section and the open stiffener can be obtained simultaneously using the ordinary transfer matrix method and the extended transfer matrix method. The extended transfer matrix is applied to consider the compatibility panel and those of the branched panel of the box-section with open stiffeners. And a technique to estimate the buckling mode shapes of these structures is also shown.

From numerical calculations, it can be concluded that the exact buckling coefficients and mode shapes of box-section with profiled sections and open stiffeners are obtained with very small computational efforts. Further it is realized that and the mode shapes obtained by the extended transfer matrix and the ordinary transfer matrix are very effective to clarify the complicated the worked plate panels and with stiffeners.

BUCKLING ANALYSIS

Transfer Matrix for Plate Panel

From the equilibrium equations of forces for plate panel subjected to in plane compressive force (shown in Figure 1) and from the relation between strains and deformations for plate panel, the partial differential equations for the state variables \( w, \varphi_y, M_y, V_y, u, N_y \) and \( N_{yx} \) are obtained. By substituting the extended state variable on the basis of the condition as the simple support by the sides \( x=0 \) and \( x=L \) of the plate into the partial differential equations,
the following ordinary differential equations \[\frac{dZ}{dy} = Z^* = \begin{bmatrix} w^*, \varphi, T, M, V, v, u, N_x, N_y, N_z \end{bmatrix}\]

referenced to the variable \(y\) only are obtained.\(^1\)

(1) \[\frac{dZ}{dy} = Z^* = A \cdot Z\]

Integrating equation (1), the transfer matrix \(F\) is obtained as follows

(2) \[Z = \exp(Ay)Z_0 = FZ_0\]

where

\[\exp(Ay) = I + (Ay) + \frac{1}{2!}(Ay)^2 + \frac{1}{3!}(Ay)^3 + \cdots\]

(3) \[I\] is the unit matrix.

**Point Matrix**

As shown in Figure 2, the state vectors for each panel are referred to the local coordinate system. Therefore, the relations between the state vectors of two consecutive panels are required, in order to proceed the transfer procedures of the state vectors over these panels.

The relation between the state vectors of the plate panels I and II (Figure 2) are described as follows:

(4) \[Z_i^L = P_i Z_j^R\]

Where \(P_i\) is the rotation matrix relating the state vectors between two consecutive plate panels.

Considering the symmetry of box-section with profiled section (Figure 3), the state vector \(Z_{14}\) and \(Z_0\) are related by means of the equation for half box-section.

(5) \[Z_{14} = \begin{bmatrix} F_{14} & P_{13} & F_{12} & P_{11} & F_{10} & P_{10} \end{bmatrix} \begin{bmatrix} F_8 & P_7 & F_6 & P_6 & F_5 & P_5 & F_4 & P_4 \end{bmatrix} \begin{bmatrix} F_3 & P_2 & F_2 & P_1 & F_1 & P_0 & F_0 \end{bmatrix} Z_0 \]

Where \(F_i\) and \(P_i\) are the field and point matrixes, which are assumed to be known.

**Stability Equation for Box-Section with Profiled Plate**
Advances in Steel Structures
ICASS'09, 16-18 December 2009, Hong Kong, China

Stability Equation for Box-Section with Stiffener

The ordinary transfer matrix method can not be applied directly to the thin-walled box-section with open stiffener.

In order to apply this method to box-section with open stiffener, the extended transfer matrix for box-section with open stiffener is presented. The compatibility and equilibrium conditions between the state vectors of the main panel and those of open panel (branched panel) of the half box section with stiffener at section[1] are described as follows(Figure 4);

$$
\begin{bmatrix}
Z^L \\
0
\end{bmatrix}_{M1} = \begin{bmatrix}
F_{P1} \\
F_{P1}^F
\end{bmatrix}^\delta - \begin{bmatrix}
P_{B1}^F \\
P_{B1}^F
\end{bmatrix}^\delta \begin{bmatrix}
Z_M \\
Z_{B1}
\end{bmatrix}_{0}
$$

(6)

Further considering the compatibility and equilibrium conditions between the state vectors of the main panel and those branched panels of half box-section with open stiffener at section [2], [3] and [4], the relation between the initial state vectors of main and branched panels, $Z_{M0}$, $Z_{B10}$, $Z_{B20}$, $Z_{B30}$, $Z_{B40}$ and the final state vector $Z_{M7}$ is described as shown equation (7).

Where superscripts $\delta$ and $F$ indicate the displacement and force components of the state vectors respectively. Superscripts L and R indicate the left and right side of the section, and subscripts M and B indicate the variables of the main and branched (stiffener) panels.

$$
\begin{bmatrix}
Z^L \\
0 \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
F_{P1}^L \\
F_{P1}^F \\
F_{P1}^F \\
F_{P1}^F
\end{bmatrix}^\delta - \begin{bmatrix}
P_{B1}^F \\
P_{B1}^F \\
P_{B1}^F \\
P_{B1}^F
\end{bmatrix}^\delta \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}_{Z_{M0}}
$$

(7)
Boundary Condition

By considering of the symmetry of the box-section with profiled section the following condition of the state vector is obtained:

$$Z_{\text{sym}} = \begin{bmatrix} w, 0, M_y, 0, 0, u, N_y, 0 \end{bmatrix}^T$$  \( (8) \)

Considering this boundary condition, the numerical calculations are performed for various thin-walled members.

NUMERICAL ANALYSIS

<table>
<thead>
<tr>
<th>TABLE1</th>
<th>PARAMETERS OF ANALYTICAL MODELS</th>
</tr>
</thead>
<tbody>
<tr>
<td>model1</td>
<td>model2</td>
</tr>
<tr>
<td>width of plate (b)</td>
<td>600(mm)</td>
</tr>
<tr>
<td>thickness of plate (t)</td>
<td>7.8(mm)</td>
</tr>
<tr>
<td>element</td>
<td>without</td>
</tr>
<tr>
<td>number of element</td>
<td>1, 2, 5, 10</td>
</tr>
<tr>
<td>height of element</td>
<td>b</td>
</tr>
<tr>
<td>width of stiffener (bs)</td>
<td>20(mm)</td>
</tr>
<tr>
<td>thickness of stiffener (ts)</td>
<td>7.8(mm)</td>
</tr>
<tr>
<td>member aspect ratio (a/b)</td>
<td>0.2 ~ 100.0</td>
</tr>
<tr>
<td>boundary condition</td>
<td>symmetry condition</td>
</tr>
<tr>
<td>Young's moduls</td>
<td>205.8(GPa)</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Figure 5: Analytical models

Figure 6: Size of profiled element
The box-section structure with various profiled sections and open stiffener are analyzed under the uniform axial load. The parameters of analytical models are shown in Table 1 and Figure 5 (a), (b), (c), (d) and (e). In Figure 6 the sizes of profiled elements (triangular form, square form and shell form) are shown.

The numbers of profiled element are 1, 2, 5 and 10 elements respectively and each element is applied for each thin-walled plate at equal spaces. The thin-walled box sections with profiled sections and open stiffener are analyzed for the half cross section by considering the symmetry.

**Buckling Coefficient**

In Figure 7 the buckling coefficients of the thin-walled box-sections with five profiled elements of triangular, square and shell forms respectively under member aspect ratios \((a/b=0.2~100.0)\). Here the vertical axis is shown for the buckling coefficient \(k=b^{2}\sigma_{x}/\pi^{2}D\) (\(D\): bending stiffness) and horizontal axis is shown for common logarithms of the member aspect ratio \(a/b\). In local buckling range, the local buckling coefficient of shell form is larger in comparison with triangular and square forms.

![Figure 7: Buckling coefficients of box-section with various profiled sections](image)

In Figure 8 the buckling coefficients of box-section with 1, 2, 5 and 10 elements of square profiled section respectively are shown. According to the figure the buckling coefficients grow with increase of number of square profiled elements.

![Figure 8:Buckling coefficients of box-section with 1, 2, 5 and 10 elements of square profiled section respectively](image)
In Figure 9 the comparison of buckling coefficients are shown for 10 elements of profiled sections and open stiffener respectively. The buckling coefficients of profiled sections are almost half in strength when compared with open stiffener. It is thought that there are almost sufficient when using this thin-walled box-section with profiled sections as compression members.

Deformation Shape

In figure 10 the local buckling deformation shape on box section with 5 elements of square profiled section is shown for member aspect ratio $a/b=5.0$ and the buckling half wave number $m=2$.

In Figure 11 shows the local buckling deformation shape on box-section with open stiffener in a very small member aspect ratio $a/b=0.7$. Accordingly, it is considered that the local deformation shape to become a quasi-joint at the position of each stiffener is generated.
In Figure 12 the local buckling deformation shape on box-section with open stiffener is shown for member aspect ratio $a/b=5.0$ and the buckling half wave number $m=2$. This local buckling behavior is equal to the local buckling mode shape of individual plate to compose box cross section in ordinary member aspect ratio range.

![Figure 11: Local deformation shape of box-section with open stiffeners in a very small member aspect ratio range](image1)

![Figure 12: Local deformation shape of box-section with open stiffeners in ordinary member aspect ratio range](image2)

**CONCLUSIONS**

An analytical procedure for the elastic buckling problems of box-section with various profiled elements by the extended and the normal transfer matrix methods was presented in this paper. A good effect of stiffener using the shell element can be obtained against the profiled sections, and in accordance with the increase of the number of elements the buckling strength has improved. However in proportion to the increment of various profiled elements the difference of the buckling strength is missing.

In a very small member aspect ratio range the first local buckling behavior to become a joint at the position of the each stiffener is generated for box-section with open stiffener, and the second local buckling behavior is the local buckling shape of the individual plate to compose box-section in the intermediate member aspect ratio range. For box-section with profiled sections the local buckling behavior is the only buckling shape of individual plate to compose box-section.

**REFERENCES**

BUCKLING ANALYSIS OF THIN-WALLED SHELL MEMBER WITH VARIOUS STIFFENERS

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KEYWORDS
thin-walled shell, transfer matrix, point matrix, open and closed stiffeners, L-type stiffener, local buckling

ABSTRACT

In recent years, a lot of structures used by thin-walled shell members are constructed. This shell structure is superior dynamically, and there are structural forms with high strength comparing with normal thin-walled plate structures. However, it is thought that there are insufficient cases because of thin-walled when using this thin-walled shell member as a compression member. Then, in order to obtain high strength structures, such as the shell member with stiffener, the extended transfer method is proposed by considering the compatibility and equilibrium conditions between the main panels (shell member) and the branched panels (stiffener) for the stiffened shell. This analytical procedure to estimate not only the buckling loads but also buckling mode shapes of thin-walled shell with open and closed stiffeners is presented. The field transfer matrix derived from differential equation of the shell element or the plate element is used. Further, the point matrix is derived from the relation of state
vectors between the adjacent panels. Since this method is analytical, exact local and overall buckling loads and buckling mode shapes are obtained systematically. And the results are given as two local buckling behaviors. The first local buckling behavior is the local buckling shape to become a joint at the position of each stiffener, and the second local buckling behavior is the local buckling shape of individual shell plate to compose the structure. From numerical calculations examined in this paper, the exact buckling coefficients and mode shapes of shell with stiffeners are obtained with very small computational efforts, and the mode shapes obtained by the transfer matrix method are very effective to clarify the complicated buckling phenomenon of the shell structure composed of shell panels and stiffeners.

INTRODUCTION

In recent years, a lot of structures used by thin-walled shell members are constructed. This shell structure is superior dynamically, and there are structural forms with high strength comparing with normal thin-walled plate structures. However, it is thought that there are insufficient cases because of thin-walled when using this thin-walled shell member as a compression member. Then, in order to obtain high strength structures, such as the shell member with stiffener, the extended transfer method is proposed by considering the compatibility and equilibrium conditions between the main panels (shell member) and the branched panels (stiffener) for the stiffened shell. This analytical procedure to estimate not only the buckling loads but also buckling mode shapes of thin-walled shell with open and closed stiffeners is presented. The results are given as two local buckling behaviors. The first local buckling behavior is local buckling shape to become a joint at position of the each stiffener, and the second local buckling is the local buckling shape of individual shell panel to compose the stiffened shell.

ANALYTIC THEORY

Transfer matrix for shell panel

Fig.1.Forces and moments of shell panel 1

Fig.2.Displacements of shell pane
From the equilibrium equations of forces for the shell panel subjected to in-plane compressive force and from the relation between strains and deflections for the shell panel (Fig.1 and 2), the partial differential equations for the state variables \( w^*, \phi^*, M^*_{\phi}, V^*, v^*, u^*, N^*_{\phi}, N^*_{\phi} \) are obtained.

By substituting the extended state variable on the basis of the condition at the simple support by the sides \( x = 0 \) and \( x = L \) of the shell plate into the partial differential equations, the following ordinary differential equations referred to the variables only obtained. \(^1\)

\[
\frac{d}{Rd\phi}Z = Z^* = A \cdot Z \quad (1)
\]

Integrating Eq.(5) the transfer matrix is obtained as follows:

\[
Z = \exp(A\phi) \cdot Z_0 = F \cdot Z_0 \quad (2)
\]

where

\[
\exp(A\phi) = I + (A\phi) + \frac{1}{2!}(A\phi)^2 + \frac{1}{3!}(A\phi)^3 + \cdots \quad (3)
\]

Where, \( I \) is the unit matrix.

**Point matrix**

As shown in Fig.3 the state vectors for each shell panel are referred to the local coordinate system. Therefore the relations between the state vectors of two consecutive shell panels are required, in order to proceed the transfer procedures of the state vectors over these shell panels.

Considering the relation between the state vectors of the shell plate panels I and II (Fig.3), the following equations is obtained:

\[
Z_{II}^i = P_i \cdot Z_{I}^i \quad (4)
\]

where \( P_i \) is the rotation matrix relating the state vectors between two consecutive shell panels.

**Extended transfer matrix for shell panel with open stiffener**

The compatibility and equilibrium conditions between the state vectors of the main shell panel and those of the branched panel at section 1 are described as follows (Fig.4):
\[ \delta Z^L_{MS} = \delta Z^R_{MS} = P_1 \delta Z_{BP} \] (5a)

\[ F Z^L_{MS} = F Z^R_{MS} = P_1 \delta Z_{BP} = 0 \] (5b)

In which superscripts \( \delta \) and \( F \) indicate the displacement and force components of the state vector, respectively; superscripts \( \text{L and R} \) indicate the left and right side of section 1, and subscripts \( \text{MS and BP} \) indicate the variables of the main shell and branched panels. The state vectors of the main shell and branched panels, at right hand of section 1 are related to the initial state vectors as follows:

\[
\begin{bmatrix}
\delta Z \\
F Z
\end{bmatrix}_{MS} = \begin{bmatrix}
\delta F_{SP} \\
F F_{SP}
\end{bmatrix} Z_{MS0}
\] (6a)

\[
\begin{bmatrix}
\delta Z \\
F Z
\end{bmatrix}_{BP} = \begin{bmatrix}
\delta F_{BP} \\
F F_{BP}
\end{bmatrix} Z_{BPO}
\] (6b)

or

\[ Z^R_{MS} = F_{SP} \cdot Z_{MS0} \] (7a)

\[ Z_{BP} = F_{BP} \cdot Z_{BPO} \] (7b)

Substituting Eq.(6) into Eq.(5a) and (5b), the extended transfer matrix relating the state vector of the main shell panel at the left hand side of section 1 and the initial state vectors of the main shell panel and branched panel can be obtained as follows:

\[
\begin{bmatrix}
Z \\
0
\end{bmatrix}_{MS} = \begin{bmatrix}
F_{SP} & P_1 \cdot F_{BP} \\
\delta F_{MS} & -P_1 \cdot F_{BP}
\end{bmatrix} \begin{bmatrix}
Z_{MS} \\
Z_{BP}
\end{bmatrix}
\] (8)

Performing the transfer procedure from section 1 to 2; the relation between the initial state vectors of the main shell and branched panels and that at section 2 described as follows:

\[
\begin{bmatrix}
Z \\
0
\end{bmatrix}_{MS2} = \begin{bmatrix}
F_{SP2} \cdot F_{SP} & F_{SP2} \cdot P_1 \cdot F_{BP} \\
\delta F_{SP} & -P_1 \cdot F_{BP}
\end{bmatrix} \begin{bmatrix}
Z_{MS0} \\
Z_{BPO}
\end{bmatrix}
\] (9)
Extended transfer matrix for the thin-walled shell with closed cross section

Assuming virtual cross section at a point on the closed panels as shown in Fig. 5, the state vector of main shell panel at right hand side of section 2, \( Z_{MS0}^R \), is related to initial state vector of main shell panel and the right hand side of the virtual cross section of the closed panel, \( Z_{MS0}^R \), \( Z_{B10}^R \) in the similar manner to that used for branched panel:

\[
\begin{pmatrix}
Z_{MS2}^R \\
0
\end{pmatrix} =
\begin{pmatrix}
F_{SP2} F_{SP1} & F_{SP2} P_{B12} F_{B12} & \delta F_{SP1} & - P_{B12} \delta F_{B12} \\
\delta F_{SP1} & - P_{B12} \delta F_{B12} & 0 & \end{pmatrix}
\begin{pmatrix}
Z_{MS0}^R \\
Z_{B10}^R
\end{pmatrix}
\] (10)

where superscript \( B \) indicates the variables of the closed panel. Furthermore, considering the compatibility and equilibrium conditions at section 2, the relation between the state vector of the main shell panel at left hand side of section 2, \( Z_{MS2}^L \), and the initial section state vectors of the main and both side of virtual cross section of the closed panel, \( Z_{MS0}^L \), \( Z_{B10}^L \) and \( Z_{B20}^L \), can be obtained as follows:

\[
\begin{pmatrix}
Z_{MS2}^L \\
0
\end{pmatrix} =
\begin{pmatrix}
F_{SP2} F_{SP1} & F_{SP2} P_{B12} F_{B12} & P_{B22} F_{B22} & \delta F_{SP1} & - P_{B12} \delta F_{B12} & 0 \\
\delta F_{SP1} & - P_{B12} \delta F_{B12} & 0 & \end{pmatrix}
\begin{pmatrix}
Z_{MS0}^L \\
Z_{B10}^L \\
Z_{B20}^L
\end{pmatrix}
\] (11)

This initial state vector of the right hand side of the virtual cross section \( Z_{B10}^R \) can be related to that of the left hand side, \( Z_{B20}^L \) as follows:

\[
\begin{pmatrix}
\delta Z_{B10}^L \\
F Z_{B20}^L
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & -1 \\
0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
Z_{MS0}^L \\
Z_{B10}^R \\
Z_{B20}^L
\end{pmatrix}
\] (12)

or

\[
Z_{B20}^L = T_{B0} \cdot Z_{B10}^R
\] (13)

where

\[
T_{B0} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & -1 \\
0 & 0 & 1 \\
\end{pmatrix}
\]

Substituting Eq.(13) into (11), and eliminating the initial state vector of the left hand side of the virtual cross section, \( Z_{B20}^L \), Eq.(11) can be written as follow:

\[
\begin{pmatrix}
Z_{MS2}^L \\
0
\end{pmatrix} =
\begin{pmatrix}
F_{SP2} F_{SP1} & F_{SP2} P_{B12} F_{B12} & P_{B22} F_{B22} T_{B0} & \delta F_{SP1} & - P_{B12} \delta F_{B12} & 0 \\
\delta F_{SP1} & - P_{B12} \delta F_{B12} & 0 & \end{pmatrix}
\begin{pmatrix}
Z_{MS0}^L \\
Z_{B10}^R
\end{pmatrix}
\] (14)

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Performing the transfer procedure from section 2 to 3, the relation between the initial state vectors, \( Z_{MS0} \), \( Z_{R10} \), and that at section 3, \( Z_{MS3} \), is described as follows:

\[
\begin{bmatrix}
Z_{MS3} \\
0
\end{bmatrix}
= \begin{bmatrix}
F_{SP3} & F_{SP2} & F_{SP1} \\
\delta & \delta & \delta
\end{bmatrix}
\begin{bmatrix}
F_{SP2}P_{B12}^F & F_{SP2}P_{B12}^F \\
0 & 0
\end{bmatrix}
+ \begin{bmatrix}
F_{B22}P_{B12}^F \\
\delta P_{B12}^F \\
-\delta P_{B12}^F \\
-\delta P_{B12}^F
\end{bmatrix}
\begin{bmatrix}
Z_{MS0} \\
Z_{R10}
\end{bmatrix}
\]

(15)

**NEWMERICAL RESULTS**

**TABLE1**

<table>
<thead>
<tr>
<th>PARAMETERS OF ANALYTICAL MODELS.</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (b)</td>
<td>600(mm)</td>
<td>600(mm)</td>
<td>600(mm)</td>
</tr>
<tr>
<td>Thickness(t)</td>
<td>7.8(mm)</td>
<td>7.8(mm)</td>
<td>7.8(mm)</td>
</tr>
<tr>
<td>Length of stiffener (l_s)</td>
<td></td>
<td>50(mm)</td>
<td></td>
</tr>
<tr>
<td>Thickness of stiffener (t_s)</td>
<td>3.9(mm), 7.8(mm), 23.4(mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance of stiffener (b_s)</td>
<td>200(mm)</td>
<td>200(mm)</td>
<td>200(mm)</td>
</tr>
<tr>
<td>Width of stiffener (w_s)</td>
<td>-</td>
<td>20(mm)</td>
<td>195(mm)</td>
</tr>
<tr>
<td>Curvature (1/r)</td>
<td>0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length-width ratio (a/b)</td>
<td>0.1~50.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young's modulus (E)</td>
<td>205.8(Gpa)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poasson's radio (\mu)</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The thin-walled shell member with various stiffeners subjected to uniform axial load is analysed. The parameters of analytical models are in Table-1. The Fig.6 shows the buckling coefficients of thin-walled shell member with open two stiffeners subjected to uniform axial load under member aspect ratios \((a/b=0.1~50.0)\) and stiffener thick ratios \((t_s/t=0.5,1.0\) and 3.0). Here the vertical axis is
shown for the buckling coefficient \( k = \frac{\sigma_x b T}{\pi^2 D} \) (D: bending stiffness) and horizontal axis is shown for common logarithms of member aspect ratio \( a/b \). As shown in Fig.6, in a very small member aspect ratio range and ordinary member aspect ratio range the thin-walled shell member with open stiffeners exhibits two local buckling behaviors. In proportion to the increase of stiffener thickness ratio \( ts/t \), the local buckling strength increases. In the stiffener thickness ratios \( ts/t = 0.5 \) and 1.0 the effect of open stiffener is small during ordinary member aspect ratio range. Fig. 7 (a), (b) show the buckling mode shape of the thin-walled shell member with open stiffener under stiffener thickness \( ts/t = 1.0 \) for member aspect ratio \( a/b = 1.02 \) and 8.26 respectively. As shown in Fig. 7 (a) the first local buckling mode shows the local deformation shape to become a joint at position of each stiffener in very small member aspect ratio range. In Fig. 7 (b) the second local buckling mode shows the local deformation shape of individual shell panel to compose the shell member. In Fig. 8 the buckling coefficients of the thin-walled shell member with L-type stiffeners subjected to compressive load under member aspect ratios \( a/b = 0.1 \sim 50.0 \) and stiffener thickness ratio \( ts/t = 0.5, 1.0 \) and 3.0. As shown in Fig. 8 the first local buckling behavior and the second local buckling phenomenon are shown during each member aspect ratio range, and similarly in the stiffener thickness ratios \( ts/t = 0.5 \) and 1.0 the effect of L-type stiffener is small during ordinary member aspect ratio range. In Fig.9 the buckling coefficients of the thin-walled shell member with the closed trapezoidal stiffener subjected to compressive load under member aspect ratios \( a/b = 0.1 \sim 50.0 \) and stiffener thickness ratios \( ts/t = 0.5, 1.0 \) and 3.0. As shown in Fig.9 the first local buckling behavior and the second local buckling phenomenon are shown during each member aspect ratio range. For stiffener thickness ratio \( ts/t = 0.5 \) the buckling coefficient of shell member with closed stiffener is small in comparison with that of shell member without stiffener. In Fig.10 the buckling mode shape of shell member with closed stiffener is shown under stiffener thickness ratio \( ts/t = 0.5 \) for member aspect ratio \( a/b = 1.22 \). In this case it is the local buckling of trapezoidal stiffener instead of shell panel. Therefore the buckling strength of shell member with closed stiffener \( ts/t = 0.5 \) is on the decline in comparison with that of shell member without stiffener. Fig.11 shows the buckling mode shapes of the shell member with the closed trapezoidal stiffener under stiffener thickness ratio \( ts/t = 1.0 \) for member aspect ratio \( a/b = 1.16 \). In this case, the first local buckling shape is shown.
CONCLUSIONS

In this paper an analytical procedure for elastic buckling problems of the thin-walled shell member with open and closed stiffeners by the extended transfer matrix method is present. In this analysis the similar tendency can be obtained for the thin-walled shell member with open and closed stiffeners. Namely the first buckling behavior is the local buckling shape to become a joint at the position of each stiffener, and the second local buckling behavior is the local buckling shape of individual shell plate to compose the shell member. Moreover, in this time the models during the analysis can be confirmed to the effect stiffener except a part. However in the small stiffener thickness ratio (ts/t) the effort of stiffener can not be confirmed due to the model of stiffener. Consequently in order to obtain the sufficient effect of stiffener for the shell member, it is to be desired that the plate thickness of stiffener enlarges.

REFERENCE

EXPERIMENTS ON THE LOCAL BUCKLING OF 420MPA STEEL EQUAL ANGLE COLUMNS UNDER AXIAL COMPRESSION

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KEYWORDS
Local buckling, high strength, steel equal angle, axial compression, Q420.

ABSTRACT
Local buckling behavior can be ignored for hot-rolled ordinary strength steel equal angle compression members, because the width-thickness ratio of the leg is small enough, not exceeding the limit value of the ratio, which is related to the steel strength. However, this may be not true for high strength steel equal angle members. With the development of steel structures, Q420 steel angle members with the nominal yield strength of 420MPa have begun to be widely used in China, especially in the transmission towers. Because of the high strength, the limit value of width-thickness ratio of the angle leg becomes smaller than that of ordinary steel strength, which causes that the width-thickness ratios of the leg for some hot-rolled steel angle sections exceed the limit value. Consequently, the local buckling behavior must be considered for the 420MPa steel equal angle members under axial compression, especially for those with small slenderness. This paper introduces an axial compression experiment of 420MPa steel equal angle columns, with the total number of 15 specimens, including 5 different sections, with small slenderness, whose width-thickness ratios of the leg all exceed the limit values specified in many countries’ steel structures design codes. The test results show that the local buckling is significant and must be considered for the 420MPa steel equal angle columns under axial compression. Compared with the design strengths by the American and European steel structures design codes, the test strengths are higher a lot, which is consistent with the existing research results.

This work was jointly supported by the National Natural Science Foundation of China (No. 50708051) and Program for Changjiang Scholars and Innovative Research Team in University (IRT00736)
INTRODUCTION

With the development of steel structures, high strength steel, which has many advantages relative to ordinary strength steel when applied in compression members, has been used in many steel structures in the world (IABSE [1], Pocock [2], Shi [3]), and Q420 steel angle members with the nominal yield strength of 420MPa have also been used in many steel structures in China, especially in the transmission tower and the trusswork (Ban [4]). However, there is lack of study on the local buckling behavior of the high strength steel equal angle members under axial compression, which makes it necessary to do the experimental study.

Local buckling behavior can be ignored for hot-rolled ordinary strength steel equal angle compression members, because the width-thickness ratio of the leg is small enough, not exceeding the limit value of the ratio. However, with the increase of the steel strength, the limit value of width-thickness ratio, which is related to the steel strength, is reduced, based on the corresponding regulations in the steel structures design codes of many countries. Consequently, the width-thickness ratios of the leg for some 420MPa steel equal angle sections exceed the limit value, which does not satisfy the requirement of the local buckling. So the local buckling behavior must be considered for the 420MPa steel equal angle members under axial compression, especially for those with small slenderness.

The experiment in this paper is just aiming at the local buckling of 420MPa steel equal angle columns under axial compression.

TEST PROGRAM

Test Specimens

The test was performed on 420MPa steel equal angle stub columns of 5 different section sizes, i.e., L125×8, L140×10, L160×10, L180×12 and L200×14. For each section size, 3 specimens were tested, so that the test program comprised a total of 15 stub column specimens. The slenderness of all the specimens, about the weak axis, was equal to 10, to ensure that the overall buckling behavior was excluded.

The test specimens have been labeled so that the section size and the specimen can be identified from the label. The first letter of the label ‘L’ represents the steel equal angle. The next two numbers following the first letter signify the width of the section and the thickness of the leg respectively. The last character of the label identifies the serial number of specimens with the same section size, and is ‘1’, ‘2’ or ‘3’, since 3 specimens were tested for each section size.

The nominal and measured dimensions of each stub column specimen are shown in Table 1, where the symbols used are defined in Figure 1. The symbols ‘l’, ‘w’ and ‘t’, whose measurements are based on the average values, signify the length of the column, the width of the section and the thickness of the leg, respectively.
TABLE 1
NOMINAL AND MEASURED DIMENSIONS OF TEST SPECIMENS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$l$ (mm) nominal</th>
<th>$l$ (mm) x=1</th>
<th>$l$ (mm) x=2</th>
<th>$l$ (mm) x=3</th>
<th>$w$ (mm) x=1</th>
<th>$w$ (mm) x=2</th>
<th>$w$ (mm) x=3</th>
<th>$t$ (mm) x=1</th>
<th>$t$ (mm) x=2</th>
<th>$t$ (mm) x=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L125×8-x</td>
<td>250</td>
<td>249.8</td>
<td>250.3</td>
<td>250.3</td>
<td>125.0</td>
<td>125.1</td>
<td>124.9</td>
<td>7.965</td>
<td>7.880</td>
<td>7.930</td>
</tr>
<tr>
<td>L140×10-x</td>
<td>278</td>
<td>277.5</td>
<td>278.2</td>
<td>277.9</td>
<td>139.9</td>
<td>140.0</td>
<td>140.1</td>
<td>10.010</td>
<td>9.990</td>
<td>10.005</td>
</tr>
<tr>
<td>L160×10-x</td>
<td>320</td>
<td>319.5</td>
<td>319.5</td>
<td>320.0</td>
<td>160.3</td>
<td>159.8</td>
<td>160.2</td>
<td>9.915</td>
<td>9.910</td>
<td>9.905</td>
</tr>
<tr>
<td>L180×12-x</td>
<td>358</td>
<td>355.9</td>
<td>359.3</td>
<td>358.2</td>
<td>179.6</td>
<td>179.6</td>
<td>179.7</td>
<td>11.880</td>
<td>11.880</td>
<td>11.885</td>
</tr>
<tr>
<td>L200×14-x</td>
<td>398</td>
<td>398.2</td>
<td>399.2</td>
<td>398.9</td>
<td>201.1</td>
<td>200.6</td>
<td>200.8</td>
<td>13.530</td>
<td>13.785</td>
<td>13.540</td>
</tr>
</tbody>
</table>

Test Configurations

For each specimen, 6 strain gauges were attached at the mid-length to measure the longitudinal strains, and their positions in the section are shown in Figure 2, labeled from ‘2-1’ to ‘2-6’. Two displacement transducers attached to the top end plate, labeled ‘1-1’ and ‘1-2’, were to measure the longitudinal displacement, and another two displacement transducers attached at the mid-length, labeled ‘1-3’ and ‘1-4’, were to measure the transversal displacement. Their positions are also shown in Figure 2.

Overall geometric imperfections in this paper are defined as the deviation of three edge lines of the angle column at the mid-length from a relevant straight line connecting the ends, and are denoted by $v_01$, $v_02$, $v_03$ and $v_04$, as shown in Figure 3. The measured geometric imperfection values of each stub column specimen are shown in Table 2.

The ends of the test specimens were milled flat before testing to allow proper seating on the end plates with the dimensions of 350mm×200mm×40mm, and then on the rigid end platens of the testing rig. The stub columns were tested between pinned end bearings in a servo-controlled test rig. The instrumentation of the pin-ended stub columns consisted of a load cell measuring the axial force, displacement transducers and strain gauges. The test schematic diagram is shown in Figure 4.
TABLE 2
MEASURED GEOMETRIC IMPERFECTION VALUES OF TEST SPECIMENS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$v_{01}$</th>
<th>$v_{02}$</th>
<th>$v_{03}$</th>
<th>$v_{04}$</th>
<th>Specimen</th>
<th>$v_{01}$</th>
<th>$v_{02}$</th>
<th>$v_{03}$</th>
<th>$v_{04}$</th>
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<tbody>
<tr>
<td>L125×8-1</td>
<td>1.020</td>
<td>1.040</td>
<td>1.900</td>
<td>0.840</td>
<td>L160×10-3</td>
<td>0.720</td>
<td>0.820</td>
<td>1.000</td>
<td>0.580</td>
</tr>
<tr>
<td>L125×8-2</td>
<td>1.180</td>
<td>0.880</td>
<td>1.480</td>
<td>0.900</td>
<td>L180×12-1</td>
<td>0.460</td>
<td>0.800</td>
<td>1.420</td>
<td>0.280</td>
</tr>
<tr>
<td>L125×8-3</td>
<td>1.380</td>
<td>1.020</td>
<td>0.860</td>
<td>0.700</td>
<td>L180×12-2</td>
<td>0.280</td>
<td>0.660</td>
<td>1.040</td>
<td>0.520</td>
</tr>
<tr>
<td>L140×10-1</td>
<td>0.800</td>
<td>1.280</td>
<td>0.820</td>
<td>0.920</td>
<td>L180×12-3</td>
<td>0.480</td>
<td>1.700</td>
<td>1.460</td>
<td>0.560</td>
</tr>
<tr>
<td>L140×10-2</td>
<td>1.180</td>
<td>0.680</td>
<td>0.720</td>
<td>1.020</td>
<td>L200×14-1</td>
<td>1.060</td>
<td>0.400</td>
<td>0.700</td>
<td>0.200</td>
</tr>
<tr>
<td>L140×10-3</td>
<td>1.320</td>
<td>1.220</td>
<td>1.480</td>
<td>0.960</td>
<td>L200×14-2</td>
<td>0.840</td>
<td>0.780</td>
<td>1.300</td>
<td>0.860</td>
</tr>
<tr>
<td>L160×10-1</td>
<td>1.000</td>
<td>0.840</td>
<td>0.830</td>
<td>0.980</td>
<td>L200×14-3</td>
<td>0.900</td>
<td>0.920</td>
<td>0.840</td>
<td>0.820</td>
</tr>
<tr>
<td>L160×10-2</td>
<td>1.040</td>
<td>1.000</td>
<td>1.940</td>
<td>1.120</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tension Coupon Tests

For each angle section, 3 tension coupons were prepared for the tension tests, to obtain the mechanical properties of the steel angles. The labels for the coupons signify the section size of angles where they were cut from. The first two letters of the label ‘TL’ mean the tension coupon of the angle. The following number represents the section size, and the last character of the label identifies the serial number of tension coupons with the same section size. The dimensions of tension coupons and their cutting locations in the angle legs are all based on the Chinese mechanical testing code (GB/T 228-2002 [5], GB/T 2975-1998 [6]).

The tension coupons were cut parallel to the rolling direction and equipped with two strain gauges on opposite sides at the mid-length to measure the longitudinal strains. An extensometer was also attached on each coupon at the mid-length to measure the longitudinal deformation, which can be used to measure accurate strains after strain gauges fail due to the excessive deformation.

The stress-strain curves obtained from tension coupon tests are shown in Figure 5, 6, 7, 8 and 9 for L125×8, L140×10, L160×10, L180×12 and L200×14 sections, respectively. In these figures, the strain $\varepsilon$ is the average of the two strain gauge readings and the stress $\sigma$ is the measured load divided by the initial area, which was calculated from the tension coupon’s dimensions measured before testing. The strains greater than 20 000$\mu$ε were obtained from the longitudinal deformation, which was calculated from the extensometer readings, divided by the standard distance 50mm of the extensometer.
The mechanical properties obtained from the tension coupon tests are shown in Table 3, where \( \sigma_y \) is the yield strength and \( \sigma_u \) is the ultimate strength. It can be seen from the table that for all the steel angles, the measured values of the yield strength exceed the nominal value 420MPa.

<table>
<thead>
<tr>
<th>Section</th>
<th>Nominal values</th>
<th>Measured values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_y ) (MPa)</td>
<td>( \sigma_y ) (MPa)</td>
</tr>
<tr>
<td>L125×8</td>
<td>420</td>
<td>442.1</td>
</tr>
<tr>
<td>L140×10</td>
<td>420</td>
<td>449.1</td>
</tr>
<tr>
<td>L160×10</td>
<td>420</td>
<td>460.7</td>
</tr>
<tr>
<td>L180×12</td>
<td>420</td>
<td>459.4</td>
</tr>
<tr>
<td>L200×14</td>
<td>420</td>
<td>448.8</td>
</tr>
</tbody>
</table>
TEST RESULTS

The measured ultimate loads $P_u$ of all the specimens are shown in Table 4, and the representative stub columns after testing are shown in Figure 10. All the specimens have the local buckling deformation.

### TABLE 4
MEASURED ULTIMATE LOADS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Measured ultimate loads $P_u$ (kN)</th>
<th>x=1</th>
<th>x=2</th>
<th>x=3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>L125×8-x</td>
<td>854.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L140×10-x</td>
<td>1181.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L160×10-x</td>
<td>1361.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L180×12-x</td>
<td>1904.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L200×14-x</td>
<td>2362.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 10: Representative columns after testing

Figure 11: Comparison between nondimensional test and design strengths

According to the Chinese steel structures design code (GB 50017-2003 [7]), the limit value of the width-thickness ratio $(b/t)_{\text{lim}}$ for hot-rolled steel equal angle compression members is determined as:

$$ (b/t)_{\text{lim}} = (10 + 0.1\lambda) \sqrt{235/f_y} $$

where $30 \leq \lambda \leq 100$ denotes the slenderness of the member and $f_y$ denotes the steel yield strength.

The width-thickness ratio $b/t$ (defined in Figure 1) of all the sections and its limit value calculated by Eqn. 1 ($\lambda=30$) are shown in Table 5, from which it can be seen that the width-thickness ratio values all exceed the limit value, so that the local buckling behavior must be considered.
The ultimate stress $\sigma$ of all the specimens, which is obtained by dividing the ultimate load $P_u$ by the section area $A$, and the comparison with the measured yield strength $\sigma_y$ (seen in Table 3) are shown in Table 6.

### TABLE 6
ULTIMATE STRESSES

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Ultimate stresses $\sigma$ (MPa)</th>
<th>$\sigma/\sigma_y$</th>
<th>x=1</th>
<th>x=2</th>
<th>x=3</th>
<th>Average</th>
<th>x=1</th>
<th>x=2</th>
<th>x=3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>L125×8-x</td>
<td>432.9 413.7 431.1 425.9</td>
<td>0.979 0.936 0.975 0.963</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L140×10-x</td>
<td>431.6 450.1 447.2 443.0</td>
<td>0.961 1.002 0.996 0.986</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L160×10-x</td>
<td>432.1 449.3 437.5 439.6</td>
<td>0.938 0.975 0.950 0.954</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L180×12-x</td>
<td>450.8 445.2 433.1 443.0</td>
<td>0.981 0.969 0.943 0.964</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L200×14-x</td>
<td>432.3 429.9 442.3 434.8</td>
<td>0.963 0.958 0.986 0.969</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table 6 it can be seen that the ultimate stresses of all the specimens, except for L140×10-2, are lower than the corresponding steel yield strength, which indicates that the local buckling appears before the steel yielding. Therefore, the local buckling is significant for the 420MPa steel equal angle members with small slenderness under axial compression, and it is the local buckling strength that determines their ultimate strength. Besides, the ratio values $\sigma/\sigma_y$ differ more or less with different width-thickness ratios. The specimens of section L160×10 have the largest width-thickness ratio value, and their ratio values $\sigma/\sigma_y$ are the smallest on average. While the specimens of section L140×10 have the smallest width-thickness ratio value, and their ratio values $\sigma/\sigma_y$ are the largest on average. It shows that with the decrease of the width-thickness ratio, the local buckling strength increases.

The comparison between the nondimensional test strengths $\sigma/\sigma_y$ and the nondimensional design strengths by the American and European steel structures design codes (ANSI/AISC 360-05 [8], Eurocode 3 [9]) is shown in Figure 11, from which it can be seen that the test strengths are all higher than the corresponding design strengths, with an average excess of 24.82% and 34.78% respectively, and with the increase of the width-thickness ratio, the excess range becomes higher. This is consistent with the existing research results. The most important reason is that the effects of imperfections such as nonstraightness and residual stresses are less severe for high strength steel members. Thus, the American and European steel structures design codes can be modified for the 420MPa steel equal angle columns under axial compression to increase the local buckling design strength, so that the advantage of high strength steel can be made full use of.
CONCLUSIONS

Experiments on the local buckling of 420MPa steel equal angle columns under axial compression have been conducted. Based on the test results, the nondimensional test strengths and the design strengths by the American and European steel structures design codes are compared. The influence of the width-thickness ratio is considered.

Based on the experimental research work above, it can be concluded that for the 420MPa steel equal angle columns under axial compression, the local buckling is significant and must be considered. For those whose ultimate strength is determined by the local buckling strength, the local buckling strength increases with the decrease of the width-thickness ratio.

According to the Chinese steel structures design code, the width-thickness ratios of some 420MPa steel equal angle sections exceed the limitation, which does not satisfy the requirement of local buckling. Compared with the nondimensional design strengths by the American and European steel structures design codes, the nondimensional test strengths are higher a lot. This is consistent with the existing research results, because the effects of imperfections such as nonstraightness and residual stresses are less severe for high strength steel members, which will increase the local buckling strength.

The conclusions above can be applied in further study on the local buckling behavior of high strength steel equal angle columns and the modification of the corresponding design codes. They can also be helpful for the application of high strength steel equal angles.

In the follow-up work, the above experimental results will be used to validate the finite element model, and then full parametric study will be conducted on the local buckling of steel equal angles by using this model. Finally the local buckling design method and calculation equations of steel equal angles considering the steel yield strength accurately will be proposed.

REFERENCES

APPLICATION OF THE GENERAL METHOD FOR THE EVALUATION OF THE STABILITY RESISTANCE OF NON-UNIFORM MEMBERS

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KEYWORDS
Eurocode 3; Steel; General Method; Stability; Numerical analysis; Tapered members.

ABSTRACT
Part 1-1 of Eurocode 3 includes a general method that allows the verification of the resistance to lateral and lateral-torsional buckling of structural components. The method uses a Merchant-Rankine type of empirical interaction expression to split the in-plane effects and the out-of-plane effects. Conceptually, the method is an interesting approach because it deals with the structural components using a unique segment length for the evaluation of the stability with respect to the various buckling modes, Müller [1]. In addition, for more sophisticated design situations that are not covered by code rules but need finite element analysis, the method simplifies this task. It is the purpose of this paper to: (i) discuss the theoretical background of this method; (ii) carry out advanced numerical simulations using GMNIA calculations for some examples considering uniform and non-uniform members; and (iii) compare and analyze the results of the numerical simulations and the procedures of clauses 6.3.1 to 6.3.3 of EC3. Finally, the safety of the method is assessed for the range of examples and some guidelines for its application are established.

INTRODUCTION

Theoretical Background

The general method, as given in EN 1993-1-1 in clause 6.3.4, EC3-1-1 [2] states that the overall resistance to out-of-plane buckling can be verified by ensuring that:

$$Z'_{op} \alpha_{ult,k} / \gamma_M \geq 1$$

(1)

where $\alpha_{ult,k}$ is the minimum load amplifier of the design loads to reach the characteristic resistance of the most critical cross section of the structural component, considering its in-plane behavior without taking lateral or lateral-torsional buckling into account however accounting for all effects due to in-
plane geometrical deformation and imperfections, global and local, where relevant. $\chi_{\text{op}}$ is the reduction factor for the non-dimensional slenderness to take into account lateral and lateral-torsional buckling and $\gamma_{M1}$ is the partial safety factor for instability effects (adopted as 1.0 in most National Annexes). The global non dimensional slenderness $\bar{\lambda}_{\text{op}}$ for the structural component, used to find the reduction factor $\chi_{\text{op}}$ in the usual way using an appropriate buckling curve, should be determined from

$$\bar{\lambda}_{\text{op}} = \sqrt{\frac{\alpha_{\text{ult,k}}}{\alpha_{\text{cr,op}}}}$$  \hspace{1cm} (2)

where $\alpha_{\text{cr,op}}$ is the minimum amplifier for the in-plane design loads to reach the elastic critical resistance of the structural component with respect to lateral or lateral-torsional buckling without accounting for in-plane flexural buckling. In the determination of $\alpha_{\text{cr,op}}$ and $\alpha_{\text{ult,k}}$, finite element analysis may be used.

According to EC3-1-1 [2], $\chi_{\text{op}}$ may be taken either as a minimum or an interpolated value between $\chi$ (for lateral buckling, according to clause 6.3.1 of EC3-1-1) or $\chi_{LT}$ (for lateral-torsional buckling, according to clause 6.3.2).

**Methodology**

Defining $\alpha_u$ as the ultimate load multiplier with respect to the applied axial force, the application of the General Method for a pinned column subjected to an arbitrary axial force $N_{Ed}$ is given by:

$$\alpha_u^{GM} = \chi_{\text{op}} \chi_y \frac{N_{pl,Ed}}{N_{Ed}} \geq 1.0$$  \hspace{1cm} (3)

For an unrestrained beam, application of the General Method exactly coincides with the application of clause 6.3.2.

Figure 1 summarizes the numerical and theoretical procedures for the calculation of the ultimate load factor, according to the General Method., for the more general case of beam-columns.

More details of the application of the General Method to columns, beams and beam-columns can be found in detail in Simões da Silva et al [3].

**NUMERICAL MODEL**

The finite element model was implemented using the commercial finite element package LUSAS, version 14 [4]. Eight-noded “semiloof” thin shell elements (QSL8) were used. S235 steel grade was considered in the reference examples, with a yield stress of 235 MPa, a modulus of elasticity of 210 GPa, and a Poisson’s ratio of 0.3. Perfect elastic-plastic behaviour of the material was considered.

Loading is applied with reference to the plastic resistance values of the smaller cross-section. Concerning imperfections, a geometrical imperfection of sinusoidal type relative both to yy and zz axis is considered with a maximum value of $L/1000$. Regarding material imperfections, residual stresses for welded cross-sections are adopted.

**DISCUSSION**

**Reduction Factor $\chi_{\text{op}}$**

According to EC3-1-1 [2], $\chi_{\text{op}}$ may be taken either as: (i) the minimum value of $\chi$ (for lateral buckling,
The General Method (Clause 6.3.4 of EC3-1-1) involves in-plane and out-of-plane resistance calculations. The in-plane resistance is evaluated analytically or numerically, depending on the clause 6.3.3 (in-plane) with \( \chi_{lt} = 1 \). Cross section resistance at end sections is calculated using GMNIA in-plane calculations and LEA calculations.

The out-of-plane elastic critical load is calculated using interaction formulae from Trahair (1993), where \( \alpha_{ult,k} \) corresponds to the critical cross section:

\[
\frac{R_{k} y}{R_{k} y} + \frac{R_{k} y}{R_{k} y} \leq 1
\]

Where

\[
\Phi = \frac{N_{Rk}}{M_{y,Rk}}
\]

It is noted that ECCS TC8 [6] recommends the use of the first option only. However, when using that option and \( \alpha_{ult,k} \) is evaluated according to clause 6.3.3, over conservative results are noticed in the extremes of the interaction curve if the buckling curve for flexural buckling is different than the buckling curve for lateral-torsional buckling.

This will occur around \( \Phi = 0 \), in case the reduction factor \( \chi_c \) is smaller than \( \chi_{lt} \), and analogously around \( \Phi = \infty \), in case reduction factor \( \chi_c \) is smaller than \( \chi_{lt} \). However, if \( \chi_{op} \) is calculated with Eqn. (6) (interpolated value between \( \chi_c \) and \( \chi_{lt} \)), the discontinuity in the interaction curve disappears, as

\[
\chi_{op} = \frac{\Phi + 1}{\Phi + \chi_{lt}}
\]

Figure 1: Application of the General Method to beam-columns according to clause 6.3.1 of EC3-1-1) or \( \chi_{lt} \) (for lateral-torsional buckling, according to clause 6.3.2); or (ii) an interpolated value between \( \chi \) and \( \chi_{lt} \) (determined as in (i)), by using the formula for \( \alpha_{ult,k} \) corresponding to the critical cross section:

\[
\frac{N_{Ed}}{N_{Rk}} + \frac{M_{y,Ed}}{M_{y,Rk}} \leq \chi_{op} \Rightarrow \chi_{op} = \frac{\Phi + 1}{\Phi + \chi_{lt}}
\]
To illustrate this, two cases are chosen such that: (a) \( \chi_z < \chi_{LT} \) and (b) \( \chi_{LT} < \chi_z \). The results are plotted in the interaction curves of Fig. 2, considering the results of clause 6.3.4 for beam-columns. For comparison, results of clause 6.3.4 considering pure axial force; results of clause 6.3.4 considering pure bending moment; and results of clause 6.3.3 are also plotted.

![Interaction curves](image)

**Figure 2:** Interaction curves regarding the value of \( \chi_{op} \)

Regarding the middle zone of the interaction curve, General Method appears to be very similar to clause 6.3.3, when considering the interpolated value between \( \chi \) and \( \chi_{LT} \). However, in [5] it was concluded that, for beam-columns, the ratio \( \alpha_{GM/6.3.3} \) can vary between approximately 80% to 115%, even when considering the minimum value of \( \chi_{op} \). Analogous to Figure 2, an example considering a IPE 200 with \( \lambda_z = 2 \), is chosen for illustration in Figure 3. In this case, considering the interpolated value between \( \chi \) and \( \chi_{LT} \) can lead to higher resistance than clause 6.3.3 mainly in the middle area of the interaction curve. Nevertheless, results of the General Method are always on the safe side relatively to GMNIA analysis.

A more detailed parametric study is needed in order to establish the limits of the application of either the minimum or the interpolated value of \( \chi \) and \( \chi_{LT} \).

**Cross-Sectional Class, Critical Cross-Section and Buckling Curve**

Upon the verification of the resistance of a tapered member, several assumptions have to be taken, which are not clear for the designer, as they are neither defined nor explained in the codes.

Firstly, assume a non-uniform member subject to a uniformly distributed loading and a relatively low axial force, see Figure 4. Assume also that for a short interval of the member it is class 3 due to the existence of axial force and to the characteristics of the cross-section in that interval.

On the safe side, an elastic verification considering class 3 cross-section should be performed, however, analyzing qualitatively a case like the example in Figure 4, the stresses in the interval corresponding to class 3 cross-section are not critical compared to the stresses in the other extreme of the tapered member. For now, only with an exhaustive evaluation of the stresses along the length of the beam-column would lead to any conclusions of a position for the assumption of the class of cross-section, but this is not practical.

On the other hand, when verifying the stability of the member, the characteristics of a certain cross-section in the member have to be defined in order to apply the rules of the codes which were derived...
for uniform members. The example of Figure 5 consists of a tapered member which varies from a modified IPE 200 with a height of $h=400\ mm$ to a IPE 200, with a length of $L=2.14\ m$. The theoretical approach of the General Method is chosen for verification of resistance. Analyzing the results of Figure 5, on the safe side, the cross-section with the smaller height of the web could be chosen but this yields very safe results. Choosing the cross-section with the maximum height can also lead to unsafe results. Finally, in literature expressions for the characteristics of an equivalent cross-section can be found, for example in Hirt and Crisinel [8] and Gálea [9]. However, there is no validation concerning the use of these equivalent properties and the buckling curves defined in EC3-1-1 [2].

Finally, if numerical results regarding in-plane GMNIA calculations and LEA (Linear Eigenvalue Analysis) are available, the verification of stability according to General Method should be performed simply by taking use of the numerical results and, at some point, of the buckling curve. However, in a tapered member, the buckling curve may vary along the member. In the parametric study of this paper, for lateral-torsional buckling, the member is composed of curve $d$ along the member and curve $c$ at the smaller extreme. One should think that curve $d$ is to be considered without any doubts, but it will be shown that the results using curve $d$ can be 70% more conservative than full GMNIA analysis. Even using curve $c$, the differences reach up to 50%. In Figure 5, the numerical results of the General Method are also illustrated for comparison.

General Method should simplify the procedure regarding all the questions which arise when verifying the buckling resistance of a tapered member, however it seems that the main problem of its application lies in the definition of the buckling curve. Therefore, to overcome this problem, at the
moment, a parametric study with a wide range of tapered members is being carried out in order to calibrate new buckling curves for tapered members, regarding flexural buckling in-plane and out-of-plane; torsional buckling; and lateral-torsional buckling.

PARAMETRIC STUDY

Definition

Only simply supported single span members with end fork conditions are studied in this paper. Concerning the cross section, taking the IPE 200 as the minimum cross-section, the longitudinal variation of the web varies according to Figure 6.

Table 1 summarizes the sub-set of cases to be compared with the advanced numerical simulations. For all cases, besides full 3D GMNIA numerical simulations, constrained in-plane GMNIA calculations and LEA (Linear Eigenvalue Analysis) are also carried out to provide data for the application of the General Method.

<table>
<thead>
<tr>
<th>Sub-set</th>
<th>Level of sub-set</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tapering ratio</td>
<td>IPE 200, ( \alpha=1.0 ) (uniform)</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Mod. IPE 200-240, ( \alpha=1.2 ) (tapered)</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Mod. IPE 200-300, ( \alpha=1.5 ) (tapered)</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Mod. IPE 200-400, ( \alpha=2.0 ) (tapered)</td>
<td>20</td>
</tr>
<tr>
<td>Level of loading</td>
<td>Beam</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Beam-column</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Column</td>
<td>52</td>
</tr>
<tr>
<td>Slenderness</td>
<td>( \bar{\alpha}_{z,IPE200} ) 0.5</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>27</td>
</tr>
<tr>
<td>Type of restraining</td>
<td>Unrestrained</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>Restrained along the tension flange</td>
<td>44</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>108</td>
</tr>
</tbody>
</table>

Regarding flexural buckling, buckling curve is \( c \) for all cases. For lateral-torsional buckling, buckling curve is \( d \), except when \( h=200 \) mm (IPE 200).

Results

Figure 7 illustrates the mean values ±1 standard deviation of the ratio \( R=\frac{\alpha_{u,GMNIA}}{\alpha_{u,GeneralMethod}} \) regarding the defined subsets of Table 1: (a) tapering ratio; (b) level of loading; (c) slenderness; and (d) type of restraining. Regarding the calculation \( \chi_{LT} \) for the General Method, both cases are considered: curve \( c \) and curve \( d \). General Case of clause 6.3.2.2 of EC3-1-1 [2] was adopted. \( \alpha_{ult,k} \) and \( \alpha_{cr,op} \) were evaluating considering the numerical results of in-plane GMNIA and LEA calculations, respectively. Regarding the level of loading, beam-columns are arranged by \( \Phi=\{0.5; 1; 2\} \) (\( \Phi \) was...
obtained considering the properties of the smaller cross-section for $M_{y,Rk}$ and $N_{Rk}$). Finally, $\chi_{op}$ is calculated considering the interpolated value between $\chi$ and $\chi_{LT}$.

Analyzing Figure x, clear trends are defined within each subset. General Method becomes more conservative with the increase of the tapering ratio; with the increase of bending moment relatively to axial force; and also with increase of the length of the member. The latter was already observed in Simões da Silva et al [3], by performing a statistical analysis on uniform members, according to Annex D of EN 1990 [7]. The type of restraining doesn’t seem to affect the results of the General Method relatively to GMNIA analysis, however more types of restraining systems have to be studied to achieve defined conclusions regarding that sub-set.

Regarding both solutions adopted considering either curve $c$ or curve $d$ for lateral-torsional buckling, the difference between these solutions increases with the increase of bending moment, achieving a maximum of 14% for beam-columns and 23% for beams. General Method always yields safer results than GMNIA analysis. For the solution in which curve $c$ is adopted, differences between the General Method and GMNIA analysis achieve a maximum of 50% on the safe side.

**CONCLUSION**

In this paper the resistance of non-uniform members according to General Method was analyzed and it was seen that, using in-plane GMNIA and LEA numerical simulations to achieve resistance according to the General Method, it is possible to avoid the difficult task of classifying the cross-
section and knowing the position of the critical cross-section for use of its properties in the verification of stability. On the other hand, it was seen that the existent buckling curves in EC3-1-1 [2], which are calibrated with uniform members, give over conservative results when clause 6.3.4 is applied, up to 50% and 70% relatively to a full 3D GMNIA analysis, if buckling curve $c$ or $d$ is used for the calculation of $\chi_{op}$, respectively. It is shown in Simões da Silva et al [3] that, for uniform members, differences up to $\pm 20\%$ between clause 6.3.3 and clause 6.3.4 (theoretical approach). Numerical results of the General Method were also within that range, and consequently, this Method gives good results for uniform members.

As a result, at the moment buckling curves for non-uniform members are being derived, hoping that the application of this method can be a reliable and simple alternative for the verification of the overall resistance of tapered members.

Finally, it was also seen that for the extremes of the interaction curve the minimum value of $\chi$ and $\chi_{LT}$ might not correspond to the real type of buckling mode. Using an interpolated value of $\chi$ and $\chi_{LT}$ (Eqn. 5) leads to more accurate results. Therefore, a more detailed parametric study is needed in order to establish the limits of the application of the reduction factor $\chi_{op}$ along the interaction curve.

ACKNOWLEDGEMENTS

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REFERENCES

STRUCTURAL BEHAVIOUR OF ELLIPITCAL HOLLOW SECTIONS UNDER COMBINED ACTIONS

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KEYWORDS
Combined bending and axial compression, Eccentric compression, Experiments, Elliptical hollow sections, Steel structures, Testing.

ABSTRACT

The structural behaviour of hot-rolled elliptical hollow sections (EHS) under axial load and bending alone has been investigated in previous studies. This paper examines the response of EHS under combined axial load plus bending at the cross-sectional level. A series of laboratory stub column tests with various load eccentricities have been conducted so as to investigate the interaction between bending and axial compression. The measured geometric and material properties of the test specimens, together with the full load-deformation histories have been reported. The generated structural performance data have been utilised to assess cross-section interaction expressions suitable for design.

INTRODUCTION

Owing to their aesthetic appeal and sound structural efficiency, hot-finished elliptical hollow sections (EHS), have been adopted in a number of recent projects including the Honda Central Sculpture in Goodwood, UK, the Society Bridge in Braemar, UK (Corus [1]), and the airport at Barajas in Madrid, Spain (Viñuela-Rueda and Martinez-Salcedo [2]). The authors of the present paper have previously conducted extensive laboratory testing, supported by parallel numerical modelling studies, to examine the behaviour of elliptical hollow sections in compression (Chan and Gardner [3]) and bending (Chan and Gardner [4]). On the basis of the findings, slenderness parameters and slenderness limits for the cross-section classification of EHS have been proposed (Gardner and Chan [5]). Further recent studies on the elastic buckling of elliptical hollow sections (Zhu and Wilkinson [6], Ruiz-Teran and Gardner [7], and Silvestre [8]), the response of filled elliptical tubes (Roufegarinejad and Bradford [9], Zhao et al. [10], Yang et al. [11] and Zhao and Packer [12]) and the behaviour of connections to EHS (Bortolotti et al. [13], Choo et al. [14], Pietrapertosa and Jaspart [15] and Willibald et al. [16]) have also been performed. However, there currently remains a lack of verified design guidance for other structural phenomena. Development of such guidance is underway, and this paper focuses on the scenario for combined bending and axial force at cross-section level. Detailed experimental
studies are described herein and design recommendations for resistance to combined bending and axial force are presented.

**EXPERIMENTAL STUDY**

**Introduction**

A series of tensile material tests, compressive stub column tests and eccentric compression tests have been carried out to investigate the structural behaviour of hot-finished elliptical hollow sections. All tests were performed in the Structures Laboratory of the School of Engineering, University of Warwick. A total of four tensile coupons, four stub columns under uniform compression and eight stub columns under eccentric compression were tested. Two section sizes were employed – EHS 150×75×5 and EHS 150×75×6.3 – both having a cross-sectional aspect ratio of two. All tested material was hot-finished carbon steel, grade 355 supplied by Corus Tubes [17]. This section summarises the testing apparatus, the experimental procedures and the test results obtained.

**Tensile coupon tests**

Tensile coupon tests were performed to establish the basic material stress-strain response; this was subsequently utilised during the analysis of the test results and in the development of numerical models. The tests were carried out in accordance with EN 10002-1 (CEN [18]). Parallel coupons were machined longitudinally from the two flattest portions of the cross-sections (i.e. along the centrelines of the minor axis). Two coupon tests, designated TC1 and TC2, were performed for each section size. The key results from the four coupon tests are summarised in Table 1.

<table>
<thead>
<tr>
<th>Tensile coupons</th>
<th>Width $b_a$ (mm)</th>
<th>Thickness $t$ (mm)</th>
<th>Young’s modulus $E$ (N/mm²)</th>
<th>Yield stress $f_y$ (N/mm²)</th>
<th>Ultimate tensile stress $f_u$ (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150×75×5.0-TC1</td>
<td>19.85</td>
<td>4.61</td>
<td>211800</td>
<td>377</td>
<td>501</td>
</tr>
<tr>
<td>150×75×5.0-TC2</td>
<td>19.85</td>
<td>4.63</td>
<td>213000</td>
<td>365</td>
<td>506</td>
</tr>
<tr>
<td>150×75×6.3-TC1</td>
<td>19.84</td>
<td>6.47</td>
<td>216300</td>
<td>410</td>
<td>529</td>
</tr>
<tr>
<td>150×75×6.3-TC2</td>
<td>19.83</td>
<td>6.38</td>
<td>216600</td>
<td>408</td>
<td>529</td>
</tr>
</tbody>
</table>

**Stub Column Tests**

Four stub columns were tested in pure axial compression to assess load carrying capacity and deformation capacity. Full load-end shortening curves were recorded, including into the post-ultimate range. The nominal lengths of the stub columns were chosen such that they were sufficiently short not to fail by overall buckling, yet still long enough to contain a representative residual stress pattern. The stub column lengths were taken as two times the larger cross-sectional dimension. The stub column tests arrangement is shown in Figure 1. The end platens of the testing arrangement were flat and parallel. Four linear variable displacement transducers (LVDTs) were used to determine the end shortening of the stub columns between the end platens of the testing machine. Four linear electrical resistance strain gauges were affixed to each specimen at mid-height, and at the ends of the major and minor axes. The strain gauges were initially used for alignment purposes, and later to modify the end shortening data from the LVDTs to eliminate the elastic deformation of the end platens. Load, strain, displacement and input voltage were all recorded using the data acquisition equipment ORION.

Measurements of major and minor axis diameters ($2a$ and $2b$, respectively), material thickness $t$ and stub column length $L$ were taken. The mean measured dimensions and maximum geometric local imperfections $\omega_0$ of the test specimens for the four stub column specimens are presented in Table 2;
cross-section geometry and notation is defined in Figure 2. The cross-sectional area for the EHS stub columns is defined using the exact formulae adopted by the authors in previous studies (Chan and Gardner [3]). Two stub column tests, designated SC1 and SC2, were performed for each section size.

![Figure 1: Stub column tests](image1)

![Figure 2: Geometry of an elliptical hollow section](image2)

### TABLE 2
MEAN MEASURED DIMENSIONS AND KEY RESULTS FROM STUB COLUMN TESTS

<table>
<thead>
<tr>
<th>Stub column designation</th>
<th>Larger outer diameter $2a$ (mm)</th>
<th>Smaller outer diameter $2b$ (mm)</th>
<th>Thickness $t$ (mm)</th>
<th>Length $L$ (mm)</th>
<th>Measured maximum local imperfection $\omega_b$ (mm)</th>
<th>Ultimate load $N_u$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150×75×5.0-SC1</td>
<td>150.25</td>
<td>75.80</td>
<td>4.85</td>
<td>300.00</td>
<td>0.099</td>
<td>671</td>
</tr>
<tr>
<td>150×75×5.0-SC2</td>
<td>150.05</td>
<td>76.00</td>
<td>4.90</td>
<td>300.05</td>
<td>0.072</td>
<td>676</td>
</tr>
<tr>
<td>150×75×6.3-SC1</td>
<td>148.15</td>
<td>76.05</td>
<td>6.72</td>
<td>300.15</td>
<td>0.078</td>
<td>973</td>
</tr>
<tr>
<td>150×75×6.3-SC2</td>
<td>148.85</td>
<td>76.05</td>
<td>6.64</td>
<td>300.15</td>
<td>0.036</td>
<td>990</td>
</tr>
</tbody>
</table>

Compression tests on stub columns reveal the average compressive response of the cross-sections. Ultimate failure is due to local buckling of the cross-section. For cross-sections comprising slender elements local buckling may occur in the elastic range. For more stocky cross-sections, local buckling may occur following significant inelastic deformation. Measured end shortening readings from the LVDTs were modified on the basis of the strain gauge readings to account for the elastic deformation of the end platens (that are present in the LVDT measurements). A summary of the key test results including ultimate test load $N_u$ is also given in Table 2. The results of the stub column tests are analysed and discussed in the following section.

**Eccentric Compression Tests**

The primary aim of the eccentric compression tests was to investigate the cross-section response of EHS under combined bending and axial compression. The load was introduced through hardened steel knife-edges fixed to the ends of the specimens. The load eccentricity was varied to provide a range of proportions of axial load to bending, with the resulting $N_u/N_{c,Rd}$, where $N_{c,Rd}$ is the cross-section compression resistance, ranging between 0.26 and 0.76. The nominal eccentricities about the minor axis were 25 mm and 75 mm whilst the nominal eccentricities about the major axis are 25 mm and 100 mm. The nominal column lengths were 300mm. The general testing configuration is depicted in Figure 3. The loading, $N$ was applied through the knife-edge at an eccentricity, $e$ to the centroidal axis of the specimen, resulting in a uniform moment ($= Ne$) along the column length, prior to lateral deformation – see Figure 4a. The loading was recorded by a 1000 kN load cell located at the top end.
of the columns. Vertical displacement was measured at the loaded end of the columns by two LVDTs, whilst two inclinometers were positioned at each end of the columns to measure end rotation. As discussed by Fujimoto et al. [19], the deformation of the specimens generate a further second order moment $M_2 = N\delta$, where $\delta$ is the lateral deflection, and the maximum moment at the mid-height is equal to $M_{1,2} = N(e+i\delta_{\text{mid}})$ (Figure 4b). An additional LVDT was located at the mid-height of the columns to measure the lateral deflection. Eight linear electrical resistance strain gauges were affixed to the section to measure the strain distribution across the section at mid-height. The strain gauge at the extreme fibre, near the lateral LVDT, was offset by 5 mm to avoid contact with the lateral LVDT. Load, strain, displacement and input voltage were all recorded using the data acquisition equipment ORION.

Figure 3: Eccentric compression test arrangement

Figure 4: Bending moment due to eccentric compression

The mean measured dimensions and maximum local geometric imperfections $\alpha_0$ are presented in Table 3. Geometric properties including cross-sectional area and section moduli for the EHS specimens are defined using the exact formulae adopted by the authors in previous studies (Chan and Gardner [3, 4]). The test specimens were labelled such that the nominal section size, type of tests, eccentricity axis and eccentricity value can be easily identified. For example, for specimen 150×75×5.0-EC-MI-25, the “150×75×5.0” designates “nominal major diameter×nominal minor diameter×nominal thickness”; “EC” represents “eccentric compression”; “MI” indicates the applied
eccentric moment about the minor axis and “25” signifies an eccentricity of 25 mm. The key results from the eccentric compression tests have been reported in Table 3.

<table>
<thead>
<tr>
<th>Eccentrically compressed columns</th>
<th>Larger outer diameter 2a (mm)</th>
<th>Smaller outer diameter 2b (mm)</th>
<th>Thickness t (mm)</th>
<th>Length L (mm)</th>
<th>Measured maximum local imp. өαb (mm)</th>
<th>Ultimate load Nu (kN)</th>
<th>Bending moment at N_u, M_{1+2} (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150×75×5.0-EC-MI-25</td>
<td>150.00</td>
<td>76.15</td>
<td>4.81</td>
<td>300.05</td>
<td>0.068</td>
<td>343</td>
<td>10.6</td>
</tr>
<tr>
<td>150×75×5.0-EC-MI-75</td>
<td>150.20</td>
<td>75.90</td>
<td>4.88</td>
<td>300.00</td>
<td>0.080</td>
<td>181</td>
<td>14.4</td>
</tr>
<tr>
<td>150×75×5.0-EC-MA-25</td>
<td>150.50</td>
<td>75.65</td>
<td>4.94</td>
<td>300.05</td>
<td>0.129</td>
<td>490</td>
<td>18.5</td>
</tr>
<tr>
<td>150×75×5.0-EC-MA-100</td>
<td>150.20</td>
<td>76.95</td>
<td>4.84</td>
<td>299.95</td>
<td>0.099</td>
<td>235</td>
<td>27.4</td>
</tr>
<tr>
<td>150×75×6.3-EC-MI-25</td>
<td>148.70</td>
<td>76.00</td>
<td>6.65</td>
<td>300.10</td>
<td>0.073</td>
<td>500</td>
<td>13.6</td>
</tr>
<tr>
<td>150×75×6.3-EC-MI-75</td>
<td>149.55</td>
<td>75.80</td>
<td>6.80</td>
<td>300.10</td>
<td>0.277</td>
<td>248</td>
<td>19.7</td>
</tr>
<tr>
<td>150×75×6.3-EC-MA-25</td>
<td>148.40</td>
<td>76.10</td>
<td>6.72</td>
<td>300.10</td>
<td>0.060</td>
<td>712</td>
<td>30.0</td>
</tr>
<tr>
<td>150×75×6.3-EC-MA-100</td>
<td>148.55</td>
<td>76.00</td>
<td>6.74</td>
<td>300.10</td>
<td>0.138</td>
<td>342</td>
<td>42.4</td>
</tr>
</tbody>
</table>

Full load-lateral deflection relationships for the eccentric compression tests are depicted in Figures 5 (150×75×5.0 specimens) and 6 (150×75×6.3 specimens). The experimental results clearly demonstrate the degradation of axial load capacity with increasing eccentricity (i.e. increasing bending moments). These results have been examined and used for the validation of design expressions in the following section.

**ANALYSIS OF RESULTS AND DESIGN PROPOSALS**

Applying the slenderness parameters and limits proposed by the authors (Chan and Gardner [3]), all tested sections are Class 1-3 (fully effective) for axial compression. The ultimate test load $N_u$ may therefore be normalised by the compression resistance $N_{c,Rd} = A f_y$, where $A$ is the cross-sectional area. The corresponding test moment $M_{1+2} (= N_u \times (e + \sigma_{mid}))$ has been normalised by either the elastic moment resistance $M_{el,Rd} = f_y W_{el}$, where $W_{el}$ is the elastic section modulus, or the plastic moment resistance $M_{pl,Rd} = f_y W_{pl}$, where $W_{pl}$ is the plastic section modulus, depending on the cross-section classification. Applying the slenderness parameters and limits proposed by the authors (Chan and Gardner [4]), all tested sections are Class 3 for bending about the minor axis and Class 1 for bending about the major axis. Therefore, for eccentric compression about the minor axis, the test moment has been normalised by $M_{el,Rd}$ and for eccentric compression about the major axis, the test moment has been normalised by $M_{pl,Rd}$. The normalised test results are summarised in Table 4 and plotted in Figure 7. The results demonstrate the degradation of bending resistance as the axial load increases.

The reduction in bending moment resistance in the presence of axial load for Class 1 and 2 EHS (those being eccentrically compressed about the major axis in this study) has been investigated by Nowzartash and Mohareb [20]; their proposal is given by Eqn. 1.

$$\left( \frac{M_{y,Ed}}{M_{pl,y,Rd}} \right)^{2} + 2\left( \frac{N_{Ed}}{N_{c,Rd}} \right)^{1.75} \leq 1.0$$

(1)

where $M_{y,Ed}$ = design bending moment about the major ($y$-$y$) axis, $M_{pl,y,Rd}$ = design plastic bending resistance about the major ($y$-$y$) axis, $N_{Ed}$ = design axial force and $N_{c,Rd}$ = design cross-section resistance under uniform compression.

For Class 3 cross-sections (those being eccentrically compressed about the minor axis in this study), Eurocode 3 Part 1-1 (CEN [21]) specifies that the maximum longitudinal stress shall less than the
material yield stress, which can be expressed by Eqn. 2. This is also applicable as a conservative treatment for Class 1 and 2 cross-sections.

\[
\left( \frac{N_{Ed}}{N_{c,Rd}} \right) + \left( \frac{M_{z,Ed}}{M_{el,z,Rd}} \right) \leq 1.0 \tag{2}
\]

where \( M_{z,Ed} \) = design bending moment about the minor \((z-z)\) axis and \( M_{el,z,Rd} \) = design elastic bending resistance about the minor \((z-z)\) axis.

The two interaction formulations (Eqns. 1 and 2) have been plotted, together with the test data, in Figure 7. The curves may be seen to provide safe-side predictions of the test data for resistance to combined bending and axial compression. It is therefore recommended that Eqns. 1 and 2 may be applied to EHS with Class 1-2 and Class 3 cross-sections, respectively. Numerical simulations are underway to extend the range of structural performance data to further confirm the applicability of the above design expressions.
TABLE 4
SUMMARY OF NORMALISED RESULTS FROM CONCENTRIC AND ECCENTRIC COMPRESSION TESTS

<table>
<thead>
<tr>
<th>Specimen designation</th>
<th>Section classification (compression)</th>
<th>Section classification (bending)</th>
<th>$N_u/N_{C,Rd}$</th>
<th>$M_{1+2}/M_{el,Rd}$ or $M_{1+2}/M_{pl,Rd}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150×75×5.0-SC1</td>
<td>Fully effective</td>
<td>-</td>
<td>1.06</td>
<td>-</td>
</tr>
<tr>
<td>150×75×5.0-SC2</td>
<td>Fully effective</td>
<td>-</td>
<td>1.06</td>
<td>-</td>
</tr>
<tr>
<td>150×75×5.0-EC-MI-25$^1$</td>
<td>Fully effective</td>
<td>3</td>
<td>0.55</td>
<td>0.87</td>
</tr>
<tr>
<td>150×75×5.0-EC-MI-75$^1$</td>
<td>Fully effective</td>
<td>3</td>
<td>0.29</td>
<td>1.17</td>
</tr>
<tr>
<td>150×75×5.0-EC-MA-25</td>
<td>Fully effective</td>
<td>1</td>
<td>0.76</td>
<td>0.70</td>
</tr>
<tr>
<td>150×75×5.0-EC-MA-100</td>
<td>Fully effective</td>
<td>1</td>
<td>0.37</td>
<td>1.05</td>
</tr>
<tr>
<td>150×75×6.3-SC1</td>
<td>Fully effective</td>
<td>-</td>
<td>1.04</td>
<td>-</td>
</tr>
<tr>
<td>150×75×6.3-SC2</td>
<td>Fully effective</td>
<td>-</td>
<td>1.07</td>
<td>-</td>
</tr>
<tr>
<td>150×75×6.3-EC-MI25$^1$</td>
<td>Fully effective</td>
<td>3</td>
<td>0.54</td>
<td>0.79</td>
</tr>
<tr>
<td>150×75×6.3-EC-MI75$^1$</td>
<td>Fully effective</td>
<td>3</td>
<td>0.26</td>
<td>1.13</td>
</tr>
<tr>
<td>150×75×6.3-EC-MA25</td>
<td>Fully effective</td>
<td>1</td>
<td>0.76</td>
<td>0.79</td>
</tr>
<tr>
<td>150×75×6.3-EC-MA100</td>
<td>Fully effective</td>
<td>1</td>
<td>0.36</td>
<td>1.12</td>
</tr>
</tbody>
</table>

$^1$ Results normalised by $M_{el,Rd}$

Figure 7: Test results and interaction curves for combined bending and axial compression

CONCLUSIONS

The cross-section response of hot-finished elliptical hollow sections (EHS) under combined bending and axial compression has been examined in this study. A total of four tensile coupon tests, four stub columns tests under pure compression and eight under eccentric compression (four about the minor axis and four about the major axis) were performed. Various load eccentricities were considered to vary the proportion of axial load to bending moment. The key material properties, geometric measurements and test results have been reported. On the basis of the experimental results, the interaction formulae for Class 1 and 2 cross-sections and Class 3 cross-sections have been assessed and found to provide safe predictions of the observed test response. Numerical simulations are underway to extend the range of structural performance data to further validate the proposed interaction formulae.
ACKNOWLEDGEMENTS

The authors would like to thank Corus Tubes for the supply of test specimens and Colin Banks, Michael Davies and Ryan Griffith (University of Warwick) for their assistance with the experiments.

REFERENCES

STRAIN HARDENING IN INDETERMINATE STEEL STRUCTURES

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KEYWORDS

Bending, Continuous beams, Deformation capacity, Indeterminate structures, Plastic design, Portal frames, Steel structures, Strain hardening.

ABSTRACT

Traditional plastic analysis and design of indeterminate steel structures is based on the formation and subsequent rotation of plastic hinges at their full plastic moment capacity $M_{pl}$. However, the occurrence of strain hardening results in moment capacities that are often significantly higher than $M_{pl}$. This paper outlines recent advances in the development of a deformation based approach to steel design, referred to the continuous strength method (CSM), focussing primarily on extension of the method from isolated elements to indeterminate structures. Comparisons are made between test results on continuous steel beams, generated as part of the present study, and the predictions of the CSM and Eurocode 3. Comparisons with the collapse load of a steel portal frame are also presented. For all cases considered, the continuous strength method, through a rational exploitation of strain hardening, offers more accurate prediction of observed physical behaviour.

INTRODUCTION

Indeterminate steel structures, such as continuous beams and portal frames, are generally designed using traditional plastic analysis methods, which are based on the formation and subsequent rotation of plastic hinges at their full plastic moment capacity. The formation of each plastic hinge causes a progressive reduction in stiffness of the structure until the final hinge forms resulting in a collapse mechanism. In reality though, plastic hinges do not rotate at a constant moment equal to $M_{pl}$ of the section due to the occurrence of strain hardening, with stockier sections often achieving resistances significantly beyond those predicted by current design approaches. A new design approach, the continuous strength method (CSM), has been developed offering a systematic means of utilising strain hardening, based on cross-section deformation capacity. The method allows the attainment of bending resistances beyond $M_{pl}$ and enables the accurate prediction of ultimate structural collapse loads. Development and application of the method to indeterminate structures are described herein.
CSM FOR DETERMINE constituted STRUCTURES

The continuous strength method (CSM) [1] is a deformation based approach to structural steel design, presented as an alternative to the current practice of cross-section classification, which is a step-wise system of allowing for the influence of local buckling. The CSM recognises that the resistance of structural cross-sections is a continuous function of their deformation capacity, as controlled by the slenderness (and hence propensity to local buckling) of the constituent plate elements. The method employs a continuous ‘base curve’, defining the relationship between cross-section slenderness and cross-section deformation capacity, together with a material model that allows for the influence of strain hardening, and applies only to fully effective (i.e. non-slender) sections. Determination of cross-section capacities in compression and bending, incorporating recent developments to the method, are summarised in the following sub-sections.

Cross-section Compression Resistance

Within the continuous strength method, cross-section slenderness is defined through Eqn. 1 by the plate slenderness of the most slender constituent element in the section:

\[ \lambda_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \]  

where \( f_y \) is the material yield strength and \( \sigma_{cr} \) is the elastic buckling stress, taking due account of element support conditions and applied stress distribution, as set out in EN 1993-1-5 [2]. The corresponding normalised deformation capacity of the cross-section \( \varepsilon_{LB}/\varepsilon_y \) is then obtained through the base curve, given by Eqn. 2.

\[ \frac{\varepsilon_{LB}}{\varepsilon_y} = \frac{1.65}{\lambda_p^{1.15+2.2\varepsilon_y}} - 2.7 \leq 15 \quad \text{for } \lambda_p < 0.748 \]  

in which \( \varepsilon_y = f_y/E \) is the yield strain of the material, where E is Young’s modulus, and \( \varepsilon_{LB} \) is the local buckling strain of the section. The base curve (Eqn. 2) was generated, as described in [1], on the basis of stub column tests. In interpreting the test data, for stocky sections, where the ultimate load \( F_u \) is greater than the yield load \( F_y \), the local buckling strain is defined as the end shortening at ultimate load \( \delta_u \) normalised by the stub column length \( L \), as given by Eqn. 3, while for slender sections (\( F_u < F_y \)), where the response is influenced by elastic post-buckling behaviour, the normalised local buckling strain \( \varepsilon_{LB}/\varepsilon_y \) is defined as the ratio of the ultimate load \( F_u \) to the yield load \( F_y \), as given by Eqn. 4. Since slender sections fail below their yield load, where stress is proportional to strain, adoption of Eqn. 4 yields a normalised relationship between deformation capacity and slenderness that is similar to that between strength and slenderness given by the familiar Winter curve.

\[ \frac{\varepsilon_{LB}}{\varepsilon_y} = \frac{\delta_u/L}{f_y/E} \quad \text{for } F_u \geq F_y \]  

\[ \frac{\varepsilon_{LB}}{\varepsilon_y} = \frac{F_u}{F_y} \quad \text{for } F_u < F_y \]  

Following recent developments, the base curve defined by Eqn. 2 now differs from that presented previously [1] due to: (1) Availability of further test data [3-4] upon which to establish the curve; (2) Element slenderness is defined using flat plate widths, in line with EN 1993-1-1 [5], rather than centreline dimensions; (3) Applicability of the method has been limited to sections where \( \lambda_p < 0.748 \), with more slender sections being covered by the existing effective width or direct strength methods;
A limitation has been placed on the normalised local buckling strain $\varepsilon_{LB}/\varepsilon_y$ of 15, which corresponds to the material ductility requirement expressed in EN 1993-1-1 [5]. Having established the local buckling strain of the section, the local buckling stress $\sigma_{LB}$ is determined directly from the strain hardening material model, for which the bi-linear elastic-linear hardening representation, with a strain hardening slope of $E/100$, as recommended in EN 1993-1-5 [2], has been adopted. Finally, the cross-section compression resistance $N_{c,Rd}$ is given by the product of the local buckling stress $\sigma_{LB}$ and the gross cross-section area $A$. While a strain hardening modulus $E_{sh}$ of $E/100$ has been adopted for all section types in the current study, this value has been found to be conservative for cold-formed sections, partly due to the influence of the enhanced strength corner regions, and a value of $E/50$ may be acceptable.

**Cross-section Bending Resistance**

In-plane bending resistance may be calculated on a similar basis to compression resistance, whereby the deformation capacity $\varepsilon_{LB}$ of the cross-section is limited either by local buckling of the web in bending or the compression flange in pure compression, and the moment resistance may be calculated by means of integration of the material model through the depth of the cross-section, assuming a linearly varying strain distribution. Integration may not be appropriate for practical design, and in order to simplify the bending resistance calculation, a direct relationship between normalised bending resistance $M_{c,Rd}/M_{pl}$ and normalised local buckling strain $\varepsilon_{LB}/\varepsilon_y$ has been developed for plated sections. Three different stages of behaviour: (1) Elastic, (2) Elastic-plastic and (3) Strain-hardening have been identified. The continuous strength method addresses stages (2) and (3); in the elastic range, where $\varepsilon_{LB}/\varepsilon_y < 1$, the moment capacity may be calculated by existing methods [5]. In the elastic-plastic range, where $1 < \varepsilon_{LB}/\varepsilon_y \leq 3$, a non-linear reduction in moment capacity from the full plastic moment $M_{pl}$ at $\varepsilon_{LB}/\varepsilon_y = 3$ to the elastic moment $M_{el}$ at $\varepsilon_{LB}/\varepsilon_y = 1$, as given by Eqn. 5 and illustrated in Figure 1, is proposed, where $M_{el,web} = t_w b_w f_y/6$ is the elastic moment capacity of the web(s) of the section. Choice of the point at which $M_{pl}$ is reached, namely $\varepsilon_{LB}/\varepsilon_y = 3$, is based on the findings of Bruneau et al. [6]; this value was verified in the present study by analytical means and further supported by the available experimental data. Alternative transitions between elastic and fully plastic responses have been proposed – Juhas [7] presented a strain based approach similar to that described herein, while Lechner et al. [8] proposed a linear transition with slenderness and also considered combined loading.

$$\frac{M_{c,Rd}}{M_{pl}} = 1 - \frac{1}{2} \frac{M_{el,web}}{M_{pl}} \left[ \frac{\varepsilon_y}{\varepsilon_{LB}} - \frac{\varepsilon_{LB} - 1}{18} \right] \quad 1 < \frac{\varepsilon_{LB}}{\varepsilon_y} \leq 3 \quad (5)$$

**Figure 1: Design model for elastic-plastic stage ($1 < \varepsilon_{LB} / \varepsilon_y \leq 3$)**

In the strain-hardening range, where $\varepsilon_{LB}/\varepsilon_y > 3$, capacities beyond the full plastic moment can be achieved. The associated strain and stress distributions are shown in Figure 2, together with the proposed design model, which comprises the full plastic moment capacity $M_{pl}$ plus the additional moment capacity due to strain hardening. The strain hardening component is derived from a linearly varying stress distribution with an outer fibre stress equal to $\sigma_{LB} f_y$. The design model is given in Eqn. 6 where $k_{sh} = E/E_{sh}$ and $a_g = W_{pl}/W_{el}$, $W_{pl}$ and $W_{el}$ being plastic and elastic section moduli, respectively.
Strain-hardening stage:

\[ \frac{M_{c,Rd}}{M_{pl}} = 1 + \left( \frac{e_{LB}}{e_y} - 3 \right) \frac{1}{a_b} \frac{1}{k_{sh}} \quad 3 < \frac{e_{LB}}{e_y} \leq 15 \]  

(6)

Figure 2: Design model for strain hardening stage \((3 < \frac{e_{LB}}{e_y} \leq 15)\)

Figure 3 shows the normalised moment capacity \(\left(\frac{M_{c,Rd}}{M_{pl}}\right)\) versus normalised local buckling strain \(\left(\frac{e_{LB}}{e_y}\right)\) for the elastic-plastic (Eqn. 5) and strain hardening (Eqn. 6) stages of the continuous strength method (CSM) model for a typical I-section beam. The corresponding analytical response of the beam, as determined by direct integration, is also shown in Figure 3, together with a previous model proposed by Kemp et al. [9] that also allows capacities beyond \(M_{pl}\) due to strain hardening.

Figure 3: CSM moment capacity model

Comparisons with Test Data

Comparison of the predictions of the CSM with the results of stub column and simple beam tests are shown in Figures 4 and 5 respectively, in which the Eurocode design model is also depicted.

**TABLE 1**

<table>
<thead>
<tr>
<th>Section type</th>
<th>No. of tests</th>
<th>(F_{u,EC3})</th>
<th>(F_{u,CSM})</th>
<th>(F_{u,CSM})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot-rolled stub columns</td>
<td>10</td>
<td>0.89</td>
<td>0.97</td>
<td>1.10</td>
</tr>
<tr>
<td>Cold-formed stub columns</td>
<td>16</td>
<td>0.86</td>
<td>0.92</td>
<td>1.08</td>
</tr>
<tr>
<td>Mean</td>
<td>26</td>
<td>0.87</td>
<td>0.94</td>
<td>1.09</td>
</tr>
<tr>
<td>COV</td>
<td>26</td>
<td>0.11</td>
<td>0.08</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 4: Stub column test data and comparison with design models
The presented test data (comprising I sections and square and rectangular hollow sections – SHS and RHS) were obtained from Akiyama [10], Wilkinson and Hancock [3, 11], Byfield and Nethercot [12] and Gardner et al. [4]. In Figure 5, the CSM design model is displayed for two geometric shape factors \( a_g = 1.14 \) and \( 1.27 \) – which correspond to the average shape factors of the presented I section and SHS/RHS test data, respectively. Numerical comparisons, including the mean and coefficient of variation (COV) of the predictions, of the CSM and Eurocode with tests are presented in Tables 1 and 2 for compression and bending, respectively. The results show that the CSM offers more accurate prediction of the test data and a reduction in scatter.

![Figure 5: Bending test data and comparison with design models](image)

### CSM FOR INDETERMINATE STRUCTURES

**Introduction and Overview of Laboratory Testing**

The importance of strain hardening in indeterminate structures has been described by Davies [13], who observed that enhanced capacity could be attained in steel frames by considering strain hardening provided local and lateral-torsional buckling were eliminated. Experiments on continuous beams were performed as part of the present study [4] to generate supplementary test data to support extension of the CSM to indeterminate steel structures. A total of 12 continuous beam tests, with two loading configurations, were performed to assess moment capacities, rotation capacities and collapse loads. The results of the tests are analysed in the following sub-sections.

<table>
<thead>
<tr>
<th>Section type</th>
<th>No. of tests</th>
<th>( M_{u,EC3} / M_{u,\text{test}} )</th>
<th>( M_{u,\text{CSM}} / M_{u,\text{test}} )</th>
<th>( M_{u,\text{CSM}} / M_{u,EC3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot-rolled I sections</td>
<td>32</td>
<td>0.85</td>
<td>0.88</td>
<td>1.04</td>
</tr>
<tr>
<td>Hot-rolled SHS/RHS</td>
<td>3</td>
<td>0.89</td>
<td>0.97</td>
<td>1.10</td>
</tr>
<tr>
<td>Cold-formed SHS/RHS</td>
<td>44</td>
<td>0.89</td>
<td>0.93</td>
<td>1.05</td>
</tr>
<tr>
<td>Mean</td>
<td>79</td>
<td>0.87</td>
<td>0.91</td>
<td>1.05</td>
</tr>
<tr>
<td>COV</td>
<td>79</td>
<td>0.08</td>
<td>0.07</td>
<td>-</td>
</tr>
</tbody>
</table>

### Design Approach

A new design approach that combines features of the traditional plastic design method and the CSM has been developed to determine the collapse loads of indeterminate steel structures, with due allowance for the influence of strain hardening. For a given collapse mechanism, the critical plastic hinge is first identified as the one that undergoes the greatest rotation relative to the deformation...
capacity of the cross-section at that location. The demands at other plastic hinge locations, i, are then
assigned in proportion to the ratio of the plastic hinge rotations in the mechanism, as shown in Figure 6,
ensuring, if variable section sizes are used, that the deformation demand $\varepsilon_{Hinge_i}/\varepsilon_y$ remains below the
deformation capacity at that location $\varepsilon_{LB_1}/\varepsilon_y$. Based on the resulting deformations, the corresponding
bending moment diagram at collapse is determined. The key design steps, applied for illustration
purposes to a continuous beam, shown in Figure 6, are summarised below:

1. Identify the locations of plastic hinges in a similar manner to traditional plastic design – see
   Figure 6.
2. Based on cross-section slenderness (Eqn. 1), calculate the corresponding cross-section
   deformation capacity $\varepsilon_{LB_1}/\varepsilon_y$ at hinge 1 (Eqn. 2).
3. Determine kinematically the deformation demands ($\varepsilon_{Hinge_i}/\varepsilon_y$) at each plastic hinge location,
i, on the basis of the aforementioned assumptions and Eqns. 7 and 8, where $\theta_1>\theta_i$.

   \[ \frac{\varepsilon_{Hinge_1}}{\varepsilon_y} = \frac{\varepsilon_{LB_1}}{\varepsilon_y} \]  \hspace{1cm} (7)

   \[ \frac{\varepsilon_{Hinge_i}}{\varepsilon_y} = \frac{\theta_i}{\theta_1} \frac{\varepsilon_{LB_1}}{\varepsilon_y} \leq \frac{\varepsilon_{LB_1}}{\varepsilon_y} \]  \hspace{1cm} (8)

4. Calculate the corresponding bending moments at the plastic hinges, $M_{Hinge_i}$, from Eqn. 5 or
   6, to yield the collapse bending moment diagram, as shown in Figure 7. Note that the two
   hinges forming in the spans undergo the same rotation, and the moments at these locations
   are equal and have both been referred to as $M_{Hinge_2}$ in Figure 7.
5. Using virtual work, determine the final collapse load by equating the external work done by
   the loads to the internal work resulting from rotation of the plastic hinges, given for the
   continuous beam shown in Figure 6 by Eqn. 9.

   \[ 2F\delta = M_{Hinge_1}\theta_1 + 2M_{Hinge_2}\theta_2 \]  \hspace{1cm} (9)

Satisfaction of the three conditions of equilibrium, mechanism and yield remains a strict requirement
in defining the unique plastic collapse load of a structure within the continuous strength method. The
key diversion from traditional plastic analysis is in the yield condition, where the moment capacity
obtained for each hinge from the CSM is used in place of $M_{pl}$. Comparisons of predicted collapse
loads from traditional plastic analysis and the CSM with those obtained from a series of continuous
beam tests and a portal frame test are made in the following sub-sections.

Comparison of Continuous Beam Test Results with Design Methods

Twelve 2-span continuous beam tests on steel SHS and RHS were conducted as part of the present
study; two configurations were considered – in Configuration 1, load was applied centrally between
the supports (i.e. $L_1=L_2$ in Figure 6), while in Configuration 2, loads were applied closer to the central
support such that $L_1=2L_2$. Both hot-rolled (HR) and cold-formed (CF) sections were tested, as
indicated by the specimen designation in Table 3. The following part of the designation (e.g. 60×40×4)
refers to the height, width and thickness of the sections, while the final number is the testing configuration (1 or 2). For each test, Table 3 contains the normalised deformation at each hinge (from Eqns. 7 and 8), the corresponding bending moments relative to $M_{pl}$, and finally the test and predicted collapse loads. The continuous strength method may be seen to provide a more accurate prediction of the test behaviour and an average increase in capacity of 9% over traditional plastic methods.

**Comparison of Portal Frame Test Result with Design Methods**

Experimental results on full scale steel portal frames are relatively scarce, though one such test has been reported by Charlton [14]. The pitched portal frame was constructed from I sections with the following key properties: $f_y=272$ N/mm$^2$, $M_{el}=23.2$ kNm and $M_{pl}=27.4$ kNm. The collapse load of the frame was predicted using traditional plastic analysis and the continuous strength method. The results, summarised in Table 4, indicate that the CSM provides a more accurate prediction of the test response, with a 7% increase in capacity over traditional plastic analysis. Further validation of the continuous strength method, based on numerically generated structural performance data, is underway.

### TABLE 3
**COMPARISON OF CONTINUOUS BEAM TEST RESULTS WITH DESIGN METHODS**

<table>
<thead>
<tr>
<th>Specimen designation</th>
<th>$\varepsilon_{Hinge1}/\varepsilon_y$</th>
<th>$\varepsilon_{Hinge2}/\varepsilon_y$</th>
<th>$M_{Hinge1}/M_{pl}$</th>
<th>$M_{Hinge2}/M_{pl}$</th>
<th>$F_{test}$ (kN)</th>
<th>$F_{EC3}$ (kN)</th>
<th>$F_{CSM}$ (kN)</th>
<th>$F_{EC3}/F_{test}$</th>
<th>$F_{CSM}/F_{test}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR60×40×4-1</td>
<td>15.0</td>
<td>11.2</td>
<td>1.10</td>
<td>1.10</td>
<td>78.1</td>
<td>67.8</td>
<td>74.2</td>
<td>0.87</td>
<td>0.95</td>
</tr>
<tr>
<td>HR60×40×4-2</td>
<td>15.0</td>
<td>11.2</td>
<td>1.10</td>
<td>1.10</td>
<td>98.4</td>
<td>84.2</td>
<td>90.7</td>
<td>0.86</td>
<td>0.92</td>
</tr>
<tr>
<td>HR40×40×4-1</td>
<td>15.0</td>
<td>11.2</td>
<td>1.10</td>
<td>1.10</td>
<td>44.6</td>
<td>37.9</td>
<td>41.6</td>
<td>0.85</td>
<td>0.93</td>
</tr>
<tr>
<td>HR40×40×3-1</td>
<td>15.0</td>
<td>11.2</td>
<td>1.10</td>
<td>1.10</td>
<td>38.1</td>
<td>32.1</td>
<td>35.3</td>
<td>0.84</td>
<td>0.93</td>
</tr>
<tr>
<td>CF60×40×4-1a</td>
<td>15.0</td>
<td>11.2</td>
<td>1.10</td>
<td>1.10</td>
<td>83.4</td>
<td>58.3</td>
<td>63.8</td>
<td>0.70</td>
<td>0.77</td>
</tr>
<tr>
<td>CF60×40×4-1b</td>
<td>15.0</td>
<td>11.2</td>
<td>1.10</td>
<td>1.10</td>
<td>83.3</td>
<td>57.9</td>
<td>63.4</td>
<td>0.70</td>
<td>0.76</td>
</tr>
<tr>
<td>CF40×40×4-1</td>
<td>15.0</td>
<td>11.2</td>
<td>1.10</td>
<td>1.10</td>
<td>40.6</td>
<td>31.0</td>
<td>34.0</td>
<td>0.76</td>
<td>0.84</td>
</tr>
<tr>
<td>CF40×40×3-1</td>
<td>15.0</td>
<td>11.2</td>
<td>1.10</td>
<td>1.10</td>
<td>34.2</td>
<td>26.4</td>
<td>29.0</td>
<td>0.77</td>
<td>0.85</td>
</tr>
<tr>
<td>Mean</td>
<td>0.80</td>
<td>0.87</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>COV</td>
<td>0.08</td>
<td>0.08</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### TABLE 4
**COMPARISON OF PORTAL FRAME TEST RESULT WITH DESIGN METHODS**

<table>
<thead>
<tr>
<th>Specimen designation</th>
<th>$F_{Test}$ (kN)</th>
<th>$F_{EC3}$ (kN)</th>
<th>$F_{CSM}$ (kN)</th>
<th>$F_{EC3}/F_{test}$</th>
<th>$F_{CSM}/F_{test}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portal frame test [14]</td>
<td>107.1</td>
<td>92.5</td>
<td>99.2</td>
<td>0.86</td>
<td>0.93</td>
</tr>
</tbody>
</table>

### CONCLUSIONS

The importance of strain hardening in the response of determinate and indeterminate steel structures has been highlighted in this paper. Refinements and developments to the continuous strength method (CSM), which offers a rational means of exploiting strain hardening in steel design, have been presented. Extension of the method to cover indeterminate structures, following the principles of traditional plastic analysis but allowing bending moments in excess of the plastic moment capacity, has also been proposed. Comparisons have been made against test results on stub columns, simple
beams, continuous beams (tested as part of the current study) and a portal frame. These comparisons show that the CSM provides a more accurate prediction of test response and enhanced structural capacity over current design methods.

REFERENCES


DESIGN OF ALUMINUM ALLOY TUBULAR SECTIONS SUBJECTED TO WEB CRIPPLING

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KEYWORDS
Aluminum, Metal structures, Proposed design equations, Reliability analysis, Square and rectangular hollow sections, Tubular sections, Web crippling.

ABSTRACT
The test strengths of aluminum square and rectangular hollow sections under End-Two-Flange (ETF) and Interior-Two-Flange (ITF) loading conditions are compared with the design strengths obtained using the current American Aluminum Design Manual, Australian/New Zealand Standard and European code for aluminum structures. Furthermore, the test strengths are also compared with the design strengths obtained using the unified web crippling equation as specified in the North American Specification for cold-formed steel structural members. It is shown that the design strengths predicted by the aforementioned specifications are either quite conservative or unconservative. Therefore, two different unified web crippling equations for aluminum square and rectangular hollow sections under ETF and ITF loading conditions are proposed. The proposed unified design equation 1 uses the same technique as the North American Specification for the unified web crippling equation with new coefficients of $C$, $CN$ and $Ch$ determined based on the test results obtained in this study. The proposed unified design equation 2 is similar to the unified web crippling equation in the NAS Specification, and the effects of the ratio $N/h$ is also considered, where $N$ is the bearing length and $h$ is the depth of the flat portion of web. Generally, it is shown that the design strengths predicted by the proposed unified web crippling equation 2 are in good agreement with the test results.

INTRODUCTION
The Aluminum Design Manual [1] for aluminum structures provides design rules for the design of flexural members against web crippling based on the reference [2]. If the edge load is concentrated over a portion of the element length, web crippling needs to be considered. This kind of failure mode is confined to the area of the web under the load and the web crippling strengths are effectively post-buckled values [2]. The web crippling is rather complicated because it involves a number of factors, such as elastic and inelastic stability of the web element, local yielding in the immediate region of the load application, initial imperfections of plate elements and other factors [3]. Two loading conditions for web crippling are classified in the AA Specification [1], namely, end loading and interior loading, depending on the concentrated load is applied at the end or away from the end of the member.
However, the loading condition of the concentrated load is acting on both flanges or through one flange only also affects the web crippling strength. There are four loading conditions specified in the North American Specification [4] for cold-formed steel structural members, namely, End-One-Flange (EOF), Interior-One-Flange (IOF), End-Two-Flange (ETF) and Interior-Two-Flange (ITF) loading conditions. It should be noted that the AA Specification [1] does not classify the one-flange loading and two-flange loading. The NAS Specification [4] specifies that when the distance from the edge of the bearing to the end of the member is less than or equal to 1.5 times the clear depth of the web is classified as end loading, otherwise it is classified as interior loading.

The Australian/New Zealand Standard [5] for aluminum structures has adopted the web crippling design rules from the AA Specification [1], and no changes are introduced into the web crippling design rules. Hence, the web crippling design strengths predicted by the AA Specification [1] and the AS/NZS Standard [5] are identical. Three loading conditions are considered in the European Code [6] for aluminum structures. The three loading conditions are as follows: (1) forces applied through one flange and resisted by shear forces in the web; (2) forces applied to one flange and transferred through the web directly to the other flange; and (3) forces applied through one flange close to an unstiffened end. The loading condition of forces applied to one flange and transferred through the web directly to the other flange is similar to the interior-two-flange loading condition considered in the NAS Specification [4]. In this study, the appropriateness of the current design rules for aluminum flexural members against web crippling is investigated. Test strengths of aluminum square and rectangular hollow sections subjected to web crippling were compared with the design strengths predicted by the AA Specification [1], AS/NZS Standard [5] and European Code [6] for aluminum structures. In addition, the test strengths were also compared with the design strengths predicted by the North American Specification [4] for cold-formed steel structural members. The web crippling design rules in the NAS Specification [4] use the unified equation approach. This unified web crippling equation for cold-formed steel was developed by Prabakaran and Schuster [7] and Beshara and Schuster [8]. Two unified web crippling equations with new coefficients for aluminum square and rectangular hollow sections are proposed in this paper.

EXPERIMENTAL INVESTIGATION

There are little tests being reported on aluminum sections subjected to web crippling. Zhou and Young [9] conducted 150 web crippling tests on square and rectangular hollow sections of normal strength material (T5) and high strength material (T6) under End-Two-Flange (ETF) and Interior-Two-Flange (ITF) loading conditions. The concentrated load or reaction force was applied by means of bearing plates. The flanges of the square and rectangular hollow section specimens were not fastened (unrestrained) to bearing plates. The test data reported by Zhou and Young [9] are used in this paper for the development of web crippling design equation for aluminum square and rectangular hollow sections.

RELIABILITY ANALYSIS

The reliability of the web crippling design rules is evaluated using reliability analysis. The reliability index (β) is a relative measure of the safety of the design. A target reliability index of 2.5 for aluminum structural members is recommended as a lower limit in the AA Specification [1]. The design rules are considered to be reliable if the reliability index is greater than 2.5. The resistance (capacity) factor (ϕm) for web crippling strength as recommended by the current AA Specification, EC9 Code and NAS Specification are shown in Tables 1 and 2. The load combinations of 1.2DL + 1.6LL and 1.35DL + 1.5LL as specified in the American Society of Civil Engineers Standard [10] and the European Code, respectively, were used in the reliability analysis, where DL is the dead load and LL is the live load. The statistical parameters are obtained from AA Specification for web crippling strength, where \( M_m = 1.10 \), \( F_m = 1.00 \), \( V_M = 0.06 \), and \( V_F = 0.05 \), which are the mean values and
coefficients of variation for material properties and fabrication factors. The statistical parameters $P_m$ and $V_p$ are the mean value and coefficient of variation of tested-to-predicted load ratio, respectively, as shown in Tables 1 and 2. In calculating the reliability index, the correction factor in the AA Specification [1] was used. The respective resistance factor ($\phi_{w1}$) and load combinations for the current AA Specification, EC9 Code and NAS Specification were used to calculate the corresponding reliability index ($\beta_1$). For the purpose of direct comparison, a constant resistant factor ($\phi_{w2}$) of 0.75 and a load combination of 1.2DL + 1.6LL as specified in the AA Specification were used to calculate the reliability index ($\beta_2$) for the EC9 Code and NAS Specification, as shown in Tables 1 and 2. Reliability analysis is detailed in the Commentary of the NAS Specification [11].

COMPARISON OF TEST STRENGTHS WITH DESIGN STRENGTHS

The web crippling loads per web obtained from the tests are compared with the nominal web crippling strengths predicted using the AA Specification [1], AS/NZS Standard [5] and EC9 Code [6] Part 1.1 for aluminum structures. The web crippling strengths predicted by the AA Specification and the AS/NZS Standard are identical. In addition, the test strengths are also compared with the nominal web crippling strengths predicted using the NAS Specification [4] for cold-formed steel structural members. Tables 1 and 2 show the comparisons of the test strengths ($P_{Exp}$) with the unfactored design strengths for ETF and ITF loading conditions. The design strengths were calculated using the measured cross-section dimensions of each specimen and the measured material properties. The measured cross-section dimensions and the measured 0.2% tensile proof stress ($\sigma_{0.2}$) are detailed in Zhou and Young [9].

In Tables 1 and 2, the design strengths predicted by the AA Specification and AS/NZS Standard are generally quite conservative but unreliable for ETF and ITF loading conditions. The mean value of tested-to-predicted load ratio is 2.06 with a large coefficient of variation (COV) of 0.513, and the reliability indices ($\beta_1$ and $\beta_2$) of 2.27 and 2.58 for ETF loading condition, as shown in Table 1. The mean value of tested-to-predicted load ratio is 1.74 with the corresponding COV of 0.403, and the values of $\beta_1 = 2.42$ and $\beta_2 = 2.79$ for ITF loading condition, as shown in Table 2.

For EC9 Code, the design strengths are generally conservative but unreliable for ETF and ITF loading conditions. The mean value of tested-to-predicted load ratio is 1.01 with a large COV of 0.523, and the values of $\beta_1 = 0.96$ and $\beta_2 = 1.35$ for ETF loading condition, as shown in Table 1. The mean value of tested-to-predicted load ratio is 1.17 with the corresponding COV of 0.263, and the values of $\beta_1 = 2.04$ and $\beta_2 = 2.69$ for ITF loading condition, as shown in Table 2.

For NAS Specification, the design strengths are generally quite conservative but unreliable for ETF loading condition. For ITF loading condition, the design strengths predicted by the NAS Specification are unconservative and unreliable. The mean value of tested-to-predicted load ratio is 2.04 with a large COV of 0.710, and the values of $\beta_1 = 1.69$ and $\beta_2 = 1.92$ for ETF loading condition, as shown in Table 1. The mean value of tested-to-predicted load ratio is 0.70 with the corresponding COV of 0.469, and the values of $\beta_1 = 0.49$ and $\beta_2 = 0.83$ for ITF loading condition, as shown in Table 2.

PROPOSED DESIGN EQUATION 1 ($P_{p1}$)

The nominal web crippling strengths calculated using the AA Specification [1], AS/NZS Standard [5], EC9 Code [6] Part 1.1 and NAS Specification [4] are either quite conservative or unconservative for ETF and ITF loading conditions, as shown in Tables 1 and 2. Hence, web crippling design equations for aluminum tubular section specimens under ETF and ITF loading conditions are proposed in this paper. The proposed equation 1 uses the same technique as the NAS Specification [4] for the unified web crippling equation. The proposed design equation 1 is the unified equation with new coefficients of $C$, $C_N$ and $C_h$ as well as using the resistance factor $\phi_w$ of 0.75 and 0.80 depending on the loading
condition, as shown in Table 3. The type of aluminum for the proposed equation is also shown in Table 3. The new coefficients are determined based on the test results obtained from Zhou and Young [9]. It should be noted that the tested specimens of square and rectangular hollow sections have no corner radius as the specimens were fabricated by extrusion. Therefore, the inside corner radius is equal to zero \( r_i = 0 \). The term \( 1 - C_R \sqrt{\frac{L}{t}} \) does not need to be considered in this study. The proposed design equation 1 is as follows,

\[
P_{p1} = Ct^2 f_y \sin \theta \left( 1 + C_N \sqrt{\frac{N}{t}} \right) \left( 1 - C_h \sqrt{\frac{h}{t}} \right)
\]

where \( C \) is the coefficient, \( C_R \) is the inside corner radius coefficient, \( C_N \) is the bearing length coefficient, \( C_h \) is the web slenderness coefficient, \( t \) is the thickness of the web, \( f_y \) is the yield stress, \( \sigma_{0.2} \) proof stress, \( \theta \) is the angle between the plane of the web and the plane of the bearing surface, \( r_i \) is the inside corner radius, \( N \) is the length of the bearing and \( h \) is the depth of the flat portion of the web measured along the plane of the web.

**COMPARISON OF TEST STRENGTHS WITH PROPOSED DESIGN STRENGTHS P1**

The experimental ultimate web crippling loads per web \( (P_{Exp}) \) are compared with the unfactored design strengths \( (P_{P1}) \) calculated using the proposed unified equation 1. The proposed design strengths were calculated using the measured cross-section dimensions and measured material properties. The resistance factors \( \phi_{w1} = 0.75 \) for ETF loading condition, and \( \phi_{w1} = 0.80 \) for ITF loading condition were obtained from reliability analysis. The load combination of \( 1.2DL + 1.6LL \) was used to determine the reliability indices \( \beta_1 \) and \( \beta_2 \) using the corresponding \( \phi_{w1} \) and \( \phi_{w2} \) factors, as shown in Tables 1 and 2.

The proposed design strengths are generally conservative and reliable for both loading conditions of ETF and ITF. The mean value of tested-to-predicted load ratio is 1.21 with the corresponding COV of 0.297, and the values of \( \beta_1 = 2.57 \) for ETF loading condition, as shown in Table 1. The mean value of tested-to-predicted load ratio is 1.07 with the corresponding COV of 0.201, and the values of \( \beta_1 = 2.61 \) and \( \beta_2 = 2.82 \) for ITF loading condition, as shown in Table 2. The reliability indices \( \beta_1 \) and \( \beta_2 \) are greater than the target value for ETF and ITF loading conditions.

**PROPOSED DESIGN EQUATION 2 \( (P_{P2}) \)**

The proposed design equation 1 uses the same technique as the NAS Specification for the unified web crippling equation with new coefficients of \( C, C_N \), and \( C_h \) determined based on the test results obtained in this study, as shown in Equation (1). The proposed design equation 1 only considers the effects of \( N/t \) and \( h/t \) ratios on the web crippling strength. However, the web crippling strength is also affected by the ratio \( N/h \). Tables 1 and 2 show that as the ratio \( N/h \) increases, the value of the tested-to-predicted load ratio calculated using the proposed design equation 1 generally trends to increase. For example, series S2T5 under ETF loading condition, the ratio \( N/h \) increases from 0.7 to 5.6, the value of the tested-to-predicted load ratio increases from 0.86 to 2.15, as shown in Table 1. Hence, the ratio \( N/h \) is introduced in the proposed design equation 2 on web crippling strength. The proposed design equation 2 is as follows,

\[
P_{P2} = Ct^2 f_y \sin \theta \left( 1 + C_N \sqrt{\frac{N}{t}} \right) \left( 1 - C_h \sqrt{\frac{h}{t}} \right) \left( 1 + C_{Nh} \sqrt{\frac{N}{h}} \right)
\]

where \( C_{Nh} \) is the coefficient considering the effects of \( N/h \) ratio on the web crippling strength. The limits for the proposed equation (2) and the new coefficients of \( C, C_N, C_h, \) and \( C_{Nh} \) as well as the resistance factor \( \phi_{w} \) of 0.80 are shown in Table 4.
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Proof Stress (MPa)</th>
<th>h/t</th>
<th>N/t</th>
<th>N/h</th>
<th>ETF</th>
<th>AA</th>
<th>EC9</th>
<th>NAS</th>
<th>Proposed-1</th>
<th>Proposed-2</th>
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<td>3.64</td>
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<td>1.59</td>
<td>3.28</td>
<td>1.66</td>
<td>1.25</td>
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</table>

| Mean, $P_{m}$          | 2.06              | 1.01| 2.04| 1.21| 1.00|
| COV, $V_{P}$           | 0.513             | 0.523| 0.710| 0.297| 0.148|
| Reliability index, $\beta_1$ | 2.27            | 0.96| 1.69| 2.57| 2.68|
| Resistance factor, $\phi_1$ | 0.90             | 0.91| 0.90| 0.75| 0.80|
| Reliability index, $\beta_2$ | 2.58            | 1.35| 1.92| 2.57| 2.92|
| Resistance factor, $\phi_2$ | 0.75             | 0.75| 0.75| 0.75| 0.75|
## TABLE 2
COMPARISON OF WEB CRIPPLING TEST STRENGTHS WITH DESIGN STRENGTHS FOR SPECIMENS UNDER ITF LOADING CONDITION

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Proof Stress $f_y$ (MPa)</th>
<th>$h/t$</th>
<th>$N/t$</th>
<th>$N/h$</th>
<th>$P_{Exp}$ (kN)</th>
<th>$P_{Exp}/P_{AA}$</th>
<th>$P_{Exp}/P_{EC9}$</th>
<th>$P_{Exp}/P_{NAS}$</th>
<th>$P_{Exp}/P_{P1}$</th>
<th>$P_{Exp}/P_{P2}$</th>
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<td>18.0</td>
<td>12.5</td>
<td>0.7</td>
<td>12.9</td>
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<td>0.74</td>
<td>0.48</td>
<td>0.76</td>
<td>0.86</td>
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<td>41.5</td>
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<td>1.09</td>
<td>0.59</td>
<td>0.95</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>ITF-R2T5-N100</td>
<td>189.1</td>
<td>33.8</td>
<td>1.1</td>
<td>47.3</td>
<td>1.42</td>
<td>1.16</td>
<td>0.64</td>
<td>0.96</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>ITF-R2T5-N150</td>
<td>189.1</td>
<td>50.5</td>
<td>1.6</td>
<td>62.7</td>
<td>1.55</td>
<td>1.31</td>
<td>0.76</td>
<td>1.04</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>ITF-R2T5-N200</td>
<td>189.1</td>
<td>67.6</td>
<td>2.1</td>
<td>81.4</td>
<td>1.72</td>
<td>1.52</td>
<td>0.92</td>
<td>1.18</td>
<td>1.15</td>
<td></td>
</tr>
</tbody>
</table>

Mean, $P_{m}$: 1.74, 1.17, 0.70, 1.07, 1.04

COV, $V_p$: 0.403, 0.263, 0.469, 0.201, 0.173

Reliability index, $\beta_1$: 2.42, 2.04, 0.49, 2.61, 2.68

Resistance factor, $\phi_1$: 0.90, 0.91, 0.90, 0.80, 0.80

Reliability index, $\beta_2$: 2.79, 2.69, 0.83, 2.82, 2.90

Resistance factor, $\phi_2$: 0.75, 0.75, 0.75, 0.75, 0.75
TABLE 3
PROPOSED WEB CRIPPLING DESIGN RULES 1 FOR ALUMINUM TUBULAR SECTIONS

<table>
<thead>
<tr>
<th>Support and Flange Conditions</th>
<th>Load Cases</th>
<th>C</th>
<th>C_N</th>
<th>C_h</th>
<th>LRFD φ_w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfastened</td>
<td>ETF</td>
<td>0.39</td>
<td>20</td>
<td>0.10</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>ITF</td>
<td>0.90</td>
<td>12</td>
<td>0.09</td>
<td>0.80</td>
</tr>
</tbody>
</table>

T5

Notes: The above coefficients apply when \(18 \leq h/t \leq 75, N/t \leq 175, N/h \leq 5.6\) and \(\theta = 90^\circ\).

TABLE 4
PROPOSED WEB CRIPPLING DESIGN RULES 2 FOR ALUMINUM TUBULAR SECTIONS

<table>
<thead>
<tr>
<th>Support and Flange Conditions</th>
<th>Load Cases</th>
<th>C</th>
<th>C_N</th>
<th>C_h</th>
<th>CNb</th>
<th>LRFD φ_w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfastened</td>
<td>ETF</td>
<td>0.21</td>
<td>20</td>
<td>0.10</td>
<td>1.0</td>
<td>0.80</td>
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<td></td>
<td>ITF</td>
<td>0.80</td>
<td>9</td>
<td>0.08</td>
<td>0.2</td>
<td>0.80</td>
</tr>
</tbody>
</table>

T5

Notes: The above coefficients apply when \(18 \leq h/t \leq 75, N/t \leq 175, N/h \leq 5.6\) and \(\theta = 90^\circ\).

COMPARISON OF TEST STRENGTHS WITH PROPOSED DESIGN STRENGTHS P2

The experimental ultimate web crippling loads per web \(P_{Exp}\) are compared with the unfactored design strengths \(P_{D2}\) calculated using the proposed unified equation 2. The resistance factor \(\phi_{w1} = 0.80\) for ETF and ITF loading conditions was obtained from reliability analysis. The load combination of 1.2DL + 1.6LL was used to determine the reliability indices \((\beta_1\) and \(\beta_2)) using the corresponding \(\phi_{w1}\) and \(\phi_{w2}\) factors, as shown in Tables 1 and 2.

The proposed design strengths are generally conservative and reliable for ETF and ITF loading conditions. The minimum mean value of tested-to-predicted load ratio is 1.00 with the corresponding COV of 0.148, and the values of \(\beta_1 = 2.68\) and \(\beta_2 = 2.92\) for ETF loading condition, as shown in Table 1. The maximum mean value of tested-to-predicted load ratio is 1.04 with the corresponding COV of 0.173, and the values of \(\beta_1 = 2.68\) and \(\beta_2 = 2.90\) for ITF loading condition, as shown in Table 2.

CONCLUSIONS

Web crippling design equations for aluminum alloy tubular sections have been proposed in this study. The test strengths of aluminum square and rectangular hollow sections subjected to web crippling were compared with the unfactored design strengths (nominal strengths) calculated using the current AA Specification [1], AS/NZS Standard [5] and EC9 Code [6] Part 1.1 for aluminum structures. The test strengths were also compared with the unfactored design strengths obtained using the NAS Specification [4] for cold-formed steel structural members. It is shown that the design strengths...
predicted by the AA Specification, AS/NZS Standard, EC9 code and NAS Specification are either quite conservative or unconservative. Therefore, unified web crippling equations for aluminum square and rectangular hollow sections under End-Two-Flange (ETF) and Interior-Two-Flange (ITF) loading conditions have been proposed in this study. It is also shown that the web crippling strength is affected by the ratio \( \frac{N}{h} \), as the ratio \( \frac{N}{h} \) increases, the value of the tested-to-predicted load ratio calculated using the proposed design equation 1 generally trends to increase. Therefore, the ratio \( \frac{N}{h} \) is included in the proposed design equation 2. The design strengths calculated using the proposed unified web crippling equation 2 are generally conservative and reliable for ETF and ITF loading conditions. The proposed design equation 2 is capable of producing reliable limit state design when calibrated with the resistance factor \( \phi_w = 0.8 \) for both loading conditions of ETF and ITF. It is recommended that the web crippling strength of aluminum square and rectangular hollow sections can be calculated using the proposed unified design equation 2.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the Asia Aluminum Manufacturing Company for supplying the test specimens.

REFERENCES

AN NUMERICAL INVESTIGATION INTO THE EFFECT OF CONSTRUCTION METHODS TO THE STRUCTURAL BEHAVIOUR OF SIMPLY SUPPORTED COMPOSITE BEAMS

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² Faculty of Engineering and Physical Sciences, University of Surrey, UK.

KEYWORDS
Composite beams, shear connectors, moment capacities, slippage of shear connectors, construction method.

ABSTRACT
This paper presents a numerical investigation into the effect of construction methods to the structural behaviour of simply supported composite beams. Based on advanced three dimensional finite element models with material, geometrical and interfacial non-linearity, the structural behaviour of composite beams under experimental investigation has been obtained by the authors and other researchers with a high level of accuracy. By considering the construction method of composite beams in practice, it is important to realize that composite action is only required to be developed during the composite stage, and to resist primarily the imposed loads of the floors. Hence, the shear connectors are not required to resist the dead loads of the composite beams, and this represents typically a 30% reduction on the forces acting onto the shear connectors. Thus, the structural demand on the shear connectors in composite beams in practice is considerably smaller than that in those composite beams which are fully supported during concrete casting. Through a numerical parametric study, the deformation characteristics of a total of 12 simply supported composite beams are presented, and the differences in the structural behaviour of composite beams due to the construction methods are discussed. It is found that the required slippage of the shear connectors in most simply supported composite beams built on sites are 4 to 5 mm smaller than that in those composite beams built in laboratory; this corresponds to a 30 to 50 % reduction. The findings are very important in quantifying the practical ranges of ductile slippage of shear studs in composite beams, and also in explaining the discrepancies between the slippage requirements of shear studs obtained from previous numerical investigations and the ductile slippages of shear studs measured from standard push-out tests.
INTRODUCTION

Composite structures are widely used in building construction for many years owing to their high constructability. The structural form of a composite beam is essentially a thin wide concrete slab connected with a steel section where the concrete slab is in compression while the steel section is largely in tension. In general, composite beams are strong and stiff flexural members with long spanning capacities. Shear connectors, usually headed studs, are welded to the top flange of the steel section and fully embedded into the concrete slab. Depending on the number of shear connectors provided along the interface between the steel section and the concrete slab which is either a solid concrete slab or a composite slab, the composite beam may operate in either full or partial shear connection (Lawson, 1989) as permitted in Eurocode 4 (2004). Hence, it exhibits a wide range of overall structural behaviour according to the load-slippage characteristics of the shear connectors.

Based on advanced three dimensional finite element models with material, geometrical and interfacial non-linearity, the structural behaviour of a number of composite beams in structural tests found in the literature have been successfully simulated by the authors (Wang & Chung, 2006; Chung & Chan, 2009) and others (Liang et al, 2004; Queiroz et al 2007) with a high level of accuracy. Moreover, important information about the “stress and strain” conditions of those beams may be analyzed for rational comparison. It is essential to acknowledge that shear connectors are not rigid but moderately deformable, and their load-slippage characteristics at both small and large deformations are very important to the overall structural behaviour of composite beams. As reported previously, very large slippages in the shear connectors, about 0.5 to 1.0 of the stud diameters in the case of headed studs, are generally required for full mobilization of the load carrying capacities of the composite beams in tests. However, such a large demand on the slippages of headed studs may not be easily met in practice.

EFFECT OF CONSTRUCTION METHOD

It should be noted that although not all, many composite beams constructed in laboratory for experimental investigation are fully supported along their lengths during concrete casting. Hence, the steel sections are not stressed, and neither are the shear connectors. However, once the composite beams are lifted into testing frames and supported at both ends, all the shear connectors are then required to resist the self weights of the composite beams as well as additional applied loads during the tests. Hence, the structural demand on the shear connectors is generally large.

However, the situation in most composite beams constructed on site is very different. In general, all steel sections are supported at both ends on site during erection, and they are required to resist their own weights as well as those of the concrete slabs during concrete casting; but the shear connectors are not. Composite action is only required to be developed during the composite stage, and to resist primarily the imposed loads of the floors. Hence, the shear connectors are not required to resist the dead loads of the composite beams, and this represents typically a 30% reduction on the forces acting onto the shear connectors. Thus, the structural demand on the shear connectors in composite beams in practice is considerably smaller than that in those composite beams which are fully supported during concrete casting.

OBJECTIVES

This paper presents a numerical investigation into the effect of construction methods to the structural behaviour of simply supported composite beams. Based on advanced three dimensional finite element models with material, geometrical and interfacial non-linearity, a numerical parametric study on a total of 12 simply supported composite beams is carried out. Differences on the predicted structural behaviour of composite beams with two different construction methods, namely, i) fully supported method, and ii) end supported method are presented. The structural implications on the slippage requirements of the shear studs is also elaborated.
Owing to limited space, the advanced finite element models are briefly described in the paper. For full details of the models, refer to Wang & Chung (2006) and Chung & Chan (2009).

FINITE ELEMENT MODELLING

Figure 1 illustrates the general arrangement of a simply supported composite beam with a span of 12 m, and details of both the geometrical dimensions and the mechanical properties of the steel sections and the solid concrete slab are also provided. No steel reinforcement is adopted in the present study for simplicity.

A three dimensional finite element model of a simply supported composite beam is established, as shown in Figure 2, using the general purpose finite element package ABAQUS (2004), and shell elements S4R, solid elements C3D8R and spring elements are employed to model the steel section, the concrete slab and the shear connectors respectively. Material and geometrical non-linearity is fully incorporated into the model. The material models of both the steel section and the concrete slab are illustrated in Figure 3.
Moreover, residual stresses in the steel section are also incorporated in the model as shown in Figure 4.

Shear Connectors

As it is essential to acknowledge that shear studs are not rigid but moderately deformable, their non-linear load-slippage with different extents of ductility at large deformations are incorporated into the models through an engineered use of spring elements. Each shear stud is modelled with one longitudinal spring, one transverse spring and one vertical spring in order to simulate the longitudinal and the transverse shear forces as well as the pull-out force of the shear stud. The assumed load-slippage curves of the shear stud under both longitudinal and transverse shear forces are shown in Figure 5. Two different types of shear connectors, namely, Types D and R, are incorporated into the model, which represent shear studs with high and reduced ductility respectively. It should be noted that there are three different arrangements of shear studs, namely, i) 2 studs per row with a spacing of 300 mm, ii) 1 stud per row with a spacing of 300 mm, and iii) 1 stud per row with a spacing of 600 mm. Refer to Figure 1 for details.

Construction Methods

Figure 6 illustrates the effect of the construction method on a simply supported composite beam when it is supported at both ends during concrete casting. Owing to the self-weights of the concrete slab and the steel section, the maximum bending moment, \( M_1 \), acting onto the steel section at mid-span is 239.4 kNm. This gives a maximum bending stress of 251 N/mm\(^2\) in the flanges of the steel section during the construction stage.

Hence, all these bending stresses are incorporated into the steel section as the initial stresses in addition to the residual stresses in the composite beam in subsequent analyses during the composite stage.
FINITE ELEMENT RESULTS

Typical deformed model of a composite beam at failure is shown in Figure 2. In order to define failure in the composite beams at large deformations, the following two check points are established according to the strain levels in the steel section and in the concrete slab:

- **Check Point 1 (or CP1)**
  Monitoring tensile strain in the steel section, $\varepsilon_{s_{\text{max}}}$, which is given by:
  $$\varepsilon_{s_{\text{max}}} = c_o \times p_y \times \sqrt{\frac{p_y}{E_s}} \times \frac{275}{E_s}$$
  where $c_o$ is the strain coefficient which is taken to be 6; $p_y$ is the yield strength of the steel section; and $E_s$ is the Young’s modulus of the steel section.

- **Check Point 2 (or CP2)**
  Limiting compressive strain in concrete, $\varepsilon_{c_{\text{max}}}$, which is equal to 0.35%.

In general, a plastic hinge is regarded to be fully developed at the composite cross-section when both the tensile and the compressive strains at the bottom and the top of the cross-section at mid-span exceed the limiting values. Figure 7 presents the load-deflection curves of all the 12 composite beams with two different construction methods while all the key numerical results are summarized in Table 1 for direct comparison. As shown in Figure 7, it is found that:

- For Beams SS-D2/R2 with a degree of shear connection, $k_s = 1.0$, there is no difference in the load–deflection curves of the two beams over the entire deformation ranges. The curves are found to be fairly ductile in both cases, allowing large tensile strains to be developed at the steel sections at mid-span of the beams. Hence, the moment capacities of the composite cross-sections according to plastic stress blocks are readily mobilized. The end slippages of the shear studs under the maximum applied loads are merely 1.2 and 0.65 mm for the fully supported and the end supported cases respectively, and hence, the structural demand on the shear studs is very small.
Figure 7: Load deflection curves of the composite beams.
However, for Beams SS-D1/R1 with $k_s = 0.65$ in the fully supported case, there is a significant difference between the load–deflection curves of the two beams. While Beam SS-D1 exhibits a fairly ductile behaviour, Beam SS-R1 is not. During the initial loading stage, both beams deflect to about 200 mm at mid-span under an end slippage of 5 mm (Point a). However, when the mid-span deflection reaches 400 mm, the end slippage in Beam SS-D1 is about 10 mm (Point b) while that in Beam SS-R1 goes beyond 15 mm (Point c). It should be noted that for Beams SS-D1/R1 in the end supported case, there is no difference between the load–deflection curves of the two beams over a large deformation range. When both beams deflect to about 400 mm at mid-span, the end slippages in both beams are merely 5 mm (Point a). Hence, the structural demand on the shear studs is considerably smaller than that in the fully supported case.

The situations in Beams SS-Da/Ra are found to be broadly similar to those in Beams SS-D1/R1. As the stud spacing is 600 mm rather than 300 mm as in all the other beams, the degree of shear connection, $k_s$, is merely 0.34. Thus, many of the shear studs are heavily stressed even in the initial loading stage in order to mobilize the resistances of the concrete slabs and the steel sections. For Beams SS-Da/Ra in the fully supported case, both beams deflect to about 100 and 200 mm at mid-span under an end slippage of merely 5 and 10 mm respectively (Points a and b). Moreover, when the mid-span deflection reaches 400 mm, the end slippage in Beam SS-Da is about 15 mm (Point c) while that in Beam SS-Ra goes beyond 20 mm (Point d). Hence, the structural demand of the shear studs is very large. However, for Beams SS-Da/Ra in the end supported case, there is no difference between the load–deflection curves of the two beams over the moderate deformation range. When both beams deflect to about 250 mm at mid-span, the end slippages in both beams are merely 5 mm (Point a). It should be noted that at Point a, over 95% of the moment capacities of the composite beams are readily mobilized.

Table 1 summarizes the predicted load carrying capacities of the composite beams. It should be noted that the load carrying capacities of the composite beams are assigned to be either the maximum applied loads, or the applied loads at failure as defined by concrete crushing, i.e. Check Point 2. It is shown that:

- In general, the structural demand on shear studs of those composite beams in the fully supported case increases with a reduction in the degree of shear connection in order to mobilize the moment capacities of the composite beams permitted by the available degree of shear connection. The same is found in those composite beams in the end supported case.
- Among the 12 composite beams covered in the present study, the required end slippage on shear studs in those composite beams in the end supported case is generally 4 to 5 mm smaller than that in those composite beams in the fully supported case.
- In order to reduce the required slippage on shear studs in those composite beams in the end supported case, it is possible to work on 95% of the moment capacities of the composite beams when it operates at low degree of shear connection, such as $k_s = 0.34$ in this case. The corresponding end slippage is found to be smaller than 5 mm, a value readily attained in practice.

**Table 1**

SUMMARY OF PREDICTED LOAD CARRYING CAPACITIES OF COMPOSITE BEAMS

<table>
<thead>
<tr>
<th>Beam</th>
<th>Fully supported case</th>
<th>End supported case</th>
<th>Back analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W$ (kN)</td>
<td>$M_{slag}$ (kNm)</td>
<td>$\Delta_{max}$ (mm)</td>
</tr>
<tr>
<td>SS-D2</td>
<td>513.1</td>
<td>769.7</td>
<td>421.9</td>
</tr>
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<td>SS-D1</td>
<td>454.2</td>
<td>681.3</td>
<td>361.0</td>
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<tr>
<td>SS-Da</td>
<td>394.2</td>
<td>591.2</td>
<td>339.5</td>
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<tr>
<td>SS-R2</td>
<td>513.1</td>
<td>769.7</td>
<td>421.9</td>
</tr>
<tr>
<td>SS-R1</td>
<td>442.3</td>
<td>663.5</td>
<td>230.7</td>
</tr>
<tr>
<td>SS-Ra</td>
<td>361.0</td>
<td>541.6</td>
<td>196.3</td>
</tr>
</tbody>
</table>
CONCLUSIONS

In practice, most composite beams are end supported during concrete casting on sites, and composite action is only required to be developed in the composite beams during the composite stage to resist primarily the imposed loads of the floors. Hence, the shear connectors are not required to resist the dead loads of the composite beams, and this represents typically a 30% reduction on the forces acting onto the shear connectors, when compared with those composite beams which are fully supported during concrete casting. Through a numerical parametric study, the deformation characteristics of a total of 12 simply supported composite beams are presented in this paper. It is found that the required end slippages of the shear studs in composite beams in the end supported case are considerably smaller than those in composite beams in the fully supported case. More specifically, the slippage requirements in the shear studs in the present study are reduced by 4 to 5 mm, and this corresponds to a 30 to 50% reduction. The findings are very important in quantifying the practical ranges of ductile slippages of shear studs in composite beams, and also in explaining the discrepancies between the slippage requirements of shear studs obtained from previous numerical investigations and the ductile slippages of shear studs measured from standard push-out tests.

REFERENCES

DEBONDING BEHAVIOR OF CFRP STRENGTHENED STEEL BEAMS UNDER STATIC AND CYCLIC LOADS

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KEYWORDS
Carbon fiber reinforced polymer, laminate, steel beam, composite, static, cyclic, bond failure.

ABSTRACT
Fiber reinforced polymer (FRP) composite materials have been widely used in upgrading and retrofitting concrete structures. The advanced mechanical characteristics of high strength FRP composites can also be utilized in strengthening and repairing steel structures. For steel structures strengthened with carbon fiber reinforced polymer (CFRP), the bond performance between CFRP laminate and steel is a crucial consideration, which will directly influence strengthening effect and determine the final capacity of the strengthened structures. This paper investigates the behaviour of debonding, i.e. the bond failure between CFRP laminate and steel surface, of CFRP-bonded steel structures. Full-scale experiments on CFRP strengthened steel beams under flexure were conducted. For specimens tested under static load, the bond failure load and the effect of laminate length were investigated. Cyclic performance of the CFRP-steel bond was also explored. The stress development, failure mode and fatigue life of the bond were analyzed. Based on the experimental results, the configuration of CFRP laminate with highest bond strength and the safe load range under cyclic load condition were recommended for steel beams strengthened by external bonding of CFRP laminates.

INTRODUCTION
With the rapid development of steel construction in the world, a considerable amount of steel infrastructures need to be strengthened or repaired due to reasons of corrosion, inappropriate use, defect in design, lack of proper maintenance and accidental damage. In the past, steel flexural members are usually retrofitted by welding or mechanically fastening additional steel plates. The traditional methods for strengthening steel structures have some disadvantages, which include the difficulty in installation of heavy steel members, increased self weight,
weld-induced residual stress, the potential for weld fatigue cracking at the cover plate ends and corrosion susceptibility. It requires some significant attention in developing new techniques for effective and economical revival of these structures.

Fibre reinforced polymer (FRP), as an advanced polymer composite material, has a lot of advanced characters in contrast to the drawbacks of steel in structural rehabilitation, such as light weight, installation facility and resistance to fatigue and corrosion. With these desirable properties, adhesive bonding of FRP has been employed to upgrade and repair concrete structures in civil engineering for many years. In recent years, the feasibility and effectiveness of using FRP to strengthen steel structures were investigated by some researchers [1-4]. For CFRP-bonded steel structures, a very important concern is the potential bond failure between the CFRP laminate and steel surface. According to the past research results about structural bonded joints [5-8], the dominant failure mode for composite-bonded metal joint is adhesive failure rather than adherend failure, which is totally different from the “rip-off” failure mode for CFRP bonded concrete structures [9]. For the later, the strength of the bond between CFRP laminate and concrete surface is generally larger than the failure strength of concrete. Therefore, in the rip-off failure the surface concrete comes off with CFRP. However, for CFRP-bonded steel structures, the failure strengths of steel and CFRP are both very large compared to that of the adhesive bond. Due to material discontinuity at the location of bond end or crack tip, the stress concentration in these areas may cause adhesive failure before steel and CFRP achieve their ultimate strengths. For this reason, the bond property of CFRP strengthened steel structures needs to be studied intensively.

In this study, the bond performance of steel flexural members strengthened with CFRP laminates under both static and cyclic loads was experimentally investigated. Full-scale experiments on CFRP strengthened steel beams under pure flexure were conducted. For specimens tested under static load, the bond failure load and the effect of laminate length were investigated. Cyclic performance of the CFRP-steel bond was also explored. The stress development, failure mode and fatigue life of the bond were analyzed.

**EXPERIMENTAL PROGRAM**

Full-scale experimental tests on CFRP strengthened steel beams were carried out in the Construction Technology Laboratory of the School of Civil and Environmental Engineering, Nanyang Technological University. A total of eleven specimens were prepared and tested under four-point bending. Five specimens were tested under static load and the others under cyclic load.

**Material Properties**

In the experimental program, wide flange low-carbon steel beams were employed for specimen fabrication. Pre-impregnated CFRP laminate was used as the strengthening material which was bonded to the bottom of the steel beams by epoxy. Material test was carried out to get the mechanical properties of both steel and CFRP. The average Young’s modulus of the steel was 196 kN/mm². The yield and ultimate stresses of the steel were 334 N/mm² and 483 N/mm², respectively. For the CFRP laminate, the tensile strength and tensile modulus were 2492 N/mm² and 150.8 kN/mm², respectively. The elongation at break is 1.65%.


**Specimen Configuration**

To prevent local failure of specimen under concentrated load, all the steel girders were reinforced with stiffeners where the supports and loading points were located. To guarantee the quality of the bond between steel girder and CFRP laminate, the bottom surface of steel girders were manually ground. Both the surfaces of CFRP laminates and steel beams were cleaned with methyl ethyl ketone. The adhesive was uniformly attached to the bottom surface of steel beams as well as CFRP laminates. After the CFRP laminates fixed to the steel beams, the laminates were pressed to even the contact with the beams and possible air bubbles in between were extruded. The schematic section view of the steel girder is shown in Figure 1 (a). The depth of the girder is \( d = 175 \text{ mm} \) and the distance between fillets is \( h = 143 \text{ mm} \). The width of the girder flange is \( b_f = 90 \text{ mm} \). The thicknesses of the flange and the web are \( t_f = 8 \text{ mm} \) and \( t_w = 5 \text{ mm} \), respectively. In Figure 1 (b), the front view of the typical specimen in the first group is illustrated. All the specimens have same length \( (l) \) of 2400 mm. The bond length of CFRP strip is \( l_0 \). To investigate the effect of number of laminate length on the static behaviour of strengthened beams, specimens tested under static load were strengthened with different laminate lengths. The first specimen tested under static load was not strengthened and was named as S1. The other specimens tested under static load were named as from S2 to S5 with the laminate lengths of 700 mm, 850 mm, 1050 mm and 1450 mm, respectively. The specimens tested under cyclic load have same configuration as specimen S2 since its failure load level was to be used as a reference for cyclic load.

**Experimental Setup and Instrumentation**

The experimental setup is shown in Figure 2. The compression force was generated from the hydraulic jack with 1000 kN capacity. Transformation from one loading point to two was realized by use of a spreader beam and two rockers between the specimen and the spreader beam. The specimen was supported by a rocker bearing and a pin bearing at the beam ends. Three linear variable displacement transducers (LVDTs) were mounted to the specimen bottom. One was positioned at the midspan and two at the points below the specimen. To investigate the phenomenon of crack initiation and growth, 2 mm long strain gauges were mounted on the face of the FRP laminate along the longitudinal centre line of the laminate. All the specimens were loaded under four-point bending. According to the research of Emberson and Mays [10], for fatigue test at frequencies much in excess of 2 Hz the beam was unable to recover fully from one load application before the arrival of the next. A high frequency may also produce undesirable heat effect on the beam. On the other hand, the test could be very time-consuming if the frequency is too low. Therefore, the frequencies of 1 Hz and 2 Hz were used for specimens tested under load levels of more than and less than 60%.
ultimate load respectively. The cyclically tested specimens were sinusoidally loaded under four-point bending at an amplitude ratio of 0.1 and at different upper boundary values from 30% up to 80% of the average static failure load of the reference specimen.

DEBONDING BEHAVIOR UNDER STATIC LOAD

The objective of current experimental program is to investigate bond performance of CFRP strengthened steel beams under static load, so that the effectiveness and the reliability of such strengthening technique on steel structures could be further examined besides the verified enhanced flexural capacity.

**Figure 2: Test setup**

**Bond Failure Load**

The load-deflection curves for specimens tested under static load are illustrated in Figure 3. Four specimens with different bond lengths were investigated. The length of the bonded laminate was increase from 700 mm to 1450 mm. With the length of the bonded FRP laminate increasing, the bond failure load of the strengthened beam was improved. The relationship between deflection at bond failure and bond length of FRP laminate is shown in Figure 4. The stress distributed in the end of the bondline played a significant role in determining the bond strength of FRP strengthened steel beams. For a beam strengthened with short FRP laminate, the stress level at the end of bondline was quite high since the large force in tension flange needed to be transferred into the laminate by means of deformation of the interfacial adhesive. The deformation of the adhesive mainly concentrated at the end of the bondline. Therefore, the stress level in the end of the bondline was very high in this case. For a beam bonded with relatively long laminate, the largely deformed zone was more widely and uniformly distributed along the bondline, thus the less the stress concentration level at the end of the bondline. Therefore for a FRP strengthened steel beam, the longer bond length means higher bond strength.
Stress Distribution

To have a fundamental understanding of the failure mechanism of the bond between the CFRP laminate and the steel beam, stress distribution along the bondline of the strengthened beams were investigated by means of numerical simulation of the tested specimens. Figure 5 shows the distributions of shear and normal stresses along the bondline of the CFRP strengthened steel beams. It can be seen that both the maximum shear and normal stresses appeared at the end of the bondline. The maximum value of normal stress was much lower than that of the shear stress. The variations of both stresses were very remarkable at the end of the bondline. The development length was less than 100 mm. In the bondline far from the end, i.e. a location with distance to the bondline end more than 100 mm, the stress magnitude was very low and there was no more variation of shear or normal stress.

DEBONDING BEHAVIOR UNDER CYCLIC LOAD

The work presented in this section aims to investigate the performance of the adhesive bond between the FRP laminate and the steel substrate under cyclic load, particularly in the regions where the stress discontinuity/concentration exists, i.e. where the CFRP laminates terminates in such structures strengthened by external bonding. This was achieved through an
experimental program in which the CFRP laminate strengthened steel beams were subjected to mechanical cyclic load with various load levels

**Fatigue Life of Bond**

The relationship between the normalized load and the logarithm of number of cycles to failure of specimens is shown in Figure 6. It is clear that the relationship is more or less linear. Linear regression analysis produced the following best-fit curve for specimens:

\[ P_N = -10.12 \log(N_f) + 98.55 \]  

where \( P_N \) is the normalized load amplitude in the fatigue test, \( N_f \) is the number of cycles to failure. The bond between the steel beam and CFRP laminate in the last specimen did not fail even after three million cycles. Furthermore, the variation of the strain at the end of laminates was still very small after three million cycles, which indicated that the specimens were not likely to fail in fatigue under such a load level. Therefore, the fatigue threshold for the tested beams is regarded as thirty percent of the ultimate failure load that will cause debonding of FRP laminate under static load.

**Stress Development**

The development of stress at different locations measured in the CFRP laminate during the cyclic tests is shown in Figure 7. It can be observed that the strains at all locations remained constant during the first thirty percent of fatigue cycles. After that the strain on the right began to drop gradually. Later on the strain on the left began to decrease as well. Therefore, bond failure on the left happened later than that on the right. The strain decrease at the centre initiated after sixty percent of total cycles. When two thirds of the fatigue life of the specimen was reached, the strain at each end approached zero indicating the bond at the end was completely invalid. The strain in the center of the laminate dropped rapidly from a constant maximum value to a much lower value close to zero, which indicated the bond between the FRP laminate and the steel beam was almost completely damaged by the cyclic load.
Failure Mode

The crack initiation and propagation in all the specimens were similar except for the specimen loaded under eighty percent of the failure load. The load amplitude on this specimen was so high that caused a sudden bond failure at one end of the strengthening laminate after only 25 cycles. Figure 8 shows the crack development track for the specimen with gradual bond failure, which was typical for all the other specimens. It can be seen that the initiations of the cracks at left and right ends were not simultaneous. Once initiated, the crack grew rapidly from the end. After one of the two cracks had advanced to certain extent, it almost stopped developing further. The typical failure mode of the specimens is shown in Figure 9. For all the specimens, the crack initiated from one end of the FRP laminate. With fatigue cycles increasing, the crack propagated along the interface between the steel beam and the adhesive and moved towards the centre of the bondline. Since there was always residual bond remained and FRP laminate and steel beam can not be completely separated, the crack stopped propagating eventually.

CONCLUSIONS

In this study, the bond performance of CFRP strengthened steel beams under both static and cyclic loads was experimentally investigated. The increase of bond length was found to have a positive effect on the bond strength. The stress along the bondline is highly concentrated at the end of the bondline. The longer the bond length is, the less the concentration level is. Therefore using several pieces combined together to strengthen a flexural member should be avoided, instead, a continuous laminate be used. The results from cyclic experiments indicate that logarithm of number of cycles at bond failure increase almost linearly with the decreasing cyclic stress range. Thirty percent of static bond failure load can be deemed as the threshold load level for bond failure, below which the debonding of CFRP from steel beams under cyclic load is very unlikely to happen.

Figure 8: Relationship between load amplitude and number of cycles
Figure 9: (a) Initiation and (b) Propagation of crack
REFERENCES


UNIFIED SLENDERNESS LIMITS FOR CIRCULAR HOLLOW SECTIONS

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London, SW7 2AZ, UK

KEYWORDS
Circular hollow sections, reliability analysis, section classification, slenderness limits, tubular construction.

ABSTRACT
Circular hollow sections (CHS) are widely used in a range of structural engineering applications. The sections may be hot-rolled or cold-formed from a variety of metallic materials with a range of yield strengths. The design of such sections is covered by all major design codes, but examination of the provisions of each reveals significant differences in the treatment of local buckling, as considered through cross-section classification. Cross-section classification criteria relate to rotation capacity and strength requirements (obtainment of the plastic or elastic moment in bending and the yield load in compression), while the relative performance of structural CHS is governed by susceptibility to local buckling and is influenced by cross-section slenderness, material stiffness and yield strength, forming process (affecting geometry, material homogeneity and residual stresses), material strain hardening characteristics and ovalization. Furthermore, the classification criteria and reliability requirements vary among the different structural design codes. This paper presents a review of 153 test results on CHS in bending, covering structural steel, aluminium, stainless steel and very high strength steel. Based on the available test data, current codified provisions in the European, North American and Australian Standards are reassessed, and new unified slenderness limits are proposed for CHS following reliability analyses.

INTRODUCTION
classes of cross-section, based upon their susceptibility to local buckling. Class 1 cross-sections, termed plastic sections in BS 5950, are capable of reaching and maintaining their full plastic moment \( M_{pl} \) in bending by forming plastic hinges with sufficient rotation capacity for plastic design. Class 2 cross-sections, referred to as compact sections in BS 5950, are also capable of reaching their full plastic moment in bending but have lower deformation capacity. In Class 3 cross-sections, termed semi-compact sections in BS 5950, local buckling prevents attainment of the full plastic moment and the bending moment resistance is limited to the yield moment \( M_{el} \). Class 4 cross-sections, commonly referred to as slender sections, exhibit local buckling before the yield stress is achieved. Bending moment resistance is determined based on an effective cross-section defined by the width-to-thickness (or diameter-to-thickness in CHS) ratios of the constituent elements. The moment-rotation characteristics of the four behavioural classes are illustrated in Figure 1. AISC 360 (2005) [5] and AS 4100 (1998) [6] effectively define three classes of cross-section: Class 1 cross-sections are referred to as compact, there is no equivalent to Class 2 sections, Class 3 sections are termed non-compact, while Class 4 cross-sections are referred to as slender sections.

FACTORS AFFECTING LOCAL BUCKLING AND STRUCTURAL RESPONSE OF CHS

Local buckling occurs in thin-walled sections when the applied compressive stress exceeds a critical value, and is characterised by local ripples in the cross-section wall. Local buckling and the structural response of CHS are influenced by a number of factors, which are discussed in this section. The elastic buckling stress of a cylindrical shell in the axis-symmetric mode is given by:

\[
\sigma_{cr} = \frac{2E}{\sqrt{3(1-\nu^2)}} \left( \frac{t}{D} \right)
\]  

(1)

where \( E \) is the Young’s modulus, \( \nu \) is Poisson’s ratio and \( D/t \) is the diameter-to-thickness ratio of the section and the geometric parameter that controls local buckling. The susceptibility to elastic buckling in preference to yielding depends on the yield strength of the material \( f_y \), and this therefore also appears in slenderness parameters. With higher yield strengths, the slenderness of the section effectively increases, i.e. the section is more susceptible to local buckling prior to yielding. Previous
studies have however shown that the slenderness limits derived for normal strength steel tubes become conservative when applied to very high strength steel [7]. This is linked to the presence and influence of initial geometric imperfections and residual stresses. Residual stresses are typically induced in structural components through plastic deformation and differential cooling during manufacture. Their influence on structural members is to cause premature yielding and loss of stiffness, often leading to deterioration of load carrying capacity. In high strength steel sections, residual stresses are a smaller fraction of the yield strength and therefore their detrimental effect is smaller than for normal strength steel [8]. Initial geometric imperfections can also have a significant influence on the strength of thin-walled sections [9] by amplifying buckling deformations and hence expediting the initiation of yield. The effect of imperfections is less detrimental to the response of high strength structural components, and a modified imperfection factor for columns, which reduces with yield strength, has been proposed to reflect this behaviour [8]; this issue has also been highlighted in the context of local buckling [10].

Local buckling and the structural response of CHS are also influenced by the stress-strain behaviour of the constituent material, which is largely controlled by its chemical composition and physical properties, but is also affected by the fabrication route through which the section is formed. Generally, there are two different types of stress-strain curves – yield point and round house. In the former, stress is linearly proportional to strain up to the yield point, after which a yield plateau and strain hardening may be observed – this behaviour is typical of hot-rolled steel sections. A round house stress-strain curve deviates from linearity at low stresses and displays a gradually yielding behaviour and no sharply defined yield point. Stainless steel and aluminium exhibit this type of behaviour as the basic material response, while cold-formed steel sections also display a rounded stress-strain response due to the Bauschinger effect, whereby residual stresses resulting from plastic deformations induced during production cause deviation of the stress-strain response from linearity upon load reversal. Resistance to local buckling depends on the stiffness of the material, and hence local buckling is promoted by any loss of stiffness due to yielding or nonlinearity. Gradual loss of stiffness as opposed to a sharp yield point is usually regarded as beneficial in terms of structural performance [11,12], with a greater degree of strain hardening enabling higher moment capacities in stocky sections of low $D/t$ ratios.

A further factor to be considered in the response of CHS in bending is ovalization; ovalization refers to the gradual flattening of a tube under bending resulting from the inclined nature of the forces in the tube wall that arise in the deformed configuration. The material and geometric properties of structural metallic tubes preclude failure by ovalization wholly in the elastic range, with yielding or local buckling being the key factors limiting structural resistance. However, ovalization may contribute to failure since hoop stresses are induced in the wall of the tube which will influence the onset of plasticity, and there is a reduction in local curvature of the most heavily compressed region of the tube, which facilitates the onset of local buckling.

**EXISTING SLENDERNESS PARAMETERS AND LIMITS**

Slenderness parameters for CHS in all structural design codes include the geometric diameter-to-thickness ratio $D/t$ and the material yield strength $f_y$ (in N/mm²), but the latter is normalised by a number of different values in the various codes, resulting in a range of measures of slenderness. These are summarised in Table 1, as is the treatment of Class 4 (slender) sections.

In order to make a direct comparison between the various design codes, the slenderness limits have been converted to a common basis, using the slenderness parameter adopted for stainless steel in EN 1993-1-4. This is appropriate since the EN 1993-1-4 slenderness parameter includes the Young’s modulus $E$ as well as the material yield strength, and can therefore reflect the different stiffnesses of the materials, with aluminium in particular having a significantly lower Young’s modulus than both steel and stainless steel. The following values for $E$ were adopted: 210000 N/mm² for structural steel,
200000 N/mm$^2$ for stainless steel and 70000 N/mm$^2$ for aluminium. The modified slenderness limits are presented in Table 2.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CHS SLENDERNESS PARAMETERS ADOPTED IN DIFFERENT STRUCTURAL DESIGN CODES</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design code</th>
<th>Cross-section slenderness parameter</th>
<th>Guidance on effective properties for Class 4 (slender) cross-sections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compression Bending</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EN 1993-1-1 (2005) Structural steel</td>
<td>$D \frac{f_y}{t}$ 235</td>
<td>Clause 3.5.6.4</td>
</tr>
<tr>
<td>BS 5950-1 (2000) Structural steel</td>
<td>$D \frac{f_y}{t}$ 275</td>
<td>Clause 3.6.6</td>
</tr>
<tr>
<td>AISC 360 (2005) Structural steel</td>
<td>$D \frac{f_y}{t}$ 275</td>
<td>Clause 3.5.6.4</td>
</tr>
<tr>
<td>AS 4100 (1998) Structural steel</td>
<td>$D \frac{f_y}{t}$ 250</td>
<td>Clause 3.5.6.4</td>
</tr>
<tr>
<td>EN 1993-1-4 (2006) Stainless steel</td>
<td>$D \frac{f_y}{t}$ 210000</td>
<td>Clause 3.5.6.4</td>
</tr>
<tr>
<td>EN 1999-1-1 (2007) Aluminium</td>
<td>$D \frac{f_y}{t}$ 250</td>
<td>Clause 3.5.6.4</td>
</tr>
</tbody>
</table>

| Note: ¹ No effective section properties are provided but designer is directed to EN 1993-1-6 for shells |

From Table 2, it may be observed that the Class 3 (yield) slenderness limits in compression are fairly consistent between the structural steel and stainless steel design codes, but a more relaxed limit is applied to aluminium. The Class 1 and 2 slenderness limits in bending are also fairly consistent across the range of design codes and materials. However, the Class 3 slenderness limits in bending show significant variation. Note that EN 1993-1-1 and EN 1999-1-1 adopt the same Class 3 slenderness limit for both compression and bending.

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SUMMARY OF CHS SLENDERNESS LIMITS IN DIFFERENT STRUCTURAL DESIGN CODES</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>Structural steel</th>
<th>Stainless steel</th>
<th>Aluminium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design code</td>
<td>EN 1993-1-1</td>
<td>BS 5950</td>
<td>AISC 360</td>
</tr>
<tr>
<td>Class 3 limit in compression</td>
<td>90.0</td>
<td>93.6</td>
<td>98.3</td>
</tr>
<tr>
<td>Class 1 limit in bending</td>
<td>50.0</td>
<td>46.8</td>
<td>62.6</td>
</tr>
<tr>
<td>Class 2 limit in bending</td>
<td>70.0</td>
<td>58.5</td>
<td>-</td>
</tr>
<tr>
<td>Class 3 limit in bending</td>
<td>90.0</td>
<td>163.</td>
<td>8</td>
</tr>
</tbody>
</table>
The Class 3 limit in bending is of particular practical significance because it represents the borderline between fully effective and slender sections, with the latter requiring additional calculation effort for designers. There are two principal reasons for the variation in this slenderness limit between the different design codes. The first relates to the pool of available structural performance data, noting that classification limits are often sensitive to the slenderness range of test data upon which they are based [13]. The Class 3 limit for CHS in bending in EN 1993-1-1 was derived on the basis of tests on stocky sections reported by Sedlacek et al. [14], whereas the same limit in AISC 360, which is significantly more relaxed was based on the results of Sherman [15], combined with other data [16-18] which included sections with a far wider range of slenderness. The second reason relates to the different regional practices in terms of structural reliability. The partial safety factors adopted in the different design codes are summarised in Table 3.

**TABLE 3**

<table>
<thead>
<tr>
<th>Material</th>
<th>Structural steel</th>
<th>Stainless steel</th>
<th>Aluminium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Code</td>
<td>EN 1993-1-1</td>
<td>BS 5950</td>
<td>AISC 360</td>
</tr>
<tr>
<td>Partial safety factor</td>
<td>1.00 1.00</td>
<td>0.90¹</td>
<td>0.90¹</td>
</tr>
</tbody>
</table>

Note: ¹ Partial factor appears in numerator, while others appear in denominator; 1/0.9=1.11

Reliability of the design provisions for cross-section resistance depends upon both the adopted slenderness limit and the partial safety factor. The target reliability index and material over-strength are also influential, as are any possible regional differences in manufacturing standards and tolerances. EN 1993-1-1 employs a partial safety factor of unity, while AISC 360 adopts a value of 0.9 (in the numerator); the EN 1993-1-1 limits would therefore be expected to be stricter, since the limit itself has to effectively compensate for the disparity in safety factors. Reliability analyses are performed in the following section and unified slenderness limits are proposed.

**EVALUATION OF TEST DATA AND PROPOSED SLENDERNESS LIMITS**

A total of 153 test results on circular hollow sections on beams of different materials and configurations under bending have been collated in this study. The following tests were performed: 52 tests on hot-rolled steel sections [11,14,16,17], 33 tests on cold-formed steel sections [17,19-21], 21 tests on fabricated steel sections [15,18,22], 12 tests on very high strength steel sections [23], 20 tests on stainless steel sections [24,25] and 15 tests on aluminium sections [26]. The tests were conducted in three different configurations: 25 in pure bending, 119 in four-point bending and 9 in three-point bending. The cross-section slenderness of the beams varied from 20.4 to 294.5 (using the EN 1993-1-4 measure of slenderness as discussed earlier – i.e. \( \varepsilon = \frac{(235/f_c)(E/210000)^{0.5}} \)); a graph of the ultimate test moment normalised by the elastic moment capacity plotted against the cross-section slenderness is shown in Figure 2. The Class 3 slenderness limits in bending for the various design codes are also shown. The available test results display the anticipated trend of decreasing normalised moment capacity with increasing slenderness, though there is significant scatter in the data, which is believed to relate largely to the factors discussed earlier in this paper. The superior performance of the very high strength steel sections is particularly evident.

In order to obtain a unified slenderness limit achieving a consistent level of safety and incorporating the uncertainty in the test results and the variability of the basic variables (material and geometric properties) in the design expression, a reliability analysis in accordance with EN 1990 (2002) [27] was performed. The analysis was performed on the 106 tests on hot-rolled, cold-formed and
fabricated structural steel sections. Since no formula for deriving effective section properties for Class 4 CHS is provided in EN 1993-1-1, a modified expression based on the BS 5 950-1 provisions was adopted in calculating the design moment capacity for these sections, as given by Equation 2:

$$W_{eff} = W_{el} \left( \frac{90}{D/t} \frac{235}{f_y} \right)$$

where $W_{eff}$ and $W_{el}$ are the effective and elastic section moduli, respectively.

![Figure 2: Normalised test moment capacity versus cross-section slenderness](image)

The parameters assumed in the statistical analyses were based on previous findings on the mechanical properties of structural steel: the ratio of mean to nominal yield strengths (i.e., the material over-strength) was taken as 1.16 and the coefficients of variation of yield strength and geometric properties were taken as 0.05 and 0.02 respectively [28,29]. These values originate from industrial data obtained from European steel producers. The results of the analysis and a summary of the key statistical parameters are presented in Table 4. The following symbols are used: $k_{d,n}=$ design (ultimate limit states) fractile factor for $n$ tests, where $n$ is the population of test data under consideration; $b=$ average ratio of experimental to model resistance based on a least squares fit to the test data; $V_{\delta}=$ coefficient of variation of the tests relative to the resistance model; $V_r=$ combined coefficient of variation incorporating both model and basic variable uncertainties; $\gamma_{M0}'=$ factor by which the mean curve should be reduced to provide a reliable design curve.

### Table 4
SUMMARY OF STATISTICAL ANALYSIS PARAMETERS FOR EN 1990

<table>
<thead>
<tr>
<th>Number of tests</th>
<th>$k_{d,n}$</th>
<th>$b$</th>
<th>$V_{\delta}$</th>
<th>$V_r$</th>
<th>$\gamma_{M0}'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>106</td>
<td>3.18</td>
<td>1.10</td>
<td>0.131</td>
<td>0.142</td>
<td>1.24</td>
</tr>
</tbody>
</table>

A least squares regression fit to the test data set is plotted in Figure 3, which is then scaled down by the required safety factor of 1.24 obtained from the reliability analysis to yield the design curve. The unified Class 3 slenderness limit (i.e., where the design curve passes through $M_s/M_{up}=1.0$) for steel sections was found to be 100 with partial factor of 1.00 adopted in the Eurocode and 135 for the AISC and Australian partial factor of 0.9, with the latter design curve being scaled down by a factor of 1.12 ($=1.24 \times 0.9$) from the mean.
CONCLUSIONS

The factors affecting local buckling in CHS and the treatment of this instability in various structural design codes have been discussed. A large disparity in the Class 3 slenderness limits in bending was observed between the different design codes. Towards the establishment of unified slenderness limits, the results of 153 bending tests on CHS were examined, and reliability analyses were performed in accordance with EN 1990 (2002). Revised Class 3 slenderness limits of 100 for EN 1993-1-1 and 135 for AISC 360 and AS 4100 were proposed. These slenderness limits provide a unified treatment across the design codes since the more relaxed slenderness limit proposed for AISC 360 and AS 4100 is offset by the inclusion of the partial safety factor of 0.90 adopted in these codes. Further investigation is underway in this area.

REFERENCES


EXPERIMENTAL STUDY ON BEHAVIOR OF SHUTTLE-SHAPED LATTICE TUBULAR COLUMNS

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KEYWORDS

Lattice tubular columns, full or diminishing scale tests, ultimate loading capacity, failure mode.

ABSTRACT

A new type of tubular column with shuttle shape composed of three circular hollow section members which are connected with some batten plates distributed at regular intervals along the column axis has been increasingly used in modern architectures. The behavior of this type of column is investigated experimentally in the paper. Tests on two columns about 18m in length based on the engineering project of Shanghai Expo are carried out under axial loading. Self-balanced loading equipment is designed and fabricated particularly for the tests. Experimental setup and results are presented and discussed. Both the shuttle-shaped lattice tubular columns show a failure mode of bending-torsional buckling in which bending deformation is dominant, accompanied by plastic yielding in some circular tubes and batten plates when the columns lose their ultimate loading capacity.

INTRODUCTION

Shanghai Expo will open in May 2010. A wide variety of attractive buildings for exhibition are under construction now, in which a lot of innovative structural forms are used. For example, a new type of column with shuttle shape composed of three circular hollow section members which are arranged in an equilateral triangle and connected with some batten plates distributed at regular intervals along the column axis is designed as a mast to support the cable-membrane structure shown in Figure 1. The design recommendations for the common type of lattice tubular column with constant cross section along its axis can be found in some design codes for steel structures ([1], [2]), but it seems no design code covers the lattice tubular column with variable cross section. In the paper, experiments on the new type of column are carried out, to investigate its behavior and check whether its design is reasonable.
TEST SPECIMENS AND SETUP

Test Specimens

Experiments focused on two different dimensions of lattice tubular columns used in real engineering. Two specimens C1 and C2 were arranged, as shown in Table 1. The specimen C2 was full-size scale, but the specimen C1 was set at a scale of 1:2 for the sake of limited area in experimental site. Both specimens were manufactured in quality as same as the actual columns. Table 2 shows the testing results of mechanical property of steel used for specimens.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Scale</th>
<th>Length (m)</th>
<th>Tube size (mm)</th>
<th>Side length at mid-span (mm)</th>
<th>Number of batten plates</th>
<th>Batten plates thickness (mm)</th>
<th>Intervals of batten plates (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1:2</td>
<td>17.86</td>
<td>Φ194×20</td>
<td>800</td>
<td>14</td>
<td>30</td>
<td>1000</td>
</tr>
<tr>
<td>C2</td>
<td>1:1</td>
<td>17.564</td>
<td>Φ325×20</td>
<td>1200</td>
<td>6</td>
<td>30</td>
<td>2000</td>
</tr>
</tbody>
</table>

Note: 1. The specimen length is the distance between the centre of pin connection at two ends (see Figure 2).
2. The side length at mid-span section is the distance between centers of every two circular tubes.

Test Set-Up

As shown in Figure 2, self-balanced loading equipment composed of four tension rods, three blocks and four 5000kN jacks was designed and fabricated specially. The specimen was installed horizontally inside the equipment by pinned connection with the blocks A and B.
The block A was fixed, but the blocks B and C could be slid through rollers beneath them when the jacks exerted force on the specimen.

A lot of displacement meters (see the symbol D in Figure 3) were used to measure the displacements $\mu_x$, $\mu_y$ and $\mu_z$ of specimen at the ends and nodes where three tubes were connected to batten plates, and many strain gauges (see the symbol S in Figure 3) were used to measure the strains or stresses in the tubes, batten plates and tension rods.

**Figure 2: Specimens and loading equipment**

**Figure 3: Layout of displacement meters and strain gauges on specimens**

**EXPERIMENTAL RESULTS AND DISCUSSION**

**Ultimate Loading Capacity**

Figure 4 shows some typical load-displacement curves for the moving ends of specimens in Z direction and the centers of batten plates in X and Y directions. The line in connection with the centers of all batten plates corresponded to the axis of the lattice tubular column. It can be
seen that the curves in Figure 4(b), (c), (e) and (f) had significant geometrical nonlinear behavior and increased to their peaks. The ultimate loading capacity of specimens C1 and C2 came to 7380kN and 13125kN respectively, which exceeded the design values of the actual columns.

![Load-displacement curves of specimens](image)

**Failure Mode**

Figure 4 shows the pictures taken after the specimens lost their ultimate loading capacity and Figure 5 presents the displacement distribution and development along the nodes (or batten plates) at several stages of axial force. Both specimens had similar failure mode, namely buckling, although they were quite different in dimension. Large bending deformation occurred in each lattice tubular column as its axial line became a sine wave roughly, not only in the direction X, but also in the direction Y. At the same time, torsional deformation around the direction Z took place, but it was quite small.

![Failure mode of specimens](image)
Advances in Steel Structures
ICASS’09, 16-18 December 2009, Hong Kong, China

Figure 6: Displacement distribution of specimens at several stages of axial force

Strains in Tubes and Batten Plates

The three tubes for each specimen were mainly on the condition of elasticity up to the specimen failure except for some locations where bending deformations were large as shown in Figure 6. Figure 7 presents load-strain curves of the tubes at these locations, which indicated the strains at final loading stage were beyond the yielding strain of the tube for the specimen C1 or C2.

Figure 8 shows the strain distribution on the batten plates when specimens were subjected to their ultimate loads. Only the batten plates close to the tubes C1-a and C1-b and close to the tube C2-a got into plasticity, where the strains were much higher than the yielding strain 1592με.

Figure 7: Load-strain curves of tubes at some locations
CONCLUSION

(1) The ultimate loading capacity of the shuttle-shaped lattice tubular columns C1 and C2 under axial force can arrive at 7380kN and 13125kN respectively, which meets needs of design requirement.

(2) The new type of lattice tubular column usually presents the failure mode of bending-torsional buckling, in which bending deformation is dominant and like sine wave roughly.

(3) The column is generally on the condition of elasticity after failure, except for the batten plates close to the connection with tubes as well as the locations where the tubes had large bending deformation.

(4) The new type of lattice tubular column is reasonable in design and can be applied in engineering.

REFERENCES


FLEXURAL MOMENT CAPACITY DESIGN RULES FOR BUILT-UP LITESTEEL BEAMS

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KEYWORDS
Built-up back to back LiteSteel beams, Cold-formed steel structures, Lateral distortional buckling.

ABSTRACT
A new cold-formed steel beam, called the LiteSteel Beam (LSB), is a channel section with two rectangular hollow flanges and a slender web, and is manufactured using a combined cold-forming and electric resistance welding process. Research on the flexural behaviour of single LSB members showed that the LSBs are susceptible to lateral distortional buckling effects and their moment capacities are significantly reduced for intermediate spans. Built-up LSB sections are expected to mitigate the effects of lateral distortional buckling and improve their flexural capacity, and hence enhance their applications in the construction industry. Many steel structures design codes have design guidelines in relation to connecting two conventional channels to form an I-section, and the required longitudinal spacing of such connections. Currently, the safe spans of built-up beams are determined by doubling the capacity of a single section. Research has shown that these guidelines are conservative. Therefore, lateral buckling tests and advanced numerical analyses were undertaken to investigate the flexural behaviour of back to back LSBs connected by fasteners at various longitudinal spacings under uniform moment conditions. The results were then used to develop suitable design rules for back to back LSBs. This paper presents the details of this investigation and the results including the new design rules developed for back to back built-up LSBs.

INTRODUCTION
LiteSteel Beam (LSB) is a new cold-formed steel beam produced by Australian Tube Mill (ATM) and marketed by LiteSteel Technologies (LST). The new beam is effectively a channel section with two rectangular hollow flanges and a slender web, and is manufactured using a combined cold-forming and electric resistance welding process (Figure 1 (a)). The LSB has a unique shape with superior torsional strength properties, provides a very high strength to weight ratio, and is on average 40% lighter than traditional hot-rolled sections of equivalent bending strength. The section depth and flange width of 13 available LSB sections vary from 125 to 300 mm and 45 to 75 mm, respectively.
while their thicknesses vary from 1.6 to 3.0 mm. LiteSteel Technologies (LST) are now promoting the use of LSBs as floor joists and bearers in residential construction, replacing hot-rolled beams.

Built-up LSB sections are expected to improve their flexural capacities and expand their usage to long span applications such as header beams, floor bearers, and hanging beams. They can be fabricated using the traditional back to back configuration as shown in Figure 1 (b) and can produce more than double the bending capacity of single LSBs. Mahaarachchi and Mahendran’s [1] research on single LSB sections showed that the LSBs are susceptible to lateral distortional buckling that reduced their moment capacity significantly for intermediate spans. The back to back built-up LSB is likely to mitigate lateral distortional buckling effects by providing additional rigidity to the weakest element of the section, the web. However, the behaviour of built-up beams is not well understood and the current design rules [2] are found to be conservative in predicting their moment capacities. Therefore new design rules were developed for back to back LSB sections based on detailed experimental and numerical studies. This paper presents the details of this research on back to back built-up LSB flexural members and the new design rules developed. It also includes a review of the current design rules for built-up beams.

**EXPERIMENTAL STUDY**

Twelve lateral buckling tests of back to back LSBs with varying fastener spacings were undertaken. Test specimens were chosen to represent all three levels of section compactness, i.e. Compact, Non-compact and Slender. Their span length selected was 3.5 m based on the current test rig capacity and was within the practical range of 12 to 24 times the section depth (d). Fastener spacings (FS) selected are the minimum spacing of span/6 as specified in AS/NZS 4600 [2], span/4, span/3, span/2, and span/1, i.e. no connections between the two end supports. Details of the test specimens are given in Table 1.

The test rig used in Mahaarachchi and Mahendran’s [1] experimental tests of single LSB sections was modified for the tests on back to back built-up LSB sections. It consists of a support system and a loading system, which are attached to an external frame structure (Figure 2 (a)). The support system was designed to ensure that the test beams were simply supported in-plane and out-plane (Figure 2 (b)). Loading arms were specially designed to apply the loads through the shear centre. The loading system was designed to prevent any restraint to the displacements and rotations of the test beam using a special wheel system. The loads were applied at the end of each overhang of the test beam to simulate a uniform moment within the span. They were applied using two hydraulic rams based on a displacement control method.
Experimental responses of back to back built-up LSBs were evaluated based on four important parameters, the ultimate moment capacity, bending deformations, failure mode and the flange separation. More details are given in Jeyaragan and Mahendran [3]. Table 1 presents only the moment capacity results and the failure mode for each tested beam while Figure 3 shows a typical failure mode of back to back LSBs. As seen in Table 1, the failure mode was governed by lateral distortional buckling for all the back to back LSB specimens. Single LSBs also exhibited lateral distortional buckling failure as shown by Mahaarachchi and Mahendran [1]. In general, test results showed that the moment capacity of back to back built-up LSBs was influenced by the fastener spacing. The beams with a fastener spacing of span/6 had an increase of 12-20% in comparison with a fastener spacing of span/l. Significant moment capacity increase was noted when compared with corresponding single LSBs. However, at present the moment capacities of back to back beams are determined by doubling the capacity of single beams. This conservative assumption was found to underestimate the capacity of back to back built-up LSB sections. The detrimental effects of lateral distortional buckling that occurs with single LSB sections appears to still remain with back to back built-up LSBs, but it is not as severe as for single LSBs.

### Table 1

<table>
<thead>
<tr>
<th>Test No</th>
<th>Specimens d × b × t</th>
<th>Span (mm)</th>
<th>FS (mm)</th>
<th>Mₜ (kNm)</th>
<th>Failure Mode</th>
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<td>20.45</td>
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<tr>
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<td>19.73</td>
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<tr>
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<td>19.84</td>
<td>LDB</td>
</tr>
</tbody>
</table>

Note: d – Depth of LSBs, b – Width of flange element, t – Thickness, FS – Fastener spacing, Mₜ – Ultimate moment capacity, LDB – Lateral distortional buckling.
FINITE ELEMENT ANALYSIS

This research entails development of two finite element models, namely ideal and experimental models using ABAQUS [4]. Experimental models (Figure 3b) were generated to validate the finite element models in comparison with experimental results whereas ideal models (Figure 4) were developed to conduct parametric studies and hence to develop design rules. Experimental and ideal finite element models are described in Jeyaragan and Mahendran [5,6].

In these models, shell elements (S4R5) with an optimum size of 5 mm x 10 mm were used based on convergence studies. A simplified bi-linear stress-strain curve with no strain hardening was used. Idealized simply supported conditions with a uniform moment within the span were implemented in the ideal models as this gives a lower bound solution. Only half the span was modeled because of symmetric loading and support conditions. Simply supported boundary conditions were applied at the support while symmetric boundary conditions were applied at mid-span. Using longitudinal compression and tension point forces a uniform moment was created about the major axis. A global imperfection of L/1000 was used based on the fabrication tolerance in AS4100 [7]. Perfect Tie MPC was used to simulate the fastener connections. Contact modelling was implemented in order to simulate the interaction between the two LSB sections connected back to back. Surface-based contact simulation was found to be adequate to represent the contact interaction between them. Small-sliding tracking approach, “hard” contact pressure-overclosure relationship, zero friction, deformable body conditions and an initial gap of 0.1 mm were used in this contact model. Both flexural and membrane residual stresses were used. The variation of flexural residual stress through the thickness was assumed to be linear, with zero stress at the centre fibre. Figure 4 shows the typical deformed shape at failure of back to back LSBs from the analysis of an ideal model.
The nonlinear experimental finite element models were validated by comparing the failure modes and moment capacities from the experimental tests (Figures 3a,b). Elastic lateral buckling moments obtained for the back to back LSBs connected continuously were compared with the predictions from a finite strip analysis program THIN-WALL. These comparisons are reported in Jayaragan and Mahendran [6] in detail. The ideal finite element model was then used to obtain the ultimate moment capacities of all 13 LSB sections with varying spans and fastener spacings (span/2, span/3, span/4 and span/6). Current design rules were reviewed using the moment capacity data, and based on which new design rules were developed for back to back built-up LSBs. These details are discussed next.

DESIGN RULES

AS 4600 [2] provides design rules for members subject to lateral distortional buckling. The member moment capacity \( M_b \) is calculated based on the effective section modulus \( Z_c \) calculated at a stress \( f_c \), determined as \( M_c/Z_f \), where \( Z_f \) is the full section modulus

\[
M_b = Z_c f_c \quad (1)
\]

The critical distortional buckling moment \( M_c \) is calculated as follows and then used in Eq. 1.

For \( \lambda_d \leq 0.59 \):

\[
M_c = M_y \quad (2a)
\]

For \( 0.59 < \lambda_d < 1.70 \):

\[
M_c = M_y \left( \frac{0.59}{\lambda_d} \right) \quad (2b)
\]

For \( \lambda_d \geq 1.70 \):

\[
M_c = M_y \left( \frac{1}{\lambda_d} \right) \quad (2c)
\]

where \( \lambda_d = \) Non-dimensional slenderness \( \sqrt{M_y/M_{od}} \), \( M_y = \) First yield moment \( (= Z_{fy}) \), \( M_{od} = \) Elastic lateral distortional buckling moment

The FEA results of back to back LSBs with fastener spacings of span/6 to span/2 were non-dimensionalised and plotted in Figure 5. The FEA results also showed that a spacing of span/6 gave the maximum moment capacities. However, span/3 may be adequate for LSBs with some spans. The comparison shown in Figure 5 confirms that the current AS/NZS 4600 [2] design rule for lateral distortional buckling is not suitable as it is quite conservative for beams of intermediate slenderness (inelastic buckling region). However, they are reasonably accurate in other regions of yielding/local buckling and elastic lateral buckling. The comparison of FEA results to AS/NZS 4600 predictions...
gave a mean value of 1.166 and a COV of 0.089. The capacity reduction factor (φ) calculated was 1.032 and confirms that the current AS/NZS 4600 design rule is quite conservative.

Anapayan and Mahendran [8] proposed new design rules within the current guidelines of AS/NZS 4600 for single LSB sections to predict their moment capacities accurately in all three regions. Their moment capacity equations for the three regions of yielding/local buckling, inelastic lateral distortional buckling and elastic lateral buckling are as follows:

For $\lambda_d \leq 0.54$:
$$M_c = M_y \left( \frac{1}{\lambda_d^2} \right) (3a)$$

For $0.54 < \lambda_d < 1.74$:
$$M_c = M_y \left( 0.28\lambda_d^2 - 1.20\lambda_d + 1.57 \right) (3b)$$

For $\lambda_d \geq 1.74$:
$$M_c = M_y \left( \frac{1}{\lambda_d^2} \right) (3c)$$

Figure 6 compares the non-dimensional member moment capacity predictions using the above equations with FEA moment capacity results of back to back LSB members. The comparison demonstrates that the moment capacities predicted by the proposed design rules of single LSB sections are also conservative. It gave a mean value of 1.104 and a COV of 0.079 while the capacity reduction factor was 0.986. This confirms that although the new design rules of single LSB sections in [8] are more accurate than the current AS/NZS 4600 design rules, further improvements are needed for back to back LSB sections.

The main aim of this research is to derive and verify appropriate design rules for back to back LSB sections. The FEA results were compiled and used to derive suitable design equations as for single LSB sections [8]. In the non-dimensionalised format, the lateral torsional buckling strength predictions given in AS/NZS 4600 [2] as well as the prediction proposed for single LSB sections are the same, and they predict the moment capacities of long span back to back LSB members accurately. Therefore only the moment capacity equation for the inelastic buckling region had to be modified based on the mean of all the results in this region. Also the limits which define the three regions had to be reviewed based on the non-dimensional moment capacity results. This design rule is given by Eq. 4b while Eqs. 4a and c provide the moment capacities in the yielding/local buckling and elastic lateral buckling regions, respectively.

For $\lambda_d \leq 0.54$:
$$M_c = M_y (4a)$$

For $0.54 < \lambda_d < 1.74$:
$$M_c = M_y \left( 0.28\lambda_d^2 - 1.29\lambda_d + 1.73 \right) (4b)$$
The new Eq. 4b gave a mean of 1.008 and a COV of 0.071 for the ratio of FEA to predicted capacity. A suitable capacity reduction factor of 0.905 was calculated for Eq. 4b using the AISI procedure [9]. The member slenderness values that separate the three regions were revised slightly. The new limits representing the three regions are 0.65 and 1.80 (see Eq. 5). Based on these new limits the mean and COV for the ratio of FEA to predicted capacities were recalculated, and they are 1.000 and 0.062, respectively. The capacity reduction factor ($\phi$) also slightly changed to 0.903, which is very close to the currently recommended reduction factor of 0.90 indicating that Eq. 5b shows a good agreement with FEA results (see Figure 7). Figure 7 compares the predicted moment capacities from Eqs. 5a to c with FEA results.

For $\lambda_d \geq 1.74$ : 
$$M_c = M_y \left( \frac{1}{\lambda_d^2} \right)$$ (4c)

\[ \text{For } \lambda_d \leq 0.65 : \quad M_c = M_y \] (5a)

\[ \text{For } 0.65 < \lambda_d < 1.80 : \quad M_c = M_y \left( 0.28\lambda_d^2 - 1.29\lambda_d + 1.73 \right) \] (5b)

\[ \text{For } \lambda_d \geq 1.80 : \quad M_c = M_y \left( \frac{1}{\lambda_d^2} \right) \] (5c)

Figure 7: Comparison of Experimental and FEA Moment Capacities with Eqs.5a-c Predictions

Figure 7 also compares the experimental results (EXP) of back to back built-up LSBs with Eq.5b predictions, which gave a mean of 1.072 and a COV of 0.081. The capacity reduction factor in this case was found to be 0.947, which is conservative. However, the comparison was made with fewer test results (12 specimens). Also the experimental tests were conducted using the overhang loading method that could have created some warping effects. Although the test results were used in non-dimensionalised moment capacity plots, the effect of possible warping effects would not have been eliminated completely.

CONCLUSION

This paper has described the details of experimental and numerical studies on the behaviour of back to back built-up LSB members and the details of the new design rules developed in this research. The results show that the back to back built-up LSB sections have greater moment capacities, allowing their use in large span applications. The moment capacities of built-up LSB sections were found to be more than twice the capacities of corresponding single LSB sections, and are a function of the section geometry, span length and fastener spacing. The current AS/NZS 4600 design rule for lateral distortional buckling was found to be quite conservative. The new design rules of single LSB sections
are also conservative although they are more accurate than the current AS/NZS 4600 design rules. Hence new design rules have been developed to predict the moment capacities of back to back LSB members.

ACKNOWLEDGEMENTS

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REFERENCES

POST-BUCKLING STRENGTH OF LITESTEEL BEAMS IN SHEAR

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KEYWORDS
LiteSteel beams, Cold-formed steel structures, Shear Strength, Direct Strength Method, Post-buckling.

ABSTRACT

This paper presents the details of experimental and numerical studies on the shear behaviour of a recently developed, cold-formed steel beam known as LiteSteel Beam (LSB). The LSB sections are produced by a patented manufacturing process involving simultaneous cold-forming and electric resistance welding. It has a unique shape of a channel beam with two rectangular hollow flanges. Recent research has demonstrated the presence of increased shear capacity of LSBs due to the additional fixity along the web to flange juncture, but the current design rules ignore this effect. Therefore they were modified by including a higher elastic shear buckling coefficient. In the present study, the ultimate shear capacity results obtained from the experimental and numerical studies of 10 different LSB sections were compared with the modified shear capacity design rules. It was found that they are still conservative as they ignore the presence of post-buckling strength. Therefore the design rules were further modified to include the available post-buckling strength. Suitable design rules were also developed under the direct strength method format. This paper presents the details of this study and the results including the final design rules for the shear capacity of LSBs.

INTRODUCTION

LiteSteel Beam (LSB) is a new cold-formed steel hollow flange channel beam produced by OneSteel Australian Tube Mills (see Figure 1). The LSB has a unique shape of a channel beam with two rectangular hollow flanges, and is manufactured using dual electric resistance welding and automated continuous roll-forming technologies. It has the beneficial characteristics of including torsionally rigid closed rectangular flanges combined with
economical fabrication processes from a single strip of high strength steel. The cross-sectional shape of the beam has been designed such that it provides a very high structural performance compared to other cold-formed steel beams produced to date. The integral benefits of lightweight, strength, and ease of constructability offer a new option of using LSBs for structural engineers. The LiteSteel beam has a wide range of applications in residential, commercial and industrial buildings (Figure 1), and is on average 40% lighter than traditional hot-rolled structural sections of equivalent bending strength [1].

In the building systems, LSB sections are commonly used as flexural members, for example, floor joists and bearers. For LSBs to be used as flexural members, their flexural and shear capacities must be known. Flexural behaviour of LSBs has been investigated recently by Mahaarachchi and Mahendran [2] by using experimental and numerical studies, and hence the moment capacities of LSBs are available. However, the shear behaviour of LSBs has not been investigated yet. Past research [3,4] was restricted to plate girders and the shear buckling behaviour of the new mono-symmetric LSB sections has not been investigated. Therefore experimental and numerical studies were undertaken to investigate the shear behaviour of LSB sections including their shear buckling characteristics, and to develop improved shear design rules that take into account the effects of additional fixity along the web to flange juncture of LSBs and post-buckling strength. This paper describes the details of these studies on the post-buckling strength of LSBs in shear. It presents the results including the new shear design rules for LSBs.

**EXPERIMENTAL STUDY**

Experimental studies were carried out to investigate the shear behaviour and strength of LSBs using a series of primarily shear tests of simply supported LSBs subjected to a mid-span load (see Figure 2). Two LSB sections were bolted back to back using three T-shaped stiffeners located at the end supports and the loading point in order to eliminate any torsional loading of test beams. In order to simulate a primarily shear condition, relatively short test beams of span based on two aspect ratios (shear span a/ clear web height d₁) of 1 and 1.5 were selected. Test specimens were chosen such that all three types of shear failure (shear yielding, inelastic and shear buckling) occurred in the tests. A 20 mm gap (see Figure 2) was included between the LSB sections to allow the test beams to behave independently while remaining together to resist torsional effects. The stiffeners were used to avoid eccentric loading and web crippling.

**FINITE ELEMENT ANALYSIS**

This section describes the development of finite element models to investigate the ultimate shear strength behaviour of LSBs (Figures 4 (a) and (c)). For this purpose, a general purpose finite element program ABAQUS Version 6.7 [5], which has the capability of undertaking nonlinear geometric and material analyses of three dimensional structures, was used. Finite element models of tested LSBs were developed with the objective of simulating the actual test members’ physical geometry, loads, constraints, mechanical properties, residual stresses and initial geometric imperfections as closely as possible. The shell element in ABAQUS called S4R5 was used to model the shear behaviour of LiteSteel beams. R3D4 rigid body elements were used to simulate the restraints and loading in the finite element models. The elastic modulus and Poisson’s ratio were taken as 200000 MPa and 0.3, respectively. Simply supported boundary conditions were implemented under a three-point loading arrangement.
Figures 4 (a) and (b) show the elastic shear buckling deformations of 200x45x1.6 LSB (Aspect Ratio = 1.5) from FEA and experiments.

Figure 1: LiteSteel Beam

Figure 2: Experimental Set-up

(a) Shear Yielding Failure Buckling  (b) Inelastic Shear Buckling  (c) Elastic Shear

Figure 3: Shear Failure Modes of LSBs

Figure 2 shows the experimental set-up used in this research. Table 1 presents the experimental results while Figure 3 shows the typical shear failure modes LSBs.

Figure 4: Elastic Shear Buckling of 200x45x1.6 LSB (Aspect Ratio = 1.5)

Figure 5: Applied Mid-span Load versus Lateral Deflection for 200x45x1.6 LSB

Figure 5 shows the applied mid-span load versus lateral deflection curve for 200x45x1.6 LSB (Aspect Ratio = 1.0). An arbitrary small initial imperfection value of d1/100,000 was included.
in an attempt to discern a bifurcation-type buckling load for 200x45x1.6 LSB (Aspect Ratio = 1.0). Point 1 in Figure 5 gives the elastic buckling load of 108 kN, which agrees well with the predicted value of 108.58 kN from the proposed design equation (Eq. 3). Figure 5 shows the presence of significant reserve capacity beyond elastic buckling for LSBs in shear. Shear tests also showed the presence of post-buckling strength. Therefore the design equations for the shear strength of LSBs should include such reserve post-buckling strength in shear.

VALIDATION OF FINITE ELEMENT MODEL USING EXPERIMENTAL RESULTS

It is necessary to validate the developed finite element model for non-linear analyses of LSBs. This was achieved by comparing the non-linear finite element analysis results with the results obtained from shear tests. Shear buckling and failure modes from finite element analyses (FEA) agreed well with experimental modes (Figure 4). The applied mid-span load versus deflection curves also agreed reasonably well [6]. Table 1 presents a summary of the ultimate shear capacity results of the non-linear static analyses using the finite element model developed in this research and a comparison of these results with corresponding experimental results. The mean and COV of the ratio of ultimate shear capacities from experiments and FEA are 0.99 and 0.028. This indicates that the finite element model predicts the ultimate shear behaviour and capacities of LSBs with very good accuracy.

<table>
<thead>
<tr>
<th>No.</th>
<th>LSB Sections</th>
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</tr>
<tr>
<td>21</td>
<td>250x75x2.5</td>
<td>1.5</td>
<td>118.9</td>
<td>0.98</td>
</tr>
<tr>
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<td>300x60x2.0</td>
<td>1.5</td>
<td>&gt;75.0</td>
<td>NA</td>
</tr>
<tr>
<td>23</td>
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<td>1.5</td>
<td>125.1</td>
<td>0.95</td>
</tr>
<tr>
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</tr>
<tr>
<td>25</td>
<td>200x60x2.5</td>
<td>1.6</td>
<td>107.9</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Mean = 0.99
COV = 0.028
PROPOSED DESIGN FORMULAE FOR THE SHEAR STRENGTH OF LITESTEEL BEAMS

New shear strength formulae ($\tau_v$) were developed for LSBs based on their experimental and FEA results and the current design equations for shear strength given in AISI [7]. Equations 1 to 3 present the new proposed design equations. The increased shear buckling coefficient given by Equation 4 ($k_{LSB}$) is included to allow for the additional fixity in the web-flange juncture [6]. It is to be noted that Equations 2 and 3 were not modified to include the reserve post-buckling strength.

$$\tau_v = \tau_{yw} = 0.6f_{yw} \quad \text{for} \quad \frac{d_1}{t_w} \leq \sqrt{\frac{E_{LSB}k_{LSB}}{f_{yw}}} \quad \text{(Shear yielding)}$$  \hspace{1cm} (1)

$$\tau_v = 0.6\sqrt{\frac{E_{LSB}f_{yw}}{\frac{d_1}{t_w}}} \quad \text{for} \quad \sqrt{\frac{E_{LSB}}{f_{yw}}} < \frac{d_1}{t_w} \leq 1.508 \sqrt{\frac{E_{LSB}}{f_{yw}}} \quad \text{(Inelastic shear buckling)}$$  \hspace{1cm} (2)

$$\tau_v = 0.905\frac{E_{LSB}}{2\left(\frac{d_1}{t_w}\right)^2} \quad \text{for} \quad \frac{d_1}{t_w} > 1.508 \sqrt{\frac{E_{LSB}}{f_{yw}}} \quad \text{(Elastic shear buckling)}$$  \hspace{1cm} (3)

For LSBs $k_{LSB} = k_{ss} + 0.87(k_{sf} - k_{ss}) \quad \text{for} \quad a/d_1 \geq 1$  \hspace{1cm} (4)

where $d_1$, $t_w$ = clear height and thickness of web; $\tau_{yw}$ = shear yield strength and $f_{yw}$ = yield strength, $k_{ss}$ and $k_{sf}$ are shear buckling coefficients of plates with simple-simple and simple-fixed boundary conditions.

Equations 5 to 7 are now proposed in which post-buckling strength is included. Here post-buckling is included in the inelastic and elastic buckling regions to replace Equations 2 and 3. New design Equations for shear strength (Eqs. 6 and 7) are based on Lee et al. [8], who used a similar approach for plate girders. The nominal shear capacities ($V_v$) can be calculated by multiplying the shear strengths ($\tau_v$) from Equations 5 to 7 by the area of web element ($d_1t_w$).

$$\tau_v = \tau_{yw} \quad \text{for} \quad \frac{d_1}{t_w} \leq \sqrt{\frac{E_{LSB}}{f_{yw}}} \quad \text{(Shear yielding)}$$  \hspace{1cm} (5)

$$\tau_v = \tau_i + 0.2(\tau_{yw} - \tau_i) \quad \text{for} \quad \sqrt{\frac{E_{LSB}}{f_{yw}}} < \frac{d_1}{t_w} \leq 1.508 \sqrt{\frac{E_{LSB}}{f_{yw}}} \quad \text{(Inelastic shear buckling)}$$  \hspace{1cm} (6)

$$\tau_v = \tau_e + 0.2(\tau_{yw} - \tau_e) \quad \text{for} \quad \frac{d_1}{t_w} > 1.508 \sqrt{\frac{E_{LSB}}{f_{yw}}} \quad \text{(Elastic shear buckling)}$$  \hspace{1cm} (7)

where $\tau_{yw} = 0.6f_{yw}$  \hspace{1cm} (8)

$$\tau_i = 0.6\sqrt{\frac{E_{LSB}f_{yw}}{\left[\frac{d_1}{t_w}\right]^2}}$$  \hspace{1cm} (9)
The new direct strength method (DSM) provides simple design procedures for cold-formed steel members. Proposed design equations (Eqs. 1 to 3 and 5 to 7) are therefore recast in the DSM format and are given as Equations 11 to 13 and 16 to 18 [9]. The ultimate shear stress \( \tau_u \) was calculated as the ultimate shear capacity from tests or FEA divided by web area of \( d_1 t_w \) whereas the slenderness \( (\lambda) \) was calculated using Equation 15. Equations 11 to 13 present the proposed direct strength method (DSM) design equations in which post-buckling strength is not included. Experimental and FEA results are compared with non-dimensional shear strength curve based on the proposed DSM equations in Figure 6. It is to be noted that in the non-dimensional shear strength curve with slenderness \( (\lambda) \) as the horizontal axis all the results can be plotted together.

\[
\tau_v = \frac{0.905 E k_{LSB}}{\left[ \frac{d_1}{t_w} \right]^2}
\]

(10)

**DIRECT STRENGTH METHOD**

\[
\frac{\tau_v}{\tau_{yw}} = 1 \quad \text{for} \quad \lambda \leq 0.815
\]

(11)

\[
\frac{\tau_v}{\tau_{yw}} = \frac{0.815}{\lambda} \quad \text{for} \quad 0.815 < \lambda \leq 1.23
\]

(12)

\[
\frac{\tau_v}{\tau_{yw}} = \frac{1}{\lambda^2} \quad \text{for} \quad \lambda < 1.23
\]

(13)

where \( k_{LSB} \) and \( \tau_{yw} \) = as defined in Eqs.4 and 8, respectively

\[
\frac{\tau_{cr}}{\tau_{yw}} = \frac{k_{LSB} \pi^2 E}{12(1-\nu^2)} \left( \frac{t_w}{d_1} \right)^2
\]

(14)

\[
\lambda = \sqrt{\frac{\tau_{yw}}{\tau_{cr}}} = 0.815 \left( \frac{d_1}{t_w} \right) \sqrt{\frac{f_{yw}}{E k_{LSB}}}
\]

(15)

Equations 16 to 18 present the proposed DSM design equations in which post-buckling strength is included.

\[
\frac{\tau_v}{\tau_{yw}} = 1 \quad \text{for} \quad \lambda \leq 0.815
\]

(16)

\[
\frac{\tau_v}{\tau_{yw}} = \frac{0.815}{\lambda} + 0.2 \left( 1 - \frac{0.815}{\lambda} \right) \quad \text{for} \quad 0.815 < \lambda \leq 1.23
\]

(17)

\[
\frac{\tau_v}{\tau_{yw}} = \frac{1}{\lambda^2} + 0.2 \left( 1 - \frac{1}{\lambda^2} \right) \quad \text{for} \quad \lambda > 1.23
\]

(18)

**COMPARISON WITH PROPOSED SHEAR STRENGTH EQUATIONS**

In this section, the shear strengths from FEA and experiments are compared with the predictions of proposed shear strength equations. Both FEA and experimental results are first
plotted in Figure 6 and compared with the new DSM based shear design equations. Figure 6 shows that there is considerable amount of post-buckling strength for LSBs subjected to shear, particularly in the case of large clear web height to thickness ratios (d₁/tw). The proposed DSM based design equations including the post-buckling strength (Eqs. 17 and 18) are able to predict the shear strengths of LSBs accurately as seen in Figure 6.

![Graph showing comparison of shear strengths](image)

Figure 6: Comparison of Shear Strengths of LSBs with DSM based Design Equations

Figures 7 (a) and (b) present the design curves based on the proposed shear strength equations within the AISI (2007) guidelines (Equations 1 to 3 and 5 to 7) with a φₕ factor of 0.95 for the aspect ratios of 1.0 and 1.5, respectively. They are compared with the ultimate shear capacities from FEA and experiments, and the current AS/NZS 4600 [10] design equations with a φₕ factor of 0.90. The shear strength design equations in AS/NZS 4600 are similar to those in AISI [7] with the main differences of 0.64fₚw in Eq.1 (instead of 0.60 fₚw) and a φₕ factor of 0.90 (instead of 0.95). The minimum measured value of LSB web yield stress was approximately 430 MPa. Therefore this value was used in plotting the design curves in these figures. Both FEA and experimental results plotted in Figures 7 (a) and (b) show that the shear strengths predicted by the current design rules in AS/NZS 4600 are very conservative whereas the proposed shear design equations including post-buckling strength are accurate in predicting the shear strengths of LSBs.

![Graph showing design shear strength curves](image)

Figure 7: Design Shear Strength of LSB versus Web Height to Thickness Ratio (d₁/tw)

fₚw = 430 MPa
Detailed parametric studies have also been undertaken using the validated finite element model to further study the shear behaviour and strengths of LSBs in shear. They have also confirmed the findings relating to post-buckling strength and the new design equations presented in this paper.

CONCLUSION

This paper has presented the details of experimental and numerical studies into the ultimate shear strength behaviour of a new cold-formed steel beam known as LiteSteel beams (LSB), and the results. Finite element models of LSBs in shear were developed and validated by comparing their results with experimental test results. Nonlinear finite element analyses were able to predict the ultimate shear capacities of LSBs with very good accuracy.

Both experiments and finite element analyses showed the presence of significant reserve strength beyond elastic buckling for LSBs in shear. Therefore new shear strength design equations were proposed within the guidelines of the current Australian and American cold-formed steel structures codes and the new Direct Strength Method. The ultimate shear capacities of LSBs from experiments and nonlinear finite element analyses were compared with the current AS/NZS 4600 design equations and the proposed shear design equations. This comparison shows that the current design rules in AS/NZS 4600 are conservative for the shear buckling design of LSBs. The proposed design equations are able to predict the shear capacities of LSBs accurately.

ACKNOWLEDGEMENTS

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FURTHER DEVELOPMENT OF STATISTICAL MOMENT-BASED DAMAGE DETECTION METHOD

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KEYWORDS

Structural damage detection, Statistical moment, Random excitation, Measurement noise, Incomplete measured responses.

ABSTRACT

A novel structural damage detection method with a new damage index has been recently proposed by the authors based on the statistical moments of dynamic responses of a shear building structure subject to white noise ground motion. The statistical moment-based damage detection (SMBDD) method is theoretically extended in this paper to include any types of structures with incomplete measured structural responses to various types of external excitations. The basic equations of the generalized SMBDD method are first given based on finite element (FE) method. The feasibility and effectiveness of the generalized method are then numerically investigated through a frame structure with incomplete measured responses. Various damage scenarios with different damage locations and damage severities are investigated by the generalized method using non-ground external excitations. The effects of measurement noise are finally investigated by contaminating the measured displacement responses with white random noise. The results show that the damage locations and their corresponding damage severities in various damage scenarios can be accurately identified even when the measurement noise is of a high level of 15% and the incomplete measured responses are considered. The main advantage of the SMBDD method that it is not only sensitive to structural damage but also insensitive to measurement noise is maintained in the generalized SMBDD method. The effectiveness and versatility of the generalized SMBDD method are therefore demonstrated.

INTRODUCTION

Civil structures begin to deteriorate once they are built due to harsh environment such as corrosion, earthquake, and typhoon. Vibration-based structural damage detection methods have thus attracted considerable attention for assessment of functionality and safety of civil structures. Nevertheless, the damage detection of civil structures still remains as a challenging task. One of the main obstacles is that the current damage detection methods are either insensitive to local structural damage or sensitive to measurement noise (Salawu [1], Farrar and Jauregui [2], Alvandi and Cremona [3], Worden et al. [4]). In this regard, the statistical moment-based damage detection (SMBDD) method has been recently proposed by the authors (Zhang et al. [5], Xu et al. [6]). The feasibility and effectiveness of
the SMBDD method have been numerically and experimentally demonstrated through a three-story shear building under ground excitations. In this paper, the SMBDD method is advanced in the following three aspects for its practical application: (1) the type of structures, (2) the number of structural responses measured, and (3) the location of external excitations. The basic equations of the generalized SMBDD method are first derived for any type of structures under various random excitations of Gaussian distribution. The generalized SMBDD method is also extended from the necessity of complete measurements of all degrees-of-freedom (DOF) responses of a shear building structure to the proper selection of measurements of incomplete DOF responses of a structure. Numerical examples are presented to demonstrate the feasibility and effectiveness of the generalized method. Various damage scenarios of a frame structure with incomplete measured responses are investigated. The effect of measurement noise on the quality of identified results is also investigated for all the damage scenarios by contaminating the measurement data with white random noises.

THE GENERALIZED SMBDD METHOD

The basic principle of the generalized SMBDD method is to first identify the stiffness parameters of a structure before and after damage occurrence through a FE model updating using the statistical moments of fully or, most probably, partially measured structure responses and then determine damage locations and damage severities by comparing the structural stiffness parameters identified at the two stages. A planar FE model of a structure with N DOFs and N_\text{e} elements is utilized here to illustrate the generalized SMBDD method. There are three DOFs at every node: the horizontal displacement \( x \), the vertical displacement \( y \) and the angular displacement \( \theta \). From practical viewpoint, the time history of dynamic angular displacement is hard to be measured. Therefore, only the horizontal displacement or the vertical displacement responses or both are assumed to be measurable and utilized to detect damage as long as the total number of measured displacement responses, denoted as \( N_\text{m} \), is larger than or at least equal to the number of unknown stiffness parameters, \( N_\text{c} \). The equation of motion in the matrix form for the structure can be expressed as

\[ \mathbf{M} \ddot{\mathbf{x}}(t) + \mathbf{C} \dot{\mathbf{x}}(t) + \mathbf{K} \mathbf{x}(t) = \mathbf{f}(t) \]  

(1)

where \( \mathbf{M} \), \( \mathbf{C} \) and \( \mathbf{K} \) are the global mass matrix, damping matrix and stiffness matrix of the structure, respectively. \( \dot{\mathbf{x}}(t) \), \( \ddot{\mathbf{x}}(t) \) and \( \mathbf{x}(t) \) are the acceleration, velocity and displacement response vectors, respectively. Only part of displacement responses are measured, that is, \( \mathbf{x}(t) = [\mathbf{x}_\text{m}(t), \mathbf{x}_\text{u}(t)]^T \), where \( \mathbf{x}_\text{m}(t) = [x_{m1}(t), x_{m2}(t), \cdots, x_{mN_\text{m}}(t)]^T \), \( \mathbf{x}_\text{u}(t) = [x_{u1}(t), x_{u2}(t), \cdots, x_{u(N-N_\text{m})}(t)]^T \), where \( N-N_\text{m} \) is the number of unmeasured displacement responses. \( \mathbf{f}(t) \) is the external excitation, \( \mathbf{f}(t) = [f_1(t), f_2(t), \cdots, f_N(t)]^T \). The Fourier transform of \( \mathbf{f}(t) \) is denoted as \( \mathbf{C}_f(\omega) \). By adopting the Rayleigh damping assumption, Eq. (1) can be decoupled through the transformation \( \mathbf{x} = \Phi \mathbf{z} \), where \( \Phi \) is the mass-normalized modal matrix of the system. The uncoupled equations of motion of the structure can then be expressed as

\[ \ddot{z}_i(t) + 2\xi_i\omega_i(K)\dot{z}_i(t) + \omega_i^2(K)z_i(t) = p_i(t) \quad i=1, 2, 3, \ldots, N \]  

(2)

where \( p_i(t) = \sum_{j=1}^{N} \phi_{ij}(\mathbf{K})f_j(t) \), the \( i \)-th generalized force; \( \phi_{ij}(\mathbf{K}) \) is the \( j \)-th component of the \( i \)-th theoretical mode shape and \( \omega_i(K) \) is the \( i \)-th theoretical circular natural frequency; \( \xi_i \) is the \( i \)-th modal damping ratio. In most cases, the first two modal damping ratios are estimated form the measured acceleration responses, while the higher modal damping ratios are derived according to the Rayleigh damping assumption. Denote the Fourier transform of \( \mathbf{x}_\text{m}(t) \) as \( \mathbf{X}_\text{m}(\omega) \),

\[ \mathbf{X}_\text{m}(\omega) = [X_{m1}(\omega), X_{m2}(\omega), \cdots, X_{mN_\text{m}}(\omega)]^T \]  

By using the mode superposition method, the Fourier
transform of the displacement response $x_{mi}(t)$ can be obtained as

$$X_m(\omega) = \sum_{k=1}^{N} C_k(\omega) \alpha_k(\omega)$$

(3)

$$\alpha_k(\omega) = \sum_{j=1}^{N} \frac{\phi_j(K) \cdot \phi_j(K)}{\omega_j(K)^2 - \omega^2 + 2i\omega\omega_j(K)\xi_j}$$

(4)

The conjugate of $X_m(\omega)$, denoted as $X^*_m(\omega)$, is calculated by

$$X^*_m(\omega) = \sum_{k=1}^{N} C_k^*(\omega) \alpha^*_k(\omega)$$

(5)

where $C_k^*(\omega)$ and $\alpha^*_mk(\omega)$ are respectively the conjugates of $C_k(\omega)$ and $\alpha_{mk}(\omega)$.

It should be noted that the relative displacement responses, denoted as $y_{mk} = [y_{m1}, y_{m2}, \ldots, y_{mN}]^T$, can also be utilized to identify structural stiffness by the generalized SMBDD method. If that is the case, the relative displacement responses can be calculated from the absolute displacement responses. For example, if $y_{mk}$ is the relative response of the $i$th absolute displacement response $x_{mi}$ to the $j$th absolute displacement response $x_{mj}$, $y_{mk}$ can be calculated as follows.

$$y_{mk} = x_{mi} - x_{mj} = Px_m$$

(6)

where $P = [0, \ldots, 0, 1, 0, \ldots, 0, -1, 0, \ldots, 0]$. In fact, when the $j$th element of $P$, denoted as $P_j$, equals 0, $y_{mk}$ represents the $i$th absolute displacement response. The Fourier transform of $y_{mk}$ can be obtained by

$$Y_{mk}(\omega) = PX_m(\omega) \quad k=1,2,3,\ldots, N_m$$

(7)

where the number of relative displacement responses measured is assumed to be equal to the number of absolute displacement responses measured although it can be different. The power spectral density (PSD) function of the $k$th relative displacement ($P_j = -1$) or the $k$th absolute displacement ($P_j = 0$) $y_{mk}$ can be uniformly expressed as

$$S_{y_{mk}}(\omega) = [PX_m(\omega)]^T \cdot PX_m(\omega)$$

(8)

where $X^*_m(\omega)$ is the conjugates of $X_m(\omega)$, $X^*_m(\omega)=[X^*_m(\omega), X^*_m(\omega), \ldots, X^*_m(\omega)]^T$. The variance of $y_{mk}$ can be calculated by

$$\sigma^2_{y_{mk}} = \int_{-\infty}^{\infty} S_{y_{mk}}(\omega) d\omega$$

(9)

The external excitations are taken as stationary Gaussian random processes in this study. Therefore, the structural responses are also stationary Gaussian random processes in terms of a linear structural system. Its statistical moments can be computed by
\[ M_{2k} = \sigma_{yak}^2, \quad M_{4k} = 3\sigma_{yak}^4, \quad M_{6k} = 15\sigma_{yak}^6, \quad k=1,2,3,\ldots, N_m \]  

The theoretical second-, fourth- and sixth-order statistical moment vectors obtained above can be expressed as

\[ \mathbf{M}_i = [M_{i1}, M_{i2}, \ldots, M_{iN_m}], \quad i=2, 4, 6 \]  

Based on the above derivation, it can be seen that the statistical moments of displacement responses are the function of the stiffness parameters of the building. Therefore, given the stiffness parameter vector an initial value \( \mathbf{k} = [k_1, k_2, \ldots, k_N] \), the \( i \)th-order statistical moment vector of the associated responses, denoted as \( \mathbf{M}_i \) \( (i=2, 4, 6) \), can be numerically computed in the frequency domain according to the aforementioned procedure. On the other hand, the actual \( i \)th-order statistical moment vector can be directly estimated from the measured displacement responses as follows, denoted as \( \hat{\mathbf{M}}_i \) \( (i=2, 4, 6) \). Denote the \( k \)th actually measured displacement response corresponding to \( y_{mk} \) as \( \hat{y}_{mk} \), \( \hat{y}_{mk} = [\hat{y}_{mk1}, \hat{y}_{mk2}, \cdots, \hat{y}_{mkN_s}] \), where \( N_s \) is the number of sampling points, the statistical moments of \( \hat{y}_{mk} \) can be calculated as

\[ \hat{M}_{2k} = \frac{1}{N_s} \sum_{i=1}^{N_s} \hat{y}_{mk}^2 - \left( \frac{1}{N_s} \sum_{i=1}^{N_s} \hat{y}_{mk} \right)^2 \]  

\[ \hat{M}_{4k} = 3(\hat{M}_{2k})^2 \]  

\[ \hat{M}_{6k} = 15(\hat{M}_{2k})^3 \]  

Therefore, the residual vector between the theoretical statistical moment vector \( \mathbf{M}_i \) calculated in terms of an assumed stiffness vector and the actual statistical moment vector \( \hat{\mathbf{M}}_i \) estimated from the actually measured building responses can be calculated and written as

\[ \mathbf{F}(\mathbf{k}) = \mathbf{M}_i(\mathbf{k}) - \hat{\mathbf{M}}_i \]  

Once the objective function has been established, the system identification of the undamaged or damaged building structure can be interpreted as a nonlinear least-squares problem, that is, giving \( \mathbf{k} \) an initial value \( \mathbf{k}_0 \) and minimizing \( \| \mathbf{F}(\mathbf{k}) \|_2^2 \) through optimization algorithms. Since it is physically impossible that the stiffness parameters of the damaged building are larger than those of the corresponding undamaged building, the constrained optimization method is utilized to identify the lateral stiffness value of the damaged building. That is, the structural stiffness parameter vector of the damaged building is identified by the nonlinear least-squares method under the constrained condition that the stiffness parameters of the damaged building shall be less than the identified stiffness parameters of the corresponding undamaged building. The structural damage including damage existence, location and severity can then be detected by comparing the identified stiffness vector \( \hat{\mathbf{k}}^d \) of the damaged building with the identified stiffness vector \( \hat{\mathbf{k}}^a \) of the undamaged building. The fourth-order moment other than the second-order or the sixth-order moment is used in the following investigation which makes a tradeoff between the sensitivity of an index to structural damage and its stability to random excitation (Zhang et al. [5]).
NUMERICAL INVESTIGATION

Numerical Model

The feasibility and robustness of the generalized SMBDD method are explored through a 2-D moment resisting one-story and one-bay steel frame structure (see Figure 1). The frame consists of two columns (W14×257 and W14×311) and one beam (W33×118). The columns are made of 345 MPA steel and the beam is made of 248 MPA steel. The bay width \( L \) is 9.15m and the height \( h \) is 3.96m. Each column or beam is divided into two elements which are numbered and marked in Figure 1. The beam element and consistent mass matrices are adopted in the finite element model. The mass density of the left column is 382.46 Kg/m, while that of the right column is 462.82 Kg/m and that of the beam is 17235.7 Kg/m. The first two damping ratios are adopted as 2%. The higher damping ratios are calculated according to the Rayleigh damping assumption. Theoretically, there is no limitation of types and locations of external excitations for the generalized SMBDD method. Since various damage scenarios have been identified by the SMBDD method using white ground noise and colored ground noise in the previous studies (Zhang et al. [5], Xu et al. [6]), non-ground excitations are utilized to demonstrate the versatility of the generalized SMBDD method in this paper. The locations of the external excitations are presented in Figure 1. The external excitations are simulated as zero-mean colored noise acceleration represented by the Kanai-Tajimi spectrum having parameters \( \omega_g = 15.6 \) rad/s and \( \zeta_g = 0.6 \) using the method of digital simulation of a random process developed by Shinozuka and Jan ([7]). The magnitude is chosen such that the maximum acceleration is 2.0 m/sec². The duration of the external excitation time history is 1000s with a sampling frequency of 256 Hz.

Numerical Results

In this study, only the horizontal and vertical displacement responses are assumed to be measured and utilized to detect damage age of the frame structure by the generalized SMBDD method. The effect of measurement noise on the quality of identified results is also considered by contaminating the measured displacement responses with white random noise. The superimposed random noises are independent and different with each other. The measurement noise intensity (MNI) of 15% is adopted. Firstly, the stiffness parameters of the undamaged frame structure are respectively identified without and with consideration of measurement noise. The identified results are presented and compared with the real values in Table 1, in which \( (EI)^a \), \( (EI)^m \) and \( (EI)^n \) stand for the real values, the identified results without consideration of measurement noise and the counterparts with consideration of measurement noise, respectively. The maximum relative errors of the two kinds of identified results compared with the actual ones are 0.69% and 1.58%, respectively. It can be seen that measurement noise has small effect on the quality of the identified results. The accurateness and stability of the identified stiffness parameters are demonstrated.
TABLE 1
IDENTIFIED RESULTS OF THE UNDAMAGED FRAME STRUCTURE

<table>
<thead>
<tr>
<th>Element</th>
<th>((EI)^u_{i\cdot} N(m^2))</th>
<th>((\hat{EI})^u_{i\cdot} N(m^2))</th>
<th>((\hat{EI})^u_{ni\cdot} N(m^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>491153082.21</td>
<td>494203217.52</td>
<td>495524388.89</td>
</tr>
<tr>
<td>2</td>
<td>491153082.21</td>
<td>489363945.78</td>
<td>489843679.41</td>
</tr>
<tr>
<td>3</td>
<td>37369.41</td>
<td>283104171.25</td>
<td>283533185.58</td>
</tr>
<tr>
<td>4</td>
<td>37369.41</td>
<td>281073515.95</td>
<td>278577259.14</td>
</tr>
<tr>
<td>5</td>
<td>360456414.57</td>
<td>360771944.64</td>
<td>361059562.66</td>
</tr>
<tr>
<td>6</td>
<td>360456414.57</td>
<td>358373464.68</td>
<td>355794388.97</td>
</tr>
</tbody>
</table>

Then four damage scenarios are examined to evaluate the effectiveness of the generalized method. The details of the four damage scenarios are presented in Table 2. For each damage scenario, the stiffness parameters of the damaged building are firstly identified by the constrained least-squares method without considering measurement noise. With reference to the identified stiffness parameters of the undamaged building with measurement noise free (see Table 1), damage locations and their corresponding damage severities of damage scenario are identified and presented in Table 3. Then the effect of measurement noise with the MNI of 15% is considered. The identified stiffness parameters with consideration of measurement noise are also obtained and presented in Table 4. All these results are presented and compared with the real values in Figure 2. As seen from Figure 2, the damage locations of the four damage scenarios are all accurately identified no matter whether measurement noise is considered or not. The identified damage severities in Table 3 and 4 are quite close to the real values for both single damage and multi-damage scenarios. The measurement noise has only marginal effect on the quality of the identified results. Even when the MNI is as high as 15% and incomplete responses are considered, the identified results are satisfactory for all the cases. The feasibility and robustness of the generalized SMBDD method are demonstrated through the frame structure with incomplete measured responses and the high level measurement noise.

TABLE 2
DETAILS OF DAMAGE SCENARIOS OF THE FRAME STRUCTURE

<table>
<thead>
<tr>
<th>Scenario No.</th>
<th>Damage severity</th>
<th>Damage location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 5%</td>
<td>3rd element</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5% 1st element</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10% 5th element</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10% 2nd element</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10% 3rd element</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20% 5th element</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5% 1st element</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10% 2nd element</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15% 4th element</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20% 5th element</td>
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</table>
Figure 2: Identified results of the frame structure with the MNI of 15%

TABLE 3
IDENTIFIED DAMAGE SEVERITIES (%) WITH NOISE FREE

<table>
<thead>
<tr>
<th>Scenario No.</th>
<th>Element 1</th>
<th>Element 2</th>
<th>Element 3</th>
<th>Element 4</th>
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<th>Element 6</th>
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<td>-0.45</td>
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<tr>
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<td>0.00</td>
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<td>-9.51</td>
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<td>-16.42</td>
<td>-20.06</td>
<td>0.00</td>
</tr>
</tbody>
</table>

TABLE 4
IDENTIFIED DAMAGE SEVERITIES OF THE FRAME STRUCTURE WITH MNI OF 15%

<table>
<thead>
<tr>
<th>Scenario No.</th>
<th>Element 1</th>
<th>Element 2</th>
<th>Element 3</th>
<th>Element 4</th>
<th>Element 5</th>
<th>Element 6</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.08</td>
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<td>0.00</td>
<td>-0.02</td>
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<td>0.00</td>
<td>-1.00</td>
<td>-9.89</td>
<td>0.00</td>
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<tr>
<td>3</td>
<td>0.00</td>
<td>-8.91</td>
<td>-9.52</td>
<td>-1.96</td>
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<td>-9.25</td>
<td>-0.32</td>
<td>-16.35</td>
<td>-20.15</td>
<td>0.00</td>
</tr>
</tbody>
</table>
CONCLUSIONS

The SMBDD method has been extended to be more versatile in this paper. Theoretically, the generalized SMBDD method can be applied to any type of structures by under any type of external excitations of Gaussian distribution. It has also the capability to cope with the incomplete measurement problem. The equations of the generalized SMBDD method were derived in the frequency domain. The generalized method was numerically investigated through a frame structure. Only the horizontal and vertical displacement responses were measured and utilized to detect structural damage. Various damage scenarios of the frame structure were explored by the proposed method using non-ground external excitations. Numerical results show that the damage locations and damage severities of the damage scenarios concerned can be accurately detected by using the colored noise random excitation. The effect of measurement noise was explored by contaminating the measurement data with white random noise. The superimposed noises are independent to and different with each other. According to the numerical results, the SMBDD method gives highly reliable results about damage locations and damage severities of the various damage scenarios of the frame structure even though the structural responses used are incomplete and the measurement noise intensity is as high as 15%. The advantage of the SMBDD method that it is sensitive to structural damage and insensitive to measurement noise is maintained in the generalized SMBDD method. The feasibility and robustness of the generalized SMBDD method are demonstrated.

REFERENCES

SAFETY AND RELIABILITY ON STEEL-CONCRETE JOINT PART OF HYBRID CABLE-STAYED BRIDGE

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KEYWORDS

Steel-concrete connection, PBL shear connector, Shear key, Hybrid cable-stayed bridge, Model test, Stress distribution, Relative slip

ABSTRACT

The innovative concept of using steel girders as main spans and concrete girders as side spans has been applied and developed world-wide in continuous girder bridges, cable-stayed bridges, and suspension bridges. Jointing two different kinds of girders at their appropriate places in a reasonable way will greatly improve the performance of bridges and will easily realize long-span bridges. The new connection structure with prefabricated shear keys for a cable-stayed bridge was adopted and introduced. The model test with the scale of 1:2 for the joint part of the hybrid girder was carried out on site to investigate the safety and reliability performance. In the experiments, distributions of normal stresses, crack resistance ability, slip between concrete and steel were carefully measured to investigate the mechanical performance, force transmission, stiffness matching of the joint part. The findings from the tests were considered to be of special significance to the design of the bridge.

INTRODUCTION

Hybrid cable-stayed bridges are composed of steel and concrete, the steel girder is used in main span and the concrete girder in the side span (Gimsing [3]). This type of bridges emanated from Germany in the 70’s last century, and widely used in Europe (Virlogeux [8]) and Japan (Eiichi [1], Ohlsson [6]). It was applied in China from the 90’s last century (He [2], Saul [7]). There are more than 20 hybrid cable-stayed bridges with large span (over 300m) in China now (Liu [4]).

The Jingyue Yangzi river cable-stayed bridge is located at Hubei province connecting highway from Suizhou to Yueyang, whose center span is 816 m, and 1444m in total length. Figure 1 shows the general arrangement of the bridge, while the cross section of main girder is shown in Figure 2. As the southern side span is shorter than the center span, PC girders are installed at the end of side span sections as counterweight girders to resist negative reaction. The main girder of bridge has a total width of 38.5m and a height of 3.8m. Flat box girders with fairings are used to ensure wind stability.
26 pairs of cables are installed in two planes with fan shape (Liu [5]). Figure 3 shows the joint part of concrete and steel girders, the steel cells in the joint part filled with concrete. The PBL connectors on the top, bottom and web plates connect concrete and steel. The steel cells has a length of 2m and height of 0.8m, which connect the concrete girder and the steel girder by the prefabricated shear keys, and the steel cells are strengthened by the T shape stiffer of 2.8m long.

Figure 1: General arrangement / m

(a) Steel girder (b) Concrete girder

Figure 2: Cross section of main girder / cm

Figure 3: Hybrid joint part / m

Model tests on key parts of the bridge are essential for design and construction. The model test with the scale of 1:2 for the joint part of the hybrid girder was carried out on site to investigate the safety and reliability performance. In the experiments, distributions of normal stresses, crack resistance ability, slip between concrete and steel were carefully measured to investigate the mechanical performance, force transmission, stiffness matching and failure mechanism of the joint part of the hybrid girder. The findings from the tests were considered to be of special significance to the design of the bridge.
MODEL TEST PROGRAM

Test Specimen

Figure 4 shows the test specimen in the bridge, which is the hybrid girder with the length of 20m at the root of first cable near the pylon in the main span. On the basis of the law of similarity, the test specimen was designed with scale of 1:2. And all the geometrical, physical and boundary conditions followed the law. Due to limitation of experiments and loading condition, one box girder was adopted as a test specimen.

Figure 4: The position of test specimen in bridge / m

Figure 5 shows the test specimen, including steel and concrete box girder, the joint part with PBL connectors and shear key. The diameter of the holes in PBL is 32.5mm, and the height of PBL is 105mm. The diameter of pre-stressed steel strands in the specimen is the same as that of the bridge, but the number of the strands reduces to half. There are 15 bundles in the top slab, 5 bundles in the bottom slab, and 9 bundles in the web. The arrangement of reinforcement in concrete girder for test specimen based on the reinforcement ratio equate to that for the research bridge.

Figure 5: Test specimen

Loading Procedure

Table 1 shows the design load of joint section for both the test specimen and the research bridge under the most adverse load combinations: the minimum bending moment. Before the loading, the preloading should be taken into account to check the good contact between the support and loading
equipment, the reliability of all the test equipments and the working condition of all the measurement instruments. The loading is applied by force control, such as, the first level being 0.1 times of the design load, the second level being 0.2 times of the design load, then monotonically increasing the load 0.1 times of the design load each up to the design load, and finally unload to zero. After this loading procedure repeats 3 times, the load is increased to 1.7 times of the design load by 0.1 times of the design load to investigate the load carrying capacity of the specimen. The bearings are set on both side and cross beam of the specimen to simulate the restriction of cables, and also produce the reaction forces. The axial load and bending moment are applied by 9 hoisting jacks on the end sections of the steel girder, as shown in Figures 6.

<table>
<thead>
<tr>
<th>Load combinations</th>
<th>Force</th>
<th>Research bridge</th>
<th>Test specimen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design load (1.0/1.7 times)</td>
<td>N (kN)</td>
<td>-160000 / -272000</td>
<td>-15290 / -26000</td>
</tr>
<tr>
<td></td>
<td>Q (kN)</td>
<td>4230 / 7191</td>
<td>430 / 740</td>
</tr>
<tr>
<td></td>
<td>M (kN·m)</td>
<td>-56400 / -95880</td>
<td>-3230 / -5500</td>
</tr>
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</table>

Note: N, Q, M are the axial, shear force and bending moment

Figure 6: Load condition

Measurement Arrangement

Twelve key sections in and around joint part are selected to measure the strains on the surface, and typical steel cells in the top, bottom and web are selected as shown in Figure 7. The strain gauges on the perforated steel plate and the penetrating reinforcement of PBL shear connectors in the steel cell are shown in Figure 8. The vibration chord strain gauges are embedded in the concrete of joint part to measure the stress distribution. The slip between steel and concrete is measured by two vibration chord strain gauges at the same position, one is welded on the steel plate, and the other is embedded in the concrete. The difference between the two values indicates the relative slip, as shown in Figures 9 (a) and 10. Also, the vibration chord strain gauges are embedded in the concrete shear keys to measure the stress, as shown in Figure 9 (b).

TEST RESULTS

Stress on Surface Plate

Figure 11 and 12 show the load-stress relationship on the surface of different key sections. It was noted that the stress can be transferred uniformly and smoothly through the joint part from the steel girder to concrete girder, and the stress increased linearly with the load. All the behavior was in elastic stage under designed load, even up to 1.7 times of the design load. Figure 13 shows the stress...
distribution on steel and concrete sections respectively under the design load and 1.7 times of the design load. It is found that the stress distribution was uniform, and the values were far lower than the allowable stress value for both steel and concrete.

(a) Elevation view
(b) Sectional view

Figure 7: Strain gauges on the surface /mm

Figure 8: Strain gauges on steel plate and penetrating reinforcement of PBL connectors / mm

(a) Steel cell
(b) Shear key

Figure 9: Embedded vibration chord strain gauges /mm

Figure 10: Vibration chord strain gauges welded on plate / mm
Figure 11: Load-stress relationship on surface of steel plate

Figure 12: Load-stress relationship on surface of concrete plate

Figure 13: Normal stress transversal distribution / MPa

Stress Distribution of Perforated Plate in Steel Cell

Figure 14(a) shows the locations of the strain gauges on the perforated plate in top cell, Figure 14(b), (c) show the load-stress relationship of perforated plate on different sections. The stresses on perforated plate are very complicated, which are considered to be affected by the concrete curing quality and the contact condition between concrete and perforated plate. The stress increased with the load, but the values were still lower than allowable value until 1.7 times of the design load.
Stress Distribution of Penetrating Reinforcement of PBL Connectors

Figure 15(a) shows the locations of the strain gauges on penetrating reinforcement of PBL connectors in top cell, Figure 15(b), (c) show stress distribution along the horizontal direction under design load. The penetrating reinforcements were subjected to tension, and the tensile value is about 5Mpa in average. And the stresses of penetrating reinforcement are very complicated, which are considered to be affected by the compressive concrete and the bond condition between concrete and reinforcement.

Concrete Stress Distribution in Cell

Figure 16 shows the normal stress of in-filled concrete in the cell under the design load and 1.7 times of the design load. The compressive stress value reduced slightly with the distance increase from the front bearing plate under both the design load and 1.7 times of the design load. However, generally the concrete stresses distribute uniformly in the steel cell.

Concrete Stress Distribution in Shear Key

The prefabricated shear keys were used to transfer vertical shear force between steel and concrete girder in the joint part. The vibration chord strain gauges are embedded in the concrete of shear keys to measure the shear stress. Figure 5(e) shows the arrangement of vibration chord strain gauges. Fig.17 shows the load-stress relationship of concrete in shear keys, the stress increased with the load, but the values are still lower than the allowable value until 1.7 times of the design load, the maximum value is about 2Mpa in the web.
Relative Slip between Concrete and Steel in Cell

Three sections are selected to measure the slip between steel and concrete in the top cell. Figure 9 and 10 show the arrangement of vibration chord strain gauges. Figure 18 shows the slips between concrete and steel at different sections under the design load and 1.7 times of the design load. It is found that the maximum value of relative slip is about 25μm, resulting in the concrete and steel in good connection performance.

![Figure 18 Slip distribution along the horizontal direction](image)

CONCLUSION AND REMARK

On the basis of the mechanical and structural property for the joint part of cable stayed bridge, the model test with scale of 1:2 was carried out on site. The representative steel cells of the joint part were selected. The stress and relative slip between steel and concrete were measured to investigate the force transmission mechanism. From the test results, it was found that the concentrated stress at the loading section can be reduced and transferred uniformly to the concrete through the joint part. In addition, the structure shows good performance and reliability under the design load. Small relative slip between concrete and steel denoted the connection soundness between two different materials.

ACKNOWLEDGEMENTS

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REFERENCES

MECHANICAL EXPERIMENT ON JOINT OF STEEL-CONCRETE HYBRID GIRDER IN CABLE-STAYED BRIDGE

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KEYWORDS
Steel-Concrete Joint, Hybrid Girder, Cable-Stayed Bridge, Model Test

ABSTRACT

New structural configuration for steel girder to concrete girder connection joint were developed in a 926m-cable-stayed bridge with hybrid girder. Pre-stressed concrete girder in side span extended into steel cells of the steel girder in main span. Huge axial loads transmitted from steel cells to the backing concrete through rear bearing plate and perforated rib shear connectors. To investigate mechanical behavior of the joint, a half-scaled girder segment model test was conducted to check stress transmission mechanism of the joint. Stress distribution of steel cells, backing concrete, and steel bars were checked. Typical deflection of the specimen was measured. Load carrying contributions between rear bearing plate and shear connectors were analyzed. Relative slip between steel and concrete was also monitored during the test. Results of the experiment showed stresses working on steel girder dispersed gently to backing concrete through rear bearing plate and shear connectors. The tiny and smooth deformation of the specimen meant a reasonable rigidity of the connection. Relative slip at interface of the connection was about 5~50 micron, which accounted for a reliable connection between steel and concrete. The rear bearing plate carried about 60% of the total axial compressive load. The experiment involving the study of mechanical behavior of this newly formed steel-concrete joint for hybrid girder proved to be encouraging. The joint could be a likely alternative for cable-stayed bridges with hybrid girder.

INTRODUCTION

Cable-stayed bridges with hybrid girder are composed of steel girder in main span and PC girder in back span. Main span and back span are connected together by steel-concrete joint. Larger spans are able to be obtained in this type of bridges where the back/main span ratio is
extremely small (Virlogeux, 1989). The Stonecutters Bridge with a 1018m main span in Hong Kong is one of these kinds (Withycombe S, 2001). Since the 90’s last century several cable-stayed bridges with large span hybrid girders have been built in Mainland China (LIU Yu-qing, 2005). Three major bridges under construction in Mainland, known as E-Dong Bridge(926m), Jin-Yue Bridge(816m) and Jiu-Jiang Bridge(nearly 900m) are also with hybrid girders.

As to the existing cable-stayed bridges with hybrid girders in China, steel envelope with padding concrete has mainly been used for the steel-concrete joint part (LIU Yu-qing, 2005). Huge axial compressive force transmitted through bearing plate and studs between steel envelope and padding concrete. In order to acquire a better workability, the three newly designed bridges have a different alternative for the joint (MOCHIZUKI, 2000). The back span concrete girder extended into multi-cell around plates of steel girder. Steel and concrete are connected whit bear plate, perforated rib shear connectors and studs. This type of joint has a different configuration and load transmission mechanism compared with Stonecutters Bridge and existing bridges in Mainland China. A half scaled model test is carried out to check the behavior of this newly applied joint. Strains of steel and concrete are measured. Load transmission mechanism is analyzed finally.

EXPERIMENTAL PROGRAM

Detail of The Connection Joint

E-Dong Bridge is a cable-stayed bridge under construction over the Yangtze River. It has a 926m main span and a back/main span ratio less than 0.3, as shown in Figure 1. In order to balance the possible uplift of the back span, the hybrid girder design has been chosen. Figure 2 illustrates cross section of the girder, which has a 38m width and 3.8m height carrying 8 vehicle lanes. Two lateral trapezoidal boxes are connected with central deck system.

Figure 1: General arrangement          Figure 2: Cross Section

Figure 3 and Figure 4 show detail of the steel-concrete joint in E-Dong Bridge. Multi-cell steel cells in the joint are filled with concrete. The joint is connected to general steel girder by reinforced steel girder. The steel cells have a 2m length and a 0.8m height. Webs of the cell are perforated steel plate with $\Phi60$mm holes and $\Phi20$mm rebar embedded in the holes to form shear connectors. Studs on top and bottom flange of the cell connect flanges and concrete. Pre-stressed tendons are anchored to the bearing plate to connect the joint and the concrete girder.
Figure 3: Longitudinal Section         Figure 4: Detail of Hybrid Joint

Specimen

Figure 5 and Figure 6 show the specimen detail of the experiment. Longitudinally the 22m length segment from pylon bearing to the first stayed cable J1 is selected. Transversely a 10m width and 3.8m height lateral box is selected. Regarding to load applying capacity, the specimen was designed with a half scale of the selected girder segment. Law of Similitude is followed in geometrical, physical and boundary condition design of the test specimen. Detail of the joint, such as perforated plate, pre-stressed tendons and headed studs are all simulated, as shown in Figure 7.

Test Setup and Loading Procedure

The test setup used in the experiment is shown in Figure 6. To simulate the vertical degree of freedom restricted by tower and cable, bearings are set beneath ends of the specimen. Axial load are applied by nine hydraulic jacks on end of steel girder. Shear force and bending moment are applied by two hydraulic jacks located between reaction beam and specimen.
Table 1 shows forces acting on the joint section of prototype bridge and model test. The adverse load combination of bending moment and axial compression are applied to the specimen. The load steps are shown in Figure 9. After three times of loading up to 1.0 times the design load combination $P_d$, the fourth loading applied up to 1.4 times $P_d$. In each load step loads are controlled by hydraulic jacks with a $0.05P_d$ increments. And the load are imposed such that failure did not occur in less than 15min.

### TABLE 1
FORCES FOR JOINT SECTION

<table>
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<tr>
<th>Force</th>
<th>Prototype</th>
<th>Model(1.0P_d)</th>
<th>Model(1.4P_d)</th>
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</thead>
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<tr>
<td>N (kN)</td>
<td>-173690</td>
<td>-16650</td>
<td>-23300</td>
</tr>
<tr>
<td>Q (kN)</td>
<td>988</td>
<td>90</td>
<td>130</td>
</tr>
<tr>
<td>M (kN·m)</td>
<td>42000</td>
<td>1850</td>
<td>2600</td>
</tr>
</tbody>
</table>

N, Q, M: Axial, shear force and bending moment. + for Tensile and – for Compressive.

Figure 8: Test setup

Figure 9: Load steps
Measurements

Electrical strain gauges were attached to the steel plate of steel cell, perforated plate and rebar to get the strain of steel plate and rebar in perforated ribs. Vibrating wire strain gauges were embedded in the padding concrete in steel cells to get the strain of concrete. Relative slip between steel and concrete were monitored by vibrating wire sensors. One end of the slip sensor was welded to steel while the other end embedded in concrete. Deflection of the girder was detected by displacement sensors.

TEST RESULTS

Stress of steel flange

Figure 10 shows the load-stress relationship of steel flange. It is noted the stress can be transferred uniformly through the joint part from the steel girder to concrete girder, the force transmission is smooth and the stress increase linearly with the load. All the behavior is in elastic stage under designed load, even up to 1.4 times of the design load.

Stress of perforated webs

The stresses on perforated plate are small compared with the general steel girder. The stresses increase with the load, but the values are still low till 1.4 times of the design load, as shown in Figure 11a). As stresses on flanges transmitted to steel webs, web stresses near the flange are higher, shown in Figure 11.
**Stress of backing concrete**

Figure 12 shows the normal stress of in-filled concrete in the cell under design load and 1.4 times of the design load. Sections near bearing plate carry loads transmit from the bearing plate. As the stress transmit from steel cells to padding concrete by shear connectors, the stresses in concrete increase gradually. The upper and bottom cells share nearly the same stress distribution, the upper has a higher compression for positive bending.

![Figure 12: Stress of backing concrete](image)

**Slip between steel and concrete**

Slip distribution between perforated steel and concrete is monitored by slip sensors arranged at the interface of the two materials. As shown in Figure 13a), slips between the two materials are very small, about 20μm even when 1.4$P_d$ was imposed. Slip at end of steel cell is monitored by dial gauges, shown in Figure 13b). Slip varies while the applied loads increased, with slips less than 10μm. The tiny slip at the interface implied a close connect between steel and concrete.

![Figure 13: Slip between steel and concrete](image)

**Deflection of the joint**

Typical load-deflection curves at different sections are shown in Figure 15. The vertical deflection of the girder is flat and smooth, with a relative small change at the steel-concrete interface due to the rigidity variety. The maximum deflection is about 0.15mm at the joint section.
The axial force transferred by one steel cell can be calculated according to the test value of stresses in the steel plate near bearing plate, T and U shape stiffer, and the axial load carrying ratio for bearing plate can also be estimated through the axial load change between the front and back sections of bearing plate. Table 2 shows the axial load carrying ratio for different parts of the steel cell in the joint part. It is noted that the bearing plate carries about half of the axial load, and the other half transfers to concrete girder through studs and perforated rib shear connectors.

<table>
<thead>
<tr>
<th>Components</th>
<th>Upper cell</th>
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<th>Lower cell</th>
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</tr>
</thead>
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<td></td>
<td>Test</td>
<td>FEM</td>
<td>Test</td>
<td>FEM</td>
</tr>
<tr>
<td>Top Flange</td>
<td>26.5</td>
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</tr>
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<td>Perforated web</td>
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</tr>
<tr>
<td>Bottom Flange</td>
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<td>Bearing plate</td>
<td>44.4</td>
<td>60.0</td>
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</table>

**CONCLUSION AND REMARK**

On the basis of the mechanical and structural property for the joint part of cable stayed bridge, the model tests with scale of 1:2 was carried out on site. The representative steel cell at the top and bottom of the joint part were selected. The stress, deflection and relative slip between steel and concrete were measured to investigate the force transmission mechanism. Test results showed that the concentrated stress at the loading section can be reduce and transfer gently to concrete through the transition region and joint part. In addition, the bearing plate carries about half of the axial load, and the other half transfers to the concrete girder through studs and perforated rib shear connectors. This leads to the conclusion that the ratio is rational and the structure shows good performance and reliability under the design load. Relatively tiny slip between concrete and steel denotes the connection soundness between two different materials connected well.
ACKNOWLEDGEMENTS

The research reported herein has been conducted as part of the research projects granted by the Authority of E-Dong Yangtze River Bridge Corp. and Ministry of Communications in China. The assistances are gratefully acknowledged.

REFERENCES

BEARING CAPACITY ANALYSIS OF A CURVILINEAR BOX GIRDER LANDSCAPE BRIDGE*

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*The key grant project of Chinese ministry of education (No.704003);
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KEYWORDS
Curvilinear bridge, steel box girder, plate and shell finite element, bearing capacity, limit loading capacity

ABSTRACT
Curvilinear steel box girder has been used as the structural form in many footbridge structures. The behavior and capacity of this type of bridge is still not well researched. In this paper, the finite element model of a curvilinear steel box girder footbridge structure is constructed making use of two different FEA softwares - MIDAS and ANSYS where issues such as the selection of element types and the effect of model simplification are discussed. The bearing capacities of the bridge under three design load scenarios, characterized by the deflection curve, rotation angle of the bridge deck and stress distribution of the steel box girder, are studied based on comparison of results from the two finite element models. Besides, the limiting capacity of the bridge structure under three design load distribution is studied with the ANSYS finite element model taking both material and geometry nonlinearities into account. In order to identify the limit load of the curvilinear steel girder bridge, the failure characteristics of this kind of bridge structure are analyzed, based on which a method of limit load identification according to the load characteristic angle curve is proposed and the limit loads and safety factors for the three load scenarios are obtained. Finally, the characteristics of limit states of the curvilinear steel box girder bridge are summarized as reference for further studies.

INTRODUCTION
Landscape bridges are those which are beautiful with good visual effects adding high aesthetic values to the view of the surrounding environment. Landscape bridges are often
designed with peculiar configurations to display relative connotations of natural sights, humanistic sights, historical sights and cultural sights, and this brings difficulties in the procedures of structural design, construction and maintenance management[1].

For the curvilinear steel box girder bridge structure, a large amount of research has been made by some domestic scholars. On the design of curvilinear bridge, Le Xiaogang et al [2] gave an illustration of a three-span continuous steel box girder bridge and introduced the construction methods, treatments of bridge deck and the shear lag effect in urban curvilinear steel box girder viaducts. Wei Yunzhen[3] studied the curvilinear steel box girder bridges which are widely used for urban viaducts and overpasses, and the structural characteristics, common occurring damages, designing criteria and the configuration arrangement principles were discussed. A feasible design method for the bridge piers and prestress was presented. On the computational algorithm, Xie Xu et al[4] proposed an analytical method for the curved box girder by introducing high order displacement functions in the axial direction. The comparison between the numerical results and those of the general finite element analysis program ANSYS indicated that the method was in good agreement even with a small number of finite elements. On the aspect of the shear lag effect, Yao Lili et al[5] analyzed the shear lag effect on elliptic steel box girder by the finite element method. However, there is a lack of research on the loading capacity of the curvilinear steel box girder bridge in the literature.

In this paper, FEA (Finite Element Analysis) is utilized to discuss the bearing capacity and limit loading capacity of the curvilinear steel box girder bridge under design loads. Based on the comparison between the two sets of results from different FEA softwares, the limit loads of the bridge under three design cases are discussed thoroughly, including the failure modes and the identification of the limit loads. The conclusions obtained can provide good references for similar studies.

PROJECT OVERVIEW

A landscape bridge located in Beijing was built in 2006. The structural type of the bridge is classified as continuous steel girder bridge the cross-section of which is shown in Figure 1. The total length of the bridge is 98.77m with three spans, and the layout of the bridge is designed as three intersecting elliptic rings (shown in Figure 2) supported by two vertical rings set at the two bridge piers, forming a five-ring configuration. The superstructure of the bridge is the curvilinear steel box girder while the substructure is made of concrete copings and pillar piers resting on a foundation of bored piles. The two tow vertical rings on the middle piers are designed as a variable cross-section steel girder. The bridge has the original structure shape formed in a beautiful figure which will be a significant bridge pass connecting two districts after completion.

Since the bridge deck has an unusual structural form, FMA method is applied for the elasto-plastic analysis of the bridge under different design cases. Calculation results of the bearing capacity and performance under limit loads are obtained from the analysis and a list of conclusions drawn after the systematic studies.
FINITE ELEMENT MODEL

Two finite element models are constructed using the large versatile FE A software ANSYS and bridge engineering analytic al software MADIS/Civil. In the following discussions, the two models are named as ANSYS model and MIDAS model for reference.

Element Selection and Model Simplification

In order to have accurate stress distribution and simulation configuration of the steel box girder joints, spatial shell elements are applied, i.e. the SHELL181 elasto-plastic thin shell element in ANSYS model and spatial four-point thin board element in MIDAS model. Both kinds of element have 6 degrees-of-freedom in each node.

The dead weight of the bridge deck is simplified as equivalent loads on the bridge in the FEM. Both models comprise dummy plate while ribbed stiffeners are not involved due to the complexity of the FEM. Flange plates only exist in the ANSYS model. The ANSYS model and MIDAS model are shown in Figure 3 and Figure 4 respectively.

Figure 1: Section of the steel box girder

Figure 2: Layout of the bridge

Figure 3: ANSYS model

Figure 4: MIDAS model
Material in FEM

Q235B steel is used for the steel box girder with a yield strength $f_y = 235$MPa. Isotropic linear elastic model is selected as default in the MIDAS model and isotropic elasto-perfectly plasticity model is used in the ANSYS model to simulate the bearing performance of the bridge after yielding.

BEARING PERFORMANCE UNDER DESIGN CASES

Design Loading Cases

Initial design loading cases are simplified into three loading cases listed in Table 1. Uniform load of 5kN/m², which exceeds the valued specified in the code considering large crowd density in future, is adopted in all the three cases. Cases 1 and 2 have the load arrangement producing the maximum deflection of the side span and mid span respectively. Case 3 is the load arrangement which generates the maximum negative moment at the support [6].

<table>
<thead>
<tr>
<th>Case</th>
<th>Distribution of uniform loads (5kN/m²) Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Half disposal of side span</td>
</tr>
<tr>
<td>2</td>
<td>Half disposal of mid span</td>
</tr>
<tr>
<td>3</td>
<td>Maximal minus moment disposal</td>
</tr>
</tbody>
</table>

Comparative Analysis of Deflection under Design Cases

Deflection curves of the steel box girder bridge under three design loading cases are obtained from the two finite element models. The layout of computation controlling points and deflection curves are illustrated in Figures 5 and 6. The following observations are made from the calculation results:

1. Deflection results from the two models are almost identical, especially for those of the Cases 1 and 2, with a maximum discrepancy of 1.96% between the two models.

2. As for Case 3, the two sets of results show little difference with larger side-span deflection in the MIDAS model and larger mid-span deflection in the ANSYS model. The displacement of point 1 (Figure 5) in the MIDAS model is 6.72% larger than that in the ANSYS model from these results; and the displacement of point 7 (Figure 5) in the ANSYS model is found 7.69% larger than that in the MIDAS model.

3. Based on the calculation results of the displacement, one can find that in the ANSYS model, the simulated stiffness of the side-support-region is larger while that of the mid-support-region is smaller than those in the MIDAS model. For engineering application, both models meet the accuracy requirements. The maximums of the two sets of results are both referred to in the design. The maximum deflection of the bridge is $38.03 \text{mm} \approx L/880$, which meets the requirements of the associated design code [6].
Stress Distributions under Three Loading Cases

Stress distributions under the three design cases are derived from the two models. In order to discuss the stress distribution characteristics of the bridge under different design cases, Von Mises stress distributions of selected regions are illustrated in Figure 7.

Figure 7 shows that the overall stress level of the bridge deck is low, with a variation of 20 - 80MPa, and the strength is not the control factor. Stress distributions derived from the two models are similar for the whole bridge deck and the main difference is found at the intersection of the upper plates and the web plates, where stress concentration belts appear in the MIDAS model and it is found in large areas in the ANSYS model. This accounts for the supporting effect of the flange plates in the steel box girder.
LIMIT LOAD ANALYSIS

Taking material and geometric nonlinearity into account in the ANSYS model, limit bearing capacity of the bridge superstructure under design loading mode is studied with discussions on the safe margin and failure mode.

Limit Load Analysis of Case 1

The load-displacement curve of Case 1 at mid span shown in Figure 8, in which the continuous line is divided into linear elastic segment and plastic development segment at about 40kN/m² level. The curve turns at about 164kN/m² stress level because the un神 m
loads in the ANSYS model are applied normal to the loading surface, causing the overturn of the box girder after excessive distortion at an extremely large load (Figure 9). But this does not conform to the reality. Consequently, the restoring section of the curve cannot be referenced.

Figure 10: Deformation of the steel box girder (case 1)

Figure 10 shows that there is a phenomenon of “crankle” in the curvilinear steel box girder under load Case 1. Therefore it is more reasonable to identify bridge limit loads using the rotation angles of section as criterion. Figure 9 shows the load-rotation angle curve along the X direction (X axis goes along the bridge span) starting at the bridge center. The curve shows distinctly three parts: static stage, transition stage and linear increasing stage. Therefore the limit state can be defined as the critical state between rotation-angle-invariance stage and rotation-angle-increase stage. Stage ① and ③ are extended to intersect at 103.8kN/m² stress level, namely the limit load, and the corresponding safety factor is 20.76.

**Limit load Analysis of Case 2**

Due to the similarity between results for the Cases 1 and 2, both the load-displacement curve at the center span and load-rotation (X) angle curve are drawn in Figures 11 and 12 respectively. The two figures look similar for Case 1, except at the transition of the curve in Figure 12, which is much more obvious. A limit load of 128kN/m² and a safety factor 25.6 are obtained in a similar as for Case 1.

**Limit load Analysis of Case 3**

Case 3 is the load arrangement with maximum negative moment at the support. According to the experience of the previous two cases, the load-rotation angle curve is selected as the limit state for this case. Here Y rotation angle of the inflection section of the side span (near point 5 in Figure 5) is taken as the rotation angle, which is maximum with applied load. The
load-rotation angle curve shown in Figure 13 shows two different stages in the curve: static stage and increase stage. The limit load is also obtained as 112.9kN/m² stress level similar to those for Cases 1 and 2 with a safety factor of 22.58.

![Figure 13: Load–inflection-section-rotation angle curve of case 3](image)

**CONCLUSIONS**

These discussions above are summarized as conclusions as follows:

1. The results of the two finite element models are fairly similar demonstrating the validity of the two models. The displacement and stress results denote that the structure can meet the strength and stiffness requirements in the design codes.
2. The characteristic rotation angle is selected as the abscissa of the load curve in this study. Critical point before the sharply-increase stage is selected to define the limit state and hence the limit loads. This approach can be used for similar research with curvilinear steel box girder bridge. However, this method still needs to be verified by further experimental research.
3. Safety factors for the three design cases are uniformly larger than 20 which can manifest the sufficient safe margin of the bridge.

**REFERENCES**

STABILITY ANALYSIS OF THE STEEL STRUCTURE OF TIANJIN BENGBU BRIDGE

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This research was sponsored by the Natural Science Foundation of Beijing (Project No. 8053021).

KEYWORDS
Bridge, buckling analysis, FEA, nonlinearity, safety factor.

ABSTRACT

Bengbu Bridge of Tianjin Haihe-River is an antisymmetric steel bridge with Beam-Arch Combination System. Its bearing capacity depends upon two groups of spatial structures which consist of longeron, crossbeam, longitudinal arches and transverse arches. The bridge is complex for mechanical and tectonic consideration. In this paper, the two methods for stability analysis including the linear and nonlinear buckling analysis have been introduced first. Then the large universal finite element program ANSYS is adopted to establish a three-dimensional finite element model of Bengbu Bridge. Through the FEM model, the elastic stability and limiting bearing capacity of the bridge under five loading cases are analyzed and the influence of nonlinearity (geometry nonlinearity and material nonlinearity), loading factor and wind factor are discussed. The results show that the nonlinearity and loading factor has great influence on the stability of the bridge, so it cannot be neglected in the design. The influence of wind loading is insignificant. The analysis results can be used as reference for practical engineering.

INTRODUCTION

Spatial Beam-Arch Composite Bridge is a novel bridge system combining spatial structure with bridge. This system of high capacity and reasonable figuration makes the best of the members as the arch and the beam worked together perfectly. These bridges are complex for mechanical and tectonic consideration, since the dead loads are apparently lessened down thus the structural behaviors have greatly enhanced. Thus the problems of stability become very important and maybe control the bridge design. So in this paper, through the FEM model, the elastic stability and limiting bearing capacity of the bridge under different loads behavior are analyzed. The influence of nonlinearity (geometry nonlinearity and material nonlinearity) and the wind loadings on the composite bridge is focused on.

DESCRIPTION OF THE BRIDGE

Tianjin Bengbu Bridge is one of the most important parts of the Haihe River Comprehensive Exploitation Project. The bridge is located in the center of Tianjin, China, and crosses the Haihe River,
connecting S hisanjing Ro ad and Beng bu Road. T he bridge is d esigned as an antis ymmetric steel bridge of Beam-Arch Combination System with a total length of 192m. The traffic lanes of t he main bridge have a total width of 23.5m , including 2× 2m pavement. Beside s, the re ar e two d etached pedestrian bridges with a width of 3.0m located in each side of the main bridge.

The bridge is steel structure with unique form and its stress status is very complex for mechanical and tectonic consideration. The main bridge is a single box and multi-cell beam structure. The pedestrian bridge is cantilevered beam structure. Its bearing capacity depends upon two groups of spatial structures which consist of longeron, crossbeam, longitudinal arches and transverse arches. The longitudinal arches are spatial contorted single box beam with a total span of 112.74m. There are abutments on the contraflexure point of all longitudinal arches. The transverse arches with variable cross-section served as important bearing members as well as connections of the main bridge and the pedestrian bridges. The picture of the bridge is followed as Figure 1.

![Figure 1: The picture of Tianjin Bengbu Bridge](image)

THE FEA MODEL

In this paper, a 3-dimensional finite element model of ANSYS is established. Since different members of the bridge own distinguishing function and attribute, the element types adopted should be various. Therefore, in order to guarantee the precision of the model and the accuracy of the analyses, four kinds of element are chosen.

The main longeron, crossbeam of the main bridge and the pedestrian bridge, the longitudinal arches and the transverse arches are modeled by Beam 188, which is suitable for analyzing slender to moderately stubby/thick beam structures. Beam 188 is based on Timoshenko beam theory. Beam 188 is a 2-node element with six or seven degrees of freedom at each node and shear deformation effects are included. This element is chosen because it is well-suited for linear, large rotation, and/or large strain nonlinear applications, which are demanded by the nonlinear analysis of stability.

The bridge deck slabs are modeled by Shell 181, which is suitable for analyzing slender to moderately stubby/thick shell structures. Shell 181 is a 4-node element with six degrees of freedom at each node. This element is chosen because it is well-suited for linear, large rotation, and/or large strain nonlinear applications.

Actually, the boundary conditions of bridges are complicated. In modeling, dead joint, hinged connection, spring and roller are introduced to simulate the boundary conditions. In the finite element model of Tianjin Bengbu Bridge, the two ends of the bridge and the abutments are jointed with hinge.

Figure 2 and Figure 3 shows the finite element model of the bridge from front view and bottom view, respectively.
THE METHODS OF ARCH STABILITY ANALYSIS

Theory of Stability and Finite Element Method

The theory of stability is aimed to determine the critical loads and deformation of structures which are associated with quantitative changes of the structure state. The actual configuration has to be taken into account in the theory of stability of structures. The stability equations are so complex that it should consider many factors, such as the geometry nonlinearity, material nonlinearity. At the same time, the study of structural stability needs complex models of structures which makes the problem of stability much more complicated, even at the level of linear analysis.

The arrival of computers and development of computational methods enable us to deal with complex problems of structural stability. The linear analysis FEM can be used to analysis a system with a large number of DOF. Also, geometrical and material nonlinearities could been considered.

This paper employs the software of ANSYS to analyze the stability of a steel composite bridge in order to discuss some influence factors of buckling of the bridge.

Two Methods of Finite Element Analysis

There are two methods available in the ANSYS for predicting the buckling load and buckling mode shape of a structure: eigenvalue (or linear) buckling analysis, and nonlinear buckling analysis. Eigenvalue buckling analysis is a traditional one and predicts the theoretical buckling strength of an ideal linear elastic structure. That means to consider the load factor of an idealize structure which has no geometric and material errors. The Eigenvalue buckling analysis provides the modes of buckling, which is a good reference for nonlinear analysis. However, imperfections and nonlinearities prevent most real-world structures from achieving their theoretical elastic buckling strength. The buckling usually is the upper limit of the structure's carrying capacity. Thus, eigenvalue buckling analysis often yields unconservative results, and should generally not be used in actual engineering analyses.

Nonlinear buckling analysis can consider features such as initial imperfections, plastic behavior, gaps, and large-deflection response. In this paper, the nonlinear analysis contains geometry nonlinearity, material nonlinearity and large-deflection response. Also, the post-buckled performance of the model can be tracked. The aim of nonlinear analysis is to track the force-displacement curves, through which the ultimate buckling load is gained.
This paper employs these two methods to analyze the stability of Bengbu Bridge under five loading cases.

**ELASTIC STABILITY ANALYSES**

The loads are applied according to the Standard of Loading for the Municipal Bridge Design (CJJ 77-98). The gravity is simulated by acceleration. The bridge deck loads, railings, vehicle loads and pedestrian loads are simulated by imposed surface load and force. The analyses including five loading cases:

1. Condition 1: dead loads.
2. Condition 2: dead loads and live loads imposed on all the bridge deck.
3. Condition 3: dead loads and live loads imposed on half of the bridge deck.
5. Condition 5: dead loads and wind loads.

The model's coefficients of stability of eigenvalue buckling under different load cases are listed in Table 1 and the first modes of eigenvalue buckling under two control load cases are shown in Figure 4.

<table>
<thead>
<tr>
<th>Load case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients of stability</td>
<td>55.0</td>
<td>14.4</td>
<td>24.7</td>
<td>22.2</td>
<td>52.8</td>
</tr>
</tbody>
</table>

Condition 2: \( \lambda = 14.4 \)  
Condition 4: \( \lambda = 22.2 \)

From the results of calculation, it can be found that:

1. The longitudinal arches and the transverse arches are buckle first. In fact, the buckling of steel can be classified in two categories: local buckling and overall buckling. When the slenderness ratio is larger than the critical slenderness ratio, the struts maybe buckle.

2. The coefficient of stability of condition 2 is 14.4 and that of condition 4 is 22.2. The two load cases are control load cases since the coefficients are less than that of other load cases. The design wind loading’s influence is insignificant.

3. All the coefficients of stability are larger than the recommend coefficient of the code of bridge, which is 4–5. That means the structure has high safety reserve. However, Eigenvalue buckling analysis is not enough in this kind of composite bridge because of the existence of...
nonlinear effect. It is only a reference to the ultimate buckling load and a nonlinear analysis is necessary.

NONLINEAR STABILITY ANALYSES

In the limit bearing capacity analyses, the material of the model are endowed with nonlinear attribute. The initial geometric imperfections which magnitude equal 1/1000 of the bridge span are imposed on according to the first modes of eigenvalue buckling. Large-deflection (large-rotation) effects or large strain effects are neglected or considered through the on-off of command “nlgeom”. Thus material nonlinear limiting bearing capacity analyses and double nonlinear limiting bearing capacity analyses were made.

The force-displacement curve of two control load cases under material nonlinear condition are shown in figure 5. The deformed shape of two control load cases under double nonlinear condition are shown in figure 6. The coefficients of stability of the structure are listed in table 2.

![Figure 5: The force-displacement curves under material nonlinear condition](image1.png)

![Figure 6: The deformed shape under double nonlinear condition](image2.png)

<table>
<thead>
<tr>
<th>Condition</th>
<th>LOAD CASE 2</th>
<th>LOAD CASE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>NODE10465横桥向位移/mm</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>加载倍数</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Condition</th>
<th>LOAD CASE 2</th>
<th>LOAD CASE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>NODE4039顺桥向位移/mm</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>加载倍数</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**TABLE 2**

| THE MODEL’S COEFFICIENTS OF STABILITY OF NONLINEAR ANALYSES UNDER DIFFERENT LOAD CASES |
|---------------------------------|-------|-------|-------|-------|-------|
| Load case | 1 | 2 | 3 | 4 | 5 |
| Material nonlinear analyses | 45.5 | 11.8 | 22.8 | 17.9 | 48.6 |
| Double nonlinear analyses | 14.3 | 4.7 | 6.6 | 6.0 | 14.6 |

The results show that:
(1) The calculated safety reserve varies with the load case. But all the coefficients of stability are larger than the recommended coefficient of the code of bridge, which is 4~5. That means the structure has high safety reserve. Besides, the design wind load does not cut down the stability of the structure.

(2) The geometrical nonlinear and the material nonlinear characteristics have great influence on the ultimate buckling load. The results of linear analysis are 200%-300% larger than those of nonlinear results, so the nonlinearity cannot be neglected in this composite bridge.

(3) When considering double nonlinear, the structure’s deformed shapes are different from those of linear analyses. The deformation characteristic is that the bridge deck buckle and some struts are yield. The structure exhibits characters of intensity control.

CONCLUSIONS

Through the FEM model, the elastic stability and limiting bearing capacity of the bridge under different load cases are analyzed. It can be concluded as follows:

(1) The longitudinal arches and the transverse arches are buckle first. In fact, the buckling of steel can be classified in two categories: local buckling and overall buckling. When the slenderness ratio is larger than the critical slenderness ratio, the struts maybe buckle. All the coefficients of stability are larger than the recommended coefficient of the code of bridge, which is 4~5. That means the structure has high safety reserve.

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REFERENCES

ON DYNAMIC STRESS AMPLIFICATION CAUSED BY SUDDEN FAILURE OF TENSION MEMBER IN STEEL TRUSS BRIDGES

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KEYWORDS

Redundancy analysis, Impact factor, Corrosion, Fatigue, Fracture, Dynamic analysis, Truss bridge

ABSTRACT

The degree of redundancy in a bridge system is one of the important factors that govern whether or not a failure of one member leads to a collapse of overall structure. In addition to the redundancy, the effect of dynamic stress amplification resulting from a sudden failure of a member may also have a considerable influence on a progress to the overall collapse. Therefore, this dynamic effect must be included in the static redundancy analysis as an impact factor. In the case of the failure of a tension member in truss bridges, the dynamic effect is classified into two types of impacts. One is the primary impact caused by the longitudinal strain wave that propagates from the failed point of the member. The other is the secondary impact resulting from a dynamic transition of overall equilibrium from a pre-failure to a post-failure state. Herein, based on a precise dynamic response analysis, the properties of these two impacts are investigated for a Warren truss bridge. From the results of the analysis, it is observed that the stress amplification due to the primary impact is small, compared with that caused by the secondary impact. Furthermore, the time difference between these two impacts is large enough for their interaction to be ignored in the evaluation of the impact factor. Therefore, it is sufficient to consider only the effect of the secondary impact in its evaluation. The calculated values of the impact factor for the respective members vary widely. However, the impact factors for the critical members in the truss bridge take almost a constant value. These values range from 1.4 to 1.8, depending on the location of the failed members.

INTRODUCTION

The necessity of structural redundancy analysis for old steel truss bridges has been increasing in Japan since the collapse of the Minneapolis I-35W Bridge in 2007. This is because Japan has a lot of old steel truss bridges and failure of a diagonal tensile member occurred in two truss bridges one...
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after another in 2007 due to corrosion (Figure 1). Fortunately, the failure of the tensile member did not lead to the overall collapse of the two Japanese bridges, being different from I-35W Bridge. As is well known, whether or not the failure of one member causes an overall collapse is mainly governed by the degree of redundancy of a bridge system. In addition to the redundancy, the dynamic stress amplification resulting from a sudden failure of a member may also have a considerable influence on a progress to the overall collapse. Therefore, this dynamic effect is included in the static redundancy analysis as an impact factor (URS Corporation [1]). The existing impact factor is determined, based on a considerably approximate bridge model with a single degree of freedom (SDOF) system. However, the validity of this impact factor is not examined enough. Since the magnitude of the impact factor plays an important role in the static redundancy analysis, it will be necessary to investigate its validity more precisely. In the case of the failure of a diagonal tension member in truss bridges, the sources of the dynamic stress amplification are of two kinds. One is an impact caused by the longitudinal strain wave that propagates from the failure point to both ends of the failed member. This impact acts on the two nodes to which both ends of the failed member are connected. The other is an impact resulting from a dynamic transition of overall equilibrium from the pre-fracture to the post-fracture state. In this case, the vertical displacements of the overall structure increase rapidly. Therefore, inertial forces due to the vertical acceleration act on all the masses in the bridge and may have a global and substantial effect on the overall failure of the bridge.

In view of the necessity to present a rational impact factor that is applied to the static redundancy analysis for truss bridges, a dynamic response analysis is carried out to examine precisely the property of the dynamic stress amplification due to a sudden failure of a diagonal tension member. In the dynamic analysis, a special consideration is paid to determine sufficient time increment as well as element division such that the convergent solutions can be obtained in terms of response stresses under impact. As a numerical example, we select one of the two existing Warren truss bridges where the failure of a diagonal tension member occurred in 2007.

**IMPACTS CAUSED BY TENSION MEMBER FAILURE**

The failure of tension members in old truss bridges normally initiates from the fatigue cracks or ductile cracks at stress concentration points caused by the corrosion on loss of material. (Figure.1)
Therefore, it may be possible that the failure is brittle and the tensile strains in a failed member are released suddenly.

At the time of the brittle failure, the longitudinal strain wave propagates with a high velocity \( c_L = \sqrt{E/\rho} \) in the longitudinal direction from the fractured surfaces to both ends of the member, where \( E \) and \( \rho \) are Young’s modulus and the density per unit volume, respectively, of the member. As a result, at the time of \( T_f = \alpha \ell / \sqrt{E/\rho} \) (\( \ell = \) member length, \( \alpha \leq 1.0 \)) after failure, compressive impact forces act on the nodes to which the ends of the fractured member are connected. This impact is herein referred to as primary impact.

At the same time when the member failure occurs, deflections of a truss bridge start to increase dynamically due to a sudden decrease in the bending rigidity of the bridge. As a result of the vertical acceleration caused by this dynamic behavior, the vertical inertia forces will have a global and substantial effect on the overall bridge system. The impact caused by these inertia forces is herein referred to as secondary impact. The secondary impact is concerned with the flexural vibration of the truss bridge with a failed member. The period of the most dominant flexural vibration mode (1st mode) for the bridge is given as \( T_{II} \approx t_i/2 \) where the maximum deflection of the 1st vibration mode occurs. Since \( T_{II} \) is much smaller than \( T_f \), the interaction between the primary and the secondary impacts may be ignored. This is because the effect of the primary impact decreases rapidly due to energy dissipation. In what follows, the effects of the primary and secondary impacts are examined precisely by carrying out a dynamic response analysis.

**BRIDGE MODEL**

Failure of a diagonal tension member occurred in two existing truss bridges in 2007. One of these two truss bridges shown in Figure 1 is selected as a model to examine the dynamic stress amplification due to a sudden failure of a diagonal.

![Figure 2: Warren truss bridge for numerical analysis](image)

**Table 1: Member details of Warren truss bridge**

<table>
<thead>
<tr>
<th>Member No.</th>
<th>Section</th>
<th>Web</th>
<th>U. flange</th>
<th>L. flange</th>
<th>( \alpha )</th>
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</thead>
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<tr>
<td>1 - 9</td>
<td>BOX</td>
<td>400</td>
<td>15</td>
<td>450</td>
<td>14</td>
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<tr>
<td>2 - 9</td>
<td>H</td>
<td>326</td>
<td>9</td>
<td>360</td>
<td>12</td>
</tr>
<tr>
<td>2 - 10</td>
<td>BOX</td>
<td>360</td>
<td>13</td>
<td>322</td>
<td>13</td>
</tr>
<tr>
<td>3 - 10</td>
<td>H</td>
<td>300</td>
<td>13</td>
<td>322</td>
<td>9</td>
</tr>
<tr>
<td>3 - 11</td>
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<td>340</td>
<td>12</td>
<td>300</td>
<td>9</td>
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<td>H</td>
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<tr>
<td>4 - 12</td>
<td>BOX</td>
<td>230</td>
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<tr>
<td>1 - 10</td>
<td>BOX</td>
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<td>9</td>
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(51x787) **ICASS’09, 16-18 December 2009, Hong Kong, China**
tension member. The details of the selected bridge model are schematically shown in Figure 2. This is a Warren truss bridge constructed in 1950s. In 1999, serious corrosion was found out at the webs of the diagonal tension members embedded in the concrete floor slab and the corroded webs were immediately repaired by bolting splice plates (Goto [2]). However, for some reason or other, some of the corroded tension members were left unrepaired and one of these unrepaired members became a cause of the member failure in 2007. The member details of the original bridge including yield stress $\sigma_y$ are summarized in Table 1. These members are assumed to be rigidly connected at truss nodes. The diagonal tension members with H section are arranged in the bridge such that their flange planes are parallel to the vertical truss plane. The rigidity of the concrete floor slab is ignored for simplicity, but its mass is modeled as equivalent concentrated masses located at the nodes of stringers as shown in Figure 3. Herein, in order to investigate the impact factor that is to be applied to the conventional static redundancy analysis, elastic dynamic behavior ($E = 2.06 \times 10^5$MPa) is assumed for the bridge model.

**PRIMARY IMPACT**

At the time of the failure of a diagonal tension member, the longitudinal strain wave starts to propagate with high velocity from the failure point toward the connected ends of the member. Then, the compressive impact force referred to as primary impact acts on both nodes of the failed member.

In the numerical model corresponding to the bridge shown in Figure 1, the member 3-10 that is to fail is cut into two parts in advance at the failure point. This point is assumed here to be the center of the member. Then, the original state before failure is physically replaced by applying a pair of external forces in the opposite directions to the both surfaces of the cross section where the cut is made to the member. The external forces are equivalent to the member forces calculated by the original bridge model. Based on the above model, the phenomenon of the member failure is expressed by reducing the external forces to zero within a short period of time. Here, in order to consider an extreme case of brittle fracture which will result in the largest impact forces, the external forces are assumed to become zero suddenly without any deformation in the failed member.

The primary impact is caused by the longitudinal strain wave with very high frequency. Therefore, in order to evaluate the dynamic stresses due to this impact accurately, the key members, that is, the failed member and the members surrounding the failed member, have to be divided into all finite elements. In addition, a very small time increment $\Delta t$ must be adopted in the numerical integration of the equation of motion. The element division and time increment are determined by comparing the numerical result with the analytical solution regarding an impact on a single element caused by the longitudinal strain wave. Herein, 400 elements per one member are used to discretize the key members, while 30 elements are used for the rest of the truss members. This element division is schematically shown in Figure 4. The time increment adopted in the analysis is $\Delta t = 3.0 \times 10^{-6}$ (sec.). The above-mentioned numerical method to assess the primary impact is here referred to as precise method.
For the respective members, distributed mass of \( \rho = 7850 \text{ (kg/m}^3 \text{)} \) is considered. In addition, concentrated masses representing the floor slab is assumed to be located at the nodes of stringers as illustrated in Figure 3. Modeling of the live load based on the Japanese code (Japan Road Assn. [3]) is shown in Figure 5. The location of this live load is determined such that the tensile axial force of the diagonal member that is assumed to fail becomes the maximum. The distributed live load is modeled as equivalent concentrated masses located at the nodes of stringers, similar to the mass distribution modeling of the floor slab. To consider the state before failure, constant gravity acceleration is applied to all the masses in the bridge model. As a damping model, Rayleigh model is adopted. Rayleigh damping coefficients \( \alpha \) and \( \beta \) are determined such that the damping coefficients of the two major vibration modes become 5%. This damping ratio is the same as the one adopted by URS Corporation [1] to calculate the impact factor. Herein, as the two major vibration modes, the modes that have the first and the second biggest modal effective mass are selected in terms of the vertical vibration mode.

As a result of the precise numerical analysis, the maximum axial stress \( \sigma_d \) response history of member 2-3 after the sudden failure of member 3-10 is shown in Figure 6. It can be observed from Figure 6 that the primary impact occurs at the time of \( t = 0.00141 \text{(sec.)} \). After the primary impact, the dynamic amplification due to this impact decreases rapidly. Then, the response stress curve gradually increases and reaches a peak point at \( t = 0.473 \text{(sec.)} \). This peak point corresponds to the secondary impact. The peak point by the secondary impact is much higher than that by the primary impact. Therefore, the effect of the primary impact may be ignored in evaluating the impact factor used for the redundancy analysis.

SECONDARY IMPACT

From the previous section regarding the precise analysis of the primary impact, it is clarified that the primary impact is much smaller than the secondary impact. In addition, the time difference between the primary and the secondary impacts is large and the effect of the primary impact
disappears before the secondary impact occurs. These facts imply that the effect of the primary impact may be ignored in evaluating the impact factor for the static redundancy analysis. The numerical method that takes into account the effect of the primary impact requires long computational time due to the fine element division and extremely small time increment required in the analysis. Therefore, it is preferable to simplify the numerical method by ignoring the effect of the primary impact.

In a simplified method used herein, first, the member that will fail is removed from the original bridge model. Then, the original state before the member failure is physically replaced by applying the external forces equivalent to the member forces at both ends of the member. The behavior of the sudden failure is expressed by removing these nodal forces at once. The simplified method completely ignores the primary impact caused by the propagation of the longitudinal strain in the failed member. Thus, it is not necessary to use fine element division or extremely small time increment. In the simplified method, the number of the element division for the members surrounding the fractured member 3-10 is reduced from 400 to 30 and the time increment is increased from $\Delta t = 3.0 \times 10^{-6}$(sec.) to $\Delta t = 2.0 \times 10^{-2}$(sec.). In Figure 6, the stress response history of the member 2-3 obtained by the simplified method is compared with that obtained by the precise method with fine element division and small time increment. It is seen from Figure 6, that the simplified dynamic analysis method predicts the stress response due to the secondary impact...
with a good accuracy. The computation time by the simplified method is reduced to approximately 1/5000 of the time required by the precise method. In what follows, the secondary impact is calculated by the simplified method.

In the calculation of the secondary impact, four cases of member failure including the failure of member 3-10 are considered as possible failure modes of the diagonal tension members. All the failure modes considered herein are illustrated in Figure 7. For each failure case, the impact factor $I_i$ for the $i$-th member is calculated by the following equation.

$$I_i = \frac{(\sigma_{idm} - \sigma_{is}^{(0)})}{\sigma_{iy}}$$

where $\sigma_{idm}$ is the maximum value of the absolute dynamic response axial stresses in the $i$-th member. This value is obtained by carrying out the simplified dynamic analysis on the secondary impact. $\sigma_{is}$ is the static stress at the same location of the $i$-th member where $\sigma_{idm}$ occurred. $\sigma_{is}$ is calculated by a static analysis on a bridge model where the failed member is removed. $\sigma_{is}^{(0)}$ is the corresponding static stress of the original bridge model prior to the member failure. Respective member impact factors $I_i$ calculated for the four failure cases are shown in Figure 8 in terms $I_i \sim (\sigma_{is} - \sigma_{is}^{(0)})/\sigma_{iy}$ relation where $\sigma_{is}$ is the yield stress of the $i$-th member. In this figure, the value of 1.854 obtained for a SDOF system with 5% damping (URS Corporation [1]) is shown as a reference value of the impact factor. In the evaluation of the impact factor, the SDOF model is so simplified that it cannot reflect either the effect of the location of the failed member or the variation of the impact factor according to the difference of the members.

Figure 8: $I_i \sim (\sigma_{is} - \sigma_{is}^{(0)})/\sigma_{iy}$ relation

![Graphs showing impact factors for different cases](image-url)
From Figure 8, it is observed for Case1~Case4 that the value of the impact factor \( I_i \) varies widely and its maximum value becomes very large when \( \left( \sigma_{is} - \sigma_{is}^{(0)} \right) / \sigma_{iy} \) is small. However, as \( \left( \sigma_{is} - \sigma_{is}^{(0)} \right) / \sigma_{iy} \) increases, the maximum value of \( I_i \) rapidly decreases and \( I_i \) converges to a certain value, except for Case 4 where \( \left( \sigma_{is} - \sigma_{is}^{(0)} \right) / \sigma_{iy} \) is small. The convergent value of the impact factor as well as the threshold value of \( \left( \sigma_{is} - \sigma_{is}^{(0)} \right) / \sigma_{iy} \) and the convergent impact factor become larger. In Case 1 where the failed member is the closest to the support, the impact factor converges to around 1.8, under \( \left( \sigma_{is} - \sigma_{is}^{(0)} \right) / \sigma_{iy} \geq 0.2 \). This convergent value 1.8 is near to 1.854 proposed by URS Corporation [1]. However, as the location of the failed member becomes farther from the support, the convergent impact factor takes smaller value, as 1.6 under \( \left( \sigma_{is} - \sigma_{is}^{(0)} \right) / \sigma_{iy} \geq 1.5 \) in Case 2 and 1.4 under \( \left( \sigma_{is} - \sigma_{is}^{(0)} \right) / \sigma_{iy} \geq 0.7 \) in Case 3. The convergent impact factor generally takes larger value in the case when the diagonal tension member carries larger force before its failure.

In the static redundancy analysis, the dynamic stress ratio \( \sigma_{idm} / \sigma_{iy} \) for the \( i \)-th member is calculated by using the impact factor \( I_i \) as

\[
\frac{\sigma_{idm}}{\sigma_{iy}} = \frac{\sigma_{is}^{(0)}}{\sigma_{iy}} + I_i \left( \frac{\sigma_{is} - \sigma_{is}^{(0)}}{\sigma_{iy}} \right) \tag{2}
\]

Although the convergent impact factor tends to take the smallest value as can been seen from Figure 8, this value is important in the evaluation of \( \sigma_{idm} / \sigma_{iy} \) given by Eq.(2) because \( \left( \sigma_{is} - \sigma_{is}^{(0)} \right) / \sigma_{iy} \) takes large values. On the other hand, the large values of the impact factor shown in Figure 8 is usually not so important because \( \left( \sigma_{is} - \sigma_{is}^{(0)} \right) / \sigma_{iy} \) is very small.

**SUMMARY AND CONCLUDING REMARKS**

The dynamic effect caused by a sudden failure of a diagonal tension member in a truss bridge is investigated in order to propose a rational impact factor that is to be used for the static redundancy analysis. The dynamic effect is classified into two types of impacts. One is the primary impact caused by the longitudinal strain wave that propagates from the failed surface of the member. The other is the secondary impact resulting from a dynamic transition of overall equilibrium from the pre-failure to the post-failure state. From the results of the numerical analysis, it is observed that the effect of the primary impact is small, compared to the secondary impact. Furthermore, the time difference between the two types of the impacts is large enough for their interaction to be ignored. Therefore, it is sufficient to consider only the effect of the secondary impact in the evaluation of the impact factor. The calculated values of the impact factors for the respective members vary widely. However, the impact factors for the critical members in a truss bridge take almost a constant value. These values range from 1.4 to 1.8, depending on the location of the failed diagonal tension members.

**References**


NUMERICAL ANALYSIS OF SEA-SALT PARTICULATE MATTER ADHESION ON BRIDGE SURFACES

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KEYWORDS
Maintenance of bridges, Corrosion Environment, Computational Fluid Dynamics

ABSTRACT

It is important to estimate corrosion environment of steel bridges for a proper corrosion prevention and maintenance program. Sea salt particulate matter (SS-PM) often plays a critical role in a corrosion process of steel bridges. Generated by wind on the ocean surface, it flies some distance over land and attached itself on the bridge surface. The generation and flyover of SS-PM have been observed and intensively recently numerically analyzed using a meso-scale weather analyzing program. However, adhesion of SS-PM on bridge surface is yet to be investigated both experimentally and numerically, as adhesion of SS-PM occurs in a highly non-uniform manner and is very important for corrosion prevention. The objective of this work is to simulate the behavior of SS-PM near a bridge using Lagrangian type multi-phase flow analysis. Numerical analysis are performed to a two box-girders bridges where we conducted in-situ observation. The results show that adhesion is governed by not only a density and wind speed but also fluctuation of wind speed. The tendency of adhesion is qualitatively similar the observed one. All these results show that the Lagrangian approach provides with a useful tool for the understandings of the adhesion behaviors, though many are still to be studied in physics and chemistry of adhesion.

INTRODUCTION

Corrosion protection and fatigue control have been one of main issues in bridge maintenance programs in developed countries[1]. In view of corrosion prevention, proper selection of materials is important in the planning and proper anti-corrosion maintenance program play a key role during a service period. The governing factors of corrosion are temperature, humidity and various dusts on the surfaces. Among them, the adhered sea-salt particulate matter(SSPM) is most important and airborne SSPM near a bridge site must be considered in its design. In Japan, airborne SSPM is generated mainly on the ocean surface and its density at any location is given by observed data[2]. Recently, sophisticated fluid mechanics approaches have been attempted to simulate transportation and diffusion of SSPM. It should be noted that simulation of airborne SSPM is now merged into meso-scale weather analyzing
simulation such as WRF[3], as generation and diffusion of ocean origin SSPM is closely related to global meteorology.

Adhesion of SSPM on a bridge surface is another problem. As air flow around bridge girders is not uniform but very complex, amount of adhesion can be highly non-uniform. Moreover, rainfalls often wash away those trapped SSPM on the outer surface. Forced wash away cleansing for corrosion prevention can be more effective if localization of SSPM adhesion is available. The objective of this work is to predict the adhesion behavior of SSPM on a bridge surface using computational fluid mechanics (CFD). Adhesion of airborne micro particles on a solid surface has drawn interests of many researchers. In early studies, this problem was approached by so-called Eulerian method where the density of particles represents the behavior of particles. Despite of its simplicity, adhesion of particles on a solid surface requires additional phenomenological method. In recent years, along with availability of huge computing ability, a rather direct method has been adopted such as Lagrangian method, where a particle is modeled as solid mass. In the latter approach, adhesion to a solid surface can be directly treated as collision of a particle to a surface. In this work, we use

**Figure 1:** Corrosion of steel bridge girders

![Figure 1](image1.jpg)

**Figure 2:** Observation at Meiji bridge

![Figure 2](image2.jpg)
Lagrangian method to simulate two phase flow and adhesion to particles on a surface. Parametric studies are performed to investigate the effects of wind speed and density. These results are also compared to in-situ observation.

OBSERVATION OF ADHERED SSPM

Location and instruments

The authors observed corrosion environment at the Meiji bridge near the coast of Sea of Japan (N36°5'16", W136°11'7") between Jan. 2006 and Mar. 2008. Observed data are temperature, humidity, wind, airborne SSPM and adhered SSPM using an atmospheric corrosion monitor (ACM) sensor. The Meiji bridge is a three spanned continuous box-girdered bridge made of weathering steel. It passes over the Hino river and the length is 270m and located from about 11km from sea shore. Wind data is acquired by an ultrasonic type windometer (Fig. 1(c)) every 1 second. The density of airborne SSPM is estimated by a trap tank set on a pier (Fig. 1(d)). As for adhered SSPM, it is monitored indirectly by using ACM sensor that probes micro current accompanied by electro-chemical reaction, i.e., corrosion. The quantity of SSPM adhesion can be estimated by amount of corrosion current and relative humidity in a given period.

Results

The observed results are summarized in Figs. 3 to 5. Figure 3(a) shows the quantity of airborne SSPM captured by a tank. Because of typical strong seasonal NW wind, more SSPM is detected in winter. Figure 4 represents corrosion rate CR (mm/year) estimated from the output of each ACM sensor. As CR is positively related to the amount of adhered SSPM in the same relative humidity, Fig. 4 actually represents relative quantity of adhered SSPM on bridge surface. It is interesting that...
advances in SSPM occurs highly non-uniform even for this simple box-girdered cross section. The more SSPM contacts to a girder surface and stay in the bay between two girders than outside, as rainfalls wash away adhered SSPM there. The quantity of adhesion is much larger in winter than in summer because of strong seasonal wind. This corresponds to the behavior of captured SSPM by a trap tank. It should be noted that corrosion rate in Feb. 2006 is much smaller than that in Feb. 2007 because the output of ACM sensors were in initial unstable state. The example of wind state in winter is given in Fig. 3(b) that shows only the frequencies of NW and SW. Although occasional strong wind comes from NW, the predominant wind is from SW. On this bridge site, a mesoscale meteorological analysis suggests that ocean origin SSPM comes from not only from NW but also SW. In order to conduct numerical simulation of adhesion, detailed meteorological data on the site is necessary. At least, simple data from public observation point is not enough, since its density is very sparse (approximately only 1200 locations in Japan).

SIMULATION OF ADHESION OF SSPM

Formulation

In order to estimate the adhesion of airborne SSPM on a bridge surface, a numerical analysis must consider (a) amount of airborne SSPM at a given site and wind data, (b) behavior of adhesion of SSPM on a solid surface, and (c) detach and wash away of adhered SSPM. These analyses depend on multi-phase fluid dynamics but each length scale is different. For example, air flow within a few kilometers area is important for (a), it is best suited to use mesoscale meteorological analysis. In fact, the transportation and diffusion of industrial origin airborne particles have been included in usual meteorological analysis. Once the distribution of airborne SSPM at a site is given, fluid dynamic analysis can be performed with a proper boundary and initial condition. In the analysis (b), a main objective is to follow particles in air flow. The detach and wash away of adhered particles poses another complex problem but cannot be ignored to discuss corrosion behavior of girders. We will discuss the analyses (c) in other literature. In this work, our efforts are focused on the analysis (b).

Transportation and adhesion of airborne micro particles have drawn interests of many researchers for various reasons. Roughly speaking, there are two major approaches. One is so-called Eulerian approach that represents the distribution of airborne particles by a continuous density function. The other is the Lagrangian approach that models particle as spherical solids and analyzes two phase fluid flow. The former method is much simpler and requires less computing resources. However, it is not suitable for adhesion of particles on a solid surface. On the other hand, the latter method uses a direct particle model and is appropriate for developing a collision-adhesion model to consider adhesion of particles. In recent years, these problems of adhesion have been investigated more by the latter method[4].

An airborne particle is subjected to the forces by particle interaction with air stream and other particles due to collision and electrostatic force. The latter interaction can be negligible for particles with very low density such as in the present case. To be exact, particles and air stream interact each other. Move of particles depend on air stream while air flow itself is affected by the existence of airborne particles. However, dilute distribution of particles renders particle to air stream influence negligible. Thus, one-way approximation is adopted hereafter. In Lagrangian approach, the equation of motion for a particle is given by

$$m_p \frac{du_p}{dt} = F_{dp} + F_p + F_{sw} + F_b$$

$$(1)$$

$m_p$ and $\vec{u}_p$ are mass and a velocity vector of a particle respectively. The right hand side represents...
drag, pressure, virtual mass, and body force term in order. The specific expressions of these terms are found in references[5], [6]. Regarding the drag term, a particle is assumed to be a sphere. In the neighborhood of bridge girders, it is necessary to deal with turbulent flow under the usual wind speed. There are several turbulent models available such as DNS, LES and $k-\varepsilon$. The most sophisticated approach of these is DNS (direct numerical simulation) but it requires too much computing resources. As our interest is focused on relatively averaged flow and no large scale unsteady vortices are important, in this work, so-called $k-\varepsilon$ turbulent model[7] is adopted. In the turbulent model, it is necessary to express a proper near fixed wall flow. To this end, a standard logarithmic wall flow function is employed. Regarding adhesion of particles to a solid wall, it depends not only on collision of particles but also static electrostatic or intermolecular force between a particle and a wall to be exact. A sort of chemical or physical reaction also involves on a solid surface. For all the research so far, no general criterion of collision related adhesion seems to exist. Instead of pursuing these adhesion behaviors that requires much more theoretical and experimental works, we simply assume that a particle is captured once it collides with a surface. At least, this assumption gives the upper bound of captured particles.

Analysis

Numerical analysis is applied to a model of the Meiji bridge. Regarding the region and cells are shown in Fig. 5. Only one layer of cells along the plane perpendicular to a bridge axis is set to avoid the complexity and computing burden. Since a bridge girder has a rod like structure, a single layer analysis can reveal qualitative adhesion behaviors of airborne particles. Finer cells are used near a girder wall. The dimension of the region is determined so that the effect of the boundaries can be ignored. The initial and boundary conditions are as follows. Inbound stream is uniform and constant along AC. The outlet boundary is set along BD where pressure gradient is equal to 0 along any stream lines and velocity is given so that continuity condition is satisfied in the region. AB is a frictionless boundary while the water surface CD is modeled as a fixed boundary. All the bridge surfaces are modeled as fixed boundaries. An 80 seconds non steady state air flow analysis makes each case unless otherwise noted. After initial 20 seconds, 100 units of particles representing SPM are injected uniformly from the boundary AC every one second. The physical property of air and airborne particles are shown in Tables 1 and 2. The size of particle is determined according to ocean origin sea salt particle. Relative humidity is set to zero as it does not have significant effects on

![Figure 5: Cell discretization](image)

![Figure 6: Distribution of velocity](image)

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adhesion. All the numerical analyses are conducted by general purpose computational fluid dynamics program STAR-CD Ver. 3.26[8] with customized subroutines.

**Results and comments**

The cell discretization is fine enough that non dimensionalized wall distance is less than 100 in the cells neighboring a bridge surface in case when average wind speed is 10m/s. Figure 6 shows that flow speed distribution when inlet flow speed on AC is 10m/s. Low velocity region suggests that large vortices appear between and behind the girders. The location of large vortices does not depend on inlet flow velocity. At the Meiji bridge yearly averaged wind speed is about 3m/s and the highest daily averaged one is around 10m/s.

Figures 7(a) and (b) show the results of adhesion of particles when inlet flow speed is 10m/s and 3m/s. In these figures circles represent aggregate of micro particles of 10μm. Since it is impractical to follow the behaviors of every single particle, particles are dealt with as a set of particles. This assumption does not affect qualitative aspects of the results. In both cases, the adhered particles are likely to be found in the upstream side of webs, on the upper deck, the lower side of main girders and the lower side of the overhang in the downstream side. Adhesion behavior is similar both in quantity and location regardless wind speed. Although not given here, these behaviors are little affected by wind speed between 3~10m/s. It should be noted that the numbers of adhered particles in these analyses do not indicate the quantity of adhesion is generally independent of wind speed. When the density of airborne particles is constant, higher velocity of airstream results in more numbers of particles on a bridge surface as expected. Figure 7(c) shows the result when wind speed is set to 3m/s but a particle size is 2μm. Table 3 summarizes the number of adhered unit particle sets for each particle size. The comparisons Figure 7(c) and (a) or (b) and Table 3 suggest that the size of particles in this range is insignificant with respect to the behaviors of adhesion. As ocean origin SSPM is as large as 2 to 10μm, the adhesion behavior can be represented with any given sized particles within this range.

As for the comparisons to in situ observation described in the previous section, only a qualitative comparisons are possible since observation points are on a pier. Moreover, the condition of wind is not anything similar to the observed one since the wind changes speed and direction at all times. Because of strong seasonal north west wind from December to March, the density of SSPM is particularly high in this period. Therefore, the numerical results are compared to the observation in winter. The numbers in Fig. 8 represents the order of the quantity of adhered particles according to the output of ACM sensors (Fig. 4). The result in Fig. 8 is counterintuitive because sea salt rich wind...
is supposed to come from the downstream side. The numerical results do not contradict to the observation if non-trivial amount of airborne sea salt are contained in the wind from the upstream side. As a matter of fact, in winter, occasional strong wind comes from ocean but the prevailing wind direction is the opposite. The distribution of airborne SSPM at a given location is dependent not only on the wind, the distance from the seashore, but also geophysical data around the bridge. Strictly speaking, the distribution of airborne SSPM should be analyzed by a sort of meteorological program to consider all these governing factors. A mesoscale meteorological program such as WRF is a useful tool. According to our analysis by WRF, the distribution of airborne SSPM is predicted in an acceptable accuracy. These predictions will be reported elsewhere.

So far, all the numerical simulations are conducted with constant inlet air flow speed. In natural environment, the wind direction and speed always varies. Figures 9 respectively show examples of power spectrum of the wind speed component perpendicular to a bridge axis of the Meiji bridge. A variation of wind speed occurs continuously and randomly. The power gets smaller when the frequency of wind speed variation becomes higher as has been widely known. In what follows, the effect of this short time wind speed variation on the adhesion behavior of particles is briefly shown. Since it is not particularly meaningful to simulate a specific observed wind variation, the following simulation focus the effects of frequency and amplitude of variation. Specifically, the variation of wind speed is given by the following sinusoidal function.

\[ v_0(t) = v_{\text{avg}} + \tilde{v} \sin(2\pi ft) \]  

The set of specific values of \( v_{\text{avg}}, \tilde{v} \) and \( f \) are summarized in Table 4. It should be noted that the assumed variation is larger than usually observed on the site. The other conditions are the same as the previous simulation. The examples of numerical results are illustrated in Figs. 10. The numbers

![Table 4: Variation of wind velocity](image)

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<td>3 1.0</td>
<td>0.5, 0.33, 0.2, 0.1, 0.05</td>
<td></td>
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<tr>
<td>6 1.0</td>
<td>0.5, 0.33, 0.2, 0.1, 0.05</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 9: Power spectrum of wind speed](image)

![Figure 10: Adhered SSPM under varied wind speed](image)

![Figure 11: Effect of wind variation on adhesion behavior](image)
of adhered particles increases sometimes by 100% even when the inlet wind speed has small variation. When closely observed, the increased adhered particles is not uniformly distributed but highly localized. Large part of increase is attributed to the bay between two main girders. This is because even a small fluctuation of wind speed causes significant stream or a large vortex, into the bay region. The dependence of adhesion behaviors on amplitude of variation and frequency are illustrated in Figs. 11. With respect to the type of cross section, the large the amplitude of variation is, the more particles are trapped on the surface. On the other hand, there exists the optimum frequency that maximize the number of adhered particles. For example, the optimum frequency is about 0.2Hz when averaged wind speed is 3m/s. The optimum frequency depends on the geometry of cross section such as distance between main girders and wind speed. These results suggest that analyses with constant wind speed do not necessarily capture the important aspects of particle adhesion. These variations partly come from the large vortices caused by a nearby structure. If so, expansion of model region and application of the Large Eddy Simulation as a turbulent flow model can be an appropriate to the present problem, in which the large scale eddy is directly simulated though LES requires much computing resources.

CONCLUSION

Adhesion behaviors of airborne SSPM are numerically simulated by the use of the Lagrangian type of modeling and discussed with in situ observation. The results are summarized as follows.

a) Adhesion behaviors of airborne particles are independent of wind speed as far as airflow is constant and less than 10m/s.

b) The effect of particle size is not significant for usual ocean origin SSPM.

c) The variation of wind speed can change adhesion behaviors both in location and quantities. Ignorance of the wind speed variation sometimes leads to unrealistic results.

As adhesion and stay of SSPM on a bridge surface plays a critical role in the process of corrosion, the present kind of simulation is very important. For a more reliable analysis, the reasonable determination of initial and boundary conditions are necessary. To this end, the combination to meso-scale meteorological simulation will be helpful.

ACKNOWLEDGMENT

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REFERENCES


SHM-BASED FATIGUE RELIABILITY EVALUATION OF STEEL BRIDGES: METHODOLOGY, EXPERIMENT, AND APPLICATION

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KEYWORDS
Steel bridge, Structural health monitoring, Fatigue reliability, Stress concentration factor, Finite element model, Finite mixture models, Statistical distribution

ABSTRACT

The present investigation is concerned with the development of a systematic method for fatigue reliability assessment of welded details at steel bridges by making use of long-term measurement data from the structural health monitoring (SHM) system. An analytical fatigue reliability model is proposed by uniquely integrating the probability distribution of hot spot stress range with a continuous probabilistic formulation of Miner’s damage cumulative rule, and applied for fatigue life and reliability evaluation of the instrumented Tsing Ma Bridge (TMB). A monitoring-based standard daily stress spectrum accounting for highway traffic, railway traffic, and typhoon effects is derived by use of the monitoring strain data, from which a multi-modal probability density function (PDF) of the stress range data is obtained using the finite mixed Weibull distributions in conjunction with a novel hybrid mixture parameter estimation approach. Then the global and local finite element models (FEMs) of the bridge are developed for numerically calculating the stress concentration factors (SCFs) at fatigue-susceptible locations, while the stochastic characterization of the SCF for a typical welded bridge T-joint is perceived by conducting full-scale model experiments of the railway beam section of TMB which reveal that the SCF for typical welded connections conforms to a normal distribution. The failure probability and reliability index versus fatigue life are achieved from the obtained joint PDF of the hot spot stress range in terms of the nominal stress range and SCF random variables.

INTRODUCTION

Long-term structural health monitoring (SHM), enabled by rapid development in sensing, data acquisition, computing, communication, data and information management, has lately been one of the most popular research areas in civil engineering community [1, 2]. An issue of great concern with a bridge SHM system is how to use the huge monitoring data effectively for structural condition and safety assessment. To this goal, advanced analytical skills and computational algorithms are sought to extract the natural features inside of the large amount of data collected by the SHM system for the purpose of reliably assessing the integrity, durability, and reliability of bridges. Fatigue is one of the main causes involved in fatal mechanical failures of civil engineering structures as well as
transportation facilities and infrastructure systems, and such devastating events with a characteristic of suddenness will result in heavy losses of life and property [3, 4]. As a paradigm of integrating SHM data into structural health and condition assessment practice, the measurement data of dynamic strain/stress are of great value in achieving the authentic field-based stress spectra for conducting stress-life fatigue evaluation of steel bridges. Due to the limitation of sensor implementation techniques and specific in-situ conditions, however, the sensors for strain monitoring usually are not deployed at the most critical locations where fatigue cracks are expected to occur and thus only the nominal strain/stress is obtained. To handle such a critical problem, the nominal stress is transformed into the value of structural stress at the hot spot by multiplying a stress concentration factor (SCF).

In recognition of uncertainty and randomness inherent in the nature of fatigue phenomenon and field measurement data, it is appropriate to conduct S-N curve-based fatigue life assessment in a probabilistic way as (i) the number of fatigue test data for derivation of the S-N curves of specific structural joints is limited, and the data are subjected to pronounced statistical scatter; (ii) there are significant disparities when using the stress-life experimental results of a specimen for component-level fatigue life assessment of real structures due to the difference of fatigue features between the test specimen and real structural components; and (iii) the material characteristics as well as the stress history and environment exhibit indeterminacy not only in the experimental process of fatigue test, but also during the service life of the real structures. In view of the above facts, investigations for probabilistic assessment of fatigue life have been carried out and most of them have focused on the usage of the S-N curve procedure [5, 6]; yet only a very limited amount of research is available on monitoring-based fatigue reliability assessment, and the lack of such studies is mainly due to the challenging nature that a continuous monitoring process for compiling external load and structural response data and updating the structural performance against fatigue is costly and may not be easily applicable or economically viable for all types of structures [7, 8]. In this study, a systematic method is developed for fatigue reliability assessment of welded details at steel bridges by making use of long-term measurement data from the SHM system. A fatigue reliability model which integrates the probability distribution of hot spot stress range with a continuous probabilistic formulation of Miner’s damage cumulative rule is proposed, which is then validated by conducting probabilistic fatigue life assessment of the suspension TMB with the use of long-term strain monitoring data. The failure probability and reliability index versus fatigue life are achieved from the obtained joint PDF of the hot spot stress range in terms of the nominal stress range and SCF random variables.

ANALYTICAL FATIGUE RELIABILITY MODEL

The classical method for predicting fatigue life is based on the S-N relationship or S-N curve. The S-N curve obtained by cycling test specimens at constant amplitude stress plots the fatigue life or number of cycles to failure, \( N \), as a function of the stress range, \( S \), and is represented by the following relationship [9]

\[
NS^m = C
\]

where \( m \) and \( C \) are positive empirical material constants.

For variable amplitude stress cycles, the fatigue damage, \( D \), can be evaluated by the Miner’s rule [10]

\[
D = \sum_{i=1}^{n} \frac{n_i}{N_i} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_n}{N_n}
\]

where \( n_i \) is the number of stress cycles for the \( i \)th stress range; and \( N_i \) is the number of stress cycles to failure in the structural component if the stress range is \( S_i \). Usually, failure is assumed to occur when the damage measured \( D = 1 \).

The nominal stress range, \( S \), is represented by an algebraic difference between the maximum stress, \( \sigma_{\text{max}} \), and the minimum stress, \( \sigma_{\text{min}} \), measured in one stress cycle; and the hot spot stress range, \( S_h \), is evaluated by multiplying the nominal stress range by an SCF. That is,

\[
S = \sigma_{\text{max}} - \sigma_{\text{min}}
\]
In the present study, the hot spot stress range is regarded as a function of two random variables: $S$ and SCF. Upon the assumption that the two random variables are independent, the joint PDF of the hot spot stress range, $f(h)$, can be obtained as

$$f(h) = f(s) \times f(t)$$

where $f(s)$ and $f(t)$ are the PDFs of the nominal stress range and the SCF, respectively.

When the joint PDF of the hot spot stress range is a continuous function, the Miner’s rule given in Eqn. 2 can be re-written as the following expression to evaluate the fatigue damage characterized by a total of $n_{tot}$ stress cycles:

$$D = \int_{S}^{T} \frac{n_{tot} f(s) f(t)}{N_f} ds dt$$

where $N_f$ is the stress cycles at failure.

Thus the fatigue life, $F$, can be expressed as

$$F = \frac{D_f}{D} = \int_{S}^{T} \frac{n_{tot} f(s) f(t)}{N_f} ds dt$$

in which,

$$n_{tot} = \sum_{i=1}^{n} \lambda_i n_i$$

$$\lambda_i = \begin{cases} \left( \frac{S_i}{S_0} \right)^2 & \text{if } S_i < S_0 \\ 1 & \text{if } S_i > S_0 \end{cases}$$

where $\lambda_i$ is the reducing factor; $S_0$ is the constant amplitude non-propagating stress range; and $D_f$ is the fatigue damage at failure.

Then the limit-state (LS) function for fatigue damage can be expressed as

$$g(s,t) = D_f - \int_{S}^{T} \frac{n_{tot} f(s) f(t)}{N_f} ds dt$$

The probability of failure, $p_f$, and the reliability index, $\beta$, can be obtained by structural reliability theory as follows

$$p_f = P\{g(s,t) \leq 0\} = P\left\{D_f - \int_{S}^{T} \frac{n_{tot} f(s) f(t)}{N_f} ds dt \leq 0 \right\}$$

$$\beta = -\Phi^{-1}(p_f)$$

where $\Phi(\cdot)$ is the standard normal cumulative density function (CDF).

Obviously, one of the key issues and challenges involved in the usage of the above fatigue reliability model for monitoring-based probabilistic fatigue life assessment is the exploration of probability distributions for both the stress range and SCF. The explicit modeling of these two probability distributions will be addressed in the following sections.

DERIVATION OF MONITORING-BASED STRESS SPECTRUM

**SHM System and Measurement Data**

The TMB in Hong Kong, as shown in Figure 1, is a suspension bridge with a main span of 1,377 m
carrying both highway and railway traffic. After completing its construction in 1997, the bridge has been instrumented with a long-term SHM system comprising 283 sensors permanently installed on the bridge [11]. As part of this monitoring system, 110 weldable foil-type strain gauges have been installed to measure dynamic strains at deck cross-sections and bearings as shown in Figure 1. Most of the strain gauges were attached to the fatigue-prone portions which were identified during the design of the SHM system. One-year (the year of 1999) monitoring data from all the 110 strain gauges have been acquired for strain-based fatigue and condition assessment. Presented in this paper are the results using the monitoring data from the strain gauge SPTLS16 located at the deck cross-section CH24662.5. This strain gauge was installed under the track plate of the railway beam composed of two inverted T-beams welded to the top flange plate.

The strain data were recorded continuously at a sample frequency of 51.2 Hz. Figure 2 illustrates two typical daily strain time histories obtained from the strain gauge SPTLS16. By examining the measured data, the following observations are made: (i) the daily strain-time curves exhibit some common characteristics in the shape of curve and magnitude of cycles, and there are almost no strain pulses from 1:30 am to 5:30 am since the airport railway ceases its daily service during this time period; (ii) many strain pulses with large amplitudes are observed in each daily strain time history, which are the strain responses caused by train traffic; and (iii) the overall drift of the strain-time curve is significant, which is attributed to the environmental factors such as temperature and solar radiation. This drift does not influence the calculation of stress range since the stress range depends only on the difference between the peak and the valley of each stress cycle, as defined in Eqn. 3.

The stress time history is obtained through simply multiplying the measured strain data by the elasticity modulus of steel. After seeking the peaks and valleys in the stress time history, one day’s stress spectrum is procured by the rainflow counting algorithm. Figure 3 shows the histograms of two typical daily stress spectra by specifying a resolution of 1 MPa for the stress range interval. The stress cycles with amplitudes less than 2 MPa are discarded because the lower valid limit of the strain gauge is 10 micro-strains. An insight into Figure 3 reveals that the daily stress spectra are similar for different days under normal traffic and wind conditions, and it is therefore reasonable to average a number of daily stress spectra resulting from different days to obtain a “standard daily stress spectrum”. Furthermore, in recognizing that the strain time history due to typhoon has a pattern different from that under normal traffic and wind conditions [12], the stress spectrum under typhoon conditions should be taken into account in derivation of the standard stress spectrum. According to the
record from the Hong Kong Observatory, there are a total of 22 days out of 365 days (one year), which were under typhoon conditions in 1999. Based on this proportion, data acquired from 20 days including one day under typhoon conditions are chosen to construct a representative data sample. Figure 4 illustrates the obtained standard daily stress spectrum using the 20 days’ daily stress spectra in consideration of highway traffic, railway traffic, and typhoon effects.

Figure 3: Histograms of daily stress spectra (a) September 23, 1999, (b) November 6, 1999

Figure 4: Histogram of standard daily stress spectrum

MULTI-MODAL MODELING OF STRESS RANGE

Structure of Finite Mixture Models

The finite mixture models are commonly employed for modeling complex probability distributions and enable the statistical modeling of random variables with multi-modal behavior where a simple parametric model fails to depict the characteristics of the observations adequately. The basic structure of finite mixture models for independent scalar or vector observations \( y \) can be written as

\[
    f(y | c, w, \theta) = \sum_{l=1}^{c} w_l f_l(y | \theta_l)
\]

(13)

where \( f(y | c, w, \theta) \) is a predictive mixture density; \( f_l(y | \theta_l) \) is a given parametric family of predictive component densities indexed by the scalar or vector parameters \( \theta_l \). The objective of the analysis is inference about the unknowns which include the number of components or groups, \( c \), the component weights, \( w_l \), summing to 1, and the component parameters, \( \theta_l \). In the present study, a hybrid mixture parameter estimation approach developed by Nagode and Fajdiga [14] is used for the unknown parameter estimation.

Multi-Modal PDF of Stress Range

The rainflow-counted stress ranges from 2 to 30 MPa are extracted for modeling the PDF of the stress range measured by the strain gauge SPTLS16, as this scope covers all the stress ranges caused by the vehicle and train traffic passing through the bridge. The total observation number of the 20 days’ stress range data is 30,986 and the number of classes is obtained as 16 according to the Sturges
classification rule (SCR) [15]. Figure 5 shows the finite mixed PDFs of the 20 days’ stress range data acquired by the strain gauge SPTLS16 when using normal, lognormal, and Weibull distributions, respectively. It can be seen that the scatter of the stress ranges is well modeled by the finite mixture distributions and easily extrapolated to the region beyond the measured stress ranges. Also, it is observed that the predicted stress range distribution is a two-modal PDF separated by 6 MPa. The stress ranges less than 6 MPa are caused by highway traffic, and the stress ranges larger than 6 MPa are mainly attributed to train traffic.

Figure 5: Mixed PDFs of 20 days’ stress range data

Figure 6: AIC values of mixed PDFs of 20 days’ stress range data

Figure 6 shows the variation of the Akaike information criterion (AIC) [16] values with the iteration number of the three mixed PDFs (normal, lognormal, and Weibull) of the 20 days’ stress range data. It is found that the AIC values for the three PDFs converge rapidly and the mixed Weibull PDF results in the lowest AIC value which helps the selection of the optimal preprocessing process, the optimal number of components, and the parametric family. The best model of the stress range distribution is determined with the lowest AIC value. As a result, the Weibull distribution is taken herein as the component distribution for modeling the measured stress ranges.

STATISTICAL DISTRIBUTION OF SCF

Numerical Analysis for Determination of SCF

A three-dimensional global FEM of TMB is established by using the general-purpose commercial software ABAQUS as illustrated in Figure 7(a). The beam and shell elements in the ABAQUS element library are chosen to model the structural components of the bridge. More than 7,375 nodes and 17,677 elements are contained in the global FEM. After verifying the predicted responses (dynamic properties and displacement/strain influence lines) of the global FEM by field measurement data, a three-dimensional local FEM of typical welded connections extracted from the section CH24662.5 where the strain gauge SPTLS16 is attached under the track plate of the railway beam, is formulated as shown in Figure 7(b). There are more than 502,879 nodes and 406,448 elements in this local FEM, and the displacement responses at the deck cross-sections obtained from the global FEM are adopted as boundary conditions of the local FEM.

Figure 7: Three-dimensional FEMs of TMB (a) Global FEM, (b) Local FEM
The dimension of the typical welded joint of the local FEM is identical to the drawing details of TMB. The weld is simply modeled as a triangle and the weld thickness and weld angle are 6 mm and 45°, respectively. A relatively fine grid mesh is used for the weld seam where stress concentration with a high stress gradient is expected, and a coarse grid mesh is used for the zone where stresses are relatively uniform. For finding a compromise between the refinement of the meshing and the size in degrees of freedom of the model, 8-node reduced integration continuum element is adopted in modeling the plate and weld. After obtaining the nominal stress at the location of strain gauge SPTLS16 and corresponding hot spot stress at the weld toe when the local FEM is subjected to a uniformly-distributed load of 1 MPa on the rail-track areas of the track plate, the SCF for the welded joint near the strain gauge SPTLS16 is determined as 1.379 according to

$$\text{SCF} = \frac{\sigma_{\text{hot}}}{\sigma_{\text{nom}}}$$  \(14\)

where \(\sigma_{\text{hot}}\) is the hot spot stress at the weld toe, and \(\sigma_{\text{nom}}\) is the nominal stress at the location of strain gauge.

**Experimental Study of Stochastic SCF Characterization**

For the purpose of exploration of the stochastic characteristics of the SCF, an experimental investigation of a typical welded bridge T-joint is conducted. A full-scale test model is designed and fabricated as shown in Figure 8, which holds a uniform profile with the railway beam section of TMB in the geometrical dimension and material property as well as the weld detail of the welded joint. The central section of the experimental model is selected for implementation of the traditional electrical strain gauges (ESGs), as shown in Figure 9. The strain data from the pre-allocated measuring points are acquired with a sampling rate of 10 Hz, and the hot stop stress at the weld toe is determined by a linear regression method [17]. The SCF is then calculated as the ratio between the hot spot stress and the nominal stress which is derived from the measured data at the location of strain gauge. With the aim of taking full account of the effect of predominant factors on the scatter of SCF, test cases under moving loading conditions are carried out with a combination of three load weight grades, two velocities of moving load in two opposite directions, and two axle-distances of the test bogie.

The statistical properties and probability distribution of the SCF at the location of strain gauge c-7 are reported herein by analyzing the combined SCF data set from all groups of experiments under moving loading conditions. Figure 10 illustrates the observed PDF and cumulative distribution function (CDF) of the SCF and the corresponding theoretical curves produced by the best-fitted normal distribution. By conducting the Kolmogorov-Smirnov goodness-of-fit test [17] on the observed CDF, the obtained test statistic is less than the critical value at a significance level of 0.05. It is therefore concluded that the SCF of the typical welded steel bridge T-joints complies statistically with a normal distribution.
RELIABILITY-BASED FATIGUE LIFE ASSESSMENT

The joint PDF of hot spot stress range can be obtained by Eqn. 5 when the PDFs of stress range and SCF have been determined. For the welded detail in concern, the SCF is regarded as a normal distribution with a mean value of 1.379 and a coefficient of variance (COV) of 0.021 [18]. The joint PDF of the hot spot stress range is subsequently obtained from the multi-modal PDF of stress range and the PDF of SCF, as illustrated in Figure 11. In general, fatigue failure is supposed to happen when the final summation of the elementary damage reaches a predetermined value known as fatigue damage index. The fatigue life at the welded detail concerned in this study is predicted to be 716 years when the damage index equals to unity as a deterministic value. In fact, the fatigue damage at failure is a random variable which may be modeled by a lognormal distribution with a mean value of 1 and a COV of 0.3 [19].

According to the proposed method in the preceding section, the failure probability and reliability index versus fatigue life are obtained as shown in Figure 12, where the range of the counted stress is between 2 and 30 MPa and the coverage of SCF is from 1 to 2. It is seen that fatigue life can be determined for a given target reliability index or failure probability. A target reliability index is defined as the value of reliability index that is acceptable for design or evaluation, and its selection should be based on economic considerations as well, involving cost of construction, inspection, repair, rehabilitation, and replacement. The reliability index in the range between 2 and 4 has been used in establishing code safety margins [20] and the value of target reliability index recommended for offshore structural components is 3 [21]. It is known from Figure 12 that the predicted fatigue life is 769 years if the value of target reliability index is taken as 3. The figure also indicates that the service fatigue life directly affects the probability of failure or reliability index of the structural component. When the service life requirement is increased, the corresponding reliability index decreases sharply.
CONCLUSIONS

A fatigue reliability model for probabilistic fatigue life assessment of steel bridges has been addressed, which is unique in integrating the probability distribution of the hot spot stress range into a continuous probabilistic formulation of Miner’s rule. The developed model has been applied for probabilistic fatigue life assessment of TMB by use of the long-term strain monitoring data from an on-line SHM system which has been permanently installed on the bridge. In consideration of highway traffic, railway traffic, and typhoon effects, a monitoring-based stress spectrum is achieved by carefully examining one-year measurement strain data. The method of finite mixture models in conjunction with a novel hybrid mixture parameter estimation approach are used to generate the PDF of stress range and the results turn out that the obtained PDF fits the measurement data fairly well. The SCF at fatigue-critical locations are calculated by means of FEM analysis. The joint PDF of hot spot stress range as well as the failure probability and reliability index versus fatigue life are obtained by structural reliability theory. The results show that the service fatigue life directly affects the probability of failure or reliability index of the structural component, and when the service life requirement is increased, the corresponding reliability index decreases sharply. In summary, the approach presented in this study is capable of conducting monitoring-based fatigue reliability assessment of steel bridges taking into account uncertainty and randomness inherent in the nature of fatigue phenomenon and measurement data. Following this approach, reliable fatigue damage condition of bridges can be provided for bridge managers to carry out appropriate decision on the inspection, maintenance, and management.

ACKNOWLEDGEMENTS

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NUMERICAL STUDY ON THE LOCAL BUCKLING OF 420MPA STEEL EQUAL ANGLE COLOUMNS UNDER AXIAL COMPRESSION

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KEYWORDS
Local buckling, high strength steel, equal angle, axial compression, Q420, finite element analysis.

ABSTRACT
High strength steel sections have been increasingly used in buildings and bridges, and Q420 steel angles with a nominal yield strength of 420MPa have also been widely used in many steel structures, especially in transmission towers and long span trusses. Compared to the conventional hot-rolled steel angles with normal strengths, local buckling is more critical for high strength steel angles. Based on a series of axial compression tests of 15 stocky equal angle columns made of 420MPa steel, a finite element model is developed to analyze the local buckling behavior of high strength steel equal angle columns under axial compression. By comparing the finite element analysis (FEA) results with the test results, it has been verified that the model can accurately simulate the axial buckling behavior of high strength steel equal angle columns based on measured initial geometric imperfections and residual stresses. A finite element parametric study is also conducted to study the influences of a range of parameters on the local buckling strength of these columns, including the initial geometric imperfections, the residual stresses and the width-to-thickness ratios of the angle leg.
INTRODUCTION

The development of steel construction is always closely linked to advancements in steel materials and their production methods. Only in the recent years, a new kind of steel with a high yield strength as well as good fabrication properties such as weldability has been introduced into the construction market (IABSE [1]). Currently, high strength steel sections which have many advantages over normal strength steel sections, have been used in buildings and bridges in many countries, such as Japan, Europe, the USA, Australia and China (Pocock [2], Shi [3]), and steel angle sections with a nominal yield strength of 420MPa have also been used in many steel structures, especially in transmission towers and long span trusses (Ban [4]).

But with the increase in the yield strengths of the steel angle sections, the limiting value of the width-to-thickness ratio against local buckling is reduced according to various steel design codes in many countries. In general, local buckling behaviour in steel equal angle columns of normal strength with typical cross-sectional dimensions are usually ignored as the width-to-thickness ratios of the angle legs are small. However, but for high strength steel angles, the width-to-thickness ratios of the angle legs often exceed the current limits, and hence, they do not satisfy the requirement of against local buckling. Hence, local buckling is more critical in high strength steel sections than in normal strength steel sections. However, there is a lack of research on the local buckling behavior of high strength steel sections. As a consequence, a systematic study on the local buckling behavior of Q420 steel equal angle columns under axial compression with both experimental and numerical investigations have been performed. The experimental investigation is presented in a companion paper (Shi [5]) whilst this paper reports on the numerical investigation.

Aiming at the local buckling behavior of high strength steel equal angle columns, a finite element model is developed with the general-purpose finite element software ANSYS with incorporation of the residual stresses and the initial geometric imperfections of the specimens. This paper presents the key details of the finite element modeling. By comparing the numerical results with the corresponding test results, it has been verified that the proposed finite element model can accurately simulate the local buckling behavior of the high strength steel equal angle columns under axial compression. Moreover, a range of key parameters have also been investigated to study their effects on the local buckling behavior of these angle columns.

FINITE ELEMENT ANALYSIS

Finite Element Model

ANSYS (Ansys Inc [6]) has been widely used in many disciplines with well-documented success in modeling of high non-linear phenomena, and it has been adopted to carry out the numerical investigation because of its robustness in non-linear analyses. Element SHELL 181, which supports non-linear buckling analyses and allows the incorporation of initial stresses, is used to develop the proposed finite element model. It is a 4-node element with 6 degrees of freedom at each node: translations in $x$, $y$, and $z$ directions, and rotations about $x$, $y$, and $z$-axes, as shown in Figure 1. In general, it is highly appropriate in linear, large rotation and large strain non-linear analyses.
In the proposed model, an axial compressive force is applied at the centroid of one end of the model. This load application will cause an artificial premature failure due to large stress concentration in the vicinity of the load application, which is clearly a numerical problem and needs to be rectified. The method to solve this problem is to provide an end plate, with a thickness of 100 mm at both ends of the model, so that the applied load will be distributed uniformly over the cross section of the angle columns. The finite element model with the end plates is shown in Figure 2.

A mesh convergence study was carried out and two finite element meshes with 8 and 16 elements along the transverse direction of the angle section had been established to compare the accuracy of the numerical models; the finite element mesh is divided into 16 elements along the longitudinal direction. It is found that the difference between these two meshes is fairly small, and so the finite element model with 8 and 16 elements along the transverse and the longitudinal directions is adopted for all subsequent analyses.

The boundary condition for both ends of the model is assigned to be an ideal pin end. At the unloaded end, three translational degrees of freedom are restrained as well as the rotation about the longitudinal axis. The loaded end is restrained as the same as that of the unloaded end except for the translation in the longitudinal direction. All the restraints are applied to the centroid node in the end plates. A total of 15 stub column test specimens (Shi [5]) are used to verify the accuracy and reliability of the finite element model, and their dimensions are shown in Table 1 of Ref. [5].

The von Mises failure criteria and a multi-linear material model, which is shown in Figure 3, is adopted for all the specimens. All the values of the material parameters are derived from tension coupon tests as they are presented in Table 1. The values of 2.06×10^5 MPa and 0.3 for the Young’s modulus, \(E\), and the Poisson ratio, \(\nu\), are adopted respectively.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>MECHANICAL PROPERTIES OF STEEL ANGLES</th>
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<tbody>
<tr>
<td>Section</td>
<td>(f_y) (MPa)</td>
</tr>
<tr>
<td>L125×8</td>
<td>442.1</td>
</tr>
<tr>
<td>L140×10</td>
<td>449.1</td>
</tr>
<tr>
<td>L160×10</td>
<td>460.7</td>
</tr>
<tr>
<td>L180×12</td>
<td>459.4</td>
</tr>
<tr>
<td>L200×14</td>
<td>448.8</td>
</tr>
</tbody>
</table>
The FEA procedure includes 3 steps. Firstly, the finite element model without any geometric imperfection or residual stress is created, and both the displacement restraints and the axial compression force are applied. Then a static solution is obtained to establish the stiffness matrix of the model. Secondly, an eigenvalue buckling analysis is conducted to obtain the geometric imperfection of the models in the subsequent non-linear buckling analyses. Thirdly, after incorporating geometric initial imperfections and residual stresses, a non-linear buckling analysis is carried out with the arc-length method to obtain the ultimate load carrying capacity of the model.

**Initial Imperfections**

The FEA conducted in this paper considers the measured initial imperfections of the specimens, including geometric imperfections and residual stresses. Geometric imperfections in the present study are defined as the deviation of the edge lines of the angle column at the mid-height from a relevant straight line connecting the ends, and are denoted by $v_{01}, v_{02}, v_{03}$, and $v_{04}$, as shown in Figure 4. The measured geometric imperfection values of all the test specimens are shown in Table 2, together with the geometric imperfection values adopted in the FEA which is given by

$$v = \max \left[ \text{abs} \left( v_{01} - v_{02} \right), \text{abs} \left( v_{03} - v_{04} \right) \right] \quad (1)$$

The specimen length and the width of the angles are also listed in Table 2. The shape of the geometric imperfections is set to match the preferred failure shape given by the critical buckling mode, which is also the first eigenvalue buckling mode of the angles as shown in Figure 5.

The residual stress distributions obtained from the residual stress tests (Ban [7]) are shown in Figure 6, where “-” denotes the compressive stress and “+” denotes the tensile stress. $\beta$ is the peak value coefficient of the residual stresses and is shown in Table 3. The distribution of the measured residual stresses is consistent with that used in GB 50017-2003 (Code for design of steel structures committee [8]), but the magnitude of the measured residual stresses is obviously smaller, as shown in Table 3. From Table 3 it is shown that the magnitude of the measured residual stresses decreases with the increase of the width-to-thickness ratios of the sections, which is consistent with other test results (Rasmussen [9]). The average value of the residual stresses of every element is applied at its integration points for convenience and simplification in the FEA. Hence, the distribution and the magnitude of the residual stresses shown in Figure 7 are adopted in the FEA.
TABLE 2
MEASURED GEOMETRIC IMPERFECTIONS OF SPECIMENS (mm)

<table>
<thead>
<tr>
<th>Specimen Length l (mm)</th>
<th>v</th>
<th>Width w (mm)</th>
<th>v/w</th>
</tr>
</thead>
<tbody>
<tr>
<td>L125×8-1 249.8 1.06</td>
<td>125.0</td>
<td>1/118</td>
<td></td>
</tr>
<tr>
<td>L125×8-2 250.3 0.58</td>
<td>125.1</td>
<td>1/216</td>
<td></td>
</tr>
<tr>
<td>L125×8-3 250.3 0.36</td>
<td>124.9</td>
<td>1/347</td>
<td></td>
</tr>
<tr>
<td>L140×10-1 277.5 0.48</td>
<td>139.9</td>
<td>1/291</td>
<td></td>
</tr>
<tr>
<td>L140×10-2 278.2 0.50</td>
<td>140.0</td>
<td>1/280</td>
<td></td>
</tr>
<tr>
<td>L140×10-3 277.9 0.52</td>
<td>140.1</td>
<td>1/269</td>
<td></td>
</tr>
<tr>
<td>L160×10-1 319.5 0.16</td>
<td>160.3</td>
<td>1/1002</td>
<td></td>
</tr>
<tr>
<td>L160×10-2 319.5 0.82</td>
<td>159.8</td>
<td>1/195</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5: Deformed shape
Figure 6: Measured residual stress distribution
Figure 7: Residual stress adopted in FEA

TABLE 3
MAGNITUDE OF RESIDUAL STRESSES ($\beta$)

<table>
<thead>
<tr>
<th>Section</th>
<th>L125×8</th>
<th>L140×10</th>
<th>L160×10</th>
<th>L180×12</th>
<th>L200×14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>0.09</td>
<td>0.13</td>
<td>0.08</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>Ref. [9]</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Verification of the Finite Element Model

A comparison between the FEA results and the test results is shown in Table 4, where $F_E$ denotes the test results and $F_A$ denotes the FEA results. It is shown that the FEA results agree well with the test results and the difference is only 2% on average. Thus, the proposed finite element models are able to analyze the local buckling behavior of high strength steel equal angle columns under axial compression after incorporating initial geometric imperfections and residual stresses, and they are readily applicable for further parametric studies.

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TABLE 4
COMPARISON BETWEEN FEA RESULTS AND TEST RESULTS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$F_E$ (kN)</th>
<th>$F_A$ (kN)</th>
<th>$\Delta = \frac{F_A}{F_E}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x=1</td>
<td>x=2</td>
<td>x=3</td>
</tr>
<tr>
<td>L125×8-x</td>
<td>854.9</td>
<td>7.1</td>
<td>851.7</td>
</tr>
<tr>
<td>L140×10-x</td>
<td>1181.3</td>
<td>1232.1</td>
<td>1224.1</td>
</tr>
<tr>
<td>L160×10-x</td>
<td>1361.2</td>
<td>1415.3</td>
<td>1378.1</td>
</tr>
<tr>
<td>L180×12-x</td>
<td>1904.4</td>
<td>1880.6</td>
<td>1829.4</td>
</tr>
<tr>
<td>L200×14-x</td>
<td>2362.2</td>
<td>2348.8</td>
<td>2417.0</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>1.032</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.021</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

PARAMETRIC STUDY AND DESIGN METHOD

After the validation of the finite element model, a series of finite element parametric study is carried out to examine the effect of the geometric imperfections, the residual stress and the width-to-thickness ratios of the angle section to the structural behavior of high strength steel equal angle columns. The nominal values of all the dimensions of the specimens and the elastic-perfectly plastic material model are used in the parametric analyses.

Three values of the geometric imperfections are included in the study: $w/250$, $w/100$ and $w/50$ according to the Chinese code for the acceptance of construction quality of steel structures (GB 50205-2001 [10]), and the FEA results are shown in Table 5, where $F_1$, $F_2$ and $F_3$ represent the ultimate loads of the angle columns with the geometric imperfections of $w/250$, $w/100$ and $w/50$, respectively. From Table 5, it is shown that the ultimate loads decrease with an increase of the geometric imperfections, but the discrepancy, which is about 6% on average, is small.

The distribution of the residual stresses is shown in Figure 7, and three values of the peak value coefficient $\beta$ are included in the study: 0.2, 0.25 and 0.3. The finite element results are shown in Table 6, where $F_1$, $F_2$ and $F_3$ represent the ultimate loads with the peak value coefficient of 0.2, 0.25 and 0.3, respectively. From Table 6, it is shown that the influence of the residual stresses on the ultimate loads of the angle columns is fairly small, which is consistent with the existing research results.

TABLE 5
FEA RESULTS WITH DIFFERENT GEOMETRIC IMPERFECTIONS

<table>
<thead>
<tr>
<th>Section</th>
<th>$F_1$ (kN)</th>
<th>$F_2$ (kN)</th>
<th>$F_3$ (kN)</th>
<th>$F_1/F_2$</th>
<th>$F_3/F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L125×8</td>
<td>802.46</td>
<td>770.22</td>
<td>695.47</td>
<td>1.042</td>
<td>0.903</td>
</tr>
<tr>
<td>L140×10 1</td>
<td>121.46</td>
<td>1105.40</td>
<td>1030.00</td>
<td>1.015</td>
<td>0.932</td>
</tr>
<tr>
<td>L160×10</td>
<td>1284.10</td>
<td>1221.00</td>
<td>1092.80</td>
<td>1.052</td>
<td>0.895</td>
</tr>
<tr>
<td>L180×12</td>
<td>1732.38</td>
<td>1681.50</td>
<td>1542.39</td>
<td>1.030</td>
<td>0.917</td>
</tr>
<tr>
<td>L200×14</td>
<td>2244.51</td>
<td>2203.20</td>
<td>2027.40</td>
<td>1.019</td>
<td>0.920</td>
</tr>
<tr>
<td>Average</td>
<td>1.032</td>
<td></td>
<td></td>
<td>0.913</td>
<td></td>
</tr>
</tbody>
</table>

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TABLE 6
FEA RESULTS WITH DIFFERENT RESIDUAL STRESSES

<table>
<thead>
<tr>
<th>Section</th>
<th>( F_1 ) (kN)</th>
<th>( F_2 ) (kN)</th>
<th>( F_3 ) (kN)</th>
<th>( F_1/F_2 )</th>
<th>( F_3/F_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>L125×8</td>
<td>773.91</td>
<td>772.19</td>
<td>770.22</td>
<td>1.002</td>
<td>0.997</td>
</tr>
<tr>
<td>L140×10</td>
<td>1107.96</td>
<td>105.94</td>
<td>105.40</td>
<td>1.002</td>
<td>1.000</td>
</tr>
<tr>
<td>L160×10</td>
<td>1226.42</td>
<td>1224.56</td>
<td>1221.00</td>
<td>1.002</td>
<td>0.997</td>
</tr>
<tr>
<td>L180×12</td>
<td>1687.29</td>
<td>1684.59</td>
<td>1681.50</td>
<td>1.002</td>
<td>0.998</td>
</tr>
<tr>
<td>L200×14</td>
<td>2207.43</td>
<td>2206.44</td>
<td>2203.20</td>
<td>1.000</td>
<td>0.999</td>
</tr>
<tr>
<td>Average</td>
<td>1.002</td>
<td></td>
<td></td>
<td>0.998</td>
<td></td>
</tr>
</tbody>
</table>

In general, the width-to-thickness ratios of the angle sections are the most important factor that influences the ultimate loads of the angle columns. The Chinese standard hot rolled section steel (GB/T 706-2008 [11]) includes 114 different angle sizes, of which the smallest width-to-thickness ratio is 5.00 (L20×4) while the largest ratio is 18.75 (L150×8). Hence, a total of 14 different angle sizes with a full range of width-to-thickness ratios between 5 and 18.75 are selected for study as presented in Table 7.

TABLE 7
WIDTH-TO-THICKNESS RATIOS OF VARIOUS STEEL ANGLES

<table>
<thead>
<tr>
<th>Section</th>
<th>L20×4</th>
<th>L100×16</th>
<th>L56×8</th>
<th>L40×5</th>
<th>L63×7</th>
<th>L200×20</th>
<th>L110×10</th>
</tr>
</thead>
<tbody>
<tr>
<td>w/t</td>
<td>5.00</td>
<td>6.25</td>
<td>7.00</td>
<td>8.00</td>
<td>9.00</td>
<td>10.00</td>
<td>11.00</td>
</tr>
<tr>
<td>Section</td>
<td>L36×3</td>
<td>L160×12</td>
<td>L140×10</td>
<td>L180×12</td>
<td>L80×5</td>
<td>L70×4</td>
<td>L150×8</td>
</tr>
<tr>
<td>w/t</td>
<td>12.00</td>
<td>13.33</td>
<td>14.00</td>
<td>15.00</td>
<td>16.00</td>
<td>17.50</td>
<td>18.75</td>
</tr>
</tbody>
</table>

The failure mode of the angle column from the FEA is shown in Figure 8, and the FEA results are plotted in Figure 9.

![Figure 8: The steel angle failure mode](image1)

![Figure 9: FEA results](image2)

It is shown that the ultimate stresses decrease with an increase in the width-to-thickness ratio. When the width-to-thickness ratio is less than 12 (8 section sizes), the ultimate stresses are rather constant. However, once the width-to-thickness ratio exceeds 12 (6 section sizes), the ultimate stresses decrease rapidly. Thus, the influence of the width-to-thickness ratio on the ultimate load of 420MPa steel equal angle columns under axial compression is significant.
CONCLUSIONS

A numerical study on the local buckling behavior of 420MPa steel equal angle columns under axial compression is presented in this paper. The proposed finite element model is capable of accurately analyzing the local buckling behavior of high strength steel equal angle columns under axial compression after incorporating both the initial geometric imperfections and the residual stresses.

The influence of geometric imperfections and residual stress on the ultimate strength of 420MPa steel equal angle short columns under axial compression is small, which is consistent with the existing research results reported in the literature. However, the influence of the width-to-thickness ratio on their local buckling strengths is significant when the value of \( w/t \) exceeds 12. It is believed that the use of high strength steel angles will increase significantly in the future, and to facilitate this, future versions of structural steel design codes should include rational and reliable design methods for high strength steel angles after extensive investigation into the structural characteristics of high strength steel sections.

REFERENCES

EXPERIMENTAL INVESTIGATIONS OF COLD-FORMED THIN WALLED C-BEAMS WITH DROP FLANGE

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²Institute of Rail Vehicles, Tabor, ul. Warszawska 181, 61-055 Poznan, Poland

KEYWORDS

Thin-walled beam, Drop flanges, Experimental investigations

ABSTRACT

Cold-formed thin-walled beams are the subject of investigations. Channel beams with open and close drop flange are considered. Because cold-formed thin-walled beams tend to buckle locally, new shapes of cross-section are searched for the higher stability and strength.

Polyurethane foam has been used as filler. It has been extruded into the free space of a gentle bends of the radius r. Obtained results confirm the correctness of the conducted investigations and the fact that the test stand and methods are valid.

INTRODUCTION

Contemporary thin-walled members make it possible to reduce the weight of structures without a loss of load capacity. The growing demand for such elements is the results of their relatively easy manufacturing and assembly. Their high load capacity is usually restricted not by the strength of material but by their stability. Because the wall thickness of such members is very small in comparison to their transverse dimensions the most common buckling modes are local ones (including interactions between them). This phenomenon is not common to hot-rolled beams. Therefore designers should consider local and global stability constraints.

Many papers and monographs are devoted to such problems. The strength, statics and especially stability of thin-walled beams is considered in monographs of Timoshenko and Gere [15] and Magnucki et al [8]. There are two types of buckling – global and local ones. Plenty of interesting cross-sections of thin-walled beams may be found in the set of norms
Eurocode 3 [2]. Critical loads obtained with the help of formulas presented in Eurocode 3 were compared with experimental results by Kesti and Davies [5]. They devoted their investigations to short beams and proposed some modifications of the formulas included in Eurocode 3. Ostwald and Magnucki [10] collected and described strength, global and local buckling and optimization problems of cold-formed thin-walled beams with open cross-sections. Magnucki and Paczos [7] described three thin-walled channel beams with corrugated, sandwich and open drop flanges. Effective cross-section shapes of these beams were determined. Macdonald et al [6] described the main types of cold-formed steel members and discussed the particular characteristics affecting their design. Experimental investigations, the stress distribution and displacements of cold-formed thin-walled beams were presented by Paczos, Magnucki and Zawodny [11]. Biegus et al [1], Cheng Yu et al [3], Paczos and Wasilewicz [12], Mahendran and Jeyaragan [9] are the examples of papers devoted to experimental investigations.

**DESCRIBE OBJECT INVESTIGATION**

Cold formed thin-walled beams are the subject of investigations. Channel beams with gentle bends are considered. They were made by Finnish company RUUKKI – Oborniki, Poland (Fig. 1). The beams were loaded at both ends with bending moments (Fig. 2). Similar investigations were conducted by Schafer [13] who considered two different kinds of load. He proposed formulas that make it possible to calculate the critical load, including also the interactions between local and global buckling.

Figure 1: The investigated beams and test stand

Figure 2: The investigated cross-section of beams
TESTING EQUIPMENT

The presented preliminary experimental investigations were conducted in order to check and validate the analytical models of the local buckling of beams. They were carried out at the testing machine EU 100 within the range of tensile load 0.2–100 kN.

Figure 3: The arrangement of strain gauges

The loads of a beam as well as its strains at test points (Fig. 3) were recorded by a computer. The following equipment was used: strain gauge bridge SPAIDER, the deflection sensor 2 (WI2) and 5 mm (WA50) HOTINGER, foil strain gauges HBM type 6/120LY11: resistance 120 Ω ± 0.35%, gauge factor 2.

EXPERIMENTAL INVESTIGATIONS

The dimensions of the investigated beams are: the total length L=1200 mm, the depth H=162 mm, the width b=80 mm, the wall thickness t=0.58 mm, the radius of a bend r=5 mm. Two kinds of beams were investigated – filled with polyurethane foam and empty in the place of a gentle bend (Fig. 2), i.e. with and without filler. It total 16 beams with were investigated (4 of each kind). They were loaded with two moments (pure bending). Some results are presented in tables and drawings (Fig. 4).

Figure 4: Experimental relationship between strains and force for beams with open drop flange and without filler
The graphs presented in Fig. 4 show strains within the full range of load. In Fig. 4 an experimental relationship between strains and load is shown. The point on the graph in which at least one strain gauge shows non-linear behaviour (the first vertical line in Fig. 4) is considered to be the first critical load. Increasing the load, the deformation of the cross-section of a beam may be observed together with different buckling modes. A difference between the readings of the strain gauges located at the outside (point A) and inside (point B) edges of a flange are equal to 50-75%. It is caused by a twisting moment that occurs in channel beams under pure bending because the shear centre of a beam does not lay on the plane of acting load.

![Graph showing force vs. deflection](image)

**Figure 5:** The Critical load of a channel beam (open and close drop flange) without filler and the deflection of a beam vs. load

The graphs presented in Fig. 5 and 7 consider beams loaded until total failure. There is a local maximum in one (sometimes in two or three) point and then after slightly decrease, load start to increase once again. This maximum is considered to be the value of critical load because it is connected with a change of the initial shape of the cross-section. However, sometimes it is hardly visible.

The upper flange of beams is compressed and at the beginning it keeps its initial shape. The flange buckles if load exceeds its critical value. This phenomenon may be observed in the graphs as a perturbation of monotonic increase of load.

![Beams before and after the experimental investigations](image)

**Figure 6:** Beams before and after the experimental investigations – the beam without filler, a shelf with waves, the beam with filler

During the test, half-waves appear along the upper flange (Fig. 6), but in the case of the beams with filler they start to be seen for bigger load. Moreover, these half-waves are smaller than half-waves of beams without filler that have lower stiffness.
### TABLE 1

**DATA FOR BEAMS BUCKLING LOAD**

<table>
<thead>
<tr>
<th>Cross section Figure 2</th>
<th>Experimental Investigation</th>
<th>Maximum Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Critical Force</td>
<td></td>
</tr>
<tr>
<td>a)</td>
<td>$F_{cr1} = 3.7 , \text{kN}$, $F_{cr2} = 9.6 , \text{kN}$</td>
<td>$F_{max} = 22 , \text{kN}$</td>
</tr>
<tr>
<td>b)</td>
<td>$F_{cr1} = 3.9 , \text{kN}$, $F_{cr2} = 11.3 , \text{kN}$</td>
<td>$F_{max} = 25.5 , \text{kN}$</td>
</tr>
<tr>
<td>c)</td>
<td>$F_{cr1} = 10.8 , \text{kN}$, $F_{cr2} = 23.5 , \text{kN}$</td>
<td>$F_{max} = 31.4 , \text{kN}$</td>
</tr>
<tr>
<td>d)</td>
<td>$F_{cr1} = 17.7 , \text{kN}$, $F_{cr2} = 27 , \text{kN}$</td>
<td>$F_{max} = 42 , \text{kN}$</td>
</tr>
</tbody>
</table>

The critical load of beams, i.e. the load causing a beam to collapse, depends on the cross-section of a beam (Table 1). Beams with close drop flanges and filler have the highest value of the critical load, whereas beams with open drop flanges and without filler have the smallest critical load.

### CONCLUSIONS

The increase of displacements of cold-formed thin-walled beams under increasing load causes local buckling that may also interact with global buckling [4]. It is a result of the low load capacity of thin-walled beams that depends on the dimensions of a beam, boundary conditions (support), load or the shape of cross-section. Differences between the critical load of beams (open and close drop flange) with and without filler varies from 15% to 25%. However, it may be seen that the maximum load of the beam with filler is bigger than for the beam without filler (Tab. 1) whereas displacements are smaller. The similar investigations of beams with other cross-sections have been done as well. The comparison of the results is presented in Fig. 6.

![Figure 7: The experimental critical load of beams with different cross-sections](image)

The buckling modes of investigated beams have been mainly local (Fig. 6) and appeared for relatively small load. Firstly, the web has buckled and then flanges started to deform (get corrugated). The interaction between buckling of web and flanges may have been observed during investigations. The experimental investigations clearly show that there is a need to search for new shapes of the cross-sections of cold-formed thin-walled beams in order to make them less prone to local buckling. The further researches will be devoted to longer beams with different cross-sections. Moreover, more precise methods of support and load of beams will be incorporated into investigations.
REFERENCES


DEFORMATION AND STRENGTH OF LIGHT GAUGE STEEL CONNECTION

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KEYWORDS

Light gauge, steel frame, local failure, buckling.

ABSTRACT

Light gauge steel elements have been often used for constructing the small buildings and the residential housings because they have a lightweight property and are suitable for providing easy constructional conditions. In this paper, the deformation characteristics and the strength of the light gauge steel connections between the end plate and the light gauge steel members are investigated numerically and experimentally. Light gauge H- and Box- cross section columns are adopted for the models and are fillet welded with the end plate. These elements are assembled to the frame structure and the structural characteristics of the frame are discussed.

INTRODUCTION

Light gauge steel elements have been often used for constructing the small buildings and the residential housings because they have a lightweight property and are suitable for providing easy constructional conditions. However, the connections between such elements are softer than those composed of normal steel sections and show a large deformation. When the frame composed of light gauge steel members is subjected to the lateral wind or earthquake load, the connections will deform disregarding the assumption of rigid joint. Also, the mechanisms of the connections are difficult to represent theoretically. Therefore, the particular numerical technique is required to design these connections. The steel frame composed of light gauge beams and columns are prone to show local deformations or local buckling of the elements. Especially, in a lateral loading condition, the steel frame represents combined geometric and material nonlinearities.

In this paper, the deformation characteristics and the strength of the frame composed of the light gauge steel members are investigated numerically and experimentally. Light gauge H- and Box- cross section columns are adopted for the members and are fillet welded with the end
plate. Then the lateral load is applied at the top of the column and structural behaviors are investigated.

In numerical analyses, FEM approach is adopted based on the isoparametric degenerate shell representation and the lateral load is applied to the model under the same conditions of the experimental results. The load deflection behaviors of the frame composed of columns and beams are examined under the assumption of the combined geometric and material nonlinear analyses. Also, the ultimate strength is examined as well. In these analyses, numerical results are obtained by using the material data based on the experimental results.

In experimental analyses, the loading tests are also performed and the load deformation characteristics and the ultimate strength of the specimen are examined under cyclic lateral loadings. Also, experimental results are compared with the numerical results.

EXPERIMENTAL ANALYSES

Figure 1 shows the experimental model. The model composed of two steel box columns (Box 105x105x3.2) and upper and lower H-beams (LH200x100x3.2x4.5). The height and the width of the frame are 3100x3100mm. When the frame is subjected to the lateral load, the shear frame is installed to prevent the shear failure of the frame.
The shear frame is also composed of two columns (Box 75x45x2.3) and the diagonal bracing (Box 75x45x2.3). Flat bars (FB6x75) are placed at the top and the bottom edges. The shear frame and the frames are connected by the bolts. The frame is fixed on the base floor and is subjected to the lateral load at the top of the column via servo actuator jack.

**The Frame under Lateral Load**

The lateral load is applied alternately in positive and negative directions under the displacement control scheme. The loading history is controlled by the shear angle, that is calculated as the ratio of lateral displacement at the column top to the column height ($\gamma=\delta/h$). The magnitude of the shear angle $\gamma$ is 1/450, 1/300, 1/200, 1/150, 1/100, 1/75 and 1/50 radian, respectively.

**Load Displacement Relation**

Figure 3 shows the load-displacement relation of the frame under cyclic loading. When the applied load is below 8kN or is above -8kN, the frame behaves almost linear. However, the load-displacement curve shows nonlinear, when the applied load exceeds this border. These phenomena are caused by follows:

1. Separation between the shear frame and the frame (see Figure 4)
2. Element local buckling (see Figure 5)
3. Material nonlinearities

![Figure 3: Load-displacement relation](image)

![Figure 4: Separation](image)

![Figure 5: Buckling](image)
Finally, the frame fails at 18kN and the maximum displacement is 81mm at the top of the column.

**Load Strain Relation**

Strains in the diagonal bracing are measured at both side of the box element shown in Figure 2. Gauges 1 and 2 are wire strain gauges pasted on the element. Figure 6 shows the load strain relation of the diagonal bracing. Gauge2 shows almost linear behavior. However, Gauge 2 shows non-symmetric behavior under positive and negative lateral loads.

Figure 6: Load strain relation

Figure 7 shows the compressive and flexural strains calculated by the experimental data shown in Figure 6. From Figure 7, the axial strain represents almost linear relations and the flexural strain shows the nonlinear behavior when the applied load is in the positive direction.

Figure 7: Load strain relation

**NUMERICAL ANALYSES**

In numerical analyses, the finite element procedure is adopted. FE program used in this paper is presented in the previous paper (Hara et. al, 2004) The outline of the program is presented here, briefly. The steel element is assembled from thin steel plates. The degenerated finite shell
elements are adopted for each plate element (see Figure 8).

The element has 9 nodes and represents Heterosis element. Each node on the element edges has 5 degrees of freedom that represent 3 translations and 2 rotations. The central node has only 2 rotations. Heterosis element is recognized as the thin shell element with small shear locking phenomena (Hinton et. al, 1984). Isoparametric representation is adopted to represent the strain and the deformation.

In numerical analyses, combined geometric and material nonlinearities are taken into account because of thin structural properties of the models. Considering the geometric nonlinearity, Green Lagrange strain expressions are introduced. Also, von Mises yield criterion is adopted to represent the yielding of the multidimensional stress status. To represent the nonlinear stress strain relationship of the steel material, the bilinear stress strain relation is defined.

Each element is subdivided into some layers to represent the progress of the material yielding through the thickness even a thin structure. So-called layered approach is adopted.

\[\text{Figure 8: Heterosis element}\]

**Numerical results of Frame Structures**

Figure 9 shows the numerical model of the frame with shear frame. Each element is defined by a beam-column element. The shear frame is connected only at nodes 6, 8, 18 and 20 in Figure 9. The lateral load is applied at the top of the column.

Figure 10 represents the deformation pattern of the frame with shear frame. The corners of the shear frame apart from the main frame as shown in experimental results (see Figure 4).

\[\text{Figure 9: Numerical model of the frame} \quad \text{Figure 10: Deformation of the frame}\]
### TABLE I

**MATERIAL PROPERTIES**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus</td>
<td>206 GPa</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>Yield Strength</td>
<td>235 MPa</td>
</tr>
<tr>
<td>Ultimate Strength</td>
<td>411 MPa</td>
</tr>
</tbody>
</table>

---

#### Figure 11: Numerical model of H-beam

#### Figure 12: Failure of H-beam

#### Figure 13: Load deformation behavior of H-beam

(a) Horizontal displacement at A

(b) Vertical displacement at B
Numerical Analysis of Local Failure of H-Beam

From Figure 10, at the upper left and the lower right corners, the large compressive failure is arisen due to the deformation of the shear frame under a lateral loading (see Figure 5). To represent these failures, the numerical model shown in Figure 11 is proposed. The model is divided into 360 shell elements and each element is subdivided into 8 layers. Therefore, a layered approach is adopted. H-beam is supported at 2 points where the fixed bolts are placed. The applied load area from the box column of the shear frame is shown in Figure 11 as the fat solid line. The material properties are obtained from the material tests and are shown in Table 1.

Figure 12 shows the failure pattern of the model. Due to the concentrate load from the box column, the local failure of the H-beam web occurs. The failure pattern is quite similar as the experimental one.

Figure 13 shows the load deflection behaviors of H-beam. From Figure 13(a), it is easy to understand the web buckling occurs when the load exceeds 60kN.

Stiffening the Beam end of H-Beam

To improve the compressive strength of H-beam, vertical stiffener is placed at the end of H-beam. From the numerical results shown in Figure 14, the local buckling does not occur. Therefore, the stiffness of H-beam is improved as shown in Figures 14 and 15.
CONCLUSIONS

From the numerical and experimental analyses, following conclusions are obtained:

(1) The experimental deformation of the frame is well simulated by the numerical analysis in spite of ignoring the residual stresses of the specimen.
(2) Light gauge frame structure is prone to show local failure. Therefore, the appropriate arrangement of the stiffeners is required.
(3) The connecting conditions and the appropriate arrangement of the elements are important to evaluate the strength of light gauge frame.

REFERENCES

LIGHT WEIGHT TENSION STRIP STRUCTURES

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KEYWORDS
Light weight structure; tension strip structure; membrane; deployable structure; cable chain structure

ABSTRACT
This paper introduces the key concept of light weight tension strip structures and presents a novel hybrid structure composed of pre-buckled strips and high strength tensile elements using cables and membrane. The potential application is for axially loaded members. The adoption of curved strips could avoid early global buckling of the axially loaded members. High strength tensile elements are applied to stabilize the axially loaded system when they are deployed and to increase its stiffness and load carrying capacity. Strips and membrane in this system are combined to minimize bending forces in the strip, which is critical for membrane structures. The basic design concept is explained first and deployment and/or folding patterns are illustrated by computer simulation. Nonlinear inelastic finite element analysis is carried out to evaluate the loading bearing capacity of the proposed light tension strip structures. Comparison with conventional tubular structures shows that the proposed structure has higher structural efficiency for structure element applications, in particular for slenderness ratio ranging between 100 and 150. Finally, the concept is expanded for larger span applications, which lead to the creation of novel deployable cable chain structures.

INTRODUCTION
Light weight structures, while meeting the requirements of safety, has always been pursued by architects and engineers. This means developing new light materials and utilizing optimally existing materials. It is well known that most structures fail from bending and, as a consequence, the procedure of optimization on forms and shapes of structures is trying to minimize the internal bending or strain energy. Rather than internal bending, conventional light weight structures take advantage of membrane actions in tensile structures consisting of pre-stressed cables and membrane structures as well as in compressive structures consisting of arches and shells. Novel structures named tension strip structures will be proposed in this paper combining the advantages of both tensile and compressive elements.

CONCEPT AND MORPHOLOGY
A large number of arch stone bridges had been built in ancient China and many dome shaped churches had been built in ancient Europe because it has been well known that arches and domes are good at resisting compressive forces with limited materials since long time ago. The secret beneath is that both forms minimize the internal bending, which is critical to most structures. One method to search for proper shape of arches or domes is
simulating hanging models. In funicular structures, e.g. cables or chains under the gravity loadings, no bending exists inside and thus the material could be optimally utilized. The optimal shape of arch under the same loading conditions could be obtained if the funicular is inversed. This could be illustrated as shown in Figure 1, where the arch is taken as an instance. For one beam with simply supported boundary conditions and under uniform load \( q \), the bending moment at the middle section is shown in Eqn. 1.

\[
M_{\text{beam}} = \frac{ql^2}{8}
\]  

Eqn. 1

In funicular with the same span \( l \) as shown on the left, the bending moment at the middle section is reduced into Eqn. 2.

\[
M_{\text{beam}} = \frac{ql^2}{8} - ph = 0
\]  

Eqn. 2

One arch shown on the right of Figure 1 could be obtained if the funicular is inversed and the bending moment at the middle section is the same as the funicular because of both geometry and loading symmetry. In fact, optimizing arch shapes by inverting the hanging funicular has been intensively used for long time. Among others, Antonio Gaudi, the Spanish architect, studied for the nave of the church by drawing on an inverted photograph of the funicular models.

Figure 1: Funicular and arch models under uniform loadings

For a slender column with boundary conditions shown in Figure 2, it will buckle like a bow under axial load as illustrated in the middle of this figure. With many springs attached along the column shown on the right of the figure, the situation is the same as one arch or funicular because of both geometry and loading symmetry. In fact, optimizing arch shapes by inverting the hanging funicular has been intensively used for long time. Among others, Antonio Gaudi, the Spanish architect, studied for the nave of the church by drawing on an inverted photograph of the funicular models.

One explicit hybrid structure is proposed as shown in Figure 3(a) consisting of compressive members (steel/aluminum strips) and tensile members (membrane and cable). Steel/aluminum strips act as the column shown in Figure 2, while membrane acts as the springs and cable is applied to pre-stress the strip and stabilize the structure under axial tensile load. Wrinkles will be produced on the membrane when the structure is under axial forces. Design with discontinuous membrane shown in Figure 3(b) could be adopted to reduce these wrinkles without affecting the structural load bearing capacity. It is worthy to note that membrane is still continuous at the interface between the membrane and strips to avoid discontinuous forces on the edge of the strips. As can be seen from Figure 3(c), relatively small section tubes could be used instead of strips as compressive members, which provide more flexible forms, e.g. 3 tubes with 3 pieces of membrane. For each compressive element, the out plane stability is enhanced by two adjacent membranes while for structures shown in Figures 3(a) and 3(b), the stability is mainly provided by the depth of the strip. One specific joint design is adopted here using node-socket connection instead of conventional pin connection, the details of which could be found in Li et al. [1].
Figure 2: Buckled configurations of slender columns under axial load

Figure 3: Tension Strip Structure
STRUCTURAL BEHAVIOURAL STUDY

The details of finite element models developed in USFOS are shown in Table 2. The strips are modeled by four node quadric shell element (QUADSHEL), while the fabric sheet is modeled by four-node membrane elements (MEMBRANE) without any bending stiffness. The struts or columns are modeled by two node beam providing non-linear capabilities like geometric, plasticity and elastic plastic column buckling, the formulation of which is based on the exact solution of the 4th order differential equation [2]. The material properties are listed in Table 1.

**TABLE 1**
MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>Material</th>
<th>Steel</th>
<th>Fabric (isotropic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E Modulus (GPa)</td>
<td>205</td>
<td>0.353</td>
</tr>
<tr>
<td>Yield Strength (MPa)</td>
<td>365</td>
<td>195</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td>0.25</td>
<td>0.3</td>
</tr>
</tbody>
</table>

In order to study the effect of slenderness, with same width of the middle section different models with different lengths ranging from 1 m to 6 m are analyzed and compared for both types a and c shown in Figure 3. Moreover, in order to study the effect of number and section shape of the tubes, other tension strip structure with 3 CHS tubes and 4 RHS tubes are also studied. The weight of all structures is kept to be the same with that of one ordinary circle tube (out diameter is 80 mm and thickness is 2 mm). For simplicity, the weight of the membrane and cable is ignored due to the small thickness or diameter.

**TABLE 2**
MODEL DETAILS

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Name</th>
<th>Model with meshes</th>
<th>Geometry Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Light Membrane 2Strips</td>
<td></td>
<td>Thickness of Membrane 0.125 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thickness of Strips 4 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Width of Strips 60 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Heights 1, 2, 4 &amp; 6m</td>
</tr>
<tr>
<td>2</td>
<td>Light Membrane 3Tubes</td>
<td></td>
<td>Thickness of Membrane 0.125 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Outer Diameter of Tubes 25 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thickness of Tubes 2.1 mm</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Heights 4 m</td>
</tr>
<tr>
<td>3</td>
<td>Light Membrane 4Tubes</td>
<td></td>
<td>Thickness of Membrane 0.125 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Outer Diameter of Tubes 20 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thickness of Tubes 2 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Heights 1, 2, 4 &amp; 6 m</td>
</tr>
<tr>
<td>4</td>
<td>Light Membrane 4 Boxer (Use rectangular hollow section box instead of CHS tubes)</td>
<td></td>
<td>Thickness of Membrane 0.125 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Height of Rectangular Box 26 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Width of Rectangular Box 6 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thickness of Box 2 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Heights 4 m</td>
</tr>
</tbody>
</table>

As shown in Figure 4, for shorter columns like 2 m and 1 m, there is no advantage for light weight membrane tubes in terms of ultimate loadings. However, the ultimate loading capacity of light membrane tubes is much higher than that of ordinary tube when the length is taken as 4 m and/or 6 m. Ultimate loading capacities of light
weight membrane strips for all of the 4 cases are quite low compared with light weight membrane tubes. The reason is that the thickness of the strips is small, which may lead to local instability under high compression force. The slenderness ratio for the ordinary tube of 6 m is bigger than 150, which is called long column in practice and should be used with caution. This means that the light weight membrane tube structures could be used for slenderness scope from 100 to 150 instead of ordinary hollow section tubes because of the higher loading capacity.

![Ultimate loading of different structures](image)

**Figure 4: Ultimate loading of different structures**

**LARGER-SPAN STRUCTURAL FORMS**

In previous paragraphs, tension strip structures are proposed for applications of one member, especially for columns and struts in compression. However, for larger structures covering long spans and/or bigger areas where many members are required, the concept should be accordingly expanded. In this case, the arch shown in Figure 2 will be divided into many struts connected to each other end to end otherwise bending will dominate because the cross section of the member is very large. In this paragraph, one deployable shelter named as deployable cable chain structure (DCCS) will be presented sharing the same concept shown in the previous sessions. The consistent tension along the arch will be replaced by tension forces at ends of struts.

![Two-dimensional cable chain structures](image)

(a) One Simple Cable Chain Structure  
(b) One Frame Structure

**Figure 5: Two-dimensional cable chain structures**

One of two-dimensional cable chain structures is shown in Figure 5(a) with basic components including hinged struts (cable chain) surrounding and cables inside. Chain can facilitate deployment speed, while cables stabilize the hinged chains outside. In the structure shown in Figure 5(a), every node obtains its equilibrium status under the support of four members: two cables and two struts. Struts tend to push the node out and cables tend to pull it back and whole structure will collapse if any member is broken. Part of CCS could be used to build a frame of protective shelter as can be seen from the Figure 5(b). Compared with the example shown in Figure 5(a), one strut and two cables connecting to this strut are removed in this frame.
A three-dimensional cable chain structure could be obtained by substituting the outside hinged struts in Figure 5(a) into 4-strut grids as shown in Figure 6. At the same time, a set of cables inside in two-dimensional cable chain structure would be replaced by two sets of cables: one set connecting the apex of pyramids shaped by the struts and the other set connecting the 4 separated nodes in the rectangular base. Front view of this structure shown in Figure 6(b) indicates that the same principle has been adopted for the three-dimensional cable chain structures compared with the previous two-dimensional cable chain structures. Each node reaches the equilibrium status under the compression forces of the struts and the tension forces of the cables, which are connected to that node. Two typical nodes are illustrated in Figure 6(a), where the thin and dash arrow mean the directions of tension forces in cables and the thick and solid arrows mean the directions of compression forces in struts.

(a) Trimetric View and the internal forces of joints A and B

(b) Front View

Figure 6: Three-dimensional cable chain structures

In order to estimate the ultimate load carrying capacity of the presented model, the structural behavior study has to be conducted. Traditional design methods use the empirical design parameters to simulate the second-order $P - \Delta$ and $P - \delta$ effects indirectly, which involves member-by-member check and leads to great computation effort as claimed in Lu et al. [3]. Second-Order inelastic analysis for steel frame design has been under intense development during the last couple of decades [4]. In this analysis, the geometrical and material nonlinearities have been taken into account for plastic hinge structural analysis of 3-dimensional frame structures [2]. The analysis deals with elasto-plastic behavior of beams and/or columns from the level of the energy state of the system [2].

Figure 7(a) shows the basic model, where dark and thick lines mean struts and thin and long lines mean cables. Dead loads and vertical imposed loads are applied directly on the nodes except the bottom ones. For membrane structures, wind or snow forces are often the predominant loading on fabric covering. Imposed load of 0.75 kN/m² is adopted for the preliminary design [5]. Two elements are applied herein: all the cables are modeled as tension elements while all of the struts are models as truss elements. The boundary condition is that all of the bottom nodes are assumed to be pinned to the ground. And the pretension (about 30% of minimum breaking load of the cables) is introduced into the cable (thin lines shown in Figure 7) to increase the stiffness of
the whole structure. Struts are made of steel of design strength 275 N/mm² and modulus of elasticity 205000 N/mm². Cables are high strength strand with breaking stress 1950 N/mm² and modulus of elasticity 195000 N/mm².

Figure 7: Analysis model of one barrel vault

Figure 8: Load deflection curve of the barrel vault

For a structure consisting struts with a tubular section of 50 mm and a thickness of 2 mm and cables with diameter of 9 mm, the loading capacity is about 2.2 times of the design loading combination as shown in Figure 8. The curve representing the relation between the deflection of the center node and the load level (applied loads/design loads) is shown in this figure.

At the beginning, the top center node moves downward linearly until the load level, which is the ratio between applied load and design load, reaches 0.9 as shown in Figure 8. After that, one set of cables are slacken, and the stiffness of the structure is decreased. Another set of cables begin to be slacken and the stiffness is decreased again when the load level reaches 1.9. The elastic phase ends because one strut yield when the applied loads reach the ultimate loading bearing capacity, which is about 2.16 times of the design load. And the deformed shape of the structure at the ultimate load is illustrated in Figure 7(b), where it could be found that top struts at the end are the most critical members.
CONCLUSION

In this paper, one novel hybrid structure named tension strip structure is proposed for axially loaded members and one evolutional structural named deployable cable chain structure is presented. The analysis shows that the light weight tension strip structures have high potential to be used for slenderness scope from 100 to 150 instead of ordinary hollow section tubes because of the higher loading capacity. Larger-span structural forms for tension strip structure are proposed and structural behavior analysis indicates that the proposed structure has desirable ductility.

REFERENCES

SHEAR BUCKLING OF THIN-WALLED CHANNEL SECTIONS WITH INTERMEDIATE WEB STIFFENER

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2 Faculty of Engineering and Information Technologies, The University of Sydney, NSW 2006, Australia

KEYWORDS
Shear buckling capacity, Lipped channel sections; Spline finite strip method, Twisting and lateral buckling mode, Intermediate web stiffener, Thin-walled channel sections.

ABSTRACT
The elastic buckling stress of the web of a thin-walled section member in shear is generally improved by the presence of flanges and lips. However, for webs with relatively large depth-to-thickness ratios, the local buckling mode in shear occurs mainly in the web. The structural efficiency of such webs can be improved by adding an intermediate stiffener cold-formed longitudinally in the middle of the web.

In this paper, the computational modelling of thin-walled steel sections is implemented by means of a spline finite strip analysis to determine the elastic buckling stresses of channel sections subject to pure shear. Lipped channels with an intermediate web stiffener are studied where the main variables are the dimensions of the stiffener in both depth and width directions. Results and comparisons of analyses are included in this paper.

INTRODUCTION
Shear buckling studies of thin-walled sections

Thin-walled sections can be subjected to axial force, bending and shear. In the cases of axial force and bending, the actions causing buckling, either Euler buckling for compression or flexural-torsional buckling for flexure, are well understood. However, for shear, the traditional approach has been to investigate shear plate buckling in the web alone and to ignore the behaviour of the whole section including the flanges. Until recently, there does not appear to be any consistent investigation of the full section buckling of thin-walled sections under shear.

Recently Pham and Hancock [1] have provided solutions to the elastic shear buckling of complete channel sections loaded in pure shear parallel with the web by using spline finite strip analysis (Lau and Hancock [2]) implemented in the program ISFSM Isoparametric Spline Finite Strip Method (Eccher [3]). Figure 1 from Pham and Hancock [1] shows the results of the buckling analyses of the
lipped channel section of length $a=1000\text{ mm}$ and the ratios of flange to web width $(b_2/b_1)$ from 0.00005 to 0.8.

The coefficient $k_v$ in Figure 1 is the shear buckling coefficient in the expression for the elastic shear buckling stress generalized by Timoshenko and Gere [4] in the following equation (1):

$$
\tau_{cr} = k_v \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b_1}\right)^2
$$

where $E$ = modulus of elasticity; $\nu$ = Poisson’s ratio; $b_1$ = width of the plate; $t$ = thickness of the plate. $k_v$ is the shear buckling coefficient, which depends on the boundary conditions and the aspect ratio of the rectangular plate $a/b_1$. In the development of Figure 1, the shear flow is that for a channel with a force parallel with the web through the shear centre. All edges of the end cross-section are simply supported and there are no lateral restraints along two longitudinal edges of the web panel. The corresponding buckling mode shapes are shown in Figure 2.

Figure 1: The Ratio of Flange and Web Widths $(b_2/b_1)$ and The Shear Buckling Coefficients $(k_v)$

The analysis results show that the lack of lateral restraint for sections with narrow flanges can lead to premature buckling of the section in a twisting and lateral buckling mode. As the ratio of $b_2/b_1$ increases to 0.4, the value of $k_v$ increases rapidly to 6.61 which is greater than the theoretical result of 5.34 for a simply supported rectangular plate in shear of infinite length (Timoshenko and Gere; Bulson; Bleich; Allen and Bulson [4-7]) and of 5.51 for a simply supported rectangular plate of length to width ratio of 5 (Pham and Hancock [1]). This is apparently due to the fact that the flanges can

Figure 2. Buckling Mode Shape of Lipped Channel Section – Length = 1000 mm, a/b_1=5
have a significant influence on improving the shear buckling capacity of thin-walled channel sections. The flanges with lips are long enough to give full lateral restraints to the lipped channel section members. For the ratio of \( b_2/b_1 \) ranging from 0.4 to 0.8, the value of \( k_v \) reduces slightly to 6.57. The buckling modes shown in Figure 2 are mainly local buckling modes in the web and the flanges. The explanation for the slight reduction of \( k_v \) is due to the effect of the slenderness of the wider flange and buckling in the flange as shown in Figure 2 for the 200x160x20 section.

**MODELLING SECTIONS WITH INTERMEDIATE STIFFENER IN SHEAR**

**Lipped Channel Geometry with Intermediate Stiffener**

The geometry of the lipped channel with an intermediate stiffener studied in this paper is shown in Figure 3. The channel section consists of a web of width 200 mm \((b_1)\), a flange of width 80 mm \((b_2)\), a lip size of 20 mm, all with thickness of 2 mm. The stiffener is positioned at the longitudinal center line of the web and the main variables are the dimensions of the stiffener. The depth of stiffener \((b_{1s})\) increases from 0.01 mm to 160 mm whereas the width of the stiffener \((b_{2s})\) varies from 0.05 mm to 50 mm. The member is subdivided into 40 longitudinal strips which include 12 strips in the web, 6 strips in each flange and 2 strips in each lip. The strip subdivision of the stiffener is 6 strips and 2 strips in the depth and width respectively. The length of the member studied is 1000 mm. The aspect ratio of the web rectangular plate is therefore \( a/b_1 = 5 \).
Shear Stress Distribution and Boundary Condition

In order to demonstrate the different ways in which a channel member with a variable size intermediate web stiffener may buckle under shear stress, the shear stress distribution in the complete channel section is firstly modelled. The shear flow distribution resulting from a shear force parallel with the web is shown in Figure 4. To simulate the variation in shear stress with the spline finite strip analysis, each strip in the cross-section is assumed to be subjected to a pure shear stress which varies from one strip to the other. The more the cross-section is subdivided into strips, the more accurately the shear stress is represented in order to match the practical shear flow distribution. The spline finite strip analysis as implemented in the Lau and Hancock [2] theory does not allow variation in shear flow across the width of a strip.

In this paper, the boundary conditions of the end cross-section assume all edges are simply supported. There are no lateral restraints along the two longitudinal edges of web panels and stiffener. Figure 5 shows the boundary conditions of the lipped channel with an intermediate web stiffener.

RESULTS OF BUCKLING ANALYSES

The results of the buckling analyses of the lipped channel section with an intermediate web stiffener are shown in Table 1. Figure 6 shows the relationship between the ratio of stiffener depth and web width \(b_{1s}/b_1\) from 0.00005 to 0.8 (\(b_{1s} = 0.01-160 \text{ mm}\)) and the shear buckling coefficients \(k_v\). Each relationship curve represents a different stiffener width \(b_{2s}\) which is in the range from 0.01 mm to 50 mm.

<table>
<thead>
<tr>
<th>(b_{1s}/b_1)</th>
<th>(b_{2s}/b_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.025</td>
</tr>
<tr>
<td>6.59</td>
<td>7.40</td>
</tr>
<tr>
<td>6.59</td>
<td>9.22</td>
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<tr>
<td>6.59</td>
<td>10.23</td>
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<tr>
<td>6.59</td>
<td>10.72</td>
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<td>6.59</td>
<td>10.95</td>
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<tr>
<td>6.59</td>
<td>10.90</td>
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<td>6.59</td>
<td>9.30</td>
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<td>6.59</td>
<td>8.56</td>
</tr>
<tr>
<td>6.59</td>
<td>7.72</td>
</tr>
</tbody>
</table>

As can be seen in Table 1, when the stiffener width \(b_{2s}\) is very small \((b_{2s} = 0.01 \text{ mm})\), the value of \(k_v\) stays unchanged (6.59) while the stiffener depth varies from 0.01 mm to 160 mm. This case is almost exactly the same as that of the plain channel section 200x80x20 shown in Figure 2. Therefore, the buckling coefficient curve \((k_v)\) with \(b_{2s} = 0.01 \text{ mm}\) is a horizontal line in Figure 6. It can be noted that the buckling mode shown in Fig 2 for this case is a local buckling mode which mainly occurs in the web. The flanges are long enough to provide elastic torsional restraint on the web. There is no distortional buckling mode in the flanges.

For a stiffener width \((b_{2s})\) of 5 mm, when the ratio of \((b_{1s}/b_1)\) is 0.00005, the value of \(k_v\) is 7.40 which is slightly greater than that of a plain channel section (6.59). As the ratio of \((b_{1s}/b_1)\) increases to 0.3, the value of \(k_v\) improves to 10. 72. This can be explained by the fact that the deeper stiffener can contribute to the shear buckling capacity of the channel section member. However, it is interesting to note that when the ratio of \((b_{1s}/b_1)\) increases further from 0.3 to 0.8, the value of \(k_v\) reduces from 10.72 to 7.74. This is due to the slenderness of longer stiffener depth which allows shear buckling in its own length. As the stiffener depth \((b_{1s})\) gets longer (60 mm to 160 mm), the slenderness is more critical.
Figure 7 shows the corresponding buckling mode shapes for different stiffener depths \((b_{1s})\) from 0.01 mm to 160 mm for lipped channel section with an intermediate web stiffener for the case of \(b_{2s}\) equal to 5 mm. When the stiffener depth \((b_{1s})\) is 0.01 mm, the local buckling mode, which occurs mainly in the web, is almost similar to that of the plain channel section shown in Figure 2. There is little distortional buckling mode in the lipped flanges. As the stiffener depth \((b_{1s})\) increases to 60 mm, local buckling still occurs in the web but is not as clear as that for the stiffener depth of \(b_{1s}\) equal to 0.01 mm. The flanges with lips start buckling in the distortional buckling mode. This is mainly due to the fact that the presence of a longer stiffener depth \((b_{1s})\) improves significantly the shear buckling capacity which causes not only local buckling in the web but also distortional buckling of the flanges and lips at higher shear load. When the stiffener depth \((b_{1s})\) is in the range from 60 mm to 160 mm, the buckling mode is mainly in the stiffener depth \((b_{1s})\). The distortional buckling in the flanges and lips becomes less severe which explains the reduction of the value of \(k_v\) when the ratio of \((b_{1s}/b_1)\) is in the range from 0.3 to 0.8.

As can also be seen in Figure 6, the relationship curves between the ratio of stiffener depths and web width \((b_{1s}/b_1)\) and the shear buckling coefficients \((k_v)\) behave similarly to that for the ratio of \((b_{1s}/b_1)\) equal to 0.00005 when the stiffener width \((b_{2s})\) increases from 0.01 mm to 50 mm. The value of \(k_v\) also increases until the ratio of \((b_{1s}/b_1)\) reaches 0.3. The value of \(k_v\) then reduces as the ratio of \((b_{1s}/b_1)\) increases further from 0.3 to 0.8. It can be seen in Figure 6 that when the stiffener width \((b_{2s})\) increases from 0.01 mm to 25 mm, the relationship curves between the ratio of \((b_{1s}/b_1)\) and the shear buckling coefficients \((k_v)\) increase with the stiffener width \((b_{2s})\). The increments of the shear buckling coefficients \((k_v)\) are significantly greater at smaller values of the stiffener width \((b_{2s})\) and decrease in the range of the stiffener width \((b_{2s})\) from 25 mm to 50 mm. Further, when the ratio of \((b_{1s}/b_1)\) increases from 0.5 to 0.8, the value of \(k_v\) drops more significantly with the ratio of \((b_{1s}/b_1)\) for larger values of the stiffener width \((b_{2s})\). The shear buckling coefficients \((k_v)\) lie slightly below those of the stiffener width \((b_{2s})\) of 35 mm as the stiffener width \((b_{2s})\) is in the range from 35 mm to 50 mm. The explanation is mainly a result of the effect of slenderness in longer stiffener width \((b_{2s})\). The longer the stiffener is, the more significantly the shear buckling capacity is affected so that for longer stiffener widths \((b_{2s})\), the slenderness is critical. Local buckling also occurs in the stiffener width \((b_{2s})\) which causes a significant reduction of the shear buckling capacity of the full channel section.

Figure 8 shows the corresponding buckling mode shapes of different stiffener depths \((b_{1s})\) from 0.01 mm to 160 mm of the lipped channel section with an intermediate web stiffener for the case \((b_{2s})\) of 50 mm. As the stiffener depths \((b_{1s})\) increases to 60 mm, local buckling in the web becomes less critical and the buckling mode is mainly a distortional buckling mode in the flanges. For the stiffener depths \((b_{1s})\) from 80 mm to 160 mm, the buckling mode is mainly local buckling in the stiffener depth and width. The line junctions of the stiffener depth and width remain almost straight. As shown in Figure 8, the relationships between the ratio of stiffener depths and web widths \((b_{1s}/b_1)\) and the shear buckling coefficients \((k_v)\) are similar to those for the ratio of \((b_{1s}/b_1)\) equal to 0.00005 when the stiffener width \((b_{2s})\) increases from 0.01 mm to 50 mm. The value of \(k_v\) also increases until the ratio of \((b_{1s}/b_1)\) reaches 0.3. The value of \(k_v\) then reduces as the ratio of \((b_{1s}/b_1)\) increases further from 0.3 to 0.8. It can be seen in Figure 6 that when the stiffener width \((b_{2s})\) increases from 0.01 mm to 25 mm, the relationship curves between the ratio of \((b_{1s}/b_1)\) and the shear buckling coefficients \((k_v)\) increase with the stiffener width \((b_{2s})\). The increments of the shear buckling coefficients \((k_v)\) are significantly greater at smaller values of the stiffener width \((b_{2s})\) and decrease in the range of the stiffener width \((b_{2s})\) from 25 mm to 50 mm. Further, when the ratio of \((b_{1s}/b_1)\) increases from 0.5 to 0.8, the value of \(k_v\) drops more significantly with the ratio of \((b_{1s}/b_1)\) for larger values of the stiffener width \((b_{2s})\). The shear buckling coefficients \((k_v)\) lie slightly below those of the stiffener width \((b_{2s})\) of 35 mm as the stiffener width \((b_{2s})\) is in the range from 35 mm to 50 mm. The explanation is mainly a result of the effect of slenderness in longer stiffener width \((b_{2s})\). The longer the stiffener is, the more significantly the shear buckling capacity is affected so that for longer stiffener widths \((b_{2s})\), the slenderness is critical. Local buckling also occurs in the stiffener width \((b_{2s})\) which causes a significant reduction of the shear buckling capacity of the full channel section.
8, there is little or no local buckling in the original flat part of the webs and little or no distortional buckling of the flanges of the full channel section.

To further understand the behaviour of the shear buckling coefficients ($k_v$) of the channel section with an intermediate web stiffener where the main variables are the stiffener depth ($b_{1s}$) and width ($b_{2s}$), Figure 9 shows the relationship between the ratio of stiffener width and web width ($b_{2s}/b_{1}$) from 0.00005 to 0.25 ($b_{2s} = 0.01-50$ mm) and the shear buckling coefficients ($k_v$). Each relationship curve represents a different stiffener depth ($b_{1s}$) which is in the range from 0.01 mm to 160 mm.

It can be seen in Figure 9 that when the stiffener width ($b_{2s}$) is 0.01 mm, the value of $k_v$ is 6.59 irrespective of the stiffener depth. For a stiffener depth ($b_{1s}$) of 20 mm, the value of $k_v$ increases dramatically from 6.59 to 28.14. As can be seen in Figure 10 which shows the corresponding buckling mode for the case of stiffener depth ($b_{1s}$) of 20 mm with increasing stiffener width ($b_{2s}$) from 0.01 mm to 50 mm, when the stiffener width ($b_{2s}$) is 0.01 mm, the buckling mode is mainly local buckling in the web. There is little distortional buckling in the lipped flanges when the stiffener width ($b_{2s}$) is 0.01 mm. As the stiffener width ($b_{2s}$) increases to 50 mm, the local buckling in the web is less critical while there is more distortional buckling in the lipped flanges. The explanation is mainly due to fact that the presence of the longer stiffener width ($b_{2s}$) improves significantly the shear buckling capacity of the web.

As can also be seen in Figure 9, when the stiffener depth ($b_{1s}$) increases to 60 mm, the relationship curves between the ratio of stiffener widths and web width ($b_{2s}/b_{1}$) and the shear buckling coefficients ($k_v$) behave similarly to those for the stiffener depth ($b_{1s}$) of 20 mm although the shear buckling coefficients ($k_v$) increase more rapidly with $b_{2s}/b_{1}$. However, it is interesting to note that the relationship curves are less straight and the increments are less when the stiffener depth ($b_{1s}$) approaches 60 mm. This can be explained mainly by the effect of the slenderness of the stiffener width ($b_{2s}$) which allows more distortional buckling in the wider stiffener. It is also interesting to note in Figure 9 that when the stiffener depth ($b_{1s}$) increases further from 60 mm to 160 mm, the latter relationship curves lie below the former ones. As discussed earlier, this is mainly due to distortional buckling in the wider stiffener.

Figure 11 shows the corresponding buckling mode shapes for different stiffener widths ($b_{2s}$) from 0.01 mm to 50 mm of the lipped channel section with an intermediate web stiffener for the case ($b_{1s}$) of 160 mm. As the stiffener width ($b_{2s}$) is 0.01 mm, the member in this case is almost exactly the same as that of plain channel section 200x8 0x20 shown in Fig 2. The buckling mode is mainly local buckling in
the web. There is no or little distortion buckling in the lipped flanges. It is interesting to note that when the stiffener width \(b_{2s}\) increases from 0.01 mm to 15 mm, the local buckling in the web is less critical and there is more distortional buckling in the flanges.

As the stiffener width \(b_{2s}\) increases further from 20 mm to 50 mm, the member buckles mainly in the stiffener depth \(b_{1s}\) in the local buckling mode. There is little or no distortional buckling in the lipped flanges and original flat part of the web. This is obviously due to the fact that when stiffener depth is longer, the greater shear stress is distributed in the stiffener depth due to the shear flow. This fact causes the reduction of the shear buckling stress of the channel section member and the buckling mode is mainly local buckling in the longer stiffener depth.
CONCLUSION

This paper has outlined buckling analyses of channel section members subject to shear stresses. Lipped channels with a variable size intermediate web stiffener were analyzed by the Isoparametric Spline Finite Strip Method program. The main variables are the dimensions of the stiffener in both depth and width directions. The boundary conditions are simply supported without lateral restraints along two longitudinal edges of web panel and line junctions of the stiffener. The assumed shear flow distribution in the whole channel section member subject to pure shear parallel with the web is used to investigate the effect of stiffener size on the shear buckling stresses.

By varying the stiffener sizes in both width and depth directions, the analysis results show that the stiffener can have a significant influence on improving the shear buckling stress of thin-walled channel sections up to a certain ratio of stiffener depth to web. The stiffener also reduces local buckling which occurs mainly in the web width. Moreover, it is also demonstrated that with longer stiffener depth and wider stiffener width the shear buckling capacity of the lipped channel section with an intermediate web stiffener reduces and the buckling mode is then mainly local buckling mode in longer stiffener depth with increasing stiffener width.

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IMPACT TESTS AND PARAMETRIC STUDIES ON DRIVE-IN STEEL STORAGE RACKS

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KEYWORDS
Steel storage rack, drive-in rack, impact force, impact sensitivity.

ABSTRACT
Extensively used in the industry to store goods, storage racks are frequently subjected to accidental impact forces from operating forklift trucks. There is currently little understanding of the nature of these impact forces, leading to occasional catastrophic failures because of inadequate structural design. International racking design codes deal with impact but use an arbitrary value of impact force with no scientific justification. This paper focuses on an impact-sensitive type of storage rack, called “drive-in racks”. Contrary to classical “selective racks”, where pallets are stored on beams and where each single pallet is always accessible; “drive-in racks” allow the forklift truck to drive into the rack to store pallets on beam rails, one after each other, on the first-in, last-out principle. This type of design leads to slender uprights in the down-aisle direction, only restrained at the base and at the top. When subjected to an impact force, the bowing of the upright triggers progressive failure by allowing the pallets to drop through. This paper briefly presents experimental test results from impact tests on a complete full-size drive-in rack structure. Six series of twenty tests each, representing three pallet loads and two rack loading configurations have been conducted by striking the drive-in rack by a forklift truck. Parametric impact studies using finite element analysis are presented. Factors affecting the sensitivity of drive-in racking structures to impact are investigated and conclusions are drawn about the parameters most significantly influencing the progressive collapse of this type of rack under impact.

INTRODUCTION
Steel storage racks are extensively used worldwide in industry for storing goods, mainly on pallets. They are predominantly made from cold-formed steel profiles and are competitively designed as lightly as possible. Due to intensive use, storage racks are frequently subjected to accidental impact forces from operating forklift trucks. There is currently little understanding of the nature of these impact forces, leading to occasional catastrophic failures.

“Selective racks”, the most common type of rack, are only one pallet deep and are separated by aisles allowing each pallet to be always accessible. When storing the same goods or in space-limited and expensive places such as industrial freezers, a more economical solution to selective racks is to store
the pallets on rail beams, one after the other, with no space between them. In this more compact type of rack called “drive-in racks”, the forklift truck drives into the rack to store the pallets on the first-in, last-out principle as illustrated in Figure 1 (a). Drive-in racks are typically 3 to 7 pallets deep and can be numerous bays wide. Other types of racks are available in the industry and are described in Pekoz and Winter [1].

To allow the forklift truck passage, drive-in racks can only be braced at the back (spine bracing) and at the top (plan bracing) in the down-aisle direction. This type of design leads to slender uprights in the down-aisle direction, only restrained at the base by the base plate assembly and at the top by the portal beam. When subjected to an impact force, the bowing of the upright may trigger progressive failure by allowing the pallets to drop through as shown in Figure 1 (b).

The main international racking design codes only deal with selective racks and do not mention drive-in racks. For selective racking systems, it is believed that the most severe damages are induced by an impact of the rear counterweight of the forklift truck between the floor and the first beam elevation; see the RMI specification (Rack Manufacturers Institute) [2] and the European Standard EN 15512 [3]. The RMI [2] only makes recommendations to “safeguard racks against the consequences of minor collisions” between the floor and the first beam elevation. For manually operated forklift trucks, the EN 15512 [3] uses an accidental impact force of 1.25 kN in the down-aisle direction and of 2.5 kN in the cross-aisle direction but only between the floor and 0.4 metres in height. On the other hand, the Australian Standard AS 4084 [4] uses an impact force applied at the most unfavourable location equal to the maximum of the unit load/15 and 0.5 kN, in both cross-aisle and down-aisle directions, for manually operated forklift trucks. However the above impact forces are arbitrary and have no scientific justification.

![Figure 1: (a) Example of drive-in rack and (b) bowing of upright and failure due to pallet dropping through during forklift truck impact on a drive-in storage racking structure (Elevation – Front view)](image)

The Fédération Européenne de la Manutention is currently developing a code dealing specifically with drive-in racks (FEM 10.2.07 [5]). In the current draft state, this code considers horizontal impact loads as in the EN 15512 [3] standard for selective rack and manually operated forklift truck. However, impact at higher elevation than 0.4 metre is considered by means of placement loads. The FEM [5] states that even through placement loads are not intended to represent accidental impact loads, it “may be assumed to include adequate allowance for or binary impact conditions”. The horizontal placement load is equal to 0.5 kN for both the cross-aisle and down-aisle directions and has to be applied at the most unfavourable beam rail elevation.

In the literature, very few investigations have been reported on impacted storage racks and they only concern selective racks with an impact at the bottom of an upright, see McConnel and Kelly [6], Bajoria [7] and Ng and al.[8]. In these studies, the impact is assumed to be strong enough to remove
the lowest section of one of the rack uprights, and the impact force, when considered, is not based on actual testing. To the authors’ knowledge, impacts on drive-rack structures have not been yet reported.

Actual impact tests on a complete full scale drive-in rack structure have been performed in the down-aisle direction in the Structures Laboratory of the School of Civil Engineering at the University of Sydney. The drive-in rack structure is 4 pallets deep, 4 bays wide and 4 stories high (i.e. featuring 3 beam rail levels) and designed to carry 2 tons per pallet. More details of the tested drive-in rack are reported in Gilbert and Rasmussen [9]. This paper briefly presents the experimental test set-up and results of the impact tests. Results from a Finite Element (FE) model of the drive-in rack under impact are found to reproduce the experimental results with satisfactory agreement. Parametric impact studies using FE analysis are presented hereafter and factors affecting the sensitivity of drive-in rack structures to impact are investigated. Conclusions are drawn about the parameters most significantly influencing the progressive collapse of this type of rack under impact.

EXPERIMENTAL TESTS AND FE MODELLING

Impact Cases

Being low, an impact by the rear counterweight of the forklift truck will not allow the upright of a drive-in rack to bow enough to trigger the failure mode shown in Figure 1 (b). For a pallet to drop through, the load has to be raised and it is the pallet carried by the forklift truck which impacts the rack as illustrated in Figure 2. Both impacts in the cross-aisle and down-aisle directions can occur but only impacts in the down-aisle direction will trigger the type of failure shown in Figure 1 (b).

Under impact in the cross-aisle direction, the forklift truck driver misjudges the location of the truck and drives straight into the rack as illustrated in Figure 2 (b). Failure due to impact in the cross-aisle direction is likely to occur due to local structural damage of the upright which is then unable to maintain its design function. Indeed, the presence of the bracing members of the upright frame allows the upright to absorb energy by local deformation rather than by global bending. Under impact in the down-aisle direction, while placing or removing a pallet on the rack, the forklift truck driver misjudges the location of the truck and starts turning before the pallet is cleared of the rack. The forklift truck, by typically rotating about its front wheel axis, causes the pallet to impact with the front upright of the rack, as shown in Figure 2 (d) and (e).

Impact Test Set-Up and Results

A full-scale drive-in rack structure (4 pallets deep, 4 bays wide, 4 stories high, i.e. featuring 3 beam rail levels and designed to carry 2 tons per pallet) has been impacted at the second rail beam elevation by a NICHYU FB20 forklift truck model (2 tons load capacity) in the Structures Laboratory of the School of Civil Engineering at the University of Sydney. The rack has been impacted in the down-
aisle direction, on the assumption that the forklift truck impacts the rack during the pallet removal as shown in Figure 2 (d). Two different pallet loading configurations (namely “Impact BC” and “Impact CD”) have been investigated. In “Impact BC”, the pallets are not loaded in the vicinity of the impacted upright allowing this upright to deform freely under impact. In “Impact CD”, the pallets are loaded in the vicinity of the impacted upright allowing the investigation of the restraining effect of the pallets on the upright deformation. Two LVDTs (Linear Velocity Displacement Transducers), LVDTs 53 and 52, record the displacement in the down-aisle direction of the impacted upright (bending of the upright and overall deformation of the rack) and of the directly opposite front upright respectively. For each loading configuration, 60 impact tests are performed representing 3 series of 20 tests each with the forklift truck carrying a 300 kgs, a 775 kgs or a 1175 kgs weight on the pallet. The increase in the load carried by the forklift is expected to increase the momentum of the forklift truck during impact. The complete test set-up and results are presented and detailed in Gilbert and Rasmussen [10].

Figure 3 plots LVDTs 52 and 53 displacements (d_{52} and d_{53} respectively) and the front bay opening against time for test 207 (representing an “Impact BC” configuration with a 1175 kgs load carried by the forklift truck). The bay opening is defined as the difference in distance between two consecutive uprights in the down-aisle direction and reflects the possibility of a pallet to drop through.

![Figure 3: Test 207 results - “Impact BC” configuration and 1175 kgs load on forklift truck: (a) LVDTs 52 and 53, (b) bay opening or LVDT 53 – LVDT 52](image)

The test results show that an impact can be separated into four distinct phases. During the first phase, the overall rack essentially stays stationary while the impacted upright starts bending, opening the bay to its maximum value. In a second phase the overall rack starts its motion while the displacement at the impact point tends to reach a maximum, causing the bay opening to reduce. At one stage during the second phase, the pallet impacting the rack loses contact with the rack while d_{52} and d_{53} may still increase due to the inertia of the overall rack and the inertia of the impacted upright. In a third transition phase the impacted upright bends back to its neutral position. The rack is then free to oscillate during the fourth and last phase. Due to the large amount of frictional damping between the pallets and the beam rails, the impacted upright does not oscillate and the bay opening is essentially equal to zero during the fourth phase. One can deduce that if the bay opens widely enough to trigger failure, then failure is likely to happen during the critical first phase of the impact where the bay opening reaches its maximum.

**Finite Element Modelling**

An Abaqus [11] finite element model is used to model the dynamic response of the tested drive-in rack under forklift truck impact. This finite element model includes nonlinear moment-rotation curves for the base plate behaviour, nonlinear moment-rotation curves for the portal beam to column connections, second-order geometric analysis, nonlinear base plate uplift behaviour, upright warping torsion, member shear centre eccentricity, pallet modelling and nonlinear friction effect between pallets and rail beams. Typically, the base plate stiffness and strength depend on the axial load in the upright (Davies and Godley [12], Baldassino and Bernuzzi [13]); this effect is also considered in the finite element model. Dynamic implicit analysis is used in Abaqus to obtain convergence.
The forklift truck impacting the rack is modelled in Abaqus using the characteristics detailed in the companion paper Gilbert and Rasmussen [14]. Figure 4 (a) shows the FE model of the forklift truck in which \( k_T \) and \( k_B \) represent the torsional and bending rotational stiffness of the forklift truck mast respectively, \( C_T \) represents the torsional damping coefficient and \( m \) represents the combined mass of the load, pallet and forks. The mass \( m \) is located at the centre of gravity of the load. The mast and the forks are modelled using rigid elements. The boundary conditions for the forklift truck are shown in Figure 4 (a) in which the base of the forklift truck is rotated by the actual rotation of the truck as detailed in the companion paper. The combined forklift truck and rack FE model is shown in Figure 4 (b). The forklift truck is considered to impact the rack at the centroid line of the upright. The Abaqus FE model of the tested drive-in rack combined with the forklift truck model is found to provide a satisfactory representation of the experimental impact tests and is used for parametric studies. Full detail of the comparison between experimental test results and FE results are given in Gilbert and Rasmussen [10].

**PARAMETRIC STUDIES**

In order to account for the diversity of drive-in racks encountered in industry, 23 drive-in racks with heights equal to 3 metres, 4.5 metres, 6 metres, 9 metres and 12 metres and unit loads varying from 200 kgs to 1200 kgs (including the pallet mass) are designed in accordance with current industry practice using the proprietary software RAD (Dematic [15]) developed in-house by Dematic Pty Ltd. Rail beams are spaced every 1.5 metres over the height of the racks. A forklift truck modelled as in Figure 4 (a) and using the characteristics detailed in the companion paper (Gilbert and Rasmussen [14]) impacts the above racks at the centroid line of a front upright at the desired rail beam elevation. The forklift truck is carrying a load equal to the design load of the impacted rack. The base of the forklift truck model is rotated by the common mean rotational value given in the companion paper, corresponding to the first phase of the impact where the bay opening is maximum. The impact force and bay opening reflecting the possibility of a pallet to drop through are extracted from Abaqus. Full detail on the parametric FE models can be found in Gilbert and Rasmussen [10].

**Impact at Mid Height**

As mentioned in section 0, the draft FEM [5] requires impact to be assumed at the most unfavourable location, which by engineering intuition would be at mid height of the drive-in racking system. Consequently, the 3 m, 6 m, 9 m and 12 m drive-in racks introduced previously are impacted at 1.5 metres, 3 metres, 4.5 metres and 6 metres height respectively. The rack is loaded with one pallet located at the second position from the front, at the impact elevation and in
the impacted bay. Opening is reported for that particular pallet. A coefficient of friction equal to 0.3 is considered between the pallet and the rail beams. Figure 5 plots the bay opening and the impact force against the rack design load for the different heights considered.

![Figure 5: Parametric study results – Impact at mid height, (a) bay openings and (b) impact forces](image)

It is observed that the bay opening and the impact force decrease with the height of the drive-in rack. This result, which may be counter intuitive, is mainly explained by the fact that the stiffness of the forklift truck decreases with the height of the impact point due to the constant rotational bending stiffness $k_B$ at the base of the truck. Figure 5 also shows that despite the increase in impact force, the bay opening decreases with increasing rack design load, this effect being more noticeable for short heights. For a given rack height, this phenomenon is related to the higher section modulus of the impacted upright and to higher friction forces between the pallets and the rail beams when increasing the design load, both effects providing more resistance to impact. Results show that impact at mid-height is not likely to be the most unfavourable impact location. Particularly for tall racks, impact below mid-height is likely to be critical.

**Impact at Different Elevations**

To further investigate the influence of the impact elevation, the 6 metres and 9 metres high drive-in racks designed to carry 200 kgs pallet loads are impacted at various locations starting from the first rail beam elevation (bottom) to the last rail beam elevation (top). The rack is loaded with only one pallet, this time located at the front position at the rail beam elevation below or above the impact elevation and in the impacted bay. Opening is reported for that particular pallet. Figure 6 plots the bay opening and the impact force against the impact elevation. It is observed that the bay opening decreases with increasing impact elevation. Furthermore, the bay opening dramatically decreases (by about 59.2%) between impact at the first and second rail beam elevations. The impact force also decreases with increasing impact elevation but slightly increases when impacting the last rail beam elevation due to the proximity of the portal frame beam connection providing additional resistance to impact. Consistent with section 0, Figure 6 demonstrates that impact at the first beam rail elevation is critical and is more likely to induce failure than impact at higher elevations.

![Figure 6: Parametric study results – Impact at various elevations, (a) bay openings and (b) impact forces](image)
Impact at Constant Elevation

The 4.5 metres, 6 metres, 9 metres and 12 metres drive-in racks are now impacted at the first beam rail elevation. The rack is loaded with only one pallet at the front position, at the second beam rail elevation and in the impacted bay. Opening is reported for that particular pallet. A coefficient of friction equal to 0.3 is considered between the pallet and the beam rails. Figure 7 shows the bay opening and the impact force against the rack design load.

![Figure 7: Parametric study results – Impact at first beam rail elevation, (a) bay openings and (b) impact forces](image)

It is observed from Figure 7 (a) that the bay opening increases dramatically (by between 36.2% and 67.2%) with the rack height changing from 4.5 metres to 6 metres in height but remains approximately constant for higher racks. Drive-in racks taller than 6 metres and designed to carry light loads are more likely to fail than short racks or racks designed to carry heavy loads. Figure 7 (a) indicates that for a given height, the bay opening tends to reach a constant value for heavy design loads.

Impact Against Friction Coefficient and Loading Configuration

The influence of the friction coefficient between the pallets and the beam rails is now investigated. The 6 metres drive-in racks are used to conduct this study with an impact at the critical first rail beam elevation. Two different loading configurations referred as “1 pallet” and “fully loaded” are considered. For the “1 pallet” configuration, a single pallet located at the front of the rack, at the second beam rail elevation and in the impacted bay is considered. For the “fully loaded” configuration, all pallets are positioned on the rack except for the front pallet at the impact elevation in the impacted bay.

Opening is reported at the pallet giving the maximum opening, which corresponds in all cases to the front pallet at the second rail beam elevation in the impacted bay. Figure 8 plots the bay opening and impact force against the friction coefficient for all rack design loads and loading configurations. In the presence of friction between the pallets and the rails, Figure 8 (a) shows that the loading configuration has a significant impact on the bay opening, this phenomenon being noticeable even for a small amount of friction. Unlike the bay opening, the impact force increases with the friction coefficient and the pallet load, this phenomenon being related to the rack offering more resistance to impact when increasing the mass and/or the friction coefficient. While the increase in the impact force for friction coefficient values between 0 and 0.3 is between 25.4% and 42.5% for the “1 pallet” configuration, it reaches 66.0% to 104.5% for the “fully loaded” configuration. As in previous sections, the bay opening decreases and the impact force increases with increasing the pallet load.

CONCLUSION

This paper briefly presents experimental test results from impact tests on a complete full-size drive-in rack structure. A finite element model is proved to satisfactorily reproduce the experimental test results and is then used for forklift truck impact parametric studies. The studies show that forklift truck
impacts at low elevation are more likely to induce failure than impacts at higher elevation, and that racks designed to carry light pallet loads are more sensitive to impact than racks designed to carry heavy loads. Moreover, the number of pallets loaded on the rack and the amount of friction between the pallets and the rail beams have a significant influence on the bay opening. Due to the horizontal bracing effect of the pallets and the stiffness of the impacted upright, a high impact load does not automatically lead to a significant bay opening.

Figure 8: Parametric study results – Impact at first beam rail elevation on 6 m racks and different loading configurations, (a) bay openings and (b) impact forces

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DETERMINATION OF ACCIDENTAL FORKLIFT TRUCK IMPACT FORCES ON DRIVE-IN STEEL RACK STRUCTURES

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KEYWORDS
Steel storage rack, drive-in rack, impact force, forklift truck behaviour, drive-in rack behaviour

ABSTRACT
The paper addresses the problem of determining the accidental forklift truck impact forces on steel storage racks. Based on first principles of mechanics, simple models of a loaded forklift truck and a drive-in racking structure are presented. Model masses, as well as stiffness and damping coefficients are calibrated against experimental results obtained from tests of a forklift truck and a drive-in racking structure. Comparisons between experimental results and solutions obtained from the simple mechanical models show that the simple models accurately reproduce the static and dynamic behaviours of their associated structures. Based on the drive-in rack impact test results presented in a companion paper (Gilbert and Rasmussen [1]) and the simple mechanical models for drive-in racks, actual impact forces are calculated and presented. Finally, using the impact test results and the simple mechanical models, the actual motion of the forklift truck body is calculated. This motion, being a common characteristic to all drive-in racking impacts, allows impact forces to be obtained for various pallet loads, impact elevations and rack characteristics. Thus, the paper concludes with a general method for calculating forces generated under forklift truck impact.

INTRODUCTION
This paper addresses the problem of determining the accidental forklift truck impact forces on steel storage racks. As presented in the companion paper (Gilbert and Rasmussen [1]), different types of forklift truck impact can occur on steel storage racks. This paper focuses on down-aisle impacts occurring on drive-in racking structures but the methodology and methods presented herein can be easily extended to any type of rack or steel structure in general.

Contrary to classical “selective racks”, by storing pallets on rail beams on the first-in, last-out principle “drive-in racks” minimise floor space allocation and are often a more economical alternative solution to selective racks. To allow the forklift passage, drive-in racks can only be braced at the back (spine bracing) and at the top (plan bracing) leading to slender uprights in the down-aisle direction, only restrained at the bottom by the base plate assembly and at the top by the portal beam. When
subjected to a forklift truck impact in the down-aisle direction, the bowing of the upright may trigger progressive failure by allowing the pallets to drop through.

The literature review shows that no investigation is available for determining the accidental forces that develop between a storage rack and a forklift truck during an impact. International racking standards only consider arbitrary impact forces which have no scientific justification. Furthermore these standards only deal with selective racks and are not applicable to drive-in racks. The present study is motivated by the high failure rate of drive-in racks compared to other types of racks and steel structures in general. Further details on the behaviour of drive-in racks under impact and available literature for this type of structure are given in the companion paper (Gilbert and Rasmussen [1]).

Based on first principles of mechanics, a simple model of a loaded forklift truck has been developed and is briefly presented. Model masses, as well as stiffness and damping coefficients were calibrated against experimental results obtained from isolated tests of an actual forklift truck. Subsequently, also based on first principles of mechanics, a simple model of a loaded drive-in rack has been developed and is briefly presented. Model masses, as well as stiffness and damping coefficients are calibrated against experimental results. Comparisons between experimental results and solutions obtained from the simple mechanical models show that the simple models accurately reproduce the static and dynamic behaviours of the forklift truck and the loaded drive-in rack. Full details of the simple mechanical model are reported in (Gilbert and Rasmussen [2]). Using the result of experimental impact tests and the simple model of the drive-in rack, actual impact forces are calculated and presented. Finally, using the experimental impact test results and the two simple mechanical models, the actual motion of the forklift truck body is determined. This motion, being a common characteristic to all drive-in racking impacts, allows impact forces to be obtained for various pallet loads, impact elevations and rack characteristics. Thus, the paper concludes with a general method for calculating forces generated under forklift truck impact.

SIMPLE MECHANICAL MODELS FOR FORKLIFT TRUCK AND DRIVE-IN RACK

Forklift Truck

Model

To measure the static and dynamic behaviour of a forklift truck, tests have been performed on a NICHIYU FB20 forklift truck with a 2 tons load capacity. Three different loads (300 kgs, 775 kgs and 1175 kgs) are placed sequentially on a pallet. The pallet is positioned at an elevation equal to the impact elevation associated with the impact tests performed on a drive-in rack structure, mentioned in Section 0 and detailed in the companion paper Gilbert and Rasmussen [1]. A hydraulic jack with a quick released system applies a force \( F \) at the impact with the rack location. The bending and torsional motion of the forklift truck mast is recorded using LVDTs (Linear Velocity Displacement Transducer) with the dynamic response of the system recorded at a rate of 50 Hz.

A photo of the tested forklift truck is given in Figure 1 (a). The simple model presented in Figure 1 (b) is found to accurately reproduce the static and dynamic behaviour of the forklift truck. In Figure 1 (b), a vertical member represents the mast and a horizontal member at a vertical distance \( H \) from the base of the mast represents the forks. A rotational stiffness \( k_B \) corresponding to the rotational stiffness of the mast bottom hinge combined with the rotational stiffness provided by the forklift truck shock absorbers restrains the mast from rotating sideways about its base. A torsional spring, stiffness \( k_T \), representing the tilt hydraulic jacks (controlling the tilting motion of the mast) restrains the torsion of the mast. A load \( F \) representing the impact load/jack load is applied on the forks at a horizontal distance \( l_1 + l_2 \) from the base.

A mass \( m \) representing the combined mass of the load, the pallet and forks is considered at the centre of gravity of the load, i.e. at a vertical distance \( H_{CoG} \) and at a horizontal distance \( l_1 + l_3 \) from the base of
the mast. The pallet mass and the mass of the forks are estimated to be 50 kgs each. The damping of the forklift truck mast is found to be a “drag force damping” type, i.e. proportional to the square of the velocity. This type of damping is found when an oscillator is immersed into a fluid and is for the forklift truck related to the action of the tilt hydraulic jacks. Being significant, this torsional damping from the tilt hydraulic jacks is considered to be the predominant source of energy loss in the system and all other sources of damping are ignored. A torsional damping coefficient $C_T$ is considered in Figure 1 (b).

The bending stiffness $k_B$ and torsional stiffness $k_T$ are calculated from the static test results and are summarised in Table 1. $C_T$ is calibrated against the dynamic test results and the obtained values for $C_T$ are given in Table 1. The simple model is found to accurately reproduce the static and dynamic behaviour of the forklift truck. Detailed test set-up, test results, the calibration of the forklift truck model and its static and dynamic validation are reported in Gilbert and Rasmussen [2]. Other test characteristics are given in Table 2.

### TABLE 1

<table>
<thead>
<tr>
<th>Load</th>
<th>$k_B$ (kN.mm/rad)</th>
<th>$k_T$ (kN.mm/rad)</th>
<th>$C_T$ (kN.mm.s²/rad²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 kg</td>
<td>915278</td>
<td>88590</td>
<td>296667</td>
</tr>
<tr>
<td>775 kg</td>
<td>835263</td>
<td>136256</td>
<td>683333</td>
</tr>
<tr>
<td>1175 kg</td>
<td>869715</td>
<td>148334</td>
<td>1266667</td>
</tr>
</tbody>
</table>

### TABLE 2

<table>
<thead>
<tr>
<th>H (mm)</th>
<th>$H_{COG}$ (mm)</th>
<th>$l_1$ (mm)</th>
<th>$l_2$ (mm)</th>
<th>$l_3$ (mm)</th>
<th>$d_W$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2425</td>
<td>2648</td>
<td>200</td>
<td>1230</td>
<td>730</td>
<td>250</td>
</tr>
</tbody>
</table>

It is observed from Table 1 that while the bending rotational stiffness $k_B$ is essentially constant with increasing pallet load, the torsional stiffness $k_T$ increases with the pallet load. The increase in stiffness $k_T$ is related to the increase in oil pressure in the tilt hydraulic jacks required to prevent the mast from tilting forward when increasing the pallet load. In view of these results, throughout this paper $k_B$ is considered constant for all pallet loads and equal to its mean value 873418 kN.mm/rad. $k_T$ is considered to vary with the pallet load and linear interpolation can be performed in Table 1 to obtain $k_T$ for pallet loads between 300 kg and 1175 kg.
Equations of motion

For the model presented in Figure 1 and for small displacements, the kinetic energy KE, potential energy PE and damping energy DE (from the generalised Rayleigh dissipation function) are given as,

\[ KE = \frac{1}{2} m \left( H_{C_{OG}} \ddot{\theta}_1 + (l_1 + l_3) \ddot{\theta}_2 \right)^2 \]  

(1)

\[ PE = \frac{1}{2} k_\theta \theta_1^2 + \frac{1}{2} k_T \theta_2^2 \]  

(2)

\[ DE = \frac{1}{3} C_T \dot{\theta}_2 \dot{\theta}_2^2 \]  

(3)

where \( \theta_1 \) and \( \theta_2 \) represent the bending and torsional angles respectively, as shown in Figure 1. The equations of motion for \( \theta_1 \) and \( \theta_2 \) under external loading are obtained using the general Lagrange equation (Bangash [3]),

\[ \frac{d}{dt} \left( \frac{\partial KE}{\partial \dot{\theta}_i} \right) - \frac{\partial KE}{\partial \theta_i} + \frac{\partial PE}{\partial \dot{\theta}_i} + \frac{\partial DE}{\partial \dot{\theta}_i} = M_i \]  

(4)

where \( M_i \) represents the external bending or torsional moment. Eqn. 4 applied to \( \theta_1 \) and \( \theta_2 \) gives,

\[ m H_{C_{OG}}^2 \dddot{\theta}_1 + k_\theta \theta_1 + m H_{C_{OG}} (l_1 + l_3) \dddot{\theta}_2 = F \cdot H \]  

(5)

\[ m (l_1 + l_3)^2 \dddot{\theta}_2 + k_T \theta_2 + C_T \dddot{\theta}_2 + m H_{C_{OG}} (l_1 + l_3) \theta_1 = F (l_1 + l_2) \]  

(6)

Drive-in Rack

Model

When impacted, a drive-in rack resists the impact by an overall deformation of the structure and by bending of the impacted upright. The simple mechanical model of a drive-in racking system given in Figure 2 is found to accurately reproduce the static and dynamic behaviour of a drive-in rack. In Figure 2, \( k_o \) represents the stiffness associated with the overall displacement of the rack and \( k_u \) represents the stiffness associated with the bending of the impacted upright at the impact point. Similarly, \( m_o \) and \( m_u \) represent the associated masses of the overall rack displacement and the impacted upright respectively, and \( C_o \) and \( C_u \) represents the damping coefficient of the overall rack and the impacted upright respectively. \( F \text{imp} \) represents the impact force on the rack, and \( x_F \) and \( x_o \) correspond to the displacement of mass \( m_u \) and \( m_o \) respectively.

Impact tests on a drive-in rack structure loaded with two different pallet configurations (namely “Impact BC” and “Impact CD”) have been performed using the forklift truck analysed in Section 0. In “impact BC”, the pallets are not loaded in the vicinity of the impacted upright allowing this upright to freely deform under impact. In “impact CD”, the pallets are loaded in the vicinity of the impacted upright allowing the investigation of the restraining effect of the pallets on the upright deformation. For each loading configuration, 60 tests have been performed representing three pallet loads (300 kgs, 775 kgs and 1175 kgs) carried by the forklift truck. The complete test results are presented and detailed in Gilbert and Rasmussen [4].
The value of \( k_u, k_o, m_u, m_o, C_o \) and \( C_u \) are calibrated against experimental results and are summarised in Table 3. The calibration of the rack model and its static and dynamic validation are reported in Gilbert and Rasmussen [2].

**TABLE 3**

<table>
<thead>
<tr>
<th>Drive-in Rack Simple Model Characteristics</th>
<th>( k_o ) (kN/mm)</th>
<th>( m_o ) (kg)</th>
<th>( C_o ) (kN/(mm/s))</th>
<th>( k_u ) (kN/mm)</th>
<th>( m_u ) (kg)</th>
<th>( C_u ) (kN/(mm/s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact BC</td>
<td>0.762</td>
<td>15051</td>
<td>0.00425</td>
<td>0.300</td>
<td>50</td>
<td>0.0111</td>
</tr>
<tr>
<td>Impact CD</td>
<td>0.694</td>
<td>14378</td>
<td>0.00492</td>
<td>1.124</td>
<td>1400</td>
<td>0.0979</td>
</tr>
</tbody>
</table>

**Equations of Motion**

Based on first principles of mechanics, the equations of motion for \( x_F \) and \( x_o \) of the system presented in Figure 2 are,

\[
\ddot{x}_F + C_o \left( \dot{x}_F - \dot{x}_o \right) + k_u \left( x_F - x_o \right) = F_{imp} \quad (7)
\]

\[
\ddot{x}_o + C_o \cdot \dot{x}_o + k_o \cdot x_o - C_u \left( \dot{x}_F - \dot{x}_o \right) - k_u \left( x_F - x_o \right) = 0 \quad (8)
\]

**IMPACT FORCE**

The impact force \( F_{imp} \) developed between the forklift truck and the drive-in rack during the experimental impact tests introduced in Section 0 and detailed in Gilbert and Rasmussen [4] is now calculated by solving eqn. 7 using the commercial software FlexPDE [5] in which values of \( x_F \) and \( x_o \) obtained from experimental test are introduced as input data. Figure 3 shows the impact force \( F_{imp} \) for 15 tests for “Impact BC” configuration and 15 tests for “Impact CD” configuration, calculated using the values of \( m_u, C_u \) and \( k_u \) given in Table 3.

While performing the impact tests, it was observed that the pallet under the 300 kgs load has a tendency to slide on the forks leading to two distinct sequences of impact. This phenomenon is detailed in Gilbert and Rasmussen [4], and is observed in Figure 3 where the impact load increases to a plateau corresponding to the pallet sliding on the forks. The sliding lasts about 0.3 sec, then the impact force increases again until the forklift truck loses contact with the rack.
If the pallet sliding effect is omitted, it is observed from Figure 3 that the impact load is rather consistent for each impact configuration and reaches a maximum between 0.1 sec and 0.12 sec for “Impact BC” configuration and between 0.08 sec and 0.1 sec for “Impact CD” configuration.

COMBINING THE MODELS FOR THE RACK AND FORKLIFT TRUCK

In order to combine the mechanical model of the forklift truck with any rack model representing different rack configurations, pallet loads and impact elevations, a common characteristic to all impacts on drive-in racking structures has to be found. Typically the impact of a forklift truck on a storage rack is induced by the rotation of the forklift truck body about its front wheel axis as shown in Figure 4. The rotation of the forklift truck body will essentially remain the same for all impacts on drive-in racking structures and can be taken as a common input into a combined forklift truck and rack model. Hence impact forces and displacement at the impact point can be calculated from the combined model.

![Figure 4: Forklift impact on drive-in racking structure](image)

Typically the front wheel axis of a forklift truck corresponds to the bottom hinge connection between the mast and the forklift truck body. Figure 5 shows the simple mechanical model of the forklift truck introduced in Figure 1 rotated about the front wheel axis by an angle \( \alpha \). The potential energy \( PE \) and damping energy \( DE \) in eqns. 2 and 3 respectively are unchanged in this position, while the kinetic energy in eqn. 1 is rewritten as,

\[
KE = \frac{1}{2} m \left( H_{COG} \dot{\alpha} + (l_1 + l_2) (\dot{\theta}_1 + \dot{\theta}_2) \right)^2
\]

![Figure 5: Rotating motion of the forklift truck mechanical model](image)

The equations of motion for \( \alpha, \theta_1 \) and \( \theta_2 \) under external loading using the Lagrange in eqn. 4,
\[ m \cdot \frac{H_{COG}^2}{H} \ddot{\theta}_1 + \ddot{F}_1 + m \cdot \frac{H_{COG}}{H} \ddot{\theta}_2 + m \cdot \frac{H_{COG}}{H} \ddot{\alpha} = -F_{imp} \cdot H \]  
(10)

\[ m \cdot (l_1 + l_3)^2 \ddot{\theta}_2 + \ddot{F}_1 + m \cdot \frac{H_{COG}}{H} \ddot{\theta}_2 + m \cdot (l_1 + l_3)^2 \ddot{\alpha} = -F_{imp} \cdot (l_1 + l_2) \]  
(11)

Using the geometrical relationship between \( x_F, \theta_1 \) and \( \theta_2 \),

\[ x_F = H \theta_1 + (l_1 + l_2) (\theta_2 + \alpha) \]  
(12)

the above equations of motion can be rewritten for \( \theta_2 \) and \( \alpha \) as,

\[ m \cdot \frac{H_{COG}}{H} \left( l_1 + l_3 - \frac{H_{COG}}{H} (l_1 + l_2) \right) \cdot \ddot{\theta}_2 + \frac{k_B}{H} (l_1 + l_2) \cdot \theta_2 + m \cdot \frac{H_{COG}^2}{H} \cdot \ddot{\alpha} + \frac{k_B}{H} x_F \]

\[ + m \cdot \frac{H_{COG}}{H} \left( l_1 + l_3 - \frac{H_{COG}}{H} (l_1 + l_2) \right) \cdot \ddot{\alpha} = -F_{imp} \cdot H \]  
(13)

\[ m \cdot (l_1 + l_3) \left( l_1 + l_3 - \frac{H_{COG}}{H} (l_1 + l_2) \right) \cdot \ddot{\theta}_2 + \frac{k_B}{H} (l_1 + l_2) \cdot \theta_2 + m \cdot \frac{H_{COG}^2}{H} \cdot \ddot{\alpha} + \frac{k_B}{H} x_F \]

\[ + m \cdot (l_1 + l_3) \left( l_1 + l_3 - \frac{H_{COG}}{H} (l_1 + l_2) \right) \cdot \ddot{\alpha} = -F_{imp} \cdot (l_1 + l_2) \]  
(14)

\( \alpha \) is obtained solving eqns. 13 and 14 using the software FlexPDE [5] and values from Table 1 and Table 2. \( x_F \) taken from experimental test results and \( F_{imp} \) calculated as per Section 0 are used as input data in FlexPDE. Figure 6 shows \( \alpha \) calculated for the 15 tests for “Impact BC” configuration and 15 tests for “Impact CD” configuration, as shown in Figure 3. \( \alpha \) is only plotted in the domain of validity of eqns. 13 and 14, i.e. when there is contact between the pallet and the rack, here considered to be until the impact force reaches zero in Figure 3. If the sliding of the pallet is omitted, results show that \( \alpha \) is consistent for all tests and reaches its maximum value at about 0.15 sec. After 0.15 sec, \( \alpha \) decreases progressively reflecting the loss of contact between the impacting pallet and the rack.

It is observed that when the maximum impact load is reached between 0.08 sec and 0.12 sec in Figure 3, \( \alpha \) is still increasing. The maximum average value of \( \alpha \) at 0.15 sec is equal to 0.023 rad.

\( \alpha \) is approximated by a multi-linear curve by averaging the results shown in Figure 6 over the first critical 0.15 sec (see Figure 7). The multi-linear approximation can be used as input in Finite Element models or in solving the eqns of motion 7, 8, 13 and 14 to obtain the impact force and rack displacement under impact.
CONCLUSION

Based on first principles of mechanics, this paper proposes simple models for a loaded forklift truck impacting with a loaded steel drive-in rack. Model masses, as well as stiffness and damping coefficients are calibrated against experimental results obtained from tests of a forklift truck and a drive-in rack structure. The impact forces developed between the forklift truck and the drive-in rack are calculated using the simple rack model and impact tests on a drive-in rack structure presented in a companion paper (Gilbert and Rasmussen [1]). Finally, combining the two simple mechanical models and using the impact test results, the actual forklift truck body rotation during impact is calculated. This motion, assumed to be a common characteristic to all drive-in racking impacts, allows impact forces to be obtained for various pallet loads, impact elevations and rack characteristics. Thus, the paper concludes with a general method for calculating forces generated under forklift truck impact.

REFERENCES

AN INVESTIGATION OF THE COMpressive STRENGTH OF COLD-FORMED STEEL BUILT-UP I SECTIONS

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KEYWORDS
Built-up, Cold-formed steel, Finite Element Method, Direct Strength Method, Effective Width Method.

ABSTRACT
Cold-formed steel members have been used extensively in low and mid-rise residential building construction. The usage of cold-formed steels as primary structural members has been increased due to its high load-to-weight ratio. Cold-formed steel built-up sections are commonly used as compression elements to carry larger loads and longer spans when a single individual section is insufficient. However, not much research has been done on built-up sections. This paper aims to investigate the compressive capacity of pin-ended cold-formed steel built-up I sections using the finite element method (FEM). In the study, cold-formed steel built up I section consists of two identical C-channels sections oriented back to back forming an I-shaped cross section and connected to each other at certain spacing along their length. A non-linear finite element model is developed and verified against theoretical and experimental results. The theoretical numerical analysis is based on the Effective Width Method and the Direct Strength Method. As for the experimental testing, the compression test is carried out on 11 specimens. It was shown that the finite element methods result correlate well with the experimental results. In addition, the analytical results by the Effective Width Method and Direct Strength Method are generally conservative for cold-formed steel built-up I sections.

INTRODUCTION
Cold-formed steel has been used widely across many countries in the construction industry. Cold-formed steel has been utilized in various forms in construction projects. The built-up section is one of the most used cold formed steel sections when single sections are no longer sufficient to cater for the advancement and complexity in construction industry. Built-up sections can be any two or more sections connected together e.g. back-to-back built-up I sections (Figure 1). These cold-formed built-up sections are commonly used as compression members such as columns, or members of roof trusses.
in buildings. However, very few studies have been carried out to study the built up cold-formed steel sections [1], [2]. The use of these built-up sections leads to complex design problems. The complexity is due to the interactive buckling characteristic of built-up members under load. In order to account for these buckling behaviors, a specific provision for design of built-up sections was introduced in section C4.5 of the 2001 edition of American Iron and Steel Institute (AISI) North American Specification for the Design of Cold Formed Steel Structural Members [3]. This specification is substantially based on the research of hot rolled steel despite the characteristics of hot rolled steel being considerably different from cold-formed steel.

DESIGN APPROACH

The three design approaches i.e. EWM, DSM I, and DSM II are based on two well known methods i.e. Effective Width Methods (EWM) and Direct Strength Methods (DSM) derived in North American Specifications (NAS) 2001 [3].

**EWM**

In this study, EWM utilizes the concept of individual elements and neglects the interaction between the plate elements where for a single C-channel, web is stiffened, flange is edge stiffened and lip is unstiffened. The degree of stiffening affects the calculation of effective area, \( A_e \). In built-up sections, NAS 2001 assumes that both channels are of the same stiffening effect. Therefore, the assumption made for effective area of built-up section is simply twice that of a single C-channel i.e. \( A_{eb} = 2A_{ec} \). In terms of slenderness ratio, the provision in Specification Section C4.5 of NAS 2001 requires that for compression members composed of two sections in contact, the nominal axial strength shall be determined by replacing \( KL/r \) with \( (KL/r)_m \). This is to account for the buckling failures that induce shear forces in the connectors between individual shapes. This spacing requirement \( a/r_i \leq 0.5(KL/r)_o \) is being used to account for ineffective and loose bolts or screws [3]. Thus, in the nominal axial strength determination, modified slenderness ratio, \( (KL/r)_m \) is used to determine buckling stress, \( F_e \).

\[
\left( \frac{KL}{r} \right)_m = \sqrt{\left( \frac{KL}{r} \right)_o^2 + \left( \frac{a}{r_i} \right)^2} \quad (1)
\]

where, \( \left( \frac{KL}{r} \right)_o = \) Overall slenderness ratio of entire section about built-up member axis, \( a = \) Intermediate fastener or spot weld spacing, \( r_i = \) Minimum radius of gyration of full unreduced cross-sectional area of an individual shape in a built-up member.

**DSM I & II**

DSM does not require complex effective area calculations as in EWM. It provides a flexible design procedure so that it simplifies the analysis of complex sections. It predicts member strength based on member’s elastic buckling loads. The first DSM approach in this study (i.e. DSM I) uses manual hand calculation from the design manual in determination of elastic buckling load. Modifications on slenderness ratio (same as EWM) were introduced to calculate critical Euler buckling stress (\( F_e \)). For DSM II, finite strip analysis software, CUFSM, is used to determine \( P_{crI} \) and \( P_{crII} \). For both DSM I and DSM II, \( P_{crI} \) and \( P_{crII} \) are simply twice the single cross-section value in the built-up cross section, thus,
analysis was done by analysing a simple C-lipped channel. However, \( P_y/P_{cre} \) differs because torsional-flexural mode is replaced by a separate torsion mode and a strong-axis flexure mode. Due to difficulties to determine \( P_y/P_{cre} \) from CUFSM curve, hand calculation methods were used. The finite strip analysis software – CUFSM used in this research was introduced by Schafer to predict the strength ratio [5]. The first minimum of the curve reveals load ratio for local buckling whereas the second minimum point shows load ratio for distortional buckling.

**Finite Element Analysis**

The general concept of finite element analysis (FEA) is the principle of discretization (sub-dividing). Complex model geometry is analysed by sub-dividing them into finite elements which connecting to each other by nodes in order to perform the analysis. In this study, cold-formed steel built-up sections were modelled using commercial finite element software, LUSAS 14.0 and the model is built based on the geometric properties of the cold-formed steel built-up sections. The self-drilling screws connecting the built-up sections were assumed as small thin steel strips. Since the thickness to width or depth ratio of the cold-formed steel section is relatively small, surface-like element was used to represent the structures. Therefore, thin shell element QSL8 is selected as suggested by Farzin et al. [6]. QSL8 is a semi-loof shell which comprising of 6 or 8 numbers of anticlockwise nodes, each with 3 degrees of freedom.

**EXPERIMENT PROGRAM**

**Specimen**

Laboratory tests were performed on 11 specimens of back-to-back built-up I sections. The test specimens were brake-pressed from high strength zinc-coated grade G450 structural steel sheets of 1.6 mm thickness. The nominal yield strength and Young’s modulus for these specimens are 450MPa and 200GPa. The test program comprised of three series of lipped back-to-back built-up columns. All these built up specimens had a standard length of 1600mm, web width of 100mm, lip of 20mm and flange width of 50mm. The variable is the screw spacing along the column length i.e. 750, 1000, 1500mm.

Figure 2 clearly illustrates the test specimen.
The three series were labelled BU750, BU1000, BU1500 where ‘‘BU’’ refers to ‘‘built-up’’ whereas 750, 1000 and 1500 refers to screw spacing. The average values of measured cross-section dimensions of the pin-ended test specimens are shown in Table 1. Not all of the built up specimen meet the fastener spacing provisions in AISI specification section D1.2.

All specimens were tested in axial compression with pinned end conditions. Compressive axial force was applied to the specimen using a 50 tonne hydraulic jack system. The specimens, end plates and ball bearings were then arranged concentrically. This is to minimise the loading imperfections. A schematic of the test setup is shown in Figure 3. Pre-load of less than 6kN was applied so that the specimen is fully in contact with the end plates. This is to hold the test setup in position and to eliminate any possible gap and movements between the end plates and the specimen. Three Low Voltage Displacement Transducer (LVDT)s were each positioned at mid span of web, mid span of flange and the steel plate extended from top of the specimen to measure deflection of web, deflection of flange and shortening of specimen respectively. Readings were recorded at every 1 second interval.

### Table 1

<table>
<thead>
<tr>
<th>Specimen</th>
<th>A' (mm)</th>
<th>B' (mm)</th>
<th>C' (mm)</th>
<th>t (mm)</th>
<th>R (mm)</th>
<th>L (mm)</th>
<th>s (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BU 750-1</td>
<td>104.0</td>
<td>48.5</td>
<td>20.0</td>
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<td>2.5</td>
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<td>750</td>
</tr>
<tr>
<td>BU 750-2</td>
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<td>48.5</td>
<td>19.5</td>
<td>1.55</td>
<td>2.5</td>
<td>1600</td>
<td>750</td>
</tr>
<tr>
<td>BU 1000-1</td>
<td>103.0</td>
<td>49.0</td>
<td>20.0</td>
<td>1.55</td>
<td>2.5</td>
<td>1600</td>
<td>1000</td>
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<tr>
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<td>49.0</td>
<td>20.0</td>
<td>1.55</td>
<td>2.5</td>
<td>1600</td>
<td>1000</td>
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<td>1500</td>
</tr>
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<td>20.0</td>
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<td>2.5</td>
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<td>2.5</td>
<td>1600</td>
<td>500</td>
</tr>
<tr>
<td>BU 1500-6</td>
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<td>1.55</td>
<td>2.5</td>
<td>1600</td>
<td>750</td>
</tr>
</tbody>
</table>

* Rounded up to the nearest 0.5mm.
Test Setup

Figure 3: Schematic of Test Setup

RESULTS & DISCUSSION

Analytical compressive strength results using finite element (LUSAS), Direct Strength Method (DSM) and Effective Width Method (EWM) for BU750, BU1000, and BU1500 series were tabulated in Table 2. The experimental local buckling load was determined using the method according to Venkataramaiah and Rorda (7). The load (N) against the square of local buckling deformation (w) graph was plotted. In this case the web deformation at mid-length was use. Then a line is subsequently fitted through the test points in the post buckling region. The interception with the load axis resulting from the line was assumed to be the experimental buckling load.

TABLE 2
ANALYTICAL RESULTS OF BUILT-UP I SECTIONS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$P_{LUSAS}$</th>
<th>$P_{DSM1}$</th>
<th>$P_{DSMII}$</th>
<th>$P_{EWM}$</th>
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</thead>
<tbody>
<tr>
<td>BU 1500</td>
<td>129.4</td>
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<td>118.4</td>
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<tr>
<td>BU 1000</td>
<td>168.6</td>
<td>131.0</td>
<td>146.5</td>
<td>152.7</td>
</tr>
<tr>
<td>BU 750</td>
<td>170.3</td>
<td>140.9</td>
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</table>
Experimental results for BU750 series are tabulated in Table 3.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$P_{\text{EXP}}$</th>
<th>$\frac{P_{\text{EXP}}}{P_{\text{LUSAS}}}$</th>
<th>$\frac{P_{\text{EXP}}}{P_{\text{DSMI}}}$</th>
<th>$\frac{P_{\text{EXP}}}{P_{\text{DSMII}}}$</th>
<th>$\frac{P_{\text{EXP}}}{P_{\text{EWM}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BU750-1</td>
<td>170.0</td>
<td>1.00</td>
<td>1.21</td>
<td>1.08</td>
<td>1.01</td>
</tr>
<tr>
<td>BU750-2</td>
<td>160.0</td>
<td>0.94</td>
<td>1.14</td>
<td>1.01</td>
<td>0.95</td>
</tr>
<tr>
<td>Average</td>
<td>0.97</td>
<td>1.17</td>
<td>1.05</td>
<td>0.98</td>
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</table>

The experimental results show good correlation with finite element method results as shown in Figure 5. However, these finite element method results by LUSAS 14.0 are un-conservative compared to Direct Strength Method. The experimental compressive strengths for this series of columns are generally in between the prediction by LUSAS and EWM results. Whereas Direct Strength results, DSM I and DSM II are relatively conservative.
BU1000 Series

Experimental results for BU1000 series are tabulated in Table 4.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( P_{\text{EXP}} )</th>
<th>( \frac{P_{\text{EXP}}}{P_{\text{LUSAS}}} )</th>
<th>( \frac{P_{\text{EXP}}}{P_{\text{DSM I}}} )</th>
<th>( \frac{P_{\text{EXP}}}{P_{\text{DSM II}}} )</th>
<th>( \frac{P_{\text{EXP}}}{P_{\text{EWM}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BU1000-1</td>
<td>158.00</td>
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<td>1.03</td>
</tr>
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<td>BU1000-2</td>
<td>164.00</td>
<td>0.97</td>
<td>1.25</td>
<td>1.12</td>
<td>1.07</td>
</tr>
<tr>
<td>BU1000-3</td>
<td>168.00</td>
<td>1.00</td>
<td>1.28</td>
<td>1.15</td>
<td>1.10</td>
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<td><strong>Average</strong></td>
<td><strong>0.97</strong></td>
<td><strong>1.25</strong></td>
<td><strong>1.11</strong></td>
<td><strong>1.07</strong></td>
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</tbody>
</table>

Figure 6: Results for BU1000 Series

Although the finite element results correlate well with the experimental results, they are generally un-conservative compared to EWM, DSM II, and DSM I results. As shown in Figure 6, Effective Width method predicts the compressive strengths for this series well whereas the Direct Strength method results, DSM I and DSM II are more conservative.
BU1500 Series

Table 5 shows that finite element method results predict well the compressive capacity of the built-up I sections compared to EWM, DSM I, and DSM II results. Figure 7 shows that finite element results are not un-conservative like in BU750 and BU1000 series. Besides, the DSM II results correlate better compared to EWM in this series.

### TABLE 5
RESULTS OF BU1500 SERIES

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$P_{EXP}$</th>
<th>$P_{EXP}$</th>
<th>$P_{EXP}$</th>
<th>$P_{EXP}$</th>
<th>$P_{EXP}$</th>
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</thead>
<tbody>
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<td>P_{DSM I}</td>
<td>P_{DSM II}</td>
<td>EWM</td>
<td></td>
</tr>
<tr>
<td>BU1500-1</td>
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<td>1.14</td>
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<td>1.08</td>
</tr>
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<td>140.0</td>
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<td>1.32</td>
<td>1.18</td>
<td>1.21</td>
</tr>
<tr>
<td>BU1500-6</td>
<td>124.0</td>
<td>0.96</td>
<td>1.17</td>
<td>1.05</td>
<td>1.07</td>
</tr>
<tr>
<td>Average</td>
<td>1.02</td>
<td>1.24</td>
<td>1.11</td>
<td>1.14</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7: Results for BU1500 Series

BU1500-2 and BU1500-5 had shown higher strength than other specimens in the series. From the laboratory observations, these sections buckled at mid-span. It is also noticed that the lipped C-channels for both BU1500-2 and BU1500-5 buckled concentrically in opposite directions. In terms of built-up I section, these two sections buckled in the strong axis. Whereas others may not be secured enough by the screws to allow the built-up I section to behave as one integral section. Therefore, reducing the compressive strength of the section

**CONCLUSION**

Theoretical analysis was carried out on 11 specimens of built-up I sections. Three design approaches (EWM, DSM I, and DSM II) based on two methods i.e. Effective Width Method (EWM) and Direct Strength Method (DSM) with the help of finite strip analysis software (CUFSM) were used to analyse the built-up I sections. In addition, finite element modelling was carried out. Finite elements method results show good correlations with the experimental results compared to EWM and DSM results. In general, DSM results are conservative as compared to EWM results for shorter build-up I sections however more study is needed for a consistent design of cold-formed steel built-up sections.
ACKNOWLEDGEMENTS
The test specimens provided by Ecosteel Pte Ltd are gratefully acknowledged.

REFERENCES
EXPERIMENTAL STUDY ON POST-BUCKLING AND POST-FAILUR BEHAVIOR OF COLD-FORMED SIGMA CONTINUOUS STEEL BEAMS AT INTERNAL SUPPORTS

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KEYWORDS
Cold-formed steel beam, Sigma section, Post-buckling, Post-failure, Plastic mechanism, Yield line, Failure load, Local buckling, Distortional buckling.

ABSTRACT
An experimental study of cold-formed (CFS) Sigma steel beams has been performed to investigate the post-buckling and the post-failure behaviour at internal supports. Load-deflection and moment-rotation curves were recorded for Sigma sections with various geometric properties during the loading test. Through the experiment, the initial buckling mode was observed and the typical plastic post-failure mechanism was identified. Based on the moment-rotation curves, the effective plastic moment resistance at the internal support, which is defined as the moment level where an effective plastic hinge can be developed, was also established. This experimental study may be used to facilitate a plastic design method for cold-formed Sigma multi-span continuous beams.

INTRODUCTION
Cold-formed steel (CFS) sections have been used in the purlin or the cladding rail system for decades. In recent years, with the advancement in CFS technology such as using higher yield strength steel and more complex cross-sections, the application has been growing continuously. The increased demand has led to a pressing need for a more rational and economic design method. The plastic design method, which allows effective moment redistribution for continuous beams, has been deemed to be more economic and therefore desirable. Unlike conventional hot rolled sections, for which the moment redistribution can be achieved with full elastic moment resistance (EMR) at internal supports, CFS sections, however, can only utilize part of it because the moment resistance will fall immediately beyond the peak value due to their large width-to-thickness ratio of cross-sectional elements, which may result in premature buckling. An effective plastic moment resistance (EPMR) at internal supports, which allows the continuous beam system to redistribute the moment to form a collapse mechanism, therefore needs to be established. EPMR is highly dependant on the post-failure behaviour of the CFS section, which is closely related to the initial buckling mode. Three most commonly observed buckling modes in CFS sections are local, distortional and lateral torsional buckling and they are described in details by Yu [1]. In order to implement the plastic design method to CFS continuous beams, the understanding of how these buckling behaviours affecting EPMA becomes imperative.
This study focuses on the load-deflection and moment-rotation behaviour of CFS beams in two stages, i.e. between the onset of the initial buckling and the failure (post-buckling stage), and after the failure (post-failure stage). In the post-buckling stage, certain buckling mode initiates and develops in the cross section, which results in a loss of the effective cross section area and hence the stiffness of the beam will be reduced gradually. The strength gain during this stage is due to the post-buckling strength. The failure of the cross section occurs when sufficient material yields, where the post failure stage starts. In this stage, the buckled section develops a local plastic mechanism and the cross section gradually loses the moment resistance while the deflection/rotation remains increasing. Large deflection/rotation will eventually develop membrane action through which the loading resistance will stop falling provided that an end anchorage system is in place. Murray [2] studied some typical forms of local plastic mechanism for unstiffened plate elements under compression and uniform bending. It is well acknowledged that both buckling mode and corresponding local plastic mechanism are highly dependent on the cross-sectional shape.

CFS sections usually have Z or C shape. Another increasingly used section type is the Sigma section which is a special form of the C section. Due to the two intermediate folds in the web, the Sigma section is able to provide higher torsional resistance by shifting the shear centre closer to the web. Another beneficial feature of this section is the increased critical local buckling stress of the web. Over the past decade, extensive investigations on buckling modes of Z sections and C sections have been conducted. A series of papers by Put et al. [3-7] investigated the lateral buckling and the biaxial bending behaviour of un-braced Z and C sections. Li et al. [8-13] investigated the influence of partial restraints provided by sheeting on both pre-buckling and buckling stresses of Z and C sections. Hancock [14], Teng et al. [15], Jiang and Davies [16], Yu and Schafer [17] studied the distortional buckling behaviour of C and Z sections. Bambach [18-19] investigated the local buckling and post-local buckling behaviour of unstiffened elements under stress gradient. Gotluru et al. [20] studied the influence of transverse eccentric loads on C sections. Considerable number of investigations on local plastic mechanisms of the section containing unstiffened plate elements have also been carried out since Murray [2] first applied rigid-plastic theory in analyzing post-failure behaviour of CFS members. Setiyono [21-22] developed an analytical method based on effective width and rigid-plastic theory to estimate the moment capacity of C sections. Kotelko [23] investigated the moment capacity of rectangular box sections, trapezoidal box sections, and C sections by using energy method based on rigid-plastic theory and taking account of strain hardening of materials. Ungureanu and Dubina [24] studied buckling performance of C sections considering plastic-elastic interaction. Most of the existing researches suggest two separate load-deflection curves to represent the pre-buckling, post-buckling and post-failure stages, respectively. Through these two curves load capacity and effective plastic moment may be determined and could be used in plastic design method.

Compared with the research on Z and C sections, only a limited number of researches on the post-buckling and post-failure behaviours of Sigma sections are available so far and the loading arrangements are only limited to pure bending or compression. The lack of researches on the Sigma sections has been hindering a wider application of this type of section. The current design code [26] only provides guidelines for the design of Z and typical C sections, which may not be suitable for Sigma sections because it does not appear to provide an appropriate estimation for the effect of the folds in webs. The newly developed Direct Strength Method (DSM) by Schafer [27] can be a reliable and efficient tool in the case of analyzing buckling behaviours for some commonly used sections under pure bending. But if it is used to analyze Sigma sections subjected to non-uniform bending, the degree of approximation by the current DSM seems to require further validation. Li and Chen [28] developed an analytical model for assessing the distortional buckling stress of Sigma sections under compression or pure bending which appears not to be suitable to analyze the behaviour at internal supports where transverse loads exist. As it has been known from literatures [21-24] the accuracy of prediction for post failure behaviours of a CFS section is greatly dependent on the correctly chosen local plastic mechanism, which are highly affected by its cross-sectional shape. However, at present there has no published literature focused on investigating local plastic mechanisms of Sigma sections.
This paper reports an experimental investigation on the post-buckling and post-failure behaviours of Sigma beams under central point load. The typical initial buckling modes and the local plastic mechanisms will be identified. The failure load and the effective plastic moment resistance of Sigma sections at the internal support will be also calculated.

TEST PROGRAMME

Consider a two span continuous beam, the bending moment diagram is shown in Figure 1. The present research is focused on the hogging zone, where the internal support is located. In this test programme, a beam of 40% span length is simply supported and torsionally restrained at both ends. A central point load is applied to represent the reaction from the support and the beam is also torsionally restrained at the loading point (see Figure 2).

![Figure 1: Bending moment diagram for a 2-span continuous beam](image1)

![Figure 2: Internal support test programme](image2)

TEST SAMPLES

A series of well-selected testing samples covering a wide range of products supplied by Albion Sections Limited [29] have been chosen to conduct the internal support test. The nominal strength of the tested Sigma sections is 450N/mm² and the real strength has been measured by carrying out material strength test for coupons cut from the same coils that purlin samples are made from. The results of the material strength tests are presented in Table 3. The variables investigated in this study include cross section size and beam length. Table 1 shows the nominal cross-sectional dimensions of the test samples. In this test programme, each test sample has been assigned a 3-part unique ID symbol, e.g. 24-20012-C1 indicates the beam span, section and thickness, type of connection (C indicates continuous connections) and number of the repeated test sample.

<table>
<thead>
<tr>
<th>Section</th>
<th>D (mm)</th>
<th>F (mm)</th>
<th>L (mm)</th>
<th>O (mm)</th>
<th>I (mm)</th>
<th>S (mm)</th>
<th>r (mm)</th>
<th>t (mm)</th>
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<td>110</td>
<td>16</td>
<td>5</td>
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<td>16</td>
<td>5</td>
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</tr>
</tbody>
</table>
**TEST SET-UP, INSTRUMENTATION AND METHOD**

The test set-up for internal support test is illustrated in Fig. 3. The load was applied through a cleat and a UB section. The former was bolted to the CFS beam and the UB section; and the latter was fixed to the loading jack. At both supports, CFS beams were also bolted to the supporting SHS sections, which were clamped by other two loading jacks to prevent them from toppling or moving [see Figure 4 (A)]. The test was conducted by using the 50T Mand testing machine. Three LVDTs were placed at quarter and middle span points to measure the vertical deflection, and two additional LVDTs/dial gauges were placed at both quarter points to measure the lateral movement. Two rotation meters were glued to the top flange of purlins with the centre aligned with those of connecting bolts (see Figure 3).

Incrementally increased loads were applied and the deflections and the rotation at each load step were recorded. Loading were continued beyond the peak load until the applied load has reduced to between 10% and 15% of its peak value or until the deflection had reached a value of six times the maximum elastic displacement. This process was repeated for each tested sample.
TEST RESULTS

Twenty internal support tests for continuous beams have been conducted, and in each test the load-deflection has been recorded during the full loading process. The load-deflection and moment-rotation curves are plotted in Figure 5.

Four different types of local plastic mechanisms are observed (as shown in Figure 6) throughout the testing program. The corresponding mechanism of each test sample is presented in Table 2 together with the recorded buckling loads and buckling modes. In Table 2, the moment resistance of the sections are obtained from the peak value of each curve in Figure 5.

The effective plastic moment resistance $M_{p,\text{eff}}$ in Table 2 is the moment level which can provide adequate rotation capacity at the internal support so that the moment redistribution can take place and eventually leads to the collapse of the system. It can be obtained as follows:

1. By the moment-rotation curve, the rotational capacity should be able to be derived by
\[ \Theta = \theta_{pl,j} - \theta_{el,j} \] [see Fig. 7(a)] at $M_i$, and therefore moment-rotational capacity curve can be plotted as in Fig. 7(b).

2. By considering the moment redistribution, the required rotational capacity at the internal support experiencing hogging moment $M$ when the system collapses is
\[ \Theta = \frac{LM_c}{6EI_{\text{eff}}} \left( 2 + 2 \sqrt{1 + \frac{M}{M_c} - 3 \frac{M}{M_c}} \right) \] (1)

where $L$ is the span of the continuous beam, $M_c$ is moment capacity of the cross section, $I_{\text{eff}}$ is the effective moment of inertia;
3. By plotting $M-\theta$ from both step (1) and (2), the crossing point can be used to find $M_{p,\text{eff}}$ and corresponding $\theta_p$.

### TABLE 2

**TEST RESULTS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Test sample</th>
<th>Material yield strength (N/mm²)</th>
<th>Buckling moment (kN.m)</th>
<th>Buckling mode*</th>
<th>Moment resistance (kN.m)</th>
<th>Local plastic mechanism</th>
<th>Effective plastic moment resistance, $M_{p,\text{eff}}$ (kN.m)</th>
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*LF-local buckling in flange; LW-local buckling in web; DS-distortional buckling in section; LL-local buckling in lip; LTB-lateral torsional buckling of member
(A) Local plastic mechanism type I     (B) Local plastic mechanism type II

(C) Local plastic mechanism type III        (D) Local plastic mechanism type IV

Figure 6: Typical local plastic mechanisms

(a) Schematic diagram for moment rotation curve         (b) Determination of $M_{p,\text{eff}}$

Rotation provided at internal support

Required rotation from Eq.(1)

Figure 7: Determination of $M_{p,\text{eff}}$
DISCUSSION OF TEST RESULTS

It can be seen from Fig.5 that all the curves have exhibited three distinct stages, i.e. linearly ascending (pre-buckling stage), nonlinear ascending (post-buckling stage) and nonlinear descending (post-failure stage). Appreciable post-buckling strength for both local buckling and distortional buckling modes was observed during the tests. Table 2 shows that in all tests, no local buckling of web firstly occurred due to the presence of two folds in the web, and the section failure did not start until the applied load exceeded 5%~20% of the buckling load. In 24-20012, 24-20016 and 24-30018 tests, it was observed that the buckling of the section was always initiated by local buckling of the top flange near the mid-span. After the load reached the failure level, two local buckling pits formed in the inner web near the mid-span, this occasionally being combined with the local buckling in the outer web, whose buckling deformation was much less. As the loading was further increased, the two pits expanded but did not connect [see Fig. 7 (A)]. In 24-20025 and 24-24023 tests, the buckling of the section was initially triggered by distortional buckling of the section at the mid-span. When the local plastic mechanism was formed, one local buckling pit appeared in the inner web at the mid-span and no local buckling of the outer web was found [see Fig. 7 (B)]. In 24-24015 tests, the observed post-buckling behaviour was very similar to 24-20012, 24-20016 and 24-30018 tests. However, the post-failure behaviour of 24-24015 tests exhibited a distinctive roof-shaped feature [see Fig. 7(D)], which significantly differed from Fig. 7 (A). The most possible reason for this discrepancy may be due to the extraordinary high material strength of 24-24015 sections [30]. 24-30025, 40-20025 and 40-24030 tests experienced similar post-buckling and post-failure behaviours, in which distortional buckling accompanied by local buckling of the lip occurred first at the cross section that approximately 300mm~500mm away from the mid-span (see Fig. 8), then one plastic pit was formed in the inner web and one plastic triangle was formed in the lip and top flange respectively [see Fig. 7 (C)]. It can be seen from Fig. 5 (B) that the moment-rotation curves of the identical sections in post-failure stages are all in very good agreement except for section 40-30030 whose post-failure curves show noticeable variation. It was found that although 40-30030-C1 and 40-30030-C2 demonstrated the similar post-buckling behaviour, they experienced different local plastic mechanisms. Observed from the tests, the local plastic mechanisms of 40-30030-C1 and 40-30030-C2 are combined type of II and IV and of III and IV, respectively. One possible reason for such a variation may be because of significant level of inherent imperfections of the testing members.

(A) Section 24-30025

(B) Section 40-30030

Figure 8: Local plastic mechanism in Type III
CONCLUSIONS

A wide range of Sigma sections have been tested under a central point load to investigate the post-buckling and post-failure behaviour of continuous beams at the internal support. The initial buckling mode and the local plastic mechanism were identified for each test. In addition, the failure load and the effective plastic moment resistance at the support were also established. It has been found that for small thickness, e.g. up to 2.0mm, the initial buckling mode normally was the local buckling in the compression flange and for sections having over 2.0mm thickness, the distortional buckling will occur first. No local buckling in the web of the Sigma section has been found and this has been attributed to the presence of intermediate folds. Sections having small thickness also usually experience type I or IV local plastic mechanism and those having large thickness will usually experience type II or III local plastic mechanism.

REFERENCES


ULTIMATE STRENGTH AND DESIGN OF LIPPED CHANNEL COLUMNS EXPERIENCING LOCAL-DISTORTIONAL MODE INTERACTION – PART I: EXPERIMENTAL INVESTIGATION

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KEYWORDS
Local-distortional interaction, Cold-formed lipped channel columns, Experimental investigation, High-strength steel, Mild steel, Initial geometrical imperfections, Ultimate strength.

ABSTRACT
This paper reports the results of an experimental investigation aimed at assessing the post-buckling behaviour and ultimate strength of fixed-ended cold-formed steel lipped channel columns experiencing local/distortional mode interaction. A total of 26 columns were tested and the specimens were carefully selected to ensure various levels of local/distortional interaction effects (more or less close local and distortional critical stresses). The experimental results presented consist of the specimen geometries, material properties, initial imperfections, non-linear equilibrium paths and ultimate strength values. Since the collapse of most columns combines local and distortional deformations, these experimental results may be used to (i) assess the relevance of local/distortional interaction, (ii) calibrate and validate numerical simulations and (iii) provide experimental data aimed at developing a Direct Strength Method (DSM) approach to design cold-formed steel lipped channel columns against local/distortional interaction – such a DSM approach is addressed in Part II of this paper.

INTRODUCTION
Most cold-formed steel members display very slender thin-walled open cross-sections, a feature making them highly susceptible to localised instability phenomena, namely local and distortional buckling. Moreover, since several commonly used members exhibit cross-section geometries (shape and/or dimensions) associated with close similar local and distortional bifurcation stresses, their overall structural behaviour is likely to be affected by the occurrence of mode interaction phenomena involving the above two buckling modes. In order to assess such overall structural behaviour, steel designers are currently faced with two alternative options: either (i) resort to highly complex and computer intensive numerical methods (an approach still prohibitive for routine applications), or (ii) take those effects into account indirectly, through their incorporation into member global analyses (e.g., FEM analyses adopting non-linear beam finite elements). The latter strategy can only be rational and fully efficient if the knowledge
about the member local and distortional post-buckling behaviour is deep enough to enable the
development of reliable and physically based models. For instance, this is the case of (i) the well-
known “plate effective width” concept, accounting for local effects, or (ii) the recently developed and
increasingly popular “Direct Strength Method” (DSM), which can handle both local and distortional effects
and takes into account the whole cross-section behaviour. However, its application to members
affected by local/distortional interaction is still under development

It is well known that experimental investigations play a key role in understanding the behaviour and
developing efficient (safe and economic) design rules for cold-formed steel members, namely uniformly
compressed columns. Careful test programs were conducted in the last few years by (i) Young and
Rasmussen [1], on plain channel columns, (ii) Kwon and Hancock [2] and Young and Rasmussen [3], on
lipped channel columns, (iii) Young and Hancock [4], on lipped channel column with sloping lips, and (iv) Yan
and Young [5], on channel columns with return lips – all these studies dealt with fixed-ended columns.
Concerning fixed-ended lipped channel columns affected by local/distortional interaction, the only tests
available were carried out by Yang and Hancock [6] and, more recently, by Yapp and Hancock [7] and
Kwon et al. [8] – some of these tests involve lipped channel columns with intermediate stiffeners.

The objective of this paper is to report the results of an experimental investigation carried out at the
University of Hong Kong to assess the non-linear (post-buckling) behaviour and ultimate strength of fixed-
extended cold-formed steel lipped channel columns affected by local/distortional mode interaction. The first step
of this work consisted of carefully selecting column geometries (cross-section dimensions and lengths)
that ensure the occurrence of various levels of local/distortional interaction effects, i.e., exhibiting more or
less close local \( f_{crl} \) and distortional \( f_{crd} \) critical stresses. This goal was achieved by means of a trial-and-
error procedure involving the performance of several buckling analyses carried out in the codes ABAQUS [9]
(shell finite elements) and GBTUL [10, 11] (beam finite elements) – it led to the identification of 26 column
geometries exhibiting commonly used cross-section dimensions and lengths, and associated with various
\( f_{crd}/f_{crl} \) ratio values. Then, lipped channel specimens with the selected geometries were fabricated, carefully
measured, in order to determine their real geometries and material properties, as well as to acquire relevant
information concerning the existing initial geometrical imperfections. Finally, the specimens were tested
as uniformly compressed fixed-ended columns. The experimental results presented consist of the specimen
geometries, material properties (stress strain curves), initial imperfections, non-linear equilibrium paths
(applied load vs. axial shortening*) and ultimate strength values. Since the collapse of most columns occurs in
modes combining local and distortional deformations, these experimental results are ideally suited to (i) draw
conclusions on the relevance of local/distortional interaction, (ii) calibrate and validate elastic-plastic shell finite
element numerical analyses and (iii) provide experimental data aimed at developing a Direct Strength
Method (DSM) approach to design cold-formed steel lipped channel columns against local/distortional
interaction – such a DSM design approach is addressed in Part II of this paper [13].

(*Other experimental non-linear column equilibrium non-linear equilibrium paths are presented in [12])

TEST SPECIMENS

The cold-formed steel lipped channel test specimens were brake-pressed from high strength (HSS) and mild
(MS) zinc-coated structural steel sheets. The specimens were cut to a specified length, ranging from 615 to
2500 mm, and their ends were first milled flat by an electronic milling machine and then welded to 25 mm
thick steel end plates to ensure full contact between the specimens and end bearings. Finally, the
specimens were uniformly compressed between fixed ends.

The test specimens had the nominal web, flange and lip widths ranging from 95 to 235 mm, 45 to 190 mm and
15 to 30 mm, respectively. The nominal plate thickness values were equal 1.0, 1.2, 1.5, 1.9 and 2.4 mm, and the
base metal thickness \( t^* \) was measured by removing the zinc coating by acid etching – the zinc
coating layer thickness was found to be equal to 18, 17, 28, 26 and 27 \( \mu \)m for the above five plate thicknesses.
Finally, the measured external corner radius \( r_c \) ranged from 2.6 to 5.5 mm. All the measured cross-section
dimensions and column length \( L \) for each test specimen are given in Table 1, following the nomenclature
defined in Figure 1 – the cross-section dimensions are the averages of values measured at both column ends.

![Nomenclature and location of the tensile coupon](image)

**Figure 1:** Nomenclature and location of the tensile coupon

### TABLE 1

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<td>$B_w$ (mm)</td>
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<td>$t^*$ (mm)</td>
<td>$r_e$ (mm)</td>
<td>$L$ (mm)</td>
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Note: 1 in. = 25.4 mm; 1 kip = 4.45 kN; L = Local buckling; D = Distortional buckling; FT = Flexural-Torsional buckling.
Specimen Labelling

The lipped channel test specimens are labelled so that the plate thickness, strength of steel material and test series can be readily identified by looking at the label – for instance, the labels “T1.5-HSS-2R” and “T1.9-MS-4” define the following specimens:

(i) The first three letters indicate the specimen plate thickness, where the prefix letter “T” refers to thickness and “1.5” and “1.9” indicate the nominal thicknesses equal to 1.5 and 1.9 mm.
(ii) The second part indicates the strength of the steel material, where “HSS” refers to high strength steel and “MS” refers to mild steel material.
(iii) The third part provides the column number – the dimensions of each column are given in Table 1.
(iv) If a test was carried out twice, the letter “R” indicates the repeated test.

Material Properties

The specimen material properties were obtained by means of tensile coupon tests. The coupons were extracted, in the longitudinal direction, from the centre of the specimen web, as shown in Figure 1 – one coupon test was carried out for each specimen batch (series). The coupon dimensions conformed to the Australian Standard AS 1391 [14] for the tensile testing of metals – 12.5 mm wide coupons of gauge length 50 mm were used. An MTS displacement controlled testing machine using friction grips was used to conduct the coupon tests, which were also performed according to the Australian Standard AS 1391. A calibrated extensometer of 50 mm gauge length was employed to measure the coupon specimen longitudinal strains. In addition, two linear strain gauges were attached at the two face centres of each coupon – their readings were used to determine the initial Young’s modulus.

A data acquisition system was used to record the load and the strain readings at regular intervals during the coupon tests. The static load was obtained by pausing the applied straining for 1.5 minutes near the 0.2% proof stress and the ultimate tensile strength, thus allowing the stress relaxation associated with plastic straining to take place. The material properties obtained from the coupon tests, summarised in Table 2.

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<th>$\sigma_{0.2}$ (MPa)</th>
<th>$\sigma_u$ (MPa)</th>
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<td>335.6</td>
<td>441.0</td>
<td>30.9</td>
</tr>
<tr>
<td>T2.4-HSS</td>
<td>212.6</td>
<td>525.9</td>
<td>544.5</td>
<td>9.5</td>
</tr>
<tr>
<td>T2.4-MS</td>
<td>205.0</td>
<td>342.7</td>
<td>463.9</td>
<td>31.6</td>
</tr>
</tbody>
</table>

Note: 1 ksi = 6.89 MPa.

Figure 2: Stress-strain curve for test series T1.0-HSS: (a) complete curve and (b) initial part.
Table 2, consist of the values of (i) Young’s modulus ($E$), (ii) static 0.2% proof stress ($\sigma_{0.2}$), (iii) static tensile strength ($\sigma_u$) and (iv) elongation after fracture ($\varepsilon_f$), based on a gauge length of 50 mm. The stress-strain curves obtained from the coupon tests are displayed in Figures 2(a)-(b) and 3(a)-(b) for the test series T1.0-HSS and T2.4-MS, respectively. While Figures 2(a) and 3(a) show the complete stress-strain curves, Figures 2(b) and 3(b) provide information about the initial parts of those curves.

**Initial Geometrical Imperfections**

Two local initial geometrical imperfection measurements were made at the column specimen mid-length cross-section prior to testing — they are indicated in Figure 4 and provide information concerning the local ($\Delta_w$) and distortional ($\Delta_d$) imperfections. The measured values are shown in Table 3, where $\Delta_w$ and $\Delta_d$ are the initial values of the (i) mid-web plate bending displacement and (ii) flange-lip junction horizontal (normal to the flange) displacement. Positive $\Delta_w$ and $\Delta_d$ values indicate inward (towards the lips) and outward deformations, respectively — i.e., those shown in Figure 4.

Moreover, the initial overall flexural geometrical imperfections (about the minor axis) were measured in all column specimens — they were measured along the specimen web-flange longitudinal edge and a theodolite was used to obtain readings at mid-length and near both specimen ends. The measured values at mid-length ($\delta$), normalised with respect to the length $L$, are also shown in Table 3. A positive $\delta/L$ value indicates that the column is curved towards the lips — i.e., $\delta$ has the sense of $\Delta_w$ in Figure 4.

**TEST RIG AND OPERATION**

The test rig and test set-up of a typical fixed-ended cold-formed steel lipped channel column test are shown in Figures 5(a) (front view) and 5(b) (side view) — they concern specimen T1.5-HSS-1. A servo-controlled hydraulic testing machine was used to apply the compressive axial force to the column specimens, which have steel plates welded to their end cross-sections ends. A rigid flat bearing plate was
connected to the upper testing machine end support, and the specimen top end plate was bolted to this bearing plate, which was restrained against warping, twist and major and/or minor axis flexural rotations – this setting may be said to correspond to a fully fixed column end support condition.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Local imperfection</th>
<th>Overall imperfection</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$\Delta_w$ (mm)</td>
<td>$\Delta_d$ (mm)</td>
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<tr>
<td>T1.0-HSS-1</td>
<td>0.20</td>
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<td>1.04</td>
<td>3.61</td>
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<td>-1.91</td>
</tr>
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<td>-2.75</td>
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<tr>
<td>T1.5-HSS-1</td>
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</tr>
<tr>
<td>T1.5-HSS-2</td>
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<td>0.96</td>
</tr>
<tr>
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<td>-0.57</td>
</tr>
<tr>
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<td>-0.32</td>
</tr>
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<td>5.96</td>
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<tr>
<td>T1.9-MS-1</td>
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<td>T1.9-MS-8</td>
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<td>6.87</td>
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</tbody>
</table>

Note: 1 in. = 25.4 mm.

A special bearing located at the lower end support was initially free to rotate in any direction. The actuator ram was moved slowly until this special bearing was in full contact with the specimen bottom end plate, for a very small applied load level (approximately 2 kN). This procedure made it possible to eliminate any gaps between the special bearing and specimen bottom end plates, which were then bolted together. The use of vertical and horizontal bolts enabled the full restraint of the bearing plate – once full contact was achieved, these bolts were used to lock the bearing plate into position, thus creating a fully fixed end support bearing. Finally, it is still worth mentioning that the use of a moveable end support allowed the performance of tests on columns with various lengths.
Three displacement transducers were used to measure the column specimen axial shortening. In addition, seven transducers were positioned around the specimen mid-length cross-section, as shown in Figure 6 – two transducers located 15 mm apart from the flange-lip junctions, four transducers located 15 mm apart from the web-flange junctions and one transducer located at the mid-web point. These transducers are able to capture the full specimen mid-length cross-section deformations. Displacement control was adopted to drive the hydraulic actuator, at a constant speed of 0.2 mm/min in all tests, thus allowing the tests to be carried out beyond the ultimate load (i.e., the post-ultimate range). A data acquisition system was used to record the applied load and displacement transducer readings at regular intervals during the tests.

![Figure 5: Test setup for specimen T1.5-HSS-1: (a) front and (b) side views](image)

![Figure 6: Transducer arrangement located at mid-length of the columns](image)

**TEST RESULTS**

The experimental ultimate loads ($P_{Exp}$) obtained from the tests are shown in Table 1, which also includes an indication of the nature of the column specimen failure – L, D and FT stand for local, distortional and flexural-torsional collapse modes. Local/distortional interaction was observed in most of the column specimens belonging to test series T1.0-HSS, T1.2-HSS, T1.5-HSS and T1.9-MS – the
exceptions were specimens T1.0-HSS-3, T1.9-MS-4 and T1.9-MS-8. Clear experimental evidence of the occurrence of Local/distortional interaction can be inspected in Figure 7, which concerns specimen T1.5-HSS-1 and is a “blow-out” of the column specimen central region depicted in Figure 5(a).

It should be noted that (i) it was perceptible in the specimen T1.0-HSS-3 failure mode the interaction between local, distortional and flexural-torsional deformations, and that (ii) the specimen T1.9-MS-4 and T1.9-MS-8 collapse mechanisms were purely distortional and global (flexural-torsional), respectively. Finally, note also that (i) no local deformations were detected in the column specimens belonging to the test series T2.4-HSS and T2.4-MS, both having a 2.4 mm wall thickness – moreover, (i) all the specimens in the test series T2.4-HSS exhibited purely distortional failure modes and (ii) the specimen T2.4-MS-1 failed in a pure flexural-torsional mode.

Figure 7: Experimental evidence of the occurrence of local/distortional interaction (specimen T1.5-HSS-1)

Figures 8 and 9 concern specimen T1.5-HSS-3 and display curves providing the relation between the applied axial load and the column (i) axial shortening and (ii) mid-length web-flange junction (distortional) displacements and (iii) mid-length mid-web (local) displacements * – it is clear that the emergence of significant local and distortional deformations starts at approximately 35 kN and that this column specimen experiences local/distortional interaction. (*It should be noted that the mid-web displacement has components stemming from is both local and distortional deformations.)

Finally, one last word to mention that two column test specimens T1.5-HSS-2 were tested and that the two ultimate loads obtained were practically identical (0.8% difference) – therefore, it seems fair to conclude that these test results are quite reliable.

Figure 8: Load vs. axial shortening curve for specimen T1.5-HSS-3

Figure 9: Load vs. mid-length cross-section displacement curves for specimen T1.5-HSS-3
CONCLUSION

This paper reported the results of an experimental investigation on the post-buckling behaviour and ultimate strength of fixed-ended cold-formed steel lipped channel columns experiencing local/distortional interaction. The 26 specimens were brake-pressed from high strength and mild zinc-coated structural steel sheets with measured 0.2% proof stresses in the 336-588 MPa range, nominal thicknesses varying from 1.0 to 2.4 mm and lengths comprised between 615 to 2500 mm. The experimental results consisted of specimen local, distortional and global initial imperfections, non-linear equilibrium paths and ultimate strengths – most specimens failed in mechanisms involving local/distortional interaction (with the consistent exception of those with 2.4 mm wall thickness). These results will be very useful to (i) calibrate and validate numerical simulations and (ii) develop a DSM approach to design cold-formed steel lipped channel columns against local/distortional interaction – see Part II of this paper and reference [12].

ACKNOWLEDGMENTS

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ULTIMATE STRENGTH AND DESIGN OF LIPPED CHANNEL COLUMNS EXPERIENCING LOCAL/DISTORTIONAL MODE INTERACTION – PART II: DSM DESIGN APPROACH

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KEYWORDS
Local/distortional mode interaction, Lipped channel columns, High strength mild steel, Ultimate strength, Direct Strength Method (DSM), Distortional and global slenderness.

ABSTRACT
This paper presents the results of an investigation on the ultimate strength and design of fixed-ended lipped channel columns experiencing local/distortional buckling mode interaction. First, a set of 26 fully fixed lipped channel columns with several cross-section dimensions and yield strengths were tested to determine their failure loads and to provide evidence of the local/distortional interaction phenomenon (see Part I of this paper). Then, a comparison between the column ultimate loads obtained from the tests and those determined from the current DSM provisions clearly indicates that the latter provide inaccurate and often very unsafe strength estimates. Based on the NLD and NDL approaches, a novel DSM including local/distortional interaction provisions is proposed and shown to provide very accurate lipped channel column strength predictions. Finally, some design recommendations are presented.

INTRODUCTION
The Direct Strength Method (DSM) has already been included in the most recent versions of the North American and Australian/New Zealander cold-formed steel design specifications. DSM provides an elegant, efficient and consistent approach to estimate the ultimate strength of cold-formed steel columns and beams experiencing global (flexural, torsional or flexural-torsional), local (L) or distortional (D) collapses, or failing in mechanisms that involve local/global interactive buckling. The current DSM version stipulates the need to perform two separate safety checks, regardless of the member critical buckling mode nature: (i) one against a distortional failure and
(ii) another against a mixed local/global failure. The DSM prescribes that the column nominal global ($f_{ne}$), mixed local/global ($f_{nle}$) and distortional ($f_{nd}$) strengths are determined by means of the expressions

$$f_{ne} = \begin{cases} f_y \left( \frac{0.658^2}{\lambda_g} \right) & \text{if } \lambda_g \leq 1.5 \\ f_y \left( \frac{0.877}{\lambda_g^2} \right) & \text{if } \lambda_g > 1.5 \end{cases}$$

where

$$\lambda_g = \frac{f_y}{f_{cre}},$$

(1)

$$f_{nle} = \begin{cases} f_{ne} & \text{if } \lambda_{le} \leq 0.776 \\ f_{ne} \left( \frac{f_{crd}}{f_{ne}} \right)^{0.4} \left[ 1 - 0.15 \left( \frac{f_{crd}}{f_{ne}} \right)^{0.4} \right] & \text{if } \lambda_{le} > 0.776 \end{cases}$$

where

$$\lambda_{le} = \frac{f_{ne}}{f_{crd}},$$

(2)

$$f_{nd} = \begin{cases} f_y & \text{if } \lambda_{dl} \leq 0.561 \\ f_y \left( \frac{f_{crd}}{f_y} \right)^{0.6} \left[ 1 - 0.25 \left( \frac{f_{crd}}{f_y} \right)^{0.6} \right] & \text{if } \lambda_{dl} > 0.561 \end{cases}$$

where

$$\lambda_{dl} = \frac{f_y}{f_{crd}},$$

(3)

where (i) $f_y$ is the yield stress and (ii) $f_{cre}$, $f_{crd}$ and $f_{crd}$ are the global (flexural or flexural-torsional), local and distortional critical buckling stresses. Since one has $f_{nle} \leq f_{ne}$, the column nominal strength always corresponds to the minimum value of the mixed local/global ($f_{nle}$) and distortional ($f_{nd}$) failure stresses.

**DSM Including Local/Distortional Buckling Interaction Provisions**

The current DSM cannot be applied to members affected by interaction phenomena involving distortional buckling modes. Following a strategy similar to the one employed to develop safety checking rules that account for local/global in interaction effects, it is possible to propose design expressions/curves to estimate the ultimate strength of columns experiencing local/distortional (L/D) interaction. In order to assess the DSM for L/D interaction, Schafer [1] tested the NLD approach, where the nominal strength against mixed local/distortional ($f_{nld}$) failure is given by

$$f_{nld} = \begin{cases} f_{nd} & \text{if } \lambda_{ld} \leq 0.776 \\ f_{nd} \left( \frac{f_{crd}}{f_{nd}} \right)^{0.4} \left[ 1 - 0.15 \left( \frac{f_{crd}}{f_{nd}} \right)^{0.4} \right] & \text{if } \lambda_{ld} > 0.776 \end{cases}$$

where

$$\lambda_{ld} = \frac{f_{nd}}{f_{crd}},$$

(4)

where $f_{nd}$ is obtained from Eq. (3). The NLD approach was then used by Yang and Hancock [2] and, very recently, also by Kwon et al. [3]. Silvestre et al. [4] presented the NDL approach, where the nominal strength against mixed local/distortional ($f_{ndl}$) failure is given by

$$f_{ndl} = \begin{cases} f_{nl} & \text{if } \lambda_{dl} \leq 0.561 \\ f_{nl} \left( \frac{f_{crd}}{f_{nl}} \right)^{0.6} \left[ 1 - 0.25 \left( \frac{f_{crd}}{f_{nl}} \right)^{0.6} \right] & \text{if } \lambda_{dl} > 0.561 \end{cases}$$

where

$$\lambda_{dl} = \frac{f_{nl}}{f_{crd}},$$

(5)
where $f_{nl}$ is obtained from Eq. (2), using $f_y$ instead of $f_{ne}$ (this is to ensure that the NDL approach does not account for the influence of global buckling). The main aim of this work is to assess the performance of these approaches in estimating the ultimate strength of fixed-ended lipped channel columns, through the comparison with the experimental results presented in detail in Part I of this paper (Young et al. [5]).

**COLUMN PROPERTIES AND TEST RESULTS**

Since the available experimental results concern mostly fixed-ended columns (rigid plates attached to their end sections), thus preventing local/global rotations and warping, one objective of this paper is to analyse a set of such lipped channel column geometries (cross-section dimensions and lengths) that are highly prone to L/D interaction. This set of lipped channel columns comprised a total of 26 specimens, with six different steel properties and sheet thickness values: (i) four high strength steel (HSS) grades with nominal thickness equal to 1.0, 1.2, 1.5 and 2.4 mm and (ii) two mild steel (MS) grades with nominal thickness equal to 1.9 and 2.4 mm. The measured values of the base metal thickness ($t^*$), Young’s modulus ($E$) and static 0.2% proof stress ($f_{0.2}$) were obtained by means of tensile coupon tests and are given in Table 1. Regarding the cross-section dimensions and column lengths, they were previously selected to (i) ensure a wide range of local and distortional slenderness values and (ii) avoid (if possible) the occurrence of global (flexural-torsional) buckling/failure. Besides selecting lipped channel specimens with nearly coincident local ($f_{crL}$) and distortional ($f_{crD}$) buckling stresses, shapes associated with very distinct $f_{crL}$ and $f_{crD}$ values were also considered.

The cross-section mid-line dimensions obtained from the lipped channel specimen measurements are also shown in Table 1 — $b_w$, $b_f$, $b_l$, $A$ and $L$ are the web height, flange width, lip width, cross-section area and column length. The adopted cross-section dimensions and steel properties fall into the $41 \leq b_w/t \leq 139$, $18 \leq b_f/t \leq 131$, $7 \leq b_l/t \leq 17$, $1.0 \leq b_w/b_f \leq 2.3$, $0.10 \leq b_l/b_f \leq 0.40$, $363 \leq E/f_y \leq 606$ ranges. Since the current DSM limits for pre-qualified columns are $b_w/t < 472$, $b_f/t < 159$, $4 < b_l/t < 33$, $0.7 < b_w/b_f < 5.0$, $0.05 < b_l/b_f < 0.41$ and $E/f_y > 340$, one readily notices that all column specimens satisfy these requirements. Also shown in Table 1 are the experimental ultimate loads and stresses ($P_{exp}$ and $f_{exp}$), as well as the observed specimen failure modes (L, D, FT). Finally, the specimen labels provide information about the test series: the first letter (T) concerns the sheet thickness and the following ones indicate whether the specimen is made of high strength (HSS) or mild (MS) steel.

**CRITICAL STRESS CALCULATION AND DSM RESULTS**

In order to evaluate the ultimate strength of a given lipped channel column, the DSM requires the calculation of its local ($f_{crL}$), distortional ($f_{crD}$) and global ($f_{cre}$) critical stresses. In simply supported columns, this task can be achieved by resorting either to the semi-analytical finite strip (CUFSM [6]) or Generalised Beam Theory (GBT — GBTUL [7, 8]) analyses. In fixed-ended columns, it is well known that semi-analytical finite strip analyses provide lower bounds for $f_{crL}$, $f_{crD}$ and $f_{cre}$ — exact values can only be obtained using other GBT or shell finite element (SFE) available commercial codes. As far as DSM applications are concerned, GBTUL has two main advantages over those SFE codes: (i) free availability and, most of all, (ii) mode selection features making it very easy to obtain accurate $f_{crL}$, $f_{crD}$ and $f_{cre}$ estimates. Indeed, SFE analyses often entail the need to consider a large number of buckling modes to determine $f_{crL}$, $f_{crD}$ and $f_{cre}$ — moreover, because the vast majority of these buckling modes exhibit mixed characteristics, it is very difficult to identify the critical (“pure”) L, D and FT buckling modes.
TABLE 1

<table>
<thead>
<tr>
<th>Specimens</th>
<th>t* (mm)</th>
<th>bw (mm)</th>
<th>bf (mm)</th>
<th>bl (mm)</th>
<th>A (mm²)</th>
<th>L (mm)</th>
<th>E (GPa)</th>
<th>f0.2 (MPa)</th>
<th>Pexp (kN)</th>
<th>fexp (MPa)</th>
<th>Failure</th>
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The GBT cross-section discretisation adopted in this work (five-node web and flanges, and two-node lips) leads to 17 deformation modes – the most relevant ones are depicted in Figure 1. Regarding the calculation of $f_{cr1}$, $f_{cr2}$ and $f_{cre}$ in a given fixed-ended column, the following strategy was adopted:

(i) Select all the symmetric local modes (7, 9, 11, 13, 15, 17) to calculate $f_{cr1}$. In all the columns considered in this study, the local buckling mode was always “critical” and had a large participation of mode 7 and minor contributions from the remaining ones. Due to the very short wavelengths, the columns had to be longitudinally discretised into 40 to 60 beam finite elements. (*It is also possible to calculate $f_{cr1}$ selecting only mode 7 - however, the error associated with neglecting the other symmetric local modes (9, 11, 13, 15, 17) may be non-negligible.)

(ii) Select only mode 5 to calculate the $f_{cr2}$. Since distortional buckling is characterised by intermediate wavelengths, column longitudinal discretisations into 10 to 20 finite elements were sufficient.

(iii) Select modes 2 and 4 to calculate $f_{cre}$. Due to the flexural-torsional buckling long wavelengths, only 5 to 10 finite elements were necessary.

![Figure 1: GBT modes used in the calculation of critical stresses ($f_{cr1}$, $f_{cr2}$ and $f_{cre}$)](473)
In order to illustrate the results obtained, consider the cross-section T1.0-HSS-1 (first row in Table 1). Figure 2(a) depicts the variation of critical stresses with the column length (logarithmic scale). The thin solid curve is the well known “cross-section signature curve”, valid for simply supported columns buckling in single half-wave modes, that may be obtained from semi-analytical finite strip or GBT analyses. The two local minima ($f_{\text{min},l}=82.2$ MPa and $f_{\text{min},d}=124.0$ MPa) are lower bounds of the fixed-ended column local and distortional critical stresses. The remaining three curves concern fixed-ended columns buckling in multi-half-wave modes: the thick solid, dotted and dashed curves provide $f_{\text{cr},l}$, $f_{\text{cr},d}$ and $f_{\text{cr},e}$, respectively – for $L=2498$ mm, one has $f_{\text{cr},l}=82.5$ MPa, $f_{\text{cr},d}=148.8$ MPa and $f_{\text{cr},e}=348.4$ MPa (three white circles in Fig. 2(a)) are the values required to apply the DSM. Figure 2(b) depicts the T1.0-HSS-1 column local (27 half-waves), distortional (3 half-waves) and flexural-torsional (1 half-wave) buckling mode shapes.

A similar procedure was used for each column length – the $f_{\text{cr},l}$, $f_{\text{cr},d}$ and $f_{\text{cr},e}$ values obtained are given in Table 2. The incorporation of these values into the current DSM expressions (Eqs. (1), (2) and (3)) leads to the flexural-torsional ($f_{\text{ft}}$), mixed local/flexural-torsional ($f_{\text{nl}}$) and distortional ($f_{\text{nd}}$) ultimate stresses also shown in Table 2. This table includes still the column ultimate strengths $f_n$ (lowest values amongst $f_{\text{nl}}$ and $f_{\text{ft}}$) and the corresponding failure mode nature ($L+\text{FT}$ or $D$) – for comparison purposes, the experimental ultimate strengths ($f_{\text{exp}}$) and observed failure mode natures are also given, together with the predicted-to-experimental ultimate strength ratio $f_n/f_{\text{exp}}$.

The close observation of Table 2 shows clearly that, with one exception, the current DSM overestimates the ultimate strength of all columns. The $f_n/f_{\text{exp}}$ values average 1.19 and exhibit a fairly high scatter (0.20 standard deviation). Moreover, the DSM and experimental failure
mode natures often do not coincide. Figure 3(a) provides the variation of $f_n/f_{\text{exp}}$ with the column distortional slenderness $\lambda_d=(f_y/f_{\text{crd}})^{0.5}$. The column ultimate strength predictions yielded by the current DSM expressions are (i) reasonably accurate for $\lambda_d<1.0$ and (ii) unsafe for $\lambda_d>1.0$ – the error grows with $\lambda_d$ and $f_n/f_{\text{exp}}$ values between 1.38 and 1.60 are reached for the seven columns T1.2-HSS-1/2/3, T1.5-HSS-3/4 and T1.9-MS-6/7 (see Table 2).

However, $f_n/f_{\text{exp}}$ is not very high for all columns with large $\lambda_d$ values (e.g., the T1.0-HSS-1/2/3 columns have $\lambda_d=1.90/1.85/1.72$ and $f_n/f_{\text{exp}}=1.11/1.09/1.06$), which raises the question: what other factor(s) affect(s) the quality of the current DSM column ultimate strength predictions?

### Table 2

**Critical Stresses and Ultimate Stresses Obtained from the Current DSM and Tests**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Yield (MPa)</th>
<th>Critical Stresses - GBT</th>
<th>DSM</th>
<th>Test</th>
<th>Failure (MPa)</th>
<th>Differ.</th>
<th>f_n/f_exp</th>
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<td>f_{crd}</td>
<td>f_{rl}</td>
<td>f_{rel}</td>
<td>f_{rd}</td>
<td>f_{rld}</td>
<td>f_{cr}</td>
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\[ \lambda_d \]

\[ f_{\text{exp}}, f_n \]

\[ \text{Figures 3(a) and 3(b)} \]

\[ \text{Figure 3: Variation of } f_n/f_{\text{exp}} \text{ with } \lambda_d: \text{(a) current DSM and (b) DSM including L/D interaction} \]
An answer to this question can be obtained by looking at the $f_{crd}/f_{crl}$ (gray) $f_{cre}/f_{crl}$ (dark grey) and $f_y/f_{crl}$ (black) values displayed in Figure 4 – these stress ratios vary between 1.1 and 2.7 ($f_{crd}/f_{crl}$), 1.5 and 51.5 ($f_{cre}/f_{crl}$) and 0.8 and 11.2 ($f_y/f_{crl}$). The seven columns mentioned before (T1.2-HSS-1/2/3; T1.5-HSS-3/4; T1.9-MS-6/7) exhibit two common features that are not jointly shared by any other column: (i) $f_{cre}/f_{crl}$ (35.0 to 52.0) much higher than $f_y/f_{crl}$ (5.0 to 11.0), i.e., low global slenderness $\lambda_g$, and (ii) $f_y/f_{crl}$ (5.0 to 11.0) much higher than $f_{crd}/f_{crl}$ (around 2.0), i.e., high distortional slenderness $\lambda_d$. The T1.0-HSS-1/2/3 columns, also mentioned before, satisfy the above second condition, but not the first one – indeed, $f_{crd}/f_{crl}$ is even lower than $f_y/f_{crl}$ (the two values are similar, both lying between 2.5 and 3.0). Then, it may be said that the occurrence of L/D interaction affects the column ultimate strength more severely as (i) the global slenderness $\lambda_g$ decreases (i.e., local/global failure becomes less relevant) and (ii) the distortional slenderness $\lambda_d$ increases (i.e., distortional failure becomes more relevant) – in columns with a given high $\lambda_d$ value, the error of the DSM estimate grows as $\lambda_g$ decreases. On the other hand, L/D interaction affects the column ultimate load most when $f_y$ is much higher than both $f_{crd}$ and $f_{cre}$ – there is room for these coupling effects to develop before yielding leads to the column failure. Concerning the column ultimate strength, the relevance of L/D interaction depends more on how high is the lowest of the yield-to-critical stress ratios $f_y/f_{crl}$ and $f_y/f_{cre}$ than on how close are $f_{crd}$ and $f_{cre}$, confirming the findings reported in [9] – in fact, this study shows that columns with local and distortional critical stresses 100% apart (e.g., $f_{crd}=2f_{crl}$) may exhibit strong L/D coupling in the elastic range, particularly if local buckling precedes distortional buckling.

![Figure 4: Values of the stress ratios $f_{crd}/f_{crl}$, $f_{cre}/f_{crl}$ and $f_y/f_{crl}$ for all tested columns.](image)

**ASSESSMENT OF NEW DSM INCLUDING LOCAL-DISTORTIONAL INTERACTION**

It is proposed here to replace the current DSM provision for distortional failure (Eq. (3)) by one concerning local/distortional interaction – either the NLD (Eq. (4)) or the ND L (Eq. (5)) approach. The ultimate stresses provided by these approaches ($f_{nld}$ and $f_{ndl}$), as well as the $f_{nle}$ values, are given in Table 3.
Observing Table 3, one concludes that both $f_{\text{NLD}}$ and $f_{\text{NDL}}$ are much more accurate and safe estimates than those provided by the current DSM — however, the $f_{\text{NDL}}$ estimates are slightly more accurate ($f_{\text{NLD}}/f_{\text{exp}}$ and $f_{\text{NDL}}/f_{\text{exp}}$ averages equal to 0.95 and 1.00) and a bit less scattered (standard deviations equal to 0.07 and 0.05). Therefore, the NDL approach is chosen to be included in the proposed DSM, together with the current provision for local/global interaction. In this new version, the column ultimate strength ($f_{n^*} = \min(f_{\text{NLD}}, f_{\text{NDL}})$) and the corresponding failure mode (L+FT or L+D) are presented in Table 3, which includes also the predicted-to-experimental ultimate strength ratio $f_{n^*}/f_{\text{exp}}$. The novel DSM clearly provides excellent predictions of all the tested column ultimate strengths. The $f_{n^*}/f_{\text{exp}}$ values average 1.00 and exhibit little scatter (0.04 standard deviation) — Figure 5(b) shows the variation of $f_{n^*}/f_{\text{exp}}$ with $\lambda_d$. There is also a better correlation between the DSM failure modes and those observed in the tests. It is also obvious that the $f_{n^*}$ error does not depend on $\lambda_d$ and $\lambda_g$ — $f_{n^*}/f_{\text{exp}}$ oscillates around unity.

Figure 5 shows the variation of $f_n/f_y$ (Figure 5(a) – white circles), $f_{n^*}/f_y$ (Figure 5(b) – white circles) and $f_{\text{exp}}/f_y$ (both figures – black circles) with $\lambda_d$. Also included is the “Winter-type” distortional strength curve given by Equation (3) — one observes that all experimental values (black circles) lie well below this curve. While the $f_n/f_y$ values (white circles in Figure 5(a)) exhibit a “vertical dispersion” that grows with $\lambda_d$, the $f_{n^*}/f_y$ values (white dots in Figure 5(b)) remain closely “aligned” with the experimental ones (black circles).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Test</th>
<th>NLD</th>
<th>NDL</th>
<th>NLE</th>
<th>DSM</th>
<th>Failure</th>
<th>$f_{n^*}/f_{\text{exp}}$</th>
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<tbody>
<tr>
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<td>140.1</td>
<td>0.96</td>
<td>156.0</td>
<td>1.03</td>
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<tr>
<td>T1.0-HSS-3</td>
<td>149.8</td>
<td>152.5</td>
<td>1.02</td>
<td>164.5</td>
<td>1.10</td>
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<td>T1.2-HSS-1</td>
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<td>119.3</td>
<td>0.94</td>
<td>206.9</td>
<td>119.3</td>
</tr>
<tr>
<td>T1.2-HSS-2</td>
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<td>113.3</td>
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<td>0.96</td>
<td>207.3</td>
<td>125.6</td>
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<tr>
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<tr>
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<td>248.0</td>
<td>216.4</td>
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<td>0.92</td>
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<td>228.4</td>
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<td>229.6</td>
<td>203.1</td>
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<td>0.94</td>
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<tr>
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<td>0.97</td>
<td>167.3</td>
<td>100.7</td>
</tr>
<tr>
<td>T1.9-MS-1</td>
<td>171.0</td>
<td>155.8</td>
<td>0.91</td>
<td>168.2</td>
<td>0.98</td>
<td>193.4</td>
<td>168.2</td>
</tr>
<tr>
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<td>173.9</td>
<td>159.3</td>
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<td>172.8</td>
<td>0.99</td>
<td>192.9</td>
<td>172.8</td>
</tr>
<tr>
<td>T1.9-MS-3</td>
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<td>155.9</td>
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<td>1.06</td>
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<td>167.7</td>
</tr>
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<td>229.9</td>
<td>1.00</td>
<td>233.1</td>
<td>1.02</td>
<td>238.4</td>
<td>233.1</td>
</tr>
<tr>
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<td>1.01</td>
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<td>237.2</td>
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<td>T1.9-MS-6</td>
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<td>156.0</td>
<td>111.7</td>
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<tr>
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<td>1.05</td>
<td>462.3</td>
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<td>371.7</td>
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<td>1.09</td>
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</table>

| Mean       | 1.00 | Mean       | 1.00 | Mean       | 1.00 |
| Sd. Dv.    | 0.07 | Sd. Dv.    | 0.05 | Sd. Dv.    | 0.04 |
CONCLUSION

An investigation on the ultimate strength and design of fixed-ended lipped channel columns experiencing L/D interaction was reported. Previous numerical studies showed clearly that the codified DSM cannot handle this interactive behaviour adequately and prompted the development of a novel DSM specifically aimed at such members. This work confirmed experimentally the relevance of L/D interaction through the testing of 26 fixed-ended lipped channel columns with several cross-section dimensions and yield stresses, and also included design recommendations drawn from comparing the experimental ultimate strengths with the various DSM estimates dealt with. The following aspects deserve to be highlighted:

(i) The relevance of the L/D interaction is not restricted to columns exhibiting close local and distortional critical stresses — indeed, columns with $f_{cr,d}$ and $f_{cr,l}$ 100% apart still experience strong coupling effects in the elastic range, particularly if local buckling precedes distortional buckling.

(ii) A column is very sensitive to L/D interaction if $\lambda_g$ (global slenderness) is low and $\lambda_d$ (distortional slenderness) is high. Regardless of the $f_{cr,d}/f_{cr,l}$ value, coupling effects are always relevant for columns with a high enough $\lambda_d$ (plenty of room for considerable L/D interaction prior to yielding).

(iii) The current DSM provides reasonable ultimate strength estimates for $\lambda_d<1.0$ and unsafe ones for $\lambda_d>1.0$ — for a given $\lambda_d$ value, the error increases as the global slenderness $\lambda_g$ decreases.

(iv) Regardless of $\lambda_d$ and $\lambda_g$, the novel DSM always yields accurate column ultimate strength estimates.

REFERENCES


THE ULTIMATE STRENGTH AND STIFFNESS OF MODERN ROOF SYSTEMS WITH HAT-SHAPED PURLINS

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KEYWORDS
Diaphragm, strength, flexibility, shear, purlins, rafter, cladding, buckling.

ABSTRACT
In this paper, the strength and stiffness of different roof structures has been investigated, in order to establish their ability to act as in-plane diaphragms for stressed skin design. In each test set-up, a roof panel of approximately 3m x 3m was constructed using top-hat purlins and standard sheeting profiles or composite panels. Different types of roofs, such as single and double skin, have been investigated, all using hat-shaped purlins. A total of 10 roof panels were examined by testing with and without shear connectors placed along the rafters. The experimental strength and stiffness of each panel was then compared against established theoretical methods and the effect of shear connectors was discussed. It was demonstrated that although it is possible to closely estimate the ultimate strength of the structure using standard calculation methods, it is often more difficult to accurately calculate its stiffness. As the panel stiffness is a function of many variables, testing is still the recommended method, in order to investigate the shear flexibility of modern roof panels.

INTRODUCTION
Stressed skin action is a well established phenomenon, based on taking into account the inherent strength and stiffness of the metal cladding in overall frame behavior. It was demonstrated by extensive research that stressed skin action can reduce or eliminate the wind bracing, reduce sway deflections under horizontal forces, and reduce the spread of the frame under vertical load (see Figure 1). A set of design recommendations was first presented in the ‘Manual of stressed skin diaphragm design’ by Davies & Bryan [1], which was then incorporated into BS5950: Part 9 [2], and the ‘European recommendation for the application of metal sheeting acting as a diaphragm’ [3].
Stressed skin design was originally investigated for traditional hot-rolled steel portal frames. In recent years, however, for frames having spans of around 12 m, the entire frame is often built from cold-formed steel members. These types of structures are often very flexible, have semi-rigid joints, and usually suffer from extensive sway deflections [4]. In such cases, implementing the stressed skin action in their analysis can offer greater benefits than for hot-rolled steel frames.

Roof systems, however, are consistently evolving, often leaving existing standards out-of-date. The typical connection detail for purlin to rafter connection includes C or Z purlins connected to the rafters through a web cleat (see Figure 2). Such an arrangement has relatively low stiffness against shear deformation. On the other hand, the use of modern top-hat shaped purlins can simplify the connection detail, act well in carrying in-plane bending, improve purlin to rafter connection flexibility, and therefore increase the diaphragm stiffness.

Figure 1: Stressed skin action in buildings to BS 5950: Part 9, pp.14 [2]

Figure 2: Different purlin to rafter connection details
Generally roof structures can fall into one of the following categories:

- Single skin roofs comprising one layer of metal cladding,
- Built-up roofs comprising liner tray, insulation layer and outer watertight panel,
- Composite panels manufactured from inner liner and outer sheeting profile with the core of insulation foam between the two layers.

**TEST ARRANGEMENT**

Each test was carried out on a cantilever panel of approximately 3m x 3m subject to shear force, as presented in Figure 3. The shear load was applied by the use of a hand-operated hydraulic jack. The panel’s displacement $\delta$, in four different directions, was measured as shown in Figure 3. To take account of rigid body movement of the panel, overall deflection was calculated from the Eqn.1 [2].

$$\Delta = \delta_1 - \delta_2 - [(a/b)(\delta_3 - \delta_4)]$$  \hspace{1cm} (1)

The test set up consisted of cold-formed steel double lipped channels of 3mm thickness for the rafters, top hat shaped members of 61 mm depth x 1mm thickness for the purlins and shear connectors placed between the purlins.

![Figure 3: Plan view of typical test arrangement](image)

**TEST SERIES**

The test panels were designed to cover entire range of sheeting profiles offered by the manufacturer. Self-drilling self-tapping screws with steel washers and E.P.D.M. seals were used, as follows:

- 5.5mm diameter for fixing the sheet to purlins.
- 6.3mm diameter for purlin to rafter connection, seam connection and shear connectors.

Three different types of sheeting profiles were tested (see Figure 4). The geometry of each profile is presented in Table 1. In each case the fixing program followed the manufacturer recommendations.
Panels with Shear Connectors (Test 1 to 9)

In the first stage of testing, 9 different roof panels were tested with shear connector placed between purlins, as shown in Figure 3. For tests 1 to 6, different single skin roof panels were assembled with end fixings placed in every trough of the roof sheeting and intermediate fixings placed in alternate troughs. The corrugated sheeting profiles in test 7 and 8 were fixed in every third crest. The seam screws and the shear connector screws were spaced at 450mm. Test 9 was carried out on 80mm thick rigid composite panels with polyisocyanurate insulation core. The sheet to end purlin fixings were spaced at approximately 243mm and the sheet to intermediate purlin fixings were spaced at 333mm centres.

Panels without Shear Connectors (Test 11 to 20)

In case of Tests 11 to 16, panels were assembled exact the same way as those in test 1 to 6, without shear connectors being present. In Tests 17 and 18 the number of sheet to end purlin fixings was increased to avoid the shear failure of fixings, observed in first stage of testing. The cladding profiles. In Test 19, the number of end fixings in composite panel was increased from 4 to 5 per each panel, by reducing the pitch of fixings to 165mm. The pitch of fixings at the intermediate purlins remained 333mm centres. In Test 20, the typical build-up roof structure was tested. The liner tray was fixed in every trough to the end purlins and in alternate troughs at intermediate purlins. The seam and shear connector screws were not present. The top water-tight sheeting was supported on twist-locked bar and bracket system and fixed in alternate trough.

![Figure 4: Different sheeting profiles](image)

<table>
<thead>
<tr>
<th>Test series</th>
<th>Sheeting Type (see Figure 4)</th>
<th>Height h (mm)</th>
<th>Thickness t\text{net}(^1) (mm)</th>
<th>Pitch d (mm)</th>
<th>Width l (mm)</th>
<th>Angle (\theta) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01&amp;02</td>
<td>Type 1</td>
<td>1</td>
<td>34</td>
<td>0.45&amp;0.65</td>
<td>167</td>
<td>23</td>
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<tr>
<td>11&amp;12</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>03&amp;04</td>
<td>Type 2</td>
<td>2</td>
<td>30</td>
<td>0.45&amp;0.65</td>
<td>200</td>
<td>30</td>
</tr>
<tr>
<td>13&amp;14</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>05&amp;06</td>
<td>Type 3</td>
<td>2</td>
<td>24</td>
<td>0.45&amp;0.65</td>
<td>167</td>
<td>20</td>
</tr>
<tr>
<td>15&amp;16</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>07&amp;08</td>
<td></td>
<td>3</td>
<td>18</td>
<td>0.45&amp;0.65</td>
<td>76</td>
<td>-</td>
</tr>
<tr>
<td>17&amp;18</td>
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<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>09&amp;19</td>
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<td>333</td>
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<td>25</td>
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<td>Bottom skin 1</td>
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<td>0.35</td>
<td>100</td>
<td>48</td>
<td>45</td>
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<tr>
<td>20</td>
<td>Top skin 2</td>
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<td>0.45</td>
<td>200</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>Bottom skin 1</td>
<td>20</td>
<td>0.65</td>
<td>200</td>
<td>80</td>
<td>45</td>
</tr>
</tbody>
</table>

\(^1\) Net thickness after fixing of screws

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The ultimate strength and flexibility of investigated cantilever diaphragms were calculated according to design rules published by Davies & Bryan [1]. The design strength of individual fixings was calculated to BS 5950: Part 5 [4] and is shown in Table 2. Flexibility values for different types of fixing were assumed according to Davies & Bryan [1] and are presented in Table 2.

### TABLE 2

<table>
<thead>
<tr>
<th>Fixing type</th>
<th>Strength (kN/mm sheet thickness)</th>
<th>Flexibility (mm/kN)</th>
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<td>5.5 mm diameter screws (sheet fixings)</td>
<td>3.2</td>
<td>0.25</td>
</tr>
<tr>
<td>6.3 mm diameter screws (seam and shear connector fixings)</td>
<td>3.7</td>
<td>0.30</td>
</tr>
<tr>
<td>6.3 mm diameter screws (purlin to rafter connection)</td>
<td>7.3</td>
<td>0.50</td>
</tr>
</tbody>
</table>

### COMPARISON OF TEST RESULTS AGAINST DESIGN METHOD

A summary of the experimental and theoretical shear strengths and flexibilities for roofing panels with and without shear connectors is shown in Table 3. The different modes of failure observed during the tests are presented in Figure 5.

- a) End collapse of the profile
- b) Shear buckling of the profile
- c) Fixing inclination
- d) End fixing shear failure
- e) Local failure of the profile
- f) Sheet to purlin connection failure
- g) Seam fixing failure
- h) Buckling of the end purlin in compression
- i) Buckling of the liner tray

1 Net sheet thickness excluding galvanizing and coating
Figure 5: Different modes of failure observed during the test programme.

TABLE 3
COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS.

<table>
<thead>
<tr>
<th>Test series n-h/t (mm/mm)</th>
<th>Experimental Strength</th>
<th>Experimental Flexibility</th>
<th>Theoretical Strength</th>
<th>Theoretical Flexibility</th>
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<tbody>
<tr>
<td></td>
<td>Strength1 (kN)</td>
<td>Flexibility2 (mm/kN)</td>
<td>Strength3 (kN)</td>
<td>Flexibility3 (mm/kN)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01-34/0.5</td>
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<td>0.27</td>
<td>17.7</td>
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<td>0.39</td>
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<tr>
<td>05-24/0.5</td>
<td>21.9</td>
<td>0.34</td>
<td>17.7</td>
<td>0.45</td>
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<td>07-13/0.5</td>
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<td>0.71</td>
<td>16.8</td>
<td>0.77</td>
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<tr>
<td>02-34/0.7</td>
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<td>25.4</td>
<td>0.39</td>
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<td>26.4</td>
<td>0.31</td>
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<td>08-13/0.7</td>
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<td>Panels without shear connectors</td>
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<td>11-34/0.5</td>
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<td>13-30/0.5</td>
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<td>0.70</td>
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<td>9.1</td>
<td>0.71</td>
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<td>19-35/(0.4/0.5)</td>
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<td>0.59</td>
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<td>0.80</td>
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<td>0.56</td>
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<td>0.63</td>
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<td>14-30/0.7</td>
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<td>0.53</td>
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<tr>
<td>16-24/0.7</td>
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<td>0.50</td>
<td>11.8</td>
<td>0.57</td>
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<td>18-13/0.7</td>
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<td>13.1</td>
<td>0.58</td>
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<tr>
<td>20-20/(0.7/0.5)</td>
<td>32.7</td>
<td>0.38</td>
<td>13.1</td>
<td>0.64</td>
</tr>
</tbody>
</table>

1 Ultimate test load
2 Shear flexibility according to Section 11.4, BS 5950: Part 9
3 Design strength adequate to experimental mode of failure

Panels with shear connectors (Test 1 to 9)

Calculated design shear capacities, based on the strength of sheet to shear connector fixings, were 10.0kN and 14.5kN for sheeting of 0.5mm and 0.7mm thickness respectively. However, in the tests, failure of the shear connectors was not observed and test load was often much higher than these design values. An interesting observation was that experimental flexibility was not as sensitive to the thickness of the cladding as indicated by the design method.

In case of Test 1, 3 and 5, the deformation of the end profile was observed (see Figure 5a). Ultimate loads calculated for this mode of failure are shown in Table 3. In Test 3, the end sheeting collapse progressed further, causing shear bucking of the sheeting (see Figure 5b). In two other cases, this phenomenon was not observed and panels failed at the seam fixings.

In Tests 2, 4 and 6, critical loads obtained from calculations for the seam failure are shown in Table 3. However, in Tests 4 and 6, panels failed due to the buckling of the end purlin in compression at higher loads (see Figure 5h). The extensive local shear distortion of the profile in Test 4 was observed in the early stage of loading, causing higher flexibility than predicted.

The theoretical analysis for profiled cladding (Test 7 and 8) was carried out for fixings located in the troughs. However, in the tests, the fixings were placed in the crest according to
the manufacturer’s recommendations. As was expected, this caused the extensive inclination of the screws along the edge members (see Figure 5c) and significantly increased the shear flexibility of the cladding panel. In case of Test 8, sudden shear failure of sheet to end purlin screw was observed (see Figure 5d).

In case of Test 9, the experimental flexibility of the composite (sandwich) panels agreed with the flexibility calculated for the bottom skin of 0.4mm thickness, as can be seen in Table 3. The panel failed at the seam connection and the experimental failure load closely agreed with the theoretical calculated for top skin of 0.5mm thickness.

**Panels without Shear Connectors (Test 11 to 20)**

According to design method shear strength of panels in Tests 11 to 19 is controlled by the strength of sheet to end purlin connection for sheeting fixed on two sides. The design strength for each of following panels is presented in Table 3. Ultimate test loads however, proved to be significantly higher from those calculated. Similar observation to this from the first stage of testing was made, that thickness of the sheeting profile does not influence significantly shear flexibility of the panel.

In case of Test 11, plastic deformation due to hole elongation in the sheet to end purlin connection (see Figure 5f) was observed around 13.8kN. The panel failed eventually at the seam connection (see Figure 5g) at a load of 18kN. The design strength for this mode of failure was calculated as 18.3kN.

Similar behaviors were observed in panels 12, 14 and 16. Again ultimate test loads were higher than design strength of seam connections. Panels 13 and 15 failed due to shear buckling of the sheeting profile (see Figure 5b) initiated by end collapse of the profile (see Figure 5a). The design strength of panels due to the end collapse of the sheeting was calculated as 16.1kN. In case of trapezoidal sheeting calculation method would on average overestimate panel’s shear flexibility by 32% for sheeting of 0.5m thickness and 14% for the sheeting of 0.7mm thickness.

Increasing the number of end fixings in tests 17 and 18 did not offer much improvement in panels’ behaviour. Although it prevented sudden shear failure of fixings experienced in test 8 (see Figure 5d) it did not stop extensive screws inclination. As an effect of this phenomenon in Tests 17 and 18, non-linearity of load-deflection relationship occurred at a very early stage of loading (around 3kN). Therefore the experimental flexibilities of tested panels were higher than those predicted by the design method. Both panels failed due to combination of extensive screws inclination and local failure of corrugated profiles (see Figure 5e and 5e).

In Test 19, the ultimate strength of the sheet to end purlin connection was calculated for the bottom skin as 6.3kN. In the load tests, signs of this mode of failure were observed slightly above this value. More load was transfer to the top skin, and therefore composite panel failed at seam connection (see Figure 5g) at a slightly lower load than calculated for the top skin seam failure. The shear flexibility value calculated for the bottom skin of 0.4mm thickness proved to be conservative to that obtained in the test.

For the built-up roof system (Test 20), it was assumed that the top skin would not add much to the strength and the stiffness, since the supporting structure had flexible brackets. Therefore, the theoretical calculations were carried out only for the liner tray. Although the panel resisted a load of 32.7kN, the liner tray started to buckle along the unsupported edge at an early stage of loading (see Fig. 5i). The failure load was close to that established theoretically for seam fixing strength, as presented in Table 3. For the initial stage of loading,
it was shown that the panel was over one and a half times stiffer than predicted by theoretical method.

**CONCLUSIONS**

For panels with shear connectors, the calculated shear capacities were governed by the strength of the shear connectors. However, this mode of failure was not observed in any of the tests and the test loads were often significantly higher. In case of panels without shear connectors it was shown that ultimate test load was always significantly higher than the ultimate strength calculated for sheet to end purlin connection.

According to the design method, changing the thickness of the trapezoidal sheeting from 0.5mm to 0.7mm decreases the panel’s flexibility by 36% and 24% for panels with and without shear connectors respectively. Tests on panels proved that test flexibility was almost the same regardless of the sheeting thickness.

In case of panels with shear connectors using trapezoidal sheeting, the theoretical method overestimates the panel’s flexibility by 39% and 13% on average for 0.5mm and 0.7mm sheeting thickness. When shear connectors were not present, the flexibility was overestimated by 32% and 14% for 0.5mm and 0.7mm sheeting thickness. For the tests on trapezoidal sheeting without shear connectors the panel’s flexibility was increased by 42% on average.

Profiled panels fixed in crests (Test 7, 8, 17, 18) are not recommended for stressed skin design. Due to extensive fixings inclination, the non-linearity of load-deflection relationship was observed in early stage of loading.

Composite panels can be used in stressed skin design as long as the fixing inclination is taken into account and sufficient number of end fixings is provided. The increase of stiffness, due to the interaction of top and bottom skin, should not be considered.

A built-up roof structure has been shown to perform well as an in-plane diaphragm as long as the seam fixings are provided in the liner tray. It was shown that the top skin, even attached to flexible supports, can significantly increase the stiffness of the whole roof structure.

**REFERENCES**

SOME EXPERIENCES ON NUMERICAL MODELLING OF COLD-FORMED STEEL LAPPED Z-SECTIONS

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KEYWORDS

Cold-formed steel; purlins; numerical modelling; Z-sections; finite element method; lapped connections.

ABSTRACT

Cold-formed steel purlins are widely adopted in building construction as supporting members of roof systems. Owing to the effective overlapping between Z-sections, they are commonly adopted to act as multi-span purlins and side rails to support building envelopes. This paper presents a systematic numerical investigation into the structural behaviour of high strength cold-formed steel lapped Z-sections under one point loads. A number of finite element models with two different element types and mesh configurations have been established with different boundary and contact conditions, and material and geometrical non-linearity has been incorporated into the finite element models together with initial geometrical imperfections. The models are calibrated against the test results of a series of lapped Z-sections with different lap lengths. It is found that the maximum applied moments and the elastic deformation characteristics predicted by the models with both thin degenerated 4-noded shell elements and continuum 8-noded shell elements are very close to the measured results. However, only the numerical models using continuum 8-noded shell elements are able to give close predictions to the deformation characteristics of continuous Z-sections at very large deformations. Hence, with suitable boundary and contact conditions, the finite element models using continuum 8-noded shell elements are shown to be highly effective in assessing the structural behaviour of lapped Z-sections with different lap lengths along their entire deformation ranges.

INTRODUCTION

Cold-formed steel purlins are widely adopted in building construction as supporting members of roof systems. Owing to the effective overlapping between Z-sections, they are commonly adopted to act as multi-span purlins and side rails to support building envelopes. Typical
yield strengths are 280 and 350 N/mm² while the thicknesses range from 1.2 to 3.0 mm. In the recent years, there is a growing interest to use high strength cold-formed steel Z-sections with yield strengths at 450 and 550 N/mm² to construct roof systems with increased load carrying and spanning capacities. In modern multi-span purlin systems, the level of section continuity across lapped sections over internal purlin-rafter supports is very important to the structural behaviour of the purlins. More specifically, the level of section continuity across lapped sections affects significantly the internal force distributions of the purlin systems, and also the section resistances against combined bending and shear at the end of the laps. While full scale tests on lapped Z-sections and multi-span purlin systems are considered to be time-consuming and expensive, finite element modelling is reckoned to be an effective alternative to examine the full range structural behaviour of the lapped Z-sections. Consequently, it is highly desirable to develop advanced finite element models to assess the structural behaviour of continuous as well as lapped Z-sections with generic bolted connections. It should be noted that a number of numerical investigations on the structural behaviour of both continuous Z-sections and lapped Z-sections under one point loads had been performed by the authors (Ho & Chung 2006, Chung et al. 2008). It is found that the deformation characteristics of the lapped Z-sections are simulated satisfactorily up to the maximum applied moments. However, the simulation beyond the maximum applied moments is found to be less satisfactory. Hence, this paper reports a systematic numerical investigation to examine the suitability of different shell elements and different mesh configurations in modelling the structural behaviour of lapped Z-sections.

SCOPE OF WORK

This paper presents a systematic numerical investigation into the structural behaviour of high strength cold-formed steel lapped Z-sections under one point loads. A number of finite element models with thin degenerated and continuum shell elements are established with different mesh configurations. Nonlinearity in mechanical properties, geometrical deformations as well as boundary and contact conditions has been carefully incorporated. The models have been calibrated against the test results of a series of continuous and lapped Z-sections with different lap lengths. Moreover, some experiences on numerical modelling are also presented.

The research work forms part of a comprehensive experimental and numerical investigation into the structural performance of cold-formed steel multi-span purlin systems. The present numerical investigation aims to improve the finite element techniques in modelling complicated behaviour of continuous and lapped Z-sections with bolted moment connections which exhibits local, distortional and overall buckling under combined bending and shear.

ONE POINT LOAD TESTS ON LAPPED Z-SECTIONS

Figure 1 illustrates the test setup of a series of simply supported Z-sections with either continuous or lapped sections at mid-span under one-point loads (Ho & Chung, 2004).

The effective span of the lapped Z-sections is 2.55 m while their nominal depth, D, and their nominal thickness, t, are 150 and 1.6 mm respectively. Moreover, interconnections are provided to each pair of Z-sections at a regular interval of 400 mm. Table 1 summarizes the details of the test series, in particular, the lap lengths, and both the measured geometrical dimensions and the measured mechanical properties of the Z-sections are also provided.
In general, section failure at the end of the laps under combined bending and shear is found in all tests with different extents of cross-section distortion. The test results of the series are adopted for calibration of the proposed finite element models established in the present investigation.

**FINITE ELEMENT MODELLING OF CONTINUOUS Z-SECTIONS**

The general finite element package ABAQUS (Version 6.5) is employed in the present investigation, and the following two shell elements are adopted to model the cold-formed steel Z-sections:

- a four-noded conventional thin degenerated shell element S4R, and
- a highly efficient eight-noded continuum shell element SC8R.
Details of the two shell elements (ABAQUS) are presented in Figure 2.

**Element S4R**

- **Conventional shell model**
  - The geometry is specified at the reference surface, and the element thickness is defined as a section property.
  - Degrees of freedom: \( u_1, u_2, u_3 \), \( \theta_1, \theta_2, \theta_3 \)
  - The integration point is located at the centroid of the element.

**Element SC8R**

- **Continuum shell model**
  - The full 3-D geometry is specified, and the element thickness is defined by the nodal geometry.
  - Degrees of freedom: \( u_1, u_2, u_3 \)
  - The integration point is located at the centroid of the element.

**Figure 2: Details of shell elements S4R and SC8R**

Two different models are established for comparison as shown in Figure 3:
- Model A0 is a full model with two continuous Z-sections interconnected with cold-formed steel strips; there are about 8300 nodes and 6250 elements in the model.
- Model E3 is a simplified model with only one continuous Z-section, and springs are provided at regular intervals to the top flange of the Z-section; there are about 4000 nodes and 3600 elements in the model.

In both models, material and geometrical non-linearity is incorporated with both measured material properties and geometrical dimensions given in Table 1. Moreover, initial geometrical imperfection is also incorporated through the use of the first eigenmode shape corresponding to the lowest possible buckling mode shape of the Z-sections. The magnitude of the out-of-flatness of the plate elements in the sections is taken to be 0.25 \( t \) where \( t \) is the nominal section thickness. It should be noted that the lateral restraint condition of the Z-section in Model E3 is carefully modelled to simulate the actual arrangement adopted in the test. The lateral restraint stiffness at 1.0 kN/ mm is adopted after a sensitivity study; refer to Ho & Chung (2009a & 2009b) for further details.

**Numerical Results**

Figure 4 plots both the measured and the predicted moment-rotation curves of the continuous Z-sections obtained with both Models A0 and E3. It is shown that although the elastic deformation characteristics obtained from both models with elements S4R and SC8R are very close to one another, the models with elements SC8R are able to give very close predictions to the measured moment-rotation curves at very large deformations. The maximum applied loads predicted by both models are also very close to one another, as shown in Table 2.
TABLE 2

<table>
<thead>
<tr>
<th>Test</th>
<th>Lap length $2L_p$ (mm)</th>
<th>Failure mode</th>
<th>Mid-span moment (kNm)</th>
<th>Model factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M_{\text{Test}}$</td>
<td>$M_{\text{S4R}}$</td>
</tr>
<tr>
<td>ZACONR</td>
<td>---</td>
<td>MV*</td>
<td>10.17</td>
<td>10.01</td>
</tr>
<tr>
<td>ZA018R</td>
<td>180</td>
<td>MV_e*</td>
<td>8.91</td>
<td>9.10</td>
</tr>
<tr>
<td>ZA024R</td>
<td>240</td>
<td></td>
<td>9.59</td>
<td>9.54</td>
</tr>
<tr>
<td>ZA030R</td>
<td>300</td>
<td></td>
<td>10.80</td>
<td>10.29</td>
</tr>
<tr>
<td>ZA060R</td>
<td>600</td>
<td></td>
<td>14.41</td>
<td>13.39</td>
</tr>
<tr>
<td>ZA090R</td>
<td>900</td>
<td></td>
<td>16.25</td>
<td>15.95</td>
</tr>
</tbody>
</table>

Notes: *MV: Section failure at mid-span under combined bending and shear; and
*MV_e: Section failure at the end of lap under combined bending and shear.
Model factor = $M_{\text{Test}} / M_{\text{FEM}}$
As there are five lapped Z-sections with different lap lengths in the test series, a total of five different models are established. For ease of presentation, Figure 5 presents the finite element model of the lapped Z-sections of Test ZA 090R which contains about 6000 nodes and 5250 elements. It should be noted that the mesh is adopted after a mesh sensitivity study with suitable mesh refinement provided locally in the mid-span region. Refer to Table 1 for details of the material properties and the geometrical dimensions of the Z-sections. Both the initial geometrical imperfections and the lateral restraint conditions of the Z-sections are incorporated in the same way as those models of the continuous Z-sections reported in the previous section.

Figure 5: Finite element model of lapped Z sections under one point loads
For the lapped Z-sections with bolted moment connections, spring elements are employed to simulate
the bolted fastenings between the two Z-sections, and the load-bearing deformation curves of
the bolted fastenings along three orthogonal axes are illustrated in Figure 6.

![Bolt fastening modelled by 3 spring elements](image)

**Figure 6: Modelling of bolted fastening with spring elements**

### Bolt fastening modelled by 3 spring elements

![Bolt fastening modelled by 3 spring elements](image)

- For bearing deformation $\delta_b$:
  - $F_b = \alpha_b \cdot \delta_b$
  - $\alpha_b = 30 \cdot \delta_b$
  - when $\delta_b \leq 0.02$ mm
  - $= 1.25(\delta_b - 0.02) + 0.6$ when $0.02 < \delta_b \leq 0.40$ mm
  - $= 0.825\left(\frac{\delta_b - 0.05}{0.35}\right) + 1.075$ when $\delta_b > 0.40$ mm

- For axial deformation $\delta_a$:
  - $k_x = k_y = k_z = k_a$ (i.e. the slope of the force-deformation curve)

- $k_a = 50 A_b$, where $A_b$ is the cross-sectional area of bolt

- $F_a = k_a \cdot \delta_a$

- $k_a = 50 A_b$

- $\delta_a$ is the axial deformation

- $F_a$ is the axial force

- $k_a$ is the spring constant

- $A_b$ is the cross-sectional area of bolt

- $F_b$ is the bearing force

- $\delta_b$ is the bearing deformation

- $k_x$, $k_y$, $k_z$ may be obtained as the slope of the force-deformation curve
Numerical Results

Failure modes

Section failure with significant cross-section distortion under combined bending and shear at the end of the laps is well predicted in the models which is highly resemble to those failure modes observed in the tests as shown in Figure 7.

![Figure 7: Typical failure modes of lapped Z sections](image)

Moment rotation curves

As shown in Figure 8, all the measured and the predicted moment-rotation curves of the lapped Z-sections are plotted onto the same graphs for direct comparison. Good comparison between the measured and the predicted curves are found not only in the elastic deformation ranges but also in the large deformation ranges.

Moment resistances

Table 2 summarizes the measured and the predicted moment resistances of the lapped Z-sections. It is shown that the predicted moment resistances are always close to but smaller than the measured values. The average model factor of the predicted moment resistances obtained from the finite element models is found to be 1.03. Hence, it is demonstrated that the finite element models are effective to predict many aspects of the structural behaviour of the lapped Z-sections with different lap lengths.
CONCLUSIONS

This paper presents a systematic numerical investigation into the structural behaviour of high strength cold-formed steel lapped Z-sections under one point loads. A number of finite element models with two different element types and mesh configurations have been established with different boundary and contact conditions, and material and geometrical non-linearity has been incorporated into the finite element models together with initial geometrical imperfections. The models are calibrated against the test results of a series of lapped Z-sections with different lap lengths.

Figure 8: Moment-rotation curves of lapped Z sections with different lap lengths
It is found that the maximum applied moments and the elastic deformation characteristics predicted by the models with both thin degenerated 4-noded shell elements and continuum 8-noded shell elements are very close to the measured results. However, only the numerical models using continuum 8-noded shell elements are able to give close predictions to the deformation characteristics of continuous Z-sections at very large deformations, when compared with those using thin degenerated 4-noded shell elements. Hence, with suitable boundary and contact conditions, the finite element models using continuum 8-noded shell elements are shown to be highly effective in assessing the structural behaviour of lapped Z-sections with different lap lengths along their entire deformation ranges.

The present numerical investigation aims to improve the finite element techniques in modelling complicated behaviour of continuous and lapped Z-sections with bolted moment connections which exhibits local, distortional and overall buckling under combined bending and shear. The experiences are expected to facilitate subsequent numerical analyses on multi-span purlin systems with lapped Z-sections with different member configurations and connection arrangements.

ACKNOWLEDGEMENT

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BENDING-SHEAR BEHAVIOR OF DEEP CONCRETE FILLED DOUBLE STEEL TUBULAR BEAM

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KEYWORDS
Deep concrete filled double steel tubular beam, Concrete filled steel tubular beam, Inner-to-outer diameter ratio, Failure mode, Bending-shear.

ABSTRACT
Concrete filled double steel tubular, abbreviated in CFDT, member has a hollow section consisting of two concentric steel tubes and filled concrete between them. It is consequently lighter than an ordinary concrete filled tubular, so-called CFT, member with a solid section. The aim of this study is to investigate their bending and shearing characteristics through eight specimens of deep CFDT beam by an asymmetrically three point loading test. Two test parameters considered were outer tube’s diameter-to-thickness ratio and inner-to-outer diameter ratio. As the results, observed failure modes were so controlled by diameter-to-thickness ratio that the modes could change from cracking of outer tubes caused by bending action to shearing compression failure of in-filled concrete as the ratio increased. The CFDT specimens of inner-to-outer diameter ratios ranging from 0.23 to 0.47 held a good deformability as same as that of the CFT specimens. Experimental strengths were almost larger than a proposed estimated equation of shearing strength of RC deep beam and their strengths with the ratios being less than 0.47 fair agreed with the bending strength based on both Bernoulli assumption and stress block technique. Finally, bi axial elasto-plastic stress behavior of inner and outer tubes under plane stress conditions was also mentioned.

INTRODUCTION
A concrete filled double steel tubular, called CFDT in subsequence, member has a hollowed cross section consisting of two concentric steel tubes and filled concrete between them as shown in Fig. 1(a). The member is, thus, lighter than an ordinary concrete filled steel tubular, so-called CFT(see Fig. 1(b), AIJ[1]). When the member is applied to a high raised bridge pier, the reduction of its self weight can draw the smaller scaled substructures’ dimensions beneath the pier. The reason is that the light weight pier can lead a considerable decreased seismic horizontal design action for the substructures design.
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Reviewing the past studies on CFDT, we can first find experimental and analytical studies on double steel tubes with filled polymer concrete (Wei et al. [2-3]). In the followings, Zhao et al. [4] carried out approximate calculation of bending strength of double square skin tubes with filled concrete, and a study on constitutive law of in-filled concrete (Tawaratani et al. [5]) were found. Moreover, detailed reviews on CFDT by Zhao et al. [6] can also be found.

Under the above-described background, we have conducted two of their experimental study with various dimensions, as an inner-to-outer diameter ratio: $D_i / D_o$ and also an outer diameter-to-outer tube's thickness ratio: $D_o / t_o$. One was as stub column members under axial action (Uenaka et al. [7]). Their obtained axial loading capacities decreased as $D_i / D_o$ increased. These findings indicate that the confinement effect of the filled concrete due to their outer tubes decreased as $D_i / D_o$ increased. The other was as beam members under a pure bending action (Uenaka et al. [8]). It is noted that their outer tube's cracking occurred owing to the local moment sustained by the end plate.

This study is devoted to investigate bending-shear characteristics of deep CFDT beam through a symmetric three point loading test of eight specimens with various $D_i / D_o$ and $D_o / t_o$. Moreover, biaxial stress behaviors of the double tubes were also mentioned.

**EXPERIMENTAL METHOD**

The details of the specimens used in this study are listed in Table 1. All the specimens commonly have

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**TABLE 1**

**LIST OF SPECIMENS**

<table>
<thead>
<tr>
<th>No.</th>
<th>Tag</th>
<th>$D_i$ (mm)</th>
<th>$t_i$ (mm)</th>
<th>$D_o$ (mm)</th>
<th>$t_o$ (mm)</th>
<th>$D_i / D_o$</th>
<th>$D_o / t_o$</th>
<th>$f_{sy}$ (MPa)</th>
<th>$f_u$ (MPa)</th>
<th>$f_{c'}$ (MPa)</th>
<th>$W_{CFDT}$</th>
<th>$W_{CFT}$</th>
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<tr>
<td>1</td>
<td>s16-000</td>
<td>0.0</td>
<td>0</td>
<td>0.0</td>
<td>0.00</td>
<td></td>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
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<td>1.6</td>
<td>0.23</td>
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<td>0.70</td>
<td>381.3</td>
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<td></td>
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</tr>
<tr>
<td>3</td>
<td>s16-750</td>
<td>75.0</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>4</td>
<td>s16-1125</td>
<td>112.5</td>
<td>1.6</td>
<td>0.70</td>
<td>0.63</td>
<td></td>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>s23-000</td>
<td>0.0</td>
<td>0</td>
<td>0.00</td>
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<td></td>
<td></td>
<td>2.3</td>
<td>69.6</td>
<td>406.8</td>
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<td>6</td>
<td>s23-375</td>
<td>37.5</td>
<td>2.3</td>
<td>0.23</td>
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<td>7</td>
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420mm in total length and 160mm in outer tube’s diameter ($D_o$). Whereas, the outer and inner tube’s thicknesses were varied as 1.6 or 2.3mm and inner tube’s diameter were 37.5, 75.0, or 112.5mm. Their $D_i / t_o$ and $D_i / D_o$ were, thus, ranging from 69.6 to 160 and 0.0 0(CFT), 0.23, 0.47 or 0.70, respectively. Yielding point: $f_{cy}$ and breaking point: $f_u$ obtained by coupon tensile test are also listed in Table 1.

![Figure 3: Position of biaxial strain gages](image)

The symmetric three point loading apparatus used herein is illustrated in Fig. 2 and Photo 1. Bending-shear action was applied to all the simple supported specimens by the universal tester. The in-filled concrete must not work as tied-arch due to lateral non-supported ends being forced the concrete out. Therefore, it may be assumed that the smaller testing value results than estimation based on the tied arch mechanism.

As shown in Fig. 3, a total of 24 biaxial strain gauges were attached on outside of double steel tubes to measure their stress distributions. Two displacement transducers were arranged at the span center to measure vertical displacement of the beam.

**RESULTS AND DISCUSSIONS**

**Failure Mode**

The observed failure mode of the specimens with $D_i / D_o$ =0.23 was cracking of outer tube as shown in Fig. 4(a). In the specimens with large inner tube’s diameter, namely, $D_i / D_o$ is larger than 0.47, in-plane deformation of inner tube was identified to restrain in-filled concrete from transforming circular into ellipse sections, so-called ovalizing(see Fig. 4(b)). Furthermore, in the specimens with $D_i / D_o$ is equal to 0.7, their compressive failure of in-filled concrete occurred within the early loading stage, afterward, their cross section gradually ovalized. No cracking of inner tube was found, though yielding of that was confirmed. The in-filled concrete between double tubes was pushed out from the
both tube's ends as shown in Fig. 5, in which no suppression were paralleled to loading direction.

Deflections

The relationship between applied load($P$) and central displacement are shown in Fig. 6. The specimens, $D_i/D_o$, of which are less than 0.47, drawn almost same behavior until the failure. This fact indicated that the deep CFDT beam has a good deformability and a large toughness under bending-shear action. The in-filled concrete crushing occurred in the initial loading stage, consequently, the specimen with $D_i/D_o=0.7$ exhibited the lowest initial rigidity and an inferior deformability.

Ultimate Strengths

Shearing Strength

Proposed shearing strength(Niwa [9]) of deep RC beam is expressed as below,

$$V_u = 0.24f'_{cu}^{1/3} \left( 1 + (100p_{w})^{1/2} \right) \left( 1 + 3.33r/d \right) b_w d$$

where $V_u$: shearing strength(N), $f'_{cu}$: concrete cylinder strength(MPa), $b_w$: thickness of web concrete, $d$: effective depth, $r$: width of loading plate(=100mm), $a$: shear span length, $p_{w}$: $A_s/b_w d$, $A_s$: equivalent quarter area of inner and outer tubes, respectively. Their predicted shearing strengths are listed in the seventh column of Table 2.

Figure 7 shows the relationship between normalized shearing strength ($V_{exp}/V_u$) and $D_i/D_o$. Although somewhat scattering was validated, experimental strengths were larger than the relevant estimations based on deep RC beam. Furthermore, their shearing strength ratio($V_{exp}/V_u$) increased as the $D_i/D_o$ increased up to 0.47, which suggested that inner tube works as the shear reinforcement. However, the shearing strength of the specimen with $D_i/D_o=0.7$ abruptly decreased. This is also because that cross section initially ovalized due to local crushing of thin-web-concrete as described in Failure mode.

Bending Strength

Longitudinal stress distribution of double tubes and in-filled concrete at ultimate state will be assumed according to Bernoulli assumption and the stress block technique were employed. Bending strength of
### TABLE 2

<table>
<thead>
<tr>
<th>No.</th>
<th>Tag</th>
<th>$D_i / D_o$</th>
<th>$P_{exp}$ (kN)</th>
<th>$V_{exp}$ (kN)</th>
<th>$M_{exp}$ (kN m)</th>
<th>$V_u$ (kN)</th>
<th>$M_u$ (kN m)</th>
<th>$V_{exp}$</th>
<th>$M_{exp}$</th>
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<tr>
<td>1</td>
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<td>0.00</td>
<td>390.0</td>
<td>195.0</td>
<td>31.2</td>
<td>90.8</td>
<td>19.1</td>
<td>2.15</td>
<td>1.63</td>
</tr>
<tr>
<td>2</td>
<td>s16-375</td>
<td>0.23</td>
<td>391.0</td>
<td>195.5</td>
<td>31.3</td>
<td>74.6</td>
<td>20.5</td>
<td>2.62</td>
<td>1.52</td>
</tr>
<tr>
<td>3</td>
<td>s16-750</td>
<td>0.47</td>
<td>347.9</td>
<td>174.0</td>
<td>27.8</td>
<td>57.8</td>
<td>23.3</td>
<td>3.01</td>
<td>1.19</td>
</tr>
<tr>
<td>4</td>
<td>s16-1125</td>
<td>0.70</td>
<td>142.1</td>
<td>71.1</td>
<td>11.4</td>
<td>39.7</td>
<td>25.4</td>
<td>1.79</td>
<td>0.45</td>
</tr>
<tr>
<td>5</td>
<td>s23-000</td>
<td>0.00</td>
<td>464.5</td>
<td>232.3</td>
<td>37.2</td>
<td>99.8</td>
<td>24.3</td>
<td>2.33</td>
<td>1.53</td>
</tr>
<tr>
<td>6</td>
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<td>497.8</td>
<td>248.9</td>
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<td>82.9</td>
<td>25.8</td>
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</tr>
<tr>
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<td>0.47</td>
<td>460.6</td>
<td>230.3</td>
<td>36.8</td>
<td>65.3</td>
<td>29.4</td>
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</tr>
<tr>
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<td>s23-1125</td>
<td>0.70</td>
<td>84.3</td>
<td>42.1</td>
<td>6.7</td>
<td>45.8</td>
<td>32.7</td>
<td>0.92</td>
<td>0.21</td>
</tr>
</tbody>
</table>

**Figure 7:** Normalized shearing strength and $D_i / D_o$

**Figure 8:** Normalized bending strength and $D_i / D_o$

CFDT beam was estimated as follows (Uenaka et al. [8]).

$$ M_u = \frac{2k f_y}{3} (R_o^3 \cos^3 \alpha_o - R_i^3 \cos^3 \alpha_i) + 4f_y (R_o^2 t_o \cos \alpha_o + R_i^2 t_i \cos \alpha_i) $$

where $f_y$: yielding point of steel tubes, $k$: equivalent stress block c coefficient (=0.85), $R_i$ and $R_o$: radii of the inner and outer tubes, $t_i$ and $t_o$: the inner and outer tube thickness and $\alpha_i$ and $\alpha_o$: angles between the horizontal centroid axis and radial lines across the neutral axis at the inner and outer tubes diameters, respectively. The summation of axial stress is expressed as
Ultimate pure bending strength ($M_u$) also can be calculated as follow steps: First, a provisional solution satisfied with no axial action condition in Eq. (3) is sought by an iteration method with prescribed incremental angle of $\alpha$. Second, the angle of $\alpha$ is introduced into Eq. (2) to obtain the ultimately pure bending strength.

The relation between normalized bending strengths ($M_{exp}/M_u$) and $D_i/D_o$ are shown in Fig. 8. In the specimens with $D_i/D_o<0.47$, the experimental values were from 1.2 to 1.6 times as large as the estimated values. It seems that strain hardening of the both tubes occurred. Bending rigidity decreased considerably owing to ovalizing. Furthermore, the experimental strength of the specimens with the largest inner tube's diameter, $D_i/D_o = 0.7$, is quite smaller than the bending estimations.

**Biaxial Stress**

**Calculation of elasto-plastic stress**

Assuming a perfectly plastic material subjected to von Mises’ yield criterion, corresponding yield condition of plane stress problem is expressed as

$$\sigma_z^2 - \sigma_r \sigma_\theta + \sigma_\theta^2 - f_{sy}^2 = 0 \quad (4)$$

where $\sigma_z$ and $\sigma_\theta$ are stress in axial and circumferential directions.

When the tubes enter elasto-plastic zone, the relationship between stress increments $d\sigma_z, d\sigma_\theta$ and
strain increments $d\varepsilon_z$, $d\varepsilon_\theta$ are obtained.

$$\begin{bmatrix} d\sigma_z \\ d\sigma_\theta \end{bmatrix} = \begin{bmatrix} E & 1 - \nu^2 \\ \nu & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \nu \end{bmatrix} \begin{bmatrix} S_1^2 & S_1 S_2 \\ S_2 S_1 & S_2^2 \end{bmatrix} \begin{bmatrix} d\varepsilon_z \\ d\varepsilon_\theta \end{bmatrix}$$

where

$$S = s_z S_1 + s_\theta S_2, \quad S_1 = \frac{E}{1 - \nu^2}(s_z + \nu s_\theta), \quad S_2 = \frac{E}{1 - \nu^2}(s_\theta + \nu s_z)$$

then, $\nu$: Poisson's ratio (=0.3), $s_\theta$ and $s_z$ are deviatoric stresses of circumferential and axial directions. From the later part of this section, compressive stresses are taken as positive.

**Stress histories**

Biaxial stress histories of the outer tube in bending-tension region are shown in Fig. 9, in which $x$ and $y$ axes are normalized by the yielding point $f_y$. After yielding point, the trajectories of both a xial and circumferential of stress flowed toward tensile region due to confined stress induced by in-filled concrete. This fact coincides with the consequence of pure bending test which had previously conducted by the authors (Uenaka et al. [8]).

Whereas, stress histories of the inner tube under tension due to bending is shown in Fig. 10. The large circumferential stress $\sigma_\theta$ occurred in an initial loading stage. Such phenomenon indicates that lower in-filled concrete supported at their lower portion. Moreover, in the specimen with $D_i/D_o=0.7$, its history oriented onto $x$-axis, which shows the largest compressive circumferential stress. This fact indicates that thinner web concrete crushing occurred the earliest of all the specimens as described in Failure mode.

Figure 11 shows stress histories of the inner tube under compression due to bending. In this figure, $x$ and $y$ axes are also divided by yielding point $f_y$. Their circumferential stress also proceeded to tensile region. Such phenomenon may suggest the in-filled concrete strictly confine circumferential direction of upper section on the inner tube.

**CONCLUDING REMARKS**

Mechanical behavior of deep concrete filled double steel tubular beam subjected to bending-shear investigated experimentally. From the results, the following remarks can be drawn.

1. The observed failure modes specimens, which were $D_i/D_o<0.23$, was cracking of outer tube in the bending-tensile. In the specimens with $D_i/D_o$ being 0.47, in-plane deformation of inner tube was identified to restrain in-filled concrete from outside-transforming.

2. The specimens, whose $D_i/D_o$ was less than 0.47, drawn almost same behavior of CFT until the failure. The specimen with $D_i/D_o=0.7$ exhibited the lowest initial rigidity and an insufficient deformability.
3. Experimental shearing strengths were larger than the corresponding estimations as deep RC beam. Up to $D_i / D_o = 0.47$, the shearing strength ratio $(V_{exp} / V_u)$ increased.

4. In the specimens with $D_i / D_o$ being up to 0.47, experimental bending strengths were up to 1.6 times as large as the estimations. Bending strength of the specimens with the largest inner tube's diameter, $D_i / D_o$ equals 0.7, is smaller than the estimations due to in-filled concrete crushing.

5. After yielding, both axial and circumferential stresses of outer tubes under bending-tension flowed toward tensile region due to confined stress induced by in-filled concrete.

6. Stress histories of inner tube under tension due to bending, the large circumferential stress $\sigma_\theta$ occurred in the initial loading. Such a phenomenon indicates that cracking of in-filled concrete pushes lower portion of the inner tubes.

7. Circumference stress of inner tube under bending-compression occurred in the initially loading stage due to confined by in-filled concrete.

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REFERENCES


EARLY-AGE SHRINKAGE AND SLAB CASTING SEQUENCES IN A LONG STEEL-CONCRETE COMPOSITE VIADUCT

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KEYWORDS

Bridges, construction phases, durability, early-age concrete, shrinkage, steel-concrete composite structures.

ABSTRACT

This paper presents a study on the effectiveness of different casting sequences to control the cracking tendency in a long continuous steel-concrete composite viaduct. The main features of the early-age behaviour of concretes are addressed and the relative modelling options are discussed. Then, the application on a real viaduct with 12 spans of different length is presented considering a continuous and two fractionated sequences of casting. The importance in the selection of the casting sequence and of the segment length is discussed. The thermal shrinkage and the weight of new segments confirmed to be the main cause of the slab premature cracking. The enhanced tensile creep may reduce stresses which remain below the concrete tensile strength even in the long term.

INTRODUCTION

The concrete cracking phenomenon during construction of continuous steel-concrete composite decks is a problem of practical interest that is influenced by the selection of the casting sequences and by the early-age concrete behaviour. In fact, in the case of long viaducts, the slab is constructed on steel beams fractionating the concrete casting. Due to the deck compliance, the weights of the concrete and travelling formworks may induce tensile stresses in the slab sections which have already hardened. The early-age concrete shrinkage constituted by thermal, endogenous (chemical) and drying components, is responsible for inducing further tensile stresses. The cracking phenomenon observed in a large number of newly constructed composite bridge decks with cast-in-situ slabs (SETRA \cite{SETRA}, Krauss and Rogalla \cite{Krauss}), affects not only the hogging regions but even the sagging regions that are usually considered uncracked, and is deemed to be strongly harming for the bridge durability. The phenomenon can be controlled by optimising the concrete pouring sequences by casting the slab sections first at the centre of each span and then over the interior supports as well as by using concretes with enhanced mechanical properties.
The authors have already studied the influence of the casting sequences and concrete early-age behaviour on continuous steel-concrete composite bridges with few spans [3]. This paper presents a study on the effectiveness of the casting techniques to control the tensile stresses in a real continuous steel-concrete composite bridge that will be constructed in Italy. In a first section, the main features regarding the behaviour of early-age concretes are presented and the modelling options that should be adopted in the analyses are discussed. Subsequently, the results obtained with dedicated analyses carried out on the “Serra Cazzola” viaduct are discussed pointing out the improvements obtained by considering a proper sequential casting instead of a continuous casting of the slab.

**BEHAVIOUR OF CONCRETE AT EARLY-AGE**

The concrete behaviour at early-age has been the object of a number of studies due to the importance of early cracking phenomena for the durability of concrete structures (Mays [4]). The mechanism of early cracking formation is due to the shrinkage when it is restrained by reinforcements, other structural elements or by the concrete itself when important thermal gradients arise between the external and the internal parts of a massive element. The concrete cracking problem is typically very delicate since it is due to the achievement of very low tensile strengths. Disregarding early shrinkage components as well as effects of stress relaxation may compromise any calculation effort leading to unrealistic results. Thus, modelling the behaviour of the concrete at the suitable level of sophistication is of primary importance to perform a proper analysis of a structure.

**Concrete shrinkage**

Even if dividing concrete shrinkage into different components is a debatable question (Sellevold and Bjøntegaard [5]), considering the following equation (Dezi et al. [6]):

\[
\varepsilon(t) = \varepsilon_T(t, t_c) + \varepsilon_E(t, t_c) + \varepsilon_D(t, t_c)
\]

in which the concrete shrinkage is given by the summation of thermal (\(\varepsilon_T\)), endogenous (\(\varepsilon_E\)) and drying (\(\varepsilon_D\)) components, each characterised by its own time evolution and duration, is quite convenient in the practice.

The cement hydration is an exothermic reaction that results in the concrete heating; the subsequent cooling induces significant volume reductions. In slender elements, like bridge slabs, the heating of the concrete happens almost completely before end setting when the concrete is in the plastic state. Thermal gradients are not very important and the subsequent volume reduction, due to cooling, may be reviewed simply as a shrinkage component. No suggestions about the temperature reduction are reported by standards except Eurocode 4 part 2 (EN 1994-2 [7]) that recommends a temperature difference of 20 °C between concrete slab and steel beam of composite bridge decks. Thermal shrinkage is the most rapid compared to the other components and its effects are fundamental for the early-age behaviour of slender concrete elements. Experimental cooling curves observed for slabs (Ducret & Lebet [8]) are almost linear and the volume reduction of the structural concrete may be calculated by multiplying the thermal reduction by the coefficient of thermal expansion. The simple following formula can be considered:

\[
\varepsilon_T(t, t_c) = -\alpha(t - t_c)\Delta T \geq -\alpha\Delta t_c \Delta T
\]

where \(\Delta T\) is the rate of cooling expressed in °C/day, \(t\) and \(t_c\) are the current and end-setting time, respectively, and \(\Delta t_c\) is the cooling duration. Usual values of \(\Delta T\) and \(\Delta t_c\) are 4-5 °C/day and 4-6 days, respectively.

Endogenous shrinkage, namely the volume reduction that occurs after end-setting due to the cement chemical reactions, is the second shrinkage component. It is the consequence of the self-desiccation
process, namely the reduction of water in the capillarity pores to hydrate the cement. It is generally deemed that this component does not depend on RH even if some recent research seems to deny this (Sellevold and Bjøntegaard [5]). The endogenous component is quite important for high strength concretes while may be neglected for normal concretes. It develops completely over a period of a few months from pouring and cannot be reversed. Nowadays the standards like Eurocode 2 (EN1992-1-1 [9]) suggest specific equations for its prediction.

Finally, drying shrinkage is the long-term component and is strongly dependent on RH. Water contained in the capillarity pores varies in time in order to achieve the hygrometric equilibrium with the environment. Generally, the water evaporates from the pores reducing the volume of concrete elements; nevertheless, the process may be reversed if they are exposed to saturated environments. The phenomenon starts after the time of end-curing \( t_s \), and develops for the entire life of the structure with higher rate exhibited during the early months. This component is somewhat important for normal concretes whereas it is less important for high strength concretes. All main codes of practice like Eurocode 2 (EN1992-1-1 [9]) suggest final values of drying shrinkage depending on RH, the cross section notional size and the concrete strength. Evolution laws are also suggested for advanced analyses.

**Tensile creep**

Creep phenomena are quite important in concrete structures. Creep functions valid for compression stresses are usually adopted in the analyses for tensile stresses even if more remarkable effects are expected to occur in this latter case. In consideration of the low tensile strength of the concrete, national and international standards suggest creep functions for compression stresses whereas tensile creep functions are available only in very specialised publications (e.g. Atrushi [10]). At early-age, slabs of composite bridges are subjected exclusively to tensile stresses and the adoption of a more suitable model is necessary to capture the real behaviour of the bridge. Because tensile creep is particularly important when stress approaches the concrete tensile strength, Bažant and Oh [11] suggested modifying the creep compliance valid for compressed concretes multiplying the creep coefficient \( \phi \) by three when the tensile stress is greater than 50% of the tensile strength \( f_{ctm} \). This introduces a non-linear behaviour for the concrete but in practical cases the problem can be simplified multiplying by 1.5 the creep coefficient for every tensile stress level. With this approach, the creep coefficients adopted in practical analyses may be those suggested by the main codes (e.g., EN1992-1-1 [9]).

**Tensile strength**

Concrete tensile strength and its time evolution are of primary importance in determining the tendency of the slab to crack during construction. The tensile strength evolution is more rapid compared to that of the compression strength. Kanstad et al. [12] proposed to modify the Model Code 90 [13] law, valid for the compression strength, according to the equation

\[
f_{ctm}(t) = f_{ctm} \left( 1 + \left( \frac{t}{t_0} \right)^s \right)
\]

where coefficient \( s \) is set equal to 0.20 for rapid hardening high-strength cements and \( n \) is a power less than 1 (typically between 0.55 and 0.70) that has to be determined from experimental results. Eurocode 2 (EN1992-1-1 [9]) suggests \( n = 1 \) for \( t < 28 \) days and \( n = 0.7 \) for \( t \geq 28 \) days.

**OPTIMIZATION OF THE CONSTRUCTION SEQUENCES OF THE “SERRA CAZZOLA” VIADUCT**

The “Serra Cazzola” viaduct will be constructed along the Porto Empedocle SS640 road in Sicily (Italy). It is a 12 span continuous steel-concrete composite bridge (Fig.1a) with span lengths varying from a minimum of 55 m (for the external spans) to a maximum of 120 m (for the central span) for a
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The total length of 0.980 km. The cross section consists of a twin-girder (Fig.1b): the slab is 26.5 m wide and 0.25 m thick and is supported by cantilevered cross beams located along the deck every 5.0 m. The steel beams have the depth variable from 2.9 m to 5.5 m according to a parabolic profile; the web and flange thicknesses are also variable as shown in Fig.1c with reference to the major span. The geometric ratio of longitudinal reinforcements in the slab is variable from 1%, at the span sections, to 2% at the inner support sections. Studs with 22 mm diameter are used for the shear connection and are placed along the deck according to four different layouts so as to fulfil local strength requirements.

The slab is poured on precast-deck elements used as formworks. With reference to the longitudinal direction, a casting speed of 43 m/day is considered in the calculation. It is assumed that the concrete setting ends at 12 hours from pouring and that the curing duration is two days. Three casting schemes are considered: a continuous sequence from one end to the other end of the deck (scheme (a) of Fig.2) and two fractionated sequences consisting in alternating the casting of a sagging segment to that of a hogging segment (schemes (b) and (c) of Fig.2). It is easy to understand that in this way the weight of every new sagging segment reduces tractions due to the early shrinkage at the hogging regions already poured. Scheme (b) differs from scheme (c) for the length of the various segments: in the first case sagging and hogging segments are chosen so as to be symmetric with respect to the axes of the relative spans (for sagging segments) and supports (for hogging segments); in the second case the sagging sections are reduced and the hogging section enlarged in a non-symmetric way.

Figure 1: “Serra Cazzola” Viaduct: (a) longitudinal view; (b) cross section; (c) steel girders’ profile
The time dependent analysis is performed considering the creep and shrinkage functions suggested by Eurocode 2 (EN1992-1-1 [9]), assuming $RH = 65\%$ and $f_{ck} = 37$ MPa. For the thermal shrinkage a reduction of $20$ °C in 6 days is considered from the time of end-setting.

**Modelling of the construction phases**

The analysis is carried out with the dedicated computer program proposed by the authors in [6] that allows taking into account the early-age behaviour of the concrete considering the features previously discussed. The composite deck is schematised as a steel-concrete composite girder in which the steel beam has variable geometry, i.e. variable depth and variable thickness of web and flanges. The connection between the concrete slab and the steel beam is considered to be flexible, permitting longitudinal relative displacement (partial shear interaction).

In order to analyse the slab casting sequences, the girder is subdivided into segments each characterised by its own instant of concrete pouring. For each segment, two phases are considered: in the first phase, for times before the concrete pouring $t_p$, only the steel section is considered and the weight of fresh concrete constitutes an applied load. In the second phase, for times after concrete end setting, the deck is assumed to act compositely and subjected to early shrinkage. Linear elastic constitutive laws are considered for the steel beams and shear connection while a linear viscoelastic law, described by means of an integral type relationship, is assumed for the concrete slab. The endogenous, thermal and drying shrinkage are introduced as non-mechanical strains varying in time. The evolution of the real geometry of the structure is captured thanks to the constitutive law of the concrete that is assumed to give zero stresses before end-setting. The numerical procedure makes it possible to capture the evolution of the complex static scheme due to the construction phases and permits following the complete time evolution of the displacements, stresses and shear flows at the beam-slab interface.

**Analysis results**

Figure 3 shows the stresses, measured at the concrete slab mid-plane at the end of the slab construction, produced by the continuous and fractionated castings. For the sake of clarity, sequences (b) and (c) are compared with the continuous casting (a) in two different pictures. All the slab segments are affected by tensile stresses for both cases of continuous and fractionated casting: in the case of continuous casting the maximum stresses are observed to occur in the hogging moment regions while in the fractionated castings these take place in the sagging moment regions. The differences between the two fractionated sequences can be summarised as follows: sequence (b) is not effective in limiting concrete tractions because the same stress levels of sequence (a) are achieved in
the sagging moment regions despite the significant stress reduction produced in the hogging moment regions; on the contrary, sequence (c), obtained with non-symmetric sections, permits a sensible reduction of the stresses. This demonstrates the importance in selecting an appropriate casting sequence and in defining a suitable length for the casting segments in which the slab is fractionated.

Figure 4 shows the stress development for cross sections A and B (see Fig.3). The time histories are compared with the curves of the tensile strength development obtained according to equation (3). It is evident that in the case of cross section A the stress remains below the strength curve only for sequence (c) while, in the case of cross section B, both sequences (b) and (c) are characterised by stresses at the limit of tensile strength. Since the model cannot capture the concrete cracking this means that the results are not valid from a quantitative viewpoint but they are still significant for a qualitative comparison useful to assess the tendency of the slab to crack.

Figure 5 shows the importance of the thermal shrinkage and demonstrates how it is responsible for more than 50% of the tensile stresses at the end of casting (30 days). Structural analyses would be strongly not realistic and non conservative in predicting the bridge cracking tendency if thermal shrinkage was neglected.

The previous results were obtained by considering the creep function valid for compression stresses and consequently the enhanced tensile creep was not taken into account. In Fig. 6 the results obtained multiplying by 1.5 the creep coefficient are reported. In such cases, the tensile strength is not achieved in the case of fractionated casting both after the slab construction and in the long term. On the contrary, considering the standard creep coefficient, stresses achieve the tensile strength in many sections of the deck in the long term. In any case, the increment of traction due to the long term drying shrinkage is partially compensated by the relaxation of stresses due to the early-age thermal shrinkage.
Figure 4: Time evolution of concrete stresses

Figure 5: Influence of the thermal shrinkage on the tensile stress state

Figure 6: Influence of the tensile creep

CONCLUSIONS

The early-cracking tendency of the slab in a long steel-concrete composite viaduct is investigated using a dedicated computer program. The effects of the early-age thermal shrinkage combined with
the endogenous and drying components are taken into account. The following conclusions may be drawn:

a. sequential casting permits reducing sensibly the stress levels during construction, when the concrete has low tensile strength; the reduction is particularly significant over the inner supports where traffic loads induce negative moments; on the contrary, continuous casting does not permit to control the stress level exhibited in the structure and may lead to significant cracking even during the construction phases;

b. the choice of the segments to be poured in the hogging and sagging moment regions is crucial; non-symmetric layouts are preferable to symmetric ones to reduce tensile stresses;

c. the thermal shrinkage is one of the main causes that may lead to premature slab cracking; disregarding its effects may result in non realistic predictions;

d. during construction, the slab is in tension within the whole deck; the enhanced tensile creep remarkably reduces stress states;

e. in the long term the increment of traction due to the drying shrinkage is partially compensated by the relaxation of stresses produced by the thermal shrinkage;

f. using concrete with advanced properties guarantees an effective control of cracking both at early-age and in the long term.

REFERENCES

A SIMPLE MODEL USED IN OPTIMUM DESIGN OF CONCRETE-FILLED TWIN STEEL TUBULAR COLUMN

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KEYWORDS
Concrete-filled, Steel Tubular Column, Axial Compression

ABSTRACT
In this paper, a simple model is proposed to predict the optimal section of concrete-filled twin steel tubular (CFTST) columns under axial compression. By this model, the optimal sizes of the standard inner and outer steel tubes can be calculated based on the assumption that the bulk modulus of the confined concrete varies with the volume of the concrete. Using this model, the effect of the thickness of the steel tubes can also be taken into account. The main advantages of the model are attributed to its simplicity and its clear physical meaning. When this model is applied to design, the outcome agrees very well with experimental tests and the predictions by existing models.

INTRODUCTION
Steel-concrete composite columns have been used widely in structures in recent years due to the significant advantages in comparison to more traditional construction methods. Up to today, there have been a number of studies conducted to investigate the behavior of concrete-filled steel tubular (CFST) columns. The main benefit of using CFST is that it utilizes the advantages of both steel and concrete, viz. steel members have high tensile strength and ductility, whereas concrete members are advantageous in compressive strength and stiffness. Compared with reinforced concrete or bare steel tubular columns, CFST columns of similar dimensions have increased section modulus, enhanced stability, better cyclic performance and higher fire resistance etc. (Han et al. 2006 [1]).

Concrete-filled twin steel tubular (CFTST) column is a typical member in the whole CFST family. It consists of an inner tubular section and an outer tubular section, fully filled with
concrete. Similar to traditional CFST, CFTST columns under axial compression have high bearing capacity due to the interaction between steel tubes and concrete. Based on existing approaches, such as the unified theory (Zhong [2], [3], Zhong et al. [4], Tao et al. [5]) and different engineering models (Han et al. 2006 [6]; Kuranovas et al. 2009 [7]), the load-bearing capacity of a particular CFST column can be predicted. The success in the use of those models shows that the same methodology can be applied to CFTST columns. However, this study aims to propose a simple method to determine the optimal sizes of the outer and inner tubes to ensure CFTST columns have the maximum ultimate strength.

METHODOLOGY

It is believed that the increase of the capacity of CFTST columns to resist axial-compression can be attributed to the confinement effect. In fact, nearly one century ago (Richart et al. [8]), it has been observed that confined concrete showed significant increase in maximum compressive strength, enhancement in stiffness and ultimate strain when the peak stress was reached. Although the concrete may have cracked or crushed when the axial load attains a certain level, a CFTST column under compression has inherent high ductility and energy dissipation ability since the concrete is confined within the steel tubes. Following this line of argument, it can be concluded that stronger confinement will result in higher strength of CFTST. On the other hand, the volume change of the concrete in CFTST can be used as an index to reflect how strong the confinement effect is. The relationship between the confinement effect and the volume change of the concrete can be summarized into one conclusion, that is, the stronger the confinement effect is, the smaller is the volume change of the concrete.

![Figure 1: Illustration of the cross-section of a concrete-filled twin steel column](image)

Fig. 1 shows a cross-section of a typical concrete-filled twin steel tubular column. Both the outer section and the inner section are assumed to be circular. The radii of the two circles are \( r_i \) and \( r_o \), respectively. For convenience, the entire area of the concrete cross-section is divided into two domains, viz. domain 1 surrounded by the inner tube and domain 2 which is sandwiched between the outer and inner tubes. For simplicity, the length of this column is set to be unity.

Denoted by \( \Delta V \), the total volume change of the concrete under axial compression can be expressed by Eq. (1):

\[
\Delta V = V_{total} - V_{initial}
\]
\[ \Delta V_i = \Delta V_1 + \Delta V_2 \]  
(1)
where \( \Delta V_1 \) is the volume change in domain 1 and \( \Delta V_2 \) is the volume change in domain 2.

Based on Eq. (1), \( \Delta V_{\text{r-min}} \), the minimum value of \( \Delta V_r \) can be calculated since \( \Delta V_i \geq 2\sqrt{\Delta V_1 \cdot \Delta V_2} \):

\[ \Delta V_{\text{r-min}} = 2\sqrt{\Delta V_1} \cdot \sqrt{\Delta V_2} , \text{ (only if } \sqrt{\Delta V_1} = \sqrt{\Delta V_2} ) \]  
(2)

Hence, the CFTST column under axial compression has the highest load-capacity when \( \Delta V_1 = \Delta V_2 \).

EXAMPLES AND VALIDATION

A convenient and simple way to calculate the volume change of concrete is to assume \( \Delta V_1 = g(V_1)V_1 \) and \( \Delta V_2 = g(V_2)V_2 \), where \( g(\cdot) \) is a function of the concrete volume. It can be seen that \( g(\cdot) \) performs like an operator which is dependent on concrete volume. Hence, according to Fig. 1, one obtains:

\[ g(V_1)r_i^2 \pi = g(V_2)(r_o^2 - r_i^2) \pi \]  
(3)
Defining \( \beta = r_i / r_o \), Eq. (3) can be rewritten as Eq. (4) where the optimal sizes of the steel tubes are determined in terms of the ratio of \( r_i / r_o \):

\[ \beta = \sqrt{\frac{g(V_2)}{g(V_1) + g(V_2)}} \]  
(4)

It must be mentioned that different forms for function \( g(\cdot) \) may be defined. Assume \( g(\cdot) \) is constant to define the simplest form, one obtains:

\[ \beta = r_i / r_o = \frac{\sqrt{2}}{2} = 0.707 \]  
(5)

Certainly, more general and complicated forms can be defined for \( g(\cdot) \). For instance, one can relate \( g(\cdot) \) to the perimeters of the cross-sections of the tubes. Under the condition that the thicknesses of the two steel tubes are the same, the shorter the perimeter is, the stronger is the confinement effect. Hence, as an example, it may be assumed that \( g(V_1) = \kappa \alpha \) and \( g(V_2) = \kappa \alpha \) (\( \alpha \geq 0 \) and \( \kappa = \text{constant} \)). As a result, one obtains:

\[ \beta^{2+\alpha} + \beta^2 - 1 = 0 \]  
(6)
The solution of Eq. (6) is the optimal ratio of $r_i / r_o$.

It should be noted that Eq. (6) is obtained based on the assumption that the two tubes are all circular. For noncircular tubes, Eq. (3) should be replaced by a more general form, that is:

$$g(V_1)A_1 = g(V_2)A_2$$

where $A_1$ and $A_2$ are the areas of domain 1 and 2, respectively.

In the case that the cross-sections of the outer and inner tubes are square and circular, respectively, Eq. (8) can be derived from Eq. (7).

$$\frac{\pi r_i^2}{a^2 - \pi r_i^2} = \frac{g(V_2)}{g(V_1)}$$

where $a$ is the edge length of the square.

If the value of function $g(\cdot)$ is still constant, one obtains:

$$\frac{r_i}{a} = \sqrt{\frac{1}{2\pi}} = 0.399$$

To verify the proposed model, some existing test results are used to compare with the predictions given in Eq. (5) and (9). Fig. 1 and 2 are two figures showing the effects of $r_i / r_o$ and $r_i / a$. The curves in the figures are obtained based on both experiments (Jiang [9], Pei [10]) and analyses (Cai [11]; Zhang et al. 2008 [12]).

![Figure 2: The effect of $r_i / r_o$ on the load-bearing capacity of CFTST column](image-url)
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0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50
72x555
518

Figure 3: The effect of $r_i/a$ on the load-bearing capacity of CFTST column

CONCLUSIONS

This study proposes a simple model to calculate the optimal sizes of the steel tubes for CFTST columns under axial compression. This model is derived from the understanding of confinement effect rather than more complex mechanics theory. Although the formulae presented in this paper implicate that the thicknesses of the two steel tubes are the same, the effect of the thicknesses of the steel tubes can also be taken into account by defining proper form of function $g(\cdot)$. In the case that the material properties of the outer and inner concrete cores are different, the proposed model can be manipulated based on the similar concept. The main advantages of the model are attributed to its simplicity and its clear physical meaning. So it can be used as a convenient tool for hand calculation in real design.

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AXIAL COMPRESSION TESTS ON FRP-JACKETED CIRCULAR CONCRETE-FILLED THIN STEEL TUBES

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KEYWORDS
FRP, Steel tubes, Concrete, Hybrid columns, Tubular columns, Strengthening, New construction

ABSTRACT
Fibre-reinforced polymer (FRP) jackets have been widely used to confine concrete columns to enhance their load-carrying capacity and ductility. More recently, the use of FRP jackets to improve the performance of hollow steel tubes and concrete-filled steel tubes has also been explored. This paper presents the results of a recent experimental study in which the behaviour of FRP-confined circular concrete-filled thin steel tubes under axial compression was examined. The experimental study included three series of tests and was focused on the effects of the diameter-to-thickness ratio of the steel tube and the confinement stiffness of the FRP jacket. The test results revealed that the FRP jacket either substantially delayed or completely suppressed the local buckling failure mode of the steel tube. As a result, the compressive behaviour of the concrete-filled steel tube as well as the concrete is significantly improved in terms of both strength and ductility.

INTRODUCTION
Fibre-reinforced polymer (FRP) jackets have been widely used to provide lateral confinement to reinforced concrete columns to enhance their load-carrying capacity and ductility [1-3]. The use of FRP jackets to suppress local outward buckling (i.e., elephant’s foot buckling) in hollow circular steel tubes and shells has also been explored [4-7]. Results presented in Teng and Hu [6] confirmed that FRP confinement of hollow circular tubes subjected to axial compression can be very effective in improving ductility. Xiao [8] recently proposed a novel form of concrete-filled steel tubular (CFT) columns, named by him as confined CFT columns ((or CCFT columns), in which the end portions are confined with steel tube segments or FRP jackets. In these columns, by providing an FRP or steel jacket at each end, the steel tube is
prevented from deforming inwards by the concrete core and outwards by the jacket, so both the ductility and strength of the steel through-tube can be substantially enhanced in the end regions. In addition to Xiao’s initial work [8], a number of other studies have been conducted by both Xiao’s group [9-11] and other researchers [12-15] on the effectiveness of FRP jacketing in improving the structural behaviour of both circular [9, 10, 12-15] and square/rectangular CFTs [11, 13].

While the advantages of FRP jacketing of CFT columns have been demonstrated with the limited test results now available, much research is still needed to develop a good understanding of the structural behavior and appropriate design methods for FRP-confined CFTs, particularly when thin steel tubes which are highly susceptible to local buckling are used in these CFTs. As part of a larger study undertaken at The Hong Kong Polytechnic University (PolyU) [16], three series of axial compression tests were conducted on concrete-filled thin steel tubes confined with glass FRP (GFRP) jackets. This paper presents and interprets the results of these tests to examine the effect of the FRP jacketing on CFTs with thin steel tubes. GFRP was used instead of carbon FRP (CFRP) in these tests as GFRP does not suffer from galvanic corrosion problems which may be a concern for CFRP directly bonded to steel and possesses a larger ultimate tensile strain which is a favorable property for ductility enhancement applications. The attention was focused on CFTs with thin steel tubes as the local buckling problem is more pronounced and the benefit of FRP jacketing is expected to be more obvious for such tubes. Thin steel tubes are also deemed to be particularly appropriate for the CCFT system where the additional confinement available in the critical regions allows the thickness of the steel through-tube to be reduced. All steel tubes used in the present experimental study had an outer diameter-to-thickness ($D_{outer}/t_s$) ratio exceeding 100.

SPECIMENS AND INSTRUMENTATION

Specimen details

In total, twelve specimens were prepared and tested in three series. Each series included one CFT specimen without FRP jacketing and three FRP-confined CFT (or CCFT) specimens with three different FRP jacket thicknesses respectively. The steel tubes used in each series had the same $D_{outer}/t_s$ ratio, but this ratio was different for different series. Steel plates of 1 mm, 1.5 mm, 2 mm in thickness respectively were used to fabricate the steel tubes for the three series. The average values of the elastic modulus ($E_s$), yield strength ($f_y$), and ultimate tensile strength ($f_u$) obtained from tensile coupon tests for the steel tubes of each series are listed in Table 1. The FRP used had an average elastic modulus of 80.1 GPa based on a nominal thickness of 0.17 mm per ply. The FRP jacket was formed via a wet lay-up process with its finishing end overlapping its starting end by 200 mm. The concrete was cast in three batches for the three series respectively, all though with the same mix ratio. The average concrete strength $f_{cu}$ from axial compression tests on three standard concrete cylinders for each series is also given in Table 1. All specimens had an outer diameter of around 200 mm and a height of 400 mm. Other details of the specimens are summarized in Table 1.
The name of each specimen starts with a letter “F” followed by the number of plies forming the FRP jacket. The last 3-digit number (e.g. 102) is used to indicate the $D_{\text{outer}}/t_s$ ratio of the steel tube. For example, specimen F1-102 is a specimen with a one-ply FRP jacket whose steel tube had a $D_{\text{outer}}/t_s$ ratio of 102.

### TABLE 1
DETAILS OF SPECIMENS

<table>
<thead>
<tr>
<th>Series</th>
<th>Specimens</th>
<th>Steel tube</th>
<th>Concrete</th>
<th>FRP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$D_{\text{outer}}$ (mm)</td>
<td>$t_s$ (mm)</td>
<td>$D_{\text{outer}}/t_s$</td>
</tr>
<tr>
<td>102</td>
<td>F0-102</td>
<td>204</td>
<td>2</td>
<td>102</td>
</tr>
<tr>
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<td>F1-102</td>
<td>0.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>F2-102</td>
<td>0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>F3-102</td>
<td>0.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>F0-135</td>
<td>203</td>
<td>1.5</td>
<td>135</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>F3-135</td>
<td>0.51</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>202</td>
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<tr>
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<td>F2-202</td>
<td>0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>F3-202</td>
<td>0.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>F4-202</td>
<td>0.68</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Figure 1: Layout of strain gauges at middle height](image)

**Instrumentation and Loading**

For each FRP-confined CFT specimen, six strain gauges in the hoop direction and two strain gauges in the axial direction were installed at the mid-height of the FRP jacket. The two axial strain gauges were installed at 180° apart from each other, both being outside the overlapping zone. Of the six hoop strain gauges, one of them was installed inside the overlapping zone while the other five were evenly distributed within the half circumference opposite the
overlapping zone. The layout of the strain gauges at mid-height is shown in Figure 1. In addition, five strain gauges in the hoop direction were installed near (20 mm from) each end of each specimen to detect any possible buckling deformation in these regions. The circumferential locations of these strain gauges were the same as the five hoop strain gauges outside the overlapping zone at mid-height. The layout of the strain gauges on bare CFT specimens was exactly the same as that for FRP-confined specimens. All the strain gauges used in FRP-confined CFT specimens had a gauge length of 20 mm while those used in CFT specimens had a gauge length of 5 mm. All the axial compression tests were conducted using an MTS machine with a displacement control rate of 0.5 mm/min until failure. The total axial shortening of the specimens was measured using three linear variable displacement transducers (LVDTs) placed at 120° apart from each other.

TEST OBSERVATIONS, RESULTS AND DISCUSSIONS

Failure Modes and Processes

Figure 2 shows the failure modes of specimens in Series 102 while the failure modes of specimens in other series are similar. In the final stage of deformation of bare CFTs, continuous lateral dilation in the mid-height region as well as continuous growth of localized outward buckling deformation of the steel tube near one or both ends were observed as the axial shortening of the specimen increased (Figure 2). The load decreased steadily after the peak load had been reached (Figure 3). All the FRP-confined CFT specimens failed by explosive rupture of the FRP jacket in the mid-height region due to the lateral expansion of concrete and this rupture caused a sudden and rapid load drop. Before this final failure, localized FRP rupture occurred near one end in some specimens (i.e. specimens F2-102, F3-102 and F2-135) due to the localized outward buckling deformation of the steel tube, but this local FRP rupture only had a small effect on the ability of the specimen to carry the applied load (see Figure 3).

Axial Load-Shortening Behavior

The axial load-axial shortening curves of all specimens are shown in Figure 3, where the axial shortening is averaged from the readings of the three LVDTs. The curves of all the bare
CFT specimens feature a smooth but relatively steep descending branch after reaching the peak load, while the FRP-confined specimens exhibited either an approximately elastic-perfectly plastic curve or an approximately bilinear curve before final failure followed by a sudden load drop.

Figure 3: Axial load-shortening curves
The key test results for all the specimens are summarized in Table 2. \( N_c \) and \( N_u \) are the peak (ultimate) load and the load at ultimate strain respectively while \( \delta_u \) is the ultimate axial shortening of a specimen. For example, \( N_{c,\text{bare}} \) and \( \delta_{u,\text{bare}} \) are the peak load and the ultimate axial shortening of a bare CFT specimen respectively. \( \varepsilon_{\text{frp,rupt}} \) is the average hoop rupture strain of the FRP jacket found from the readings of the five hoop strain gauges outside the overlapping zone. For a bare CFT specimen, the ultimate shortening is defined as the value at a load which is 80% of the peak load. For an FRP-confined CFT specimen, the ultimate shortening is reached when explosive rupture of the FRP jacket in the mid-height region occurs. The load at the ultimate shortening of an FRP-confined CFT specimen is seen to be either the same as or on slightly lower than its ultimate load. The short descending branch before the rupture failure of the FRP jacket found for some of the FRP-confined CFT specimens was due to either the local rupture of the FRP jacket near an end (specimens F2-102, F3-102 and F2-135, see Figure 3) or the use of a thin FRP jacket (specimen F1-102, see Figure 3).

### Table 2

<table>
<thead>
<tr>
<th>Series</th>
<th>Specimens</th>
<th>( N_c ) (kN)</th>
<th>( N_u ) (kN)</th>
<th>( N_u ) / ( N_{c,\text{bare}} )</th>
<th>( \delta_u ) (mm)</th>
<th>( \delta_u ) / ( \delta_{u,\text{bare}} )</th>
<th>( \varepsilon_{\text{frp,rupt}} )</th>
<th>( k_{\varepsilon 1} )</th>
<th>( k_{\varepsilon 2} )</th>
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</thead>
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<td>F0-102</td>
<td>1864 14</td>
<td>91</td>
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<td>3.72</td>
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<td>N/A</td>
<td>N/A</td>
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<tr>
<td></td>
<td>F1-102</td>
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<td>78</td>
<td>1.01</td>
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<td>1.42</td>
<td>-0.0179</td>
<td>0.693</td>
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<tr>
<td></td>
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<td>2.27</td>
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<td>0.961</td>
<td>0.910</td>
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<tr>
<td></td>
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<td>2427 22</td>
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<td>9.43</td>
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<td>1.004</td>
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<tr>
<td><strong>Series 135</strong></td>
<td>F0-135</td>
<td>1699 13</td>
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<tr>
<td></td>
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<td>F4-135</td>
<td>2561 25</td>
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<tr>
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<tr>
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<td>F4-202</td>
<td>2265 22</td>
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<td>2.12</td>
<td>-0.0192</td>
<td>0.891</td>
<td>0.944</td>
</tr>
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</table>

The effect of confinement from the FRP jacket is obvious in Figure 3 and Table 2. Of the specimens tested, the load-carrying capacity was increased by up to 64% while the axial shortening capacity was more than doubled in some of the cases; the energy dissipation capacity was also substantially enhanced. As expected, such enhancement in performance was greater when a thicker FRP jacket was used. When the same FRP jacket thickness was used, the enhancement in load-carrying capacity is seen to be more pronounced for CFTs with a thinner steel tube where the contribution of the steel tube to the load-carrying capacity was smaller.

### Rupture Strain of the FRP Jacket

At the tensile rupture of the FRP jacket, the distribution of hoop strains in the jacket was found to be highly non-uniform. In studies on FRP-confined concrete columns, an FRP efficiency factor, denoted by \( k_{\varepsilon} \), has been defined to evaluate the efficiency of an FRP jacket [17, 18]. This factor is equal to the ratio of the average FRP hoop rupture strain in a confined column to the ultimate tensile strain obtained from flat coupon tests. The FRP efficiency factor \( k_{\varepsilon} \) can be found as the product of two components [17]: \( k_{\varepsilon 1} \) which is the ratio of the average hoop rupture strain to the maximum measured hoop strain in the FRP jacket at
rupture, and $k_{e_2}$ which is the ratio of the maximum measured hoop rupture strain in the FRP jacket to the ultimate tensile strain from flat coupon tests.

The values of $k_{e_1}$ and $k_{e_2}$ for all the FRP-confined CFT specimens are listed in Table 2. In calculating $k_{e_1}$, only readings from those strain gauges outside the overlapping zone were used. The values of $k_{e_1}$ are seen to vary from 0.693 to 0.961, with a mean value of 0.865 for all specimens. The values of $k_{e_2}$ are seen to vary from 0.824 to 1.133, with a mean value of 0.948 for all specimens. The $k_{e_1}$ values are found to lie around the average value (i.e. 0.908) found from tests on GFRP-confined concrete cylinders [18], while the $k_{e_2}$ values are all higher than the average value (i.e. 0.820) of GFRP-confined concrete cylinders [18]. The larger $k_{e_2}$ values observed in the present tests may be attributed to the less severe stress concentration on the surface of a steel tube than that of a concrete cylinder. That is, the maximum strain reading recorded by the limited number of hoop strain gauges on an FRP jacket is believed to be closer to the real maximum strain occurring when the FRP jacket is bonded to a steel tube than when it is bonded to a concrete cylinder.

CONCLUSIONS

This paper has presented an experimental study on the behaviour of FRP-confined concrete-filled thin steel tubes under axial compression. The experimental program included three series of tests where the main parameters examined were the thickness (or the diameter-to-thickness ratio) of the steel tube and the stiffness of the FRP jacket. The test results showed that the FRP jacket is very effective in improving the axial compressive behaviour of concrete-filled thin steel tubes in terms of both the load-carrying capacity and the ductility. The local buckling of the steel tube in a concrete-filled steel tubular column can be either substantially delayed or completely suppressed by the FRP jacket. The benefit of an FRP jacket is more pronounced for CFT specimens with a thinner steel tube than for those with a thicker steel tube.

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EXPERIMENTAL AND ANALYTICAL INVESTIGATIONS OF TRUSSES COMPOSED OF BARE AND COMPOSITE RHS

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KEYWORDS
Steel tube, Concrete-filled steel Tube, Second-order analysis, Rectangular hollow section, Truss, Effective length, Eurocode 3, Eurocode 4

ABSTRACT:
This paper presents the experimental and analytical investigations on bare and composite rectangular hollow sections (RHS) which used as members of trusses. The load resistances of the trusses consisted of RHS steel and concrete-filled RHS steel tube members are compared to indicate the beneficial effects due to the in-filled concrete. The maximum load on the trusses is predicted using the design method in Eurocode 3 and Eurocode 4, and the results showed that the use of effective length method in linear analysis and design method is inconvenient and inaccurate. Finally, the results are further predicted by the second-order analysis and design method and compared with test results. The second-order analysis and design method not only gives a more accurate prediction than the linear analysis, but it also provides an efficient design as assumption of effective length is not needed.

INTRODUCTION
Experimental studies of concrete-filled steel tube members under different end conditions have been extensive conducted and the works include those by Knowles and Park [1], Bridge [2], Shakir-Khalil [3] and Lu and Kennedy [4] who investigated the axial and flexural behavior. Most experiments were focused on the behavior of single member with ends restrained against lateral movement. In this paper, the end movement of the member were allowed but only connected to and restrained by other truss members. The ends movement of the member induces the P-Δ effect which complicates the behavior of the members and inclusion of this moment is important in the analysis and design. Several design codes provide different design methods on composite member such as Eurocode 4 [5], BS5400 [6] and CoPHK [7]. These codes contain various design methods for several common types of composite columns which allow engineers to use the first order analysis and effective length method to check the strength and stability of each member. The accuracy of this design method depends highly on the error
of the effective length factor which is not quite possible to estimate since the idealized assumption for simple end conditions like pin and rigid end are unrealistic in most practical structures. In this paper, two effective length factors which are the upper and lower bounds to the actual buckling length of the failure member were used to predict the design load, and the results indicated the inconsistency, inconvenience and inaccuracy of using this approach. Due to the limitations of the first order analysis, the second-order analysis and design method for steel tube and concrete-filled steel tube members is introduced in this paper and the predicted results by using the second-order analysis and design method would be compared with experimental results for verification.

EXPERIMENTAL PROGRAM

Specimens

Two trusses were tested and their dimensions are shown in Figure 1. One truss was composed of RHS and SHS (square hollow section) steel tube in all members and another truss was composed of concrete-filled RHS and SHS steel tube in compression members and SHS steel tube in tension members. Each three-dimensional truss consisted of 19 members which included two 50x30x3 RHS tubes and 17 number of 60x60x3 SHS tubes. The two target failure members were 50x30x3 RHS tube and the rest of the members were 60x60x3 SHS tube. The length of each truss member was 2m approximately and the tie members used to connect the two 2-D truss was 0.8m approximately. The ends of the members were connected rigidly by using 8mm butt weld.

![Figure 1: The dimension of the trusses](image)

The average width, depth and thickness of both sections are listed in Table 1. The coupon test was carried out to determine the stress-strain curve of the steel section. The average yield stress ($f_y$), ultimate tensile stress ($f_u$) and the Young’s modulus ($E_s$) are summarized below.

| Steel section | B (mm) | D (mm) | t (mm) | Yield stress ($f_y$) N/mm$^2$ | Ultimate tensile stress ($f_u$) N/mm$^2$ | Young’s modulus ($E_s$) kN/mm$^2$
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>50x30x3</td>
<td>50.00</td>
<td>30.08</td>
<td>2.96</td>
<td>399.17</td>
<td>448.30</td>
<td>203.87</td>
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<tr>
<td>60x60x3</td>
<td>60.58</td>
<td>60.53</td>
<td>3.25</td>
<td>376.12</td>
<td>439.91</td>
<td>217.50</td>
</tr>
</tbody>
</table>

The high strength concrete was used in-filling in the steel tubes, and the design strength of concrete
was 90MPa at 28 days. The concrete mix was water (238.1kg/m³), Ordinary Portland Cement (479.5kg/m³), coarse aggregate (862.5kg/m³), fine aggregate (709kg/m³) and Pulverized Fly Ash (205.5kg/m³). The average compressive cube and cylinder strength were 91.65N/mm² and 89.87N/mm² respectively and the modulus of elasticity of concrete was 37.45kN/mm².

**Test Results**

The trusses were simply supported at two ends and loaded in pairs by using the hydraulic jack with the maximum capacity of 400kN shown in Figure 1. Totally 12 displacement transducers were used to measure the deflection of the target failure members and movement in the truss. 18 strain gauges were placed at 3 locations including the top, middle and bottom of each target failure member and six strain gauges were mounted at each location. The locations of displacement transducers and strain gauges are shown in Figure 2.

The applied load on top of the truss against mid-span in-plane deflection of the failure members are plotted in Figure 3a for RHS steel and concrete-filled RHS steel tube. For steel tube, the member deflection increased linearly with the applied load until it reached 36.85kN, and the load-deflection relationship then became non-linear. The maximum applied load on the steel truss was 76.61kN. For concrete-filled steel tube, the member deflection under applied load was similar to the steel tube that a linear relationship was observed before the applied load reached 26.20kN. After the load, the deflection is increased with applied force non-linearly and the maximum applied load was found to be
90.00kN. The flexural buckling about the principal minor axis of the failure member took place and shown in Figures 4a and 4b for both trusses. The maximum load taken by the member in composite truss was 17.5% higher than the bare steel truss. The applied load against out-of-plane mid-span deflection of the failure members is also plotted in Figure 3b. The curves showed that the out-of-plane deflection was small compared with the in-plane deflection at maximum applied load in both trusses.

Figure 4a: Steel member truss Figure 4b: Composite member truss

Figure 4: The failure shape of the trusses

Figure 5a: Steel RHS tube member

Figure 5b: Concrete-filled RHS steel tube member

Figure 5: Load-Strain curve for the failure member

The applied load against the strain plots at the mid-length of the failure members are shown in Figure 5. A non-linear relationship was observed as expected and the post-failure behaviour could be observed in both RHS steel and concrete-filled RHS steel tubes. The RHS steel and concrete-filled RHS steel tube gave similar behaviour in strain as shown in Figures 5a and 5b. The variation on the strain alone the top fiber (SG1 and SG2) and alone the bottom fiber (SG4 and SG5) is small, hence the average strain in top and bottom was plotted against the applied load and large compressive strains was developed at bottom fiber which gave a consistent result with displacement transducers. The readings
from SG6 and SG3 were identically up and close to the failure load for concrete-filled RHS steel tube and the slightly different in RHS steel tube, due to the out-of straightness imperfections in major axis direction. The result implied that the out-of-plane deflection was insignificant before failure load and, after the failure load, the out-of-plane deflection increased significant with decreasing load, hence the load-strain curve started to diverge.

CODE AND SECOND-ORDER ANALYSIS AND DESIGN METHOD

Predicted results from Eurocode 3 and Eurocode 4

The predicted results by using the design methods in Eurocode 3 [8] and Eurocode 4 [5] for RHS steel and concrete-filled RHS steel tube are summarized in Table 2. The effective length factor equal to 0.5 (Lₑ=1000) and 1.0 (Lₑ=2000) were assumed to simulate the fix end with translational fixed and free conditions which are the close to the true behavior of the members in the truss. Predicted maximum applied loads according to the design codes were 31.77kN and 106.14kN for RHS steel tube, and 36.10kN and 130.49kN for concrete-filled RHS steel tube under these two different effective length factors. The ratios of test to predict load was around 0.7 and 2.4 for effective length factors equal to 0.5 and 1.0 respectively. The results indicate that the true effective length of the members should be between these two values.

<table>
<thead>
<tr>
<th>Truss member</th>
<th>Maximum Applied Force (kN)</th>
<th>Test Load/Predicted Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test result P_t</td>
<td>Second-order analysis P_a</td>
</tr>
<tr>
<td>RHS Steel Tube</td>
<td>76.61</td>
<td>77.8</td>
</tr>
<tr>
<td>Concrete-filled RHS Steel Tube</td>
<td>90</td>
<td>90.35</td>
</tr>
</tbody>
</table>

Second-order design and analysis method

Second-order analysis and design method not only simplifies the design process, but also gives an accurate result because the non-linear effects, which including member imperfection, P-δ and P-Δ moments. When these nonlinear factors are included in analysis, the complicate and individual member design using code tables and charts is eliminated. The formulation of the element, tangent stiffness and secant stiffness matrix for use of steel and composite members have been detailed by Chan and Zhou [9] [10] and Chan etc. [11] and will not be repeated here.

Section capacity check

In the second-order analysis and design method, the section capacity check is used in place of individual member design. For steel member, the equation below is adopted.

\[
\frac{P}{P_p} + \frac{M_y + P(\delta_y + \Delta_y)}{M_{py}} + \frac{M_z + P(\delta_z + \Delta_z)}{M_{pz}} = \phi \leq 1
\]
For composite member, two section capacity equations are used for two different load conditions. When the applied force is larger than the section capacity of concrete section (i.e., \( P > P_{pm} \)), Eq. (2) will be used and it allows for the effects of axial force and moments in the section capacity equation. When the applied force is less than the capacity of concrete section (i.e., \( P \leq P_{pm} \)), only applied moments are considered since the axial force does not reduce the failure load and Eq. (3) is then used for section capacity check. These two sets of section capacity equations are given as follows.

For \( P > P_{pm} \)
\[
\frac{P - P_{pm}}{P_{cp} - P_{pm}} \left( \frac{M_y + P(\delta_y + \Delta_y)}{M_{cpy}} + \frac{M_z + P(\delta_z + \Delta_z)}{M_{cz}} \right) = \phi \leq 1
\]  

For \( P \leq P_{pm} \)
\[
\frac{M_y + P(\delta_y + \Delta_y)}{M_{cpy}} + \frac{M_z + P(\delta_z + \Delta_z)}{M_{cz}} = \phi \leq 1
\]  
in which \( P \) is the applied force, \( P_{p}, P_{pm}, P_{cp} \) is compressive capacity of steel, concrete and composite cross-section, \( M_y \) and \( M_z \) are the external moments about the y and z axis, \( P(\delta_y + \Delta_y) \) and \( P(\delta_z + \Delta_z) \) are the P-\( \Delta \) and P-\( \delta \) moment about the y and z axes, \( M_{py}, M_{cpy}, M_{pz}, M_{cz} \) is the moment capacity of composite cross-section about the y and z axes.

As shown in the section capacity check equations that the P-\( \Delta \) and P-\( \delta \) effects have been included such that the assumption of effective length is no longer required. Further, the inclusion of initial imperfection has been directly done in analysis that the concept of section capacity check for imperfect columns can be applied directly in the integrated analysis and design model.

**Analysis Results**

The analytical model and the deformed shape of the truss are shown in Figure 6. The average yield stress and Young’s Modulus of steel and concrete from tested material was used in computer model. The initial imperfection of the member was taken as \( L/300 \), where \( L \) is the member length, according to Table 5.1 in Eurocode 3 [8] and Table 6.5 in Eurocode 4 [5] for both steel and composite column respectively. The member center to center length was used and rigid connection between each member was assumed. Two point loads were applied to the top of the truss at each side and load increment of 0.05kN was used in analysis until the section capacity factor was larger than 1. The analysis results were presented together with test results in Table 2. The failure loads of steel and composite truss were 77.80kN and 90.35kN respectively and the ratios of test to analysis result are 0.98 and 1.0 for RHS steel and concrete-filled RHS steel tube members. Nevertheless, the accuracy is expected and it is envisaged that a larger variation, but still within the practical application, between the analysis and tested results should be expected. The analysis results show that the use of second-order analysis gave accurate results on prediction the loading capacity of the steel and concrete-filled steel tube members.
CONCLUSION

The experimental investigation on the behaviors of RHS steel and concrete-filled RHS steel tubes act as the members of truss was presented in the paper. The load capacity of the concrete-filled RHS steel tube member is 17.5% higher than the RHS steel tube member. The results predicted from the Eurocode 3 [8] and Eurocode 4 [5] show that the assumption of effective length factor to be 0.5 over-estimates the resistance of the failure member which leads to an unsafe design. On the other hand, the assumption of the effective length factor as 1.0 under-estimates the member capacity which leads to an uneconomic design. Therefore, the use of second-order analysis and design method is proposed in which the assumption of effective length is eliminated as the non-linear effects are directly included in analysis. The use of the second-order analysis and design method in design of steel and composite members was demonstrated and accurate results was predicted compared with test results which confirms the accuracy of second-order analysis and design method.

ACKNOWLEDGE

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REFERENCES


INFLUENCE OF LONG-TERM LOADING ON THE PERFORMANCE OF CONCRETE-FILLED DOUBLE SKIN STEEL TUBULAR COLUMNS: EXPERIMENTS

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KEYWORDS
Concrete-filled double skin tubes (CFDST), long-term sustained load, ultimate load tests, load-bearing capacity.

ABSTRACT
Concrete-filled double skin steel tubes (CFDST) have been studied by several researchers in the past few years. However, these members when subjected to sustained loading have not been addressed satisfactorily. This paper carried out a series of tests on CFDST columns with circular section inner, square or circular section outer performed under long-term sustained loading condition. The test process included two stages, i.e., long-term service test and ultimate strength test. The long-term deformation of the CFDST columns, as well as the influence of long-term sustained loading on the ultimate strength of the specimens, was investigated experimentally.

INTRODUCTION
Concrete-filled double skin steel tube (CFDST) is a new type of composite construction, which combines the advantages of the well-known concrete-filled steel tube (CFST) and the conventional hollow reinforced concrete (RC) columns. Thus, CFDST columns have a series of advantages, such as high strength and bending stiffness, good seismic and fire performance, and favorable construction ability. Figure 1 shows two typical profiles of the CFDST sections, where hollow ratio ($\chi$) of the CFDST sections was defined as Eqn.1 [1, 2]:

$$\chi = \frac{D_i}{(B_o - 2t_o)} \quad \text{or} \quad \frac{D_i}{(D_o - 2t_i)}$$  \hspace{2cm} (1)

where $B_o$ is the width of outer square steel tube, $D_o$ and $D_i$ are the diameters of outer and inner circular steel tube, $t_o$, and $t_i$ are the thicknesses of outer and inner steel hollow sections.
CFDSTs have great potential to be used as columns or piers in structures. In the past, the behaviour of CFDST columns under short-term loading has been the subject of investigation, such as Yagishita [3], Zhao and Han [4], Tao and Han [5]. Just like conventional CFST columns, the innovative CFDST columns under service loads, in a building or bridges will also suffer to the effects of creep and shrinkage of the in-filled concrete.

In the past, experimental observation of creep in normal CFST columns were reported by Ichinose et al. [6], Terrey et al. [7], Morino et al. [8], Uy [9], Han and Yang [10], Han et al. [11], Kwon [12, 13], Yang et al. [14]. However, it appears that little attention has previously been given to the influence of long-term sustained load on the behaviour of the CFDST columns, which indicates a need for further research in this area.

The present study is thus an attempt to study the time-dependent behaviour of CFDST columns. A series of tests were conducted to investigate the behaviour of CFDST columns under long-term sustained loading. Based on the test results, the long-term behaviour of the CFDST columns, as well as the influence of long-term sustained loading on the ultimate strength of the specimens was investigated experimentally.

**EXPERIMENTAL PROGRAM**

**Specimen Preparation**

Six tests, including two circular CFDST specimens (both the outer and inner steel skins are circular hollow sections, shown in Figure 1(a)), two square CFDST specimens (the inner steel skin is a circular hollow section, and the outer steel skin is a square hollow section, shown in Figure 1(b)), and two conventional CFST specimens, under concentrically sustained long-term loading were carried out. These specimens were tested under two stages, including the long-term service testing and ultimate load testing. In addition, ten CFDST and CFST specimens without experiencing long-term loading were also tested to measure their ultimate loads for comparison purposes.

The outer steel tubes of all tested specimens have a sectional size of \( D_o(B_o) \times t_o = 120 \times 1.96 \) mm, whilst the inner steel tube size of the CFDST specimen is \( D_i \times t_i = 60 \times 1.96 \) mm. A summary of the specimens is presented in Table 1, where \( L \) is the length of the specimen; \( t \) is the sustaining time for long-term loading; \( N_L \) is the sustained long-term load; and \( n \) is the long-term sustained load level, and can be determined as \( n = N_L / N_u \), where \( N_u \) is the ultimate strength of the CFDST columns at the short-term loading condition, which can be calculated by using the method presented by Han et al. [1] and Tao et al. [2]. In Table 1, specimen designations starting with DC, DS, C and S represent that the specimens are circular CFDST,
square CFDST, circular CFST and square CFST, respectively. A following letter c refers to their counterpart specimens, which were only tested under short-term loading. Two CFDST specimens were tested at a concrete curing age of 28 days, which are marked with stars in Table 1.

<table>
<thead>
<tr>
<th>Sectional Type</th>
<th>Number</th>
<th>Specimen label</th>
<th>$L$ (mm)</th>
<th>$t$ (day)</th>
<th>$N_L$ (kN)</th>
<th>$n$</th>
<th>$\varphi$</th>
<th>$N_{ue}$ (kN)</th>
</tr>
</thead>
<tbody>
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<td>1 D</td>
<td>C-1</td>
<td>1324</td>
<td>1050</td>
<td>177</td>
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<td>0.66</td>
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<tr>
<td></td>
<td>2 D</td>
<td>C-2</td>
<td>1324</td>
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<td>354</td>
<td>0.6</td>
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<td></td>
<td>3 D</td>
<td>Cc-0*</td>
<td>1324</td>
<td>—</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>578.0</td>
</tr>
<tr>
<td></td>
<td>4 D</td>
<td>Cc-1</td>
<td>1324</td>
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<tr>
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<td>0</td>
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<tr>
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<td>0</td>
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<td>10 D</td>
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<tr>
<td></td>
<td>11 D</td>
<td>Sc-0*</td>
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<td>—</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>800.0</td>
</tr>
<tr>
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<td></td>
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<td>0</td>
<td>1149.5</td>
</tr>
</tbody>
</table>

* Test conducted at a concrete age of 28 days

Self-consolidating concrete (SCC) was used to fabricate the specimens. The mix proportions by mass were as follows: Cement: Blast furnace slag: Water: Sand: Coarse aggregate: Additional high-range water reducer = 300: 200: 181: 994: 720: 3.6 (kg/m$^3$). The fresh properties of the SCC mixture were given as follows: Slump flow (mm): 270; Concrete temperature (°C): 20; Flow time (s): 13; Flow speed (mm/s): 62; and Flow distance (mm): 640. In preparing the composite specimens, the self-consolidating concrete (SCC) was filled in layers without any vibration.

**Material Properties**

To determine the steel material properties, tension coupons were cut from a randomly selected steel sheet. The average yield strength ($f_{sy}$) and elastic modulus ($E_s$) of the outer steel tubes for both circular and square CF DST specimens are 311 MPa and 205500 MPa, respectively. The yield strength ($f_{sy}$) and elastic modulus ($E_s$) of the inner steel tubes are 380 MPa and 206000 MPa, respectively. The material properties of the steel tubes for the CFST specimens are the same as those of the outer steel tubes for the CFDST specimens, since they were fabricated from the same original steel sheets.

The elastic modulus ($E_c$) of the sandwiched concrete at 28 days and at the time of ultimate...
load tests are 33010 MPa and 31080 MPa respectively, whilst the corresponding average cube strength \( f_{cu} \) are 39.3 MPa and 66.4 MPa respectively.

**Experimental Methodology**

The long-term service load tests started at 28 days after concrete casting. Figure 2 shows the test set-up for the long-term service load tests. The long-term sustained load \( N_L \) was applied by pre-stressing bars and controlled by means of a load cell. The load was kept constant with adjustments of these bars during the measurement. The axial deformation of the specimen were obtained by the deformation transducers, whilst eight gauges were mounted on the tube surface at mid-height to record the axial and transverse strains of the outer steel tube, shown in Figure 2. The long-term service load tests lasted for 1050 days and were conducted under the room temperature and humidity of the laboratory.

At the completion of the long-term service load tests, all the specimens were removed from the creep-loading devices and tested to failure under axial compression. Eight corresponding specimens without long-term service load applied were also tested at the same time, while two related specimens (DCc-0* and DSc-0*) were tested earlier at the concrete age of 28 days. All tests were performed on a 5000 kN capacity testing machine in pure compression conditions, as shown in Figure 3. Eight strain gauges were used for each specimen to measure the longitudinal and transverse strains at the middle height. Two displacement transducers were used to measure the axial deformation, and three transducers were used to measure the lateral deflection. A load interval of less than one-tenth of the estimated load capacity was used. Each load interval level was maintained for about 2 min.

**TEST RESULTS AND DISCUSSIONS**

**Long-term Service Test**

Figure 4 shows the axial strain \( \varepsilon_o \) versus sustaining time \( t \) relations for all tested specimens. It was found that, under the long-term sustained loads, the axial deformation increased relatively fast at the preliminary stage with the axial strain \( \varepsilon_o \) at 1 month attaining approximate 60% of that at 4 months. After that, \( \varepsilon_o \) developed much slower, and the process is tending to stabilise after about one year. This feature of the long-term deformation for CFDST specimens is generally similar as that for CFST specimens, as shown in Figure 4.
Since the concrete in a CFST or CFDST column was isolated from the environment, the shrinkage strain was actually very small and can be considered negligible (Ichinose et al. [6], Uy [9]). Therefore, in this paper, the creep coefficient ($\phi$) was calculated as $(\varepsilon_{cr}\varepsilon_{sh})/\varepsilon_L$, where $(\varepsilon_{cr}\varepsilon_{sh})$ is the measured long-term strain (creep strain plus shrinkage strain) and $\varepsilon_L$ is the measured short-term strain. The creep coefficients ($\phi$) of all tested CFDST and CFST specimens are presented in Table 1. It was found that, $\phi$ of the CFDST columns were
generally lower than those of the CFST column. This is mainly due to the fact that the concrete sectional area for the CFDST column is smaller than that of its CFST counterpart, which resulted in the less remarkable effect of the long-term loading.

**Ultimate Strength Test**

During the ultimate strength tests, all specimens behaved in a relatively ductile manner and testing proceeded in a smooth and controlled way. The failure pattern up to and beyond the load-bearing capacity is almost the same. Figure 5 shows the final failure appearance of all specimens. It can be seen that, both the CF DST and CFST members exhibited an overall bucking failure model, and the long-term loading had no obvious influence on the column failure modes.

![Specimens with long-term loading](image1)

![Specimens without long-term loading](image2)

Figure 5: Failure modes of the tested specimens

The ultimate strengths ($N_{ue}$) of all tested specimens are summarised in Table 1. It was found that, $N_{ue}$ of the specimens (DCc-0* and DSc-0*) tested at the concrete age of 28 days is much lower than those of the corresponding specimens tested after the long-term service load tests, due to the fact that concrete strength increased with an increase of its age. For the CFST specimens, the load-carrying capacity of the specimens subjected to long-term sustained loads were slightly lower than those of the specimens without long-term sustained loads. The strength decrease is 7.5% for the circular CFST column, and 2.0% for the square CFST column. This trend is different from the test results reported by Han and Yang [10] and Han et al. [11]. It may be explained by the fact that the sustaining time of the long-term loading in the current tests is much longer than that in the previous tests (in current tests, the sustaining time is three years, whilst they were 120 days and 180 days respectively for the tests reported by Han and Yang [10] and Han et al [11]). At the same time, the columns tested in this paper are slender than those presented previously. However, for the CFDST specimens, there is no obvious trend for the influence of the long-term loading on the ultimate strength. After experiencing the long-term loading, the circular CFDST columns had a 7.4% strength increase on average, whilst the square CFDST columns had a 5.4% strength decrease on average. It seems more tests should be conducted to clarify this.

Figures 6 and 7 present the axial load ($N$) versus lateral deflection ($u_m$) curves, respectively, for the CFST and CFDST specimens, where $u_m$ is the lateral deflection at mid-height. It was found that, in general, the sustained long-term load has a moderate effect on the shape of $N$-$u_m$ curves of the CFST and CFDST columns.

**CONCLUSIONS**

In this paper, tests on the behaviour of concrete-filled double skin tubular (CFDST) columns...
under long-term sustained loading were performed. The following conclusions are made based on the limited research reported in the paper.

Figure 6: Axial load (N) versus lateral deflection (\(u_m\)) relations of CFST specimens

(1) Circular section
(2) Square section

Figure 7: Axial load (N) versus lateral deflection (\(u_m\)) relations of CFDST specimens

(1) For CFDST columns, the long-term deformation increased relatively fast at the preliminary stage, and the process tended to stabilise after one year. This feature is generally similar to that for normal concrete-filled steel tube (CFST) columns.

(2) The creep coefficients (\(\phi\)) of the CFDST columns are generally lower than those of the CFST columns, mainly due to the fact that the concrete section area of a CFDST column is smaller than that for its CFST counterpart.

(3) The sustained long-term load has a moderate effect on the failure modes of the CFST and CFDST columns, as well as the shape of their \(N-\ u_m\) curves.

It should be noted that, further numerical study is needed to extend the range of test data, and to clarify the effect of changing variables that has not been investigated in this paper. The experimental work presented in this paper has provided a good basis for the development of theoretical models.
ACKNOWLEDGEMENTS

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REFERENCES

EXPERIMENTAL BEHAVIOUR OF SLENDER CIRCULAR CONCRETE-FILLED STAINLESS STEEL TUBULAR COLUMNS UNDER AXIAL COMPRESSION

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\textsuperscript{3} Department of Civil Engineering, Tsinghua University, Beijing, China

KEYWORDS
Concrete-filled steel tubes (C FST), stainless steel, slender columns, axial compression, strength, design, code comparison.

ABSTRACT
Stainless steel has many desirable properties which can be taken advantage of in structural applications. A total of 12 test results of circular concrete-filled stainless steel tubular columns under axial compression were presented in this paper. Several existing design codes, including the Australian design code AS 5100, American code AISC, Chinese code DBJ 13-51-2003 and Eurocode 4, are used to predict the column strength and to compare with the test results. It seems that all these codes underestimate the column strength ranged from 10% to 24% on average. Compared with conventional composite columns comprised of carbon steel, the predictions are a little more conservative.

INTRODUCTION
The past few decades have seen the accelerating interest in the use of stainless steel in construction throughout the world. This is due to the fact that stainless steel has many desirable properties, such as excellent corrosion resistance, decorative qualities, ease of maintenance and fire resistance. Concrete-filled stainless steel tubular (CFSST) columns combine the advantages of stainless steel and concrete filled steel tubes (CFST), and are considered promising to be used as structural members. So far, only limited studies have been conducted to investigate the behaviour of CFSST short columns [1-3]. In practice, columns are usually subjected to the influence of slenderness, and a slenderness reduction factor in this case should be applied for design purposes.
Against the above background, a test program was developed recently to investigate the behaviour of slender CFSST columns, where six test results of square columns have been reported by Uy et al. [4], and the test results of circular columns will be given in this paper. Several existing design codes, including the Australian design code AS 5100 [5], American code AISC [6], Chinese code DBJ 13-51-2003 [7] and Eurocode 4 [8], are used to predict the column strength and to compare with the test results. These codes are to be referred to as “AS 5100”, “AISC”, “DBJ” and “EC4” in the following.

EXPERIMENTAL PROGRAM

General

Twelve tests were planned as shown in Table 1, in which diameter-to-thickness ratio \( D/t \) (40.6, 68.2), slenderness ratio \( \lambda \) (17.1-103.5) and concrete cylinder strength \( f_c' \) (36.3, 75.4 MPa) were selected as the investigating parameters. In order to distinguish specimens with different parameters, the specimen labels were assigned according to (1) section shape (C represents specimen with circular section); (2) diameter-to-thickness ratio (1 and 2 used to distinguish specimens with different tube thicknesses); (3) slenderness ratio (1, 2 and 3 used to distinguish specimens with different slenderness ratios); and (4) concrete strength (“a” for normal strength concrete and “b” for high strength concrete). For circular columns, slenderness ratio \( \lambda \) is defined as

\[
\lambda = \frac{4L_e}{D}
\]

where \( L_e \) is the effective length of a column, and \( D \) is the overall sectional diameter. According to EC4, the shortest specimen in each test series is classified as short column, i.e. the strength reduction factor \( \phi \) is taken as unity, whilst the remains are classified as slender columns.

Material Properties

All specimens were fabricated from cold-formed tubes. Tensile coupons cut from the same original steel tubes were conducted to measure the material properties of the stainless steel. The measured properties of the steel tubes from the tests are shown in Table 2.

Two types of concrete, with cylinder compressive strengths \( f_c' \) at 28 days of 30.2 MPa and 58.6 MPa, respectively, were used to fill the hollow sections. The achieved average compressive strengths at the time of testing were 36.3 MPa and 75.4 MPa, respectively. The corresponding modulus of elasticity \( E_c \) were 33900 MPa and 37900 MPa, respectively.

Instrumentation and Loading Setup

All the axial loading tests were undertaken using a 3000 kN capacity Instron testing machine. In order to simulate pin-ended supports, two spherical hinges with a diameter of 140 mm were installed at both ends of each column before testing. The instrumentation used during the tests includes both load-lateral deflection as well as strain readings. Further details can be found in [4].

EXPERIMENTAL PROGRAM
TABLE 1
DETAILS OF SPECIMENS AND TEST RESULTS

<table>
<thead>
<tr>
<th>No.</th>
<th>Specimen label</th>
<th>D×t (mm)</th>
<th>Length L (mm)</th>
<th>λ</th>
<th>σ₀.₂ (MPa)</th>
<th>f’c’ (MPa)</th>
<th>Nₓₑ (kN)</th>
<th>uₓₘ,ᵤᵢₜ (mm)</th>
<th>εₓₑ (με)</th>
<th>εₓ (με)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C1-1a</td>
<td>113.6×2.8</td>
<td>485 17.1</td>
<td></td>
<td>288.6</td>
<td>36.3</td>
<td>738.0</td>
<td>0.39 -19297</td>
<td>13402</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>C1-1b</td>
<td>113.6×2.8</td>
<td>485 17.1</td>
<td></td>
<td>288.6</td>
<td>75.4</td>
<td>1137.1</td>
<td>0.24 -7819</td>
<td>5349</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>C1-2a</td>
<td>113.6×2.8</td>
<td>1540 54.2</td>
<td></td>
<td>288.6</td>
<td>36.3</td>
<td>578.9</td>
<td>2.07 -5443</td>
<td>3278</td>
<td></td>
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<tr>
<td>4</td>
<td>C1-2b</td>
<td>113.6×2.8</td>
<td>1540 54.2</td>
<td></td>
<td>288.6</td>
<td>75.4</td>
<td>851.1</td>
<td>2.58 -3197</td>
<td>1304</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>C1-3a</td>
<td>113.6×2.8</td>
<td>2940 103.5</td>
<td></td>
<td>288.6</td>
<td>36.3</td>
<td>357.6</td>
<td>11.80 -1630</td>
<td>479</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>C1-3b</td>
<td>113.6×2.8</td>
<td>2940 103.5</td>
<td></td>
<td>288.6</td>
<td>75.4</td>
<td>731.8</td>
<td>3.65 -2014</td>
<td>646</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>C2-1a</td>
<td>101×1.48</td>
<td>440 17.4</td>
<td></td>
<td>320.6</td>
<td>36.3</td>
<td>501.3</td>
<td>1.28 -10205</td>
<td>8191</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>C2-1b</td>
<td>101×1.48</td>
<td>440 17.4</td>
<td></td>
<td>320.6</td>
<td>75.4</td>
<td>819.0</td>
<td>0.28 -5339</td>
<td>3530</td>
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<tr>
<td>9</td>
<td>C2-2a</td>
<td>101×1.48</td>
<td>1340 53.1</td>
<td></td>
<td>320.6</td>
<td>36.3</td>
<td>446.0</td>
<td>1.70 -3653</td>
<td>2100</td>
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<td>10</td>
<td>C2-2b</td>
<td>101×1.48</td>
<td>1340 53.1</td>
<td></td>
<td>320.6</td>
<td>75.4</td>
<td>692.9</td>
<td>1.94 -3048</td>
<td>1256</td>
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<td>2540 100.6</td>
<td></td>
<td>320.6</td>
<td>36.3</td>
<td>383.0</td>
<td>1.25 -1813</td>
<td>626</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>C2-3b</td>
<td>101×1.48</td>
<td>2540 100.6</td>
<td></td>
<td>320.6</td>
<td>75.4</td>
<td>389.7</td>
<td>13.69 -2462</td>
<td>840</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 2
MATERIAL PROPERTIES OF STAINLESS STEEL

<table>
<thead>
<tr>
<th>Steel thickness (mm)</th>
<th>Initial elastic modulus E₀ (GPa)</th>
<th>0.2% proof stress σ₀.₂ (MPa)</th>
<th>Strain-hardening exponent n</th>
<th>Poisson’s ratio</th>
<th>Yield strain (10⁻⁶)</th>
<th>Ultimate strength (MPa)</th>
<th>Elongation percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8</td>
<td>173.9</td>
<td>288.6</td>
<td>7.6</td>
<td>0.262</td>
<td>3648</td>
<td>689.5</td>
<td>74.5</td>
</tr>
<tr>
<td>1.48</td>
<td>184.2</td>
<td>320.6</td>
<td>7.2</td>
<td>0.293</td>
<td>3740</td>
<td>708.0</td>
<td>54.9</td>
</tr>
</tbody>
</table>

TEST RESULTS AND DISCUSSIONS

Test Observations

The test results of the columns are summarised in Table 1, where uₘ,ᵤᵢₜ is the mid-height deflection at peak load (Nₓₑ), εₓₑ and εₓ are the corresponding axial and lateral strains, which were measured from the strain gauges located at the extreme compression fibre. Figure 1 shows a general view of typical specimens after testing. It was observed that all slender columns failed in a typical flexural mode with large lateral deflections. For those short columns, their failure was characterised by axial compression. Despite this, obvious bending deformation has also developed at the end of the testing. Since compact circular hollow sections were used, local buckling was only observed for those short columns in each test series after their peak loads had been attained, as shown in Figure 1.

Lateral deflections were measured along the column height for slender columns. Except the two slender specimens, i.e. C1-3a, C2-3b, no obvious lateral deformation was observed before peak load was reached for the other specimens. The exception for specimens C1-3a and C2-3b may be attributed to the more significant global imperfections compared with other columns. After peak load reached, the lateral deformation developed progressively with load reduction, and the strain distribution around the cross-section was no longer even. At the initial loading stage, generally, the deflection curve of a specimen was not completely symmetrical owing to the random distribution of initial global imperfections. After reaching the peak loads, the deflection curve was approximately in the shape of a half-sine wave.
Effect of Slenderness Ratio

The effect of slenderness ratio ($\lambda$) on the axial load ($N$) versus mid-height lateral deflection ($u_m$) curves is shown in Figure 2. As expected, the larger the slenderness ratio, the smaller the peak load is. Generally, the post-peak curves become steeper with decreasing $\lambda$. According to EC4, the short columns have attained their section capacities, whilst those slender columns only attained 50.8%-82.1% their section capacities, respectively. It is thus expected that slenderness reduction factors should be applied in designing slender CFSST columns.

Effect of Concrete Strength

The effect of concrete strength on the $N-u_m$ curves is depicted in Figure 3. It can be seen that the higher the concrete strength, the higher the peak load is. At the same time, less ductile behaviour can be seen from the $N-u_m$ curves as high strength concrete was used.

Effect of Diameter-to-Thickness Ratio

All specimens presented were fabricated from two kinds of steel tubes with a diameter-to-thickness ratio ($D/t$) of 40.6 and 68.2, respectively. The effect of $D/t$ ratio on the $N-u_m$ curves is plotted in Figure 4. Clearly, specimens with a smaller $D/t$ ratio reveal more ductile behaviour. But this effect is only apparent for those short columns. It seems that the $D/t$ ratio has no significant influence on the shape of $N-u_m$ curves for those slender columns. This is attributed to the fact that confinement of steel tube on concrete is less effective with an increasing of slenderness ratio.
Strain Analysis

Based on the strain readings from strain gauges located at different locations of mid-height, it seems that each column was under nearly axial compression before the peak load was reached. After that, part of the cross-section of a slender column reversed from compression to tension, which was resulted from the second-order effect.

The effects of slenderness ratio and concrete strength on the ultimate strain ($\varepsilon_{cu}$) are shown in Figure 5. It is observed from this figure that the value of $\varepsilon_{cu}$ decreases with an increase of slenderness ratio. This is because the column with a larger slenderness ratio failed at a...
smaller load level due to the second-order effect, and the materials had not been fully utilised. For a short column, it has developed ε_{cu} beyond the steel yield strain, whilst the value of ε_{cu} for a slender column is only close to or even below the steel yield strain. From Figure 5, it can also be seen that a specimen comprised of high strength concrete had a smaller ε_{cu} compared with the corresponding specimen in-filled with normal strength concrete, except those longest columns in the tests. This is attributed to the fact that high-strength concrete dilates much slower under high axial loading than normal strength concrete, thus the effect of confinement from steel tube is more pronounced for normal strength concrete. This is proved by the comparison of the lateral strains (ε_L) measured at peak loads, as shown in Table 1. As can also be seen from Figure 5 and Table 1, a shorter specimen with a smaller D/t ratio tends to develop a higher ε_{cu}. But the influence of D/t ratio on ε_{cu} for slender columns is not significant.

![Figure 5: Effect of slenderness ratio and concrete strength on ε_{cu}](image)

LOAD-CARRYING CAPACITY PREDICTION

Nowadays, no standard is available for the design of CFSST columns. However, there are several well-known national codes mentioned before that can be used for the design of carbon steel CFST columns [9]. Based on the test results presented in this paper, the design codes of AS 5100, AISC, DBJ and EC4 are compared with the current test results to evaluate their applicability in calculating the strength of CFSST columns. To fulfill this comparison, all material partial safety factors specified in the design codes have been taken as unity when comparing code calculations with the tests. At the same time, all code limitations are ignored with a purpose to check the feasibility of those design codes in predicting the load-carrying capacities of the test specimens.

The comparison between the test results N_{ue} and code predictions N_{uc} is shown in Table 3 and Figure 6. For brevity, the predicted results using AS 5100, AISC, DBJ and EC4 are designated as N_{ASS5100}, N_{AISC}, N_{DBJ} and N_{EC4} in Table 3, respectively. The obtained average values (μ) of N_{ue}/N_{uc} and standard deviations (σ) are also given in Table 3. Obviously, all codes are generally conservative and underestimate the column strength by about 17-26% on average.

In order to further evaluate the above prediction accuracy of CFSST columns, the comparison results (μ and σ of N_{ue}/N_{uc}) presented by Tao et al. (2008) for conventional circular carbon steel CFST columns are given in Table 4, where a total of 420 test results was compared.
After comparing the predictions for the CFSS T and carbon steel CFST columns, it can be seen that all code predictions for the CFSST columns are a little more conservative.

### TABLE 3

**MATERIAL PROPERTIES OF STAINLESS STEEL**

<table>
<thead>
<tr>
<th>No. Specimen label</th>
<th>$N_{ue}$</th>
<th>$N_{AS5100}$</th>
<th>$N_{ue}/N_{AS5100}$</th>
<th>$N_{AISC}$</th>
<th>$N_{ue}/N_{AISC}$</th>
<th>$N_{DBJ}$</th>
<th>$N_{ue}/N_{DBJ}$</th>
<th>$N_{EC4}$</th>
<th>$N_{ue}/N_{EC4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 C1-1a</td>
<td>738.0</td>
<td>709.0</td>
<td>1.041</td>
<td>589.6</td>
<td>1.252</td>
<td>642.9</td>
<td>1.148</td>
<td>703.9</td>
<td>1.048</td>
</tr>
<tr>
<td>2 C1-1b</td>
<td>1137.1</td>
<td>1038.6</td>
<td>1.095</td>
<td>921.2</td>
<td>1.234</td>
<td>951.1</td>
<td>1.196</td>
<td>1032.4</td>
<td>1.101</td>
</tr>
<tr>
<td>3 C1-2a</td>
<td>578.9</td>
<td>532.0</td>
<td>1.088</td>
<td>524.3</td>
<td>1.104</td>
<td>523.5</td>
<td>1.106</td>
<td>549.5</td>
<td>1.053</td>
</tr>
<tr>
<td>4 C1-2b</td>
<td>851.1</td>
<td>804.7</td>
<td>1.058</td>
<td>785.2</td>
<td>1.084</td>
<td>708.4</td>
<td>1.201</td>
<td>818.6</td>
<td>1.040</td>
</tr>
<tr>
<td>5 C1-3a</td>
<td>357.6</td>
<td>361.2</td>
<td>0.990</td>
<td>371.3</td>
<td>0.963</td>
<td>376.6</td>
<td>0.950</td>
<td>357.8</td>
<td>0.999</td>
</tr>
<tr>
<td>6 C1-3b</td>
<td>731.8</td>
<td>476.5</td>
<td>1.536</td>
<td>491.2</td>
<td>1.490</td>
<td>478.9</td>
<td>1.528</td>
<td>426.3</td>
<td>1.717</td>
</tr>
<tr>
<td>7 C2-1a</td>
<td>501.3</td>
<td>469.2</td>
<td>1.068</td>
<td>402.4</td>
<td>1.246</td>
<td>419.0</td>
<td>1.196</td>
<td>466.0</td>
<td>1.076</td>
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<td>8 C2-1b</td>
<td>819.0</td>
<td>744.0</td>
<td>1.101</td>
<td>674.2</td>
<td>1.215</td>
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<td>1.235</td>
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<td>1.110</td>
</tr>
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<td>9 C2-2a</td>
<td>446.0</td>
<td>364.3</td>
<td>1.224</td>
<td>353.3</td>
<td>1.262</td>
<td>340.5</td>
<td>1.310</td>
<td>373.4</td>
<td>1.194</td>
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<td>10 C2-2b</td>
<td>692.9</td>
<td>588.0</td>
<td>1.178</td>
<td>561.9</td>
<td>1.233</td>
<td>492.8</td>
<td>1.406</td>
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<td>1.183</td>
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<td>11 C2-3a</td>
<td>383.0</td>
<td>245.6</td>
<td>1.559</td>
<td>242.1</td>
<td>1.582</td>
<td>239.7</td>
<td>1.598</td>
<td>232.8</td>
<td>1.645</td>
</tr>
<tr>
<td>12 C2-3b</td>
<td>389.7</td>
<td>342.2</td>
<td>1.139</td>
<td>331.0</td>
<td>1.177</td>
<td>327.1</td>
<td>1.191</td>
<td>284.3</td>
<td>1.371</td>
</tr>
</tbody>
</table>

| $\mu$             | 1.173    | 1.237        | 1.255                | 1.21      | 1                  |
| $\sigma$          | 0.185    | 0.166        | 0.181                | 0.241     |                    |

Figure 6: Column strength based on different code predictions

### TABLE 4

**COMPARISON RESULTS OF CODE PREDICTIONS WITH TEST RESULTS OF CARBON STEEL CFST COLUMNS**

<table>
<thead>
<tr>
<th></th>
<th>AS 5100</th>
<th>AISC</th>
<th>DBJ 13-51-2003</th>
<th>EC4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>1.163</td>
<td>0.170</td>
<td>1.195</td>
<td>0.172</td>
<td>1.080</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.172</td>
<td>0.108</td>
<td>0.189</td>
<td>0.133</td>
<td>0.162</td>
</tr>
</tbody>
</table>
CONCLUSIONS

Twelve circular concrete-fill ed stainless steel tubular (CFSST) columns were tested under axial compression to investigate the influence of global slenderness. Comparisons of the test results were also m ade with several existing design methods for conventional carbon steel CFST columns. The following conclusions can be drawn within the limitation of this study:

1. Slenderness reduction factors should be applied in designing slender CFSST columns.
2. The higher the concrete strength, the higher the peak load is. Short columns with a smaller diameter-to-thickness ratio show more ductile behaviour. But this effect is less significant for slender columns.
3. All codes used in this paper underestimate the CFSST column strength by about 10-24%. Compared with carbon steel CFST columns, the predictions are a little more conservative.

ACKNOWLEDGEMENTS

This research work is part of the Research Grant Scheme and the International Research Initiatives Scheme supported by the University of Western Sydney, and the project is supported by the National Basic Research Program of China (973 Program) (Grant No. 2009CB623200). This financial support is gratefully acknowledged.

REFERENCES

CLOSED FORM SOLUTIONS FOR THE LONG-TERM ANALYSIS OF COMPOSITE STEEL-CONCRETE MEMBERS SUBJECTED TO NON-UNIFORM SHRINKAGE DISTRIBUTIONS

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KEYWORDS

Composite action, composite members, partial interaction, shrinkage, time effects.

ABSTRACT

This paper presents analytical solutions for the long-term analysis of composite steel-concrete members considering different shrinkage profiles through the cross-section of the slab. This situation is particularly important for members in the presence of steel deck. In fact, the non-uniform shrinkage profile develops through the slab thickness because of the lack of moisture egress on its bottom side due to the presence of the sheeting.

In practice, the main advantage of composite flooring systems relies on the ability of the profiled sheeting to act as permanent formwork and, once the concrete has hardened, as external reinforcement.

The weak form of the partial interaction problem is derived by means of the principle of virtual work and is integrated by parts to produce the local balance condition. The long-term behaviour of the concrete is modeled by means of the simplified methods while the remaining materials are assumed to remain linear-elastic.

The general solution is then applied to the particular cases of simply supported and propped cantilever for which closed form solutions are presented.

INTRODUCTION

The use of steel deck for building application has been gaining popularity thanks to its ability to act as permanent formwork and, once the concrete has hardened, as external reinforcement.

This paper proposes an analytical model capable of handling non-uniform shrinkage distributions which can occur in composite steel-concrete beams when constructed with profiled sheeting. In these cases drying can only occur on the top side of the slab as moisture egress is prevented by the presence of the sheeting.
MODEL DESCRIPTION

Model Assumptions

A prismatic steel-concrete composite beam is made of a reinforced concrete or composite slab and a steel beam, as shown in Figure 1. In its undeformed state, the composite beam occupies the cylindrical region \( V = A \times [0,L] \) generated by translating its cross-section \( A \), with regular boundary \( \partial A \), along a rectilinear axis orthogonal to the cross-section and parallel to the \( Z \) axis of an ortho-normal reference system \( \{O;X,Y,Z\} \); \( \mathbf{i}, \mathbf{j}, \mathbf{k} \) are the unit vectors of axis \( X, Y, Z \). The composite cross-section domain is formed by the slab, referred to as \( A_1 \), and by the steel beam, referred to as \( A_2 \). The two components of the composite cross-section, \( A_1 \) and \( A_2 \), are assumed to be symmetric about the \( YZ \) plane. Loads are symmetric with respect to the \( YZ \) plane which represents the plane of bending.

Both slab component and steel joist are assumed to deform based on the assumptions of the Euler-Bernoulli beam model, i.e., small displacements and strains, plane sections perpendicular to the beam axis remain plane, rigid and perpendicular to the beam axis after deformation. Perfect bond occurs between reinforcement and concrete, and between the profiled sheeting and concrete. The composite action between the two layers is provided by a continuous deformable interface along a rectilinear line \( \mathbf{I} \) at the interface between the two layers, whose domain consists of the points in the \( YZ \) plane with \( y = y_{sc} \) and \( z \in [0,L] \), \( y_{sc} \) being defined in Figure 1. The connection is assumed to permit only discontinuities parallel to the beam axis.

Displacement and Strain Fields

The displacement field of a generic point \( P(x,y,z) \) of the composite beam is defined by vector \( \mathbf{d} \):

\[
\mathbf{d}(y,z;t) = \begin{cases} 
\mathbf{d}_1(y,z;t) = v(z;t)\mathbf{j} + (w(z;t) - y_{sc})v'(z;t) - s(z;t)\mathbf{k} & \forall (x,y) \in A_1, \; z \in [0,L] \\
\mathbf{d}_2(y,z;t) = v(z;t)\mathbf{j} + (w(z;t) - y_{sc})v'(z;t)\mathbf{k} & \forall (x,y) \in A_2, \; z \in [0,L] 
\end{cases}
\]

where \( w(z;t) \) is the axial displacement measured at the arbitrary reference fibre of the steel joist referred to as \( y_r \) (Figure 2), \( v(z;t) \) represents the deflection of both components as no vertical separation can take place, \( s(z;t) \) is the slip between the two components which depicts the relative movement between the slab and joist, \( t \) is the time measured from an arbitrary instant in time (usually defined as the instant of the concrete pour) and the prime represents a
derivative with respect to $z$. The functions defining the displacement field can be grouped in the vector

$$\mathbf{u}^T(z;t) = [w(z;t) \ v(z;t) \ s(z;t)] \ (2)$$

Figure 2: Displacement field

Based on the assumed displacement field, the non zero components of the strain field are:

$$\varepsilon_z(y,z;t) = \frac{\partial d}{\partial z} = \begin{cases} \varepsilon_{z1}(y,z;t) = w' - \left(y - y_r\right)\nu'' - s' & \forall (x,y) \in A_1, z \in [0,L] \\ \varepsilon_{z2}(y,z;t) = w' - \left(y - y_r\right)\nu'' & \forall (x,y) \in A_2, z \in [0,L] \end{cases} \ (3)$$

where $\varepsilon_{z1}$ and $\varepsilon_{z2}$ are the axial strains of the two components respectively. The functions describing the strain field can be collected in the vector

$$\varepsilon^T(z;t) = [\varepsilon(z;t) \ \kappa(z;t) \ -s'(z;t) \ s(z;t)] \ (4)$$

where $\varepsilon(z;t) = w'$ is the axial strain at the levels of the reference fibre and $\kappa(z;t) = -\nu''$ is the curvature. The vector of strain functions can be obtained from the vector of displacement functions by means of the relation:

$$\varepsilon = \mathcal{D}\mathbf{u} \ (5)$$

where the matrix operator is defined as

$$\mathcal{D}^T = \begin{bmatrix} \partial & 0 & 0 & 0 \\ 0 & -\partial^2 & 0 & 0 \\ 0 & 0 & -\partial & 1 \end{bmatrix} \ (6)$$

being $\partial$ the derivative with respect to $z$. For ease of notation, space and time variable dependency is omitted when clear from the context.

**MATERIAL PROPERTIES**
The time-dependent behaviour of the concrete is modeled accounting for creep and shrinkage effects based on the integral-type creep law [1] as

\[
\varepsilon_{sh}(t) - \varepsilon_{sh}(t_0) = \sigma_c(t_0) J(t, t_0) + \int_{t_0}^{t} J(t, \tau) d\sigma_c(\tau) (7)
\]

where \( t \) is the time from casting of the concrete, \( t_0 \) is the time of first loading, \( \sigma_c(\tau) \) is the concrete stress calculated at time \( \tau \), \( \varepsilon_{sh}(t) \) is the shrinkage strain, and \( J(t, \tau) \) is the creep function defined as the strain at time \( t \) due to a constant unit stress acting from time \( \tau \) to time \( t \). The superposition integral of equation (7) is here approximated by means of the algebraic methods, i.e. using the AEMM and MS methods ([1][1-3]). These have been recommended by Dezi et al. [4,5] for the analysis of composite beams based on a parametric study carried out benchmarking the results obtained using different algebraic representations against those using the step-by-step method. In particular, they recommended the use of the AEMM method to model the time-dependent behaviour of the concrete when the structural system is subjected to external loads, while using the MS method to consider shrinkage effects. It is also assumed that the time-dependent behaviour of the concrete is identical in both compression and tension, as recommended by Gilbert [6] for stress levels in compression less than about one half of the compressive strength of the concrete, and for tensile stresses less than about one half of the tensile strength of the concrete. Based on this, the results obtained using the proposed approach are assumed to be acceptable from a qualitative and quantitative viewpoint when the calculated stresses remain in this stress range. Nevertheless, when the calculated stresses are outside this range the results might still be meaningful from a qualitative viewpoint, for example in comparing the effects of different cross-sectional properties.

It is assumed that the reinforcing bars, the steel joist and the profiled sheeting behave in a linear-elastic fashion, and \( E_r, E_s \) and \( E_d \) are the relevant elastic moduli for the reinforcement, steel joist and steel deck respectively. The constitutive relationship for the shear connection is expressed by

\[
g(z; t) = k_s(z; t) (8)
\]

in which the connection stiffness \( k \) relates the longitudinal force per unit length \( g \) to the corresponding slip \( s \).

**WEAK AND STRONG FORMULATIONS**

The weak formulation of the partial interaction problem, derived by means of the principle of virtual work, can be expressed in compact form for material accounting for the assumed material properties as

\[
\left[ (TD u - f_d) \cdot D \hat{u} \right] dz = \left[ (q \cdot H \hat{u}) dz + [Q \cdot H \hat{u}]_{,L} \right] \quad \forall \hat{u} (9)
\]

and

\[
T = \begin{bmatrix} AE & BE & AE_1 & 0 \\ BE & IE & BE_1 & 0 \\ AE_1 & BE_1 & AE_1 & 0 \\ 0 & 0 & 0 & k \end{bmatrix} \quad H^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} (10a,b)
\]
where the virtual displacements and strains have been identified by a hat (^) placed above the variable considered, \( T \) depicts the cross-sectional properties, \( AE \), \( BE \) and \( IE \) represent the axial rigidity, the rigidity related to the first moment of area and the flexural rigidity respectively, \( AE_1 \) and \( BE_1 \) define the rigidities related to the top layer only, \( b \) and \( p \) represent body and surface forces respectively and the solution of the problem is then sought in the spaces of the regular functions fulfilling the kinematic boundary conditions. Without loss of generality, the shrinkage variations are assumed to remain constant along the length of the member while the non-uniform variation across the thickness is defined by the following cubic function:

\[
\varepsilon_{sh}(y) = a_{sh0} + a_{sh1}y + a_{sh2}y^2 + a_{sh3}y^3 \quad (13)
\]

Obviously, other expressions could have been easily adopted in the derivation. Integrating the weak form by parts produces the local balance conditions of the partial interaction problem which can be expressed as

\[
H \cdot (TD \mathbf{u} - \mathbf{f}_{sh}) + H \cdot \mathbf{q} = 0 \quad (14)
\]

with the relevant boundary conditions at the two end sections of the segment being obtained as

\[
[A \ (TD \mathbf{u} - \mathbf{f}_{sh}) + B \cdot \mathbf{q}] \cdot \mathbf{u}_{b,0,L} - Q_{L} \cdot \mathbf{H} \cdot \mathbf{u}_{L} - Q_{0} \cdot \mathbf{H} \cdot \mathbf{u}_{b} = 0 \quad \forall \mathbf{u}_{b,0,L} \quad (15)
\]

which represents the kinematic conditions when \( \mathbf{u} = 0 \) and the dual static conditions when \( \mathbf{u} \neq 0 \), and the rest of the notation is defined as:

\[
D^* = \begin{bmatrix}
\partial^4 & 0 & 0 & 0 \\
0 & \partial^4 & 0 & 0 \\
0 & 0 & \partial^4 & 1
\end{bmatrix} ; H^* = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \partial^4 & 0 & 0 \\
0 & 0 & \partial^4 & 0
\end{bmatrix} ; A = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & \partial^4 & 0 & 0
\end{bmatrix} ; B = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

(16a,b,c,d)
CLOSED FORM SOLUTIONS

Closed form solutions can be obtained based on the system of differential equations and corresponding boundary conditions expressed in Equations (14-15) and these can be expressed for the generalised displacements as follows

\[ v = \frac{C_1}{6} z^3 + \frac{C_2}{2} z^2 + C_3 z + C_4 + \frac{\beta_1 e^{\alpha z}}{\alpha} C_5 - \frac{\beta_2 e^{-\alpha z}}{\alpha} C_6 + \frac{\beta_3}{6} p_z + \frac{\beta_4}{24} p_y + \frac{\beta_5}{6} z^3 p_m \]  

(17a)

\[ s = -\frac{\beta_3}{k} C_1 + C_5 e^{\alpha z} + C_6 e^{-\alpha z} + \frac{1}{k} p_z \frac{\beta_1}{k} p_y - \frac{\beta_2}{k} z \frac{\beta_4}{k} p_y + \frac{\beta_5}{k} p_m \]  

(18b)

\[ w = \int \left( \frac{B k}{AE} v'' + \frac{AE}{AE} s' - \frac{p_z}{AE} \right) dz \]  

(19c)

where the introduced constants \( C_j \) (with \( j = 1, \ldots, 8 \)) vary depending on the boundary conditions of the problem while all remaining notation is defined in Appendix.

In the following the actual closed form solutions describing the time-dependent response of a simply supported beam and a propped cantilever are provided considering non-uniform shrinkage only. In fact, the principle of superposition can be utilised to combine these with other expressions readily available in the literature. [7]

Applying the appropriate boundary condition in the case of a simply supported beam leads to the following expressions for the eight constants as

\[ C_1 = 0 \quad \text{;} \quad C_2 = \frac{\beta_3 \beta_{sh} - \beta_{sh2}}{\beta_0 \beta_1} \quad \text{;} \quad C_3 = -\frac{L}{2} C_2 \quad \text{;} \quad C_4 = \frac{\beta_3 (-C_4 + C_6)}{\alpha} \]  

(20a,b,c)

\[ C_5 = \beta_{sh1} \frac{e^{\alpha z} - 1}{\alpha \beta_0 \beta_1 (e^{\alpha z} - e^{-\alpha z})} \quad \text{;} \quad C_6 = \beta_{sh1} \frac{e^{-\alpha z} - 1}{\alpha \beta_0 \beta_1 (e^{\alpha z} - e^{-\alpha z})} \]  

(20d,e)

\[ \beta_{sh1} (\beta_2 BE + AE_1) - \int_{A_1} E_e e_{sh}(y) \, da AEBE_1 (BE_1 - BE) - \int_{A_2} (y - y_r) E_e e_{sh}(y) \, da AEAE_1 BE_2 \]  

(20f)

\[ C_7 = \frac{AE \beta_0 \beta_1}{C_5 + C_6 (BE \beta_2 + AE_1)} \]  

(20g)

while for propped cantilevers these are expressed as

\[ C_1 = -\frac{3 k}{2 \alpha} \frac{2 e_3 \beta_2 \beta_{sh1} + e_2 \alpha^2 L^2 (\beta_2 \beta_{sh1} - \beta_{sh2})}{\beta_0 \beta_1 \left[ -e_3 \alpha L (k L^2 + 3 \beta_3 \beta_3) + 3 e_2 \beta_2 \beta_1 \right]} \]  

(21a)

\[ C_2 = \frac{1}{3} \frac{e_3 \alpha^2 C_1 \beta_7 + 6 \alpha \beta_2 \beta_3 e_4 C_1 \beta_8 + 6 k \beta_2 e_3 \left( A E_2 \int_{A_1} E_e e_{sh}(y) \, da + C_1 L \beta_8 \right)}{k \left( -e_3 \alpha^2 L^2 \beta_8 - 2 e_2 \beta_2 \beta_1 \right)} \]  

(21b)

\[ C_3 = -\frac{\beta_3 \beta_2 C_7}{k} \quad \text{;} \quad C_4 = \frac{1}{3} \frac{3 \beta_3 \beta_2 C_3 (2 L \alpha - e_2) - k \alpha L^2 (C_1 L + 3 C_2)}{k e_3} \]  

(21c,d)
\[ C_5 = \frac{1}{6} 6 \beta_2 C_0 e^{-\alpha t} - C_1 L^3 \alpha - 3C_1 L^2 \alpha - 6C_1 L \alpha - 6C_1 \alpha \] (21e)

\[ C_6 = \frac{1}{6} k \alpha L^2 (C_1 + 3C_2) + 6k \alpha (C_3 L + C_4) + 6 \beta_2 C_2 \alpha e^{\alpha t} \] (21f)

\[ BEC_2 + \int \frac{E_v e_{sh}(y)}{AE} \, dy \]

\[ C_7 = -\beta_2 C_1 \frac{BE \beta_2 + AE \epsilon}{AEk} \] (21g)

and the remaining notation is defined in Appendix.

**VALIDATION OF THE CLOSED FORM SOLUTIONS**

The proposed closed form solutions have been validated against results obtained using the finite element method considering the three non-uniform shrinkage profiles illustrated in Figure 3. These consist of a linear and two parabolic ones referred to in the following as Case L, Case PA and Case PB respectively. For this purpose the refined 16DOF element, first proposed by Dall'Asta and Zona [8], has been utilised. Due to limited space only comparisons for the case of a simply supported beam are proposed in Figure 4. The beam dimensions and cross-sectional properties of the beam are those utilised in experimental work carried out by the authors and reported in reference [9]. The proposed results have been calculated for an \( \alpha L \) value equal to 5 which is well representative of the partial interaction response.

![Figure 3: Definition of three shrinkage profiles for the application: one linear and two parabolic ones](image)

![Figure 4: Variation of the deflection and slip along the member length of a simply supported beam](image)
CONCLUSIONS

This paper presented closed form solutions for the analysis of composite beams with partial interaction subjected to non-uniform shrinkage profiles. This loading condition becomes important when using steel decking as formwork. In fact, in this case drying can only occur on one side of the slab due to the presence of the profiled sheeting.

The weak and strong forms of the partial interaction problem are presented. Analytical solutions are derived for the generalised displacements considering the cases of simply supported beams and propped cantilevers. Their accuracy has been validated against results obtained using the finite element method.

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REFERENCES


APPENDIX

\[
\begin{align*}
\beta_0 &= AEIE - BE^2 ; \quad \beta_1 = \left( AE, AE, IE - BE, IE - BE, IE \right) / \beta_0 ; \quad \beta_2 = \left( BE, AE, IE - AE, BE \right) / \beta_0 \\
\beta_3 &= \beta_0 \beta_2 / AE ; \quad \beta_4 = \left( BE, IE - AE, IE \right) / \beta_0 ; \quad \beta_5 = -BE / \beta_0 ; \quad \beta_6 = AE / \beta_0 \\
\beta_7 &= kL \beta_8 - 6 \beta_3 \beta_4 \left( \beta_2 \beta_6 + AE, IE \right) ; \quad \beta_8 = AE, AE, IE + \beta_2 \left( BE, IA, IE - AE, BE \right) \\
\beta_9 &= BE, IE - AE, IE ; \quad \epsilon_1 = e^{at} + e^{-at} ; \quad \epsilon_2 = e^{at} - e^{-at} ; \quad \epsilon_3 = 2 - e^{at} - e^{-at} \\
\alpha &= \sqrt{\frac{1}{\beta_1}} ; \quad \beta_{a1} = \int_{a}^{b} E_e, e_a(y) \, dy \, d \alpha \beta_0 (1 + \beta_2) + \int(y - y) E_e, e_a(y) \, dy \, d \alpha \beta_0 \beta_2 \\
\beta_{a2} &= \int_{a}^{b} E_e, e_a(y) \, dy \, d \alpha (\beta_0 \beta_2 - AE, IE, IE) + \int(y - y) E_e, e_a(y) \, dy \, d \alpha AE, IE, IE
\end{align*}
\]
STRESS ANALYSIS OF STEEL FIBER REINFORCED CONCRETE ENCASED TUBULAR STEEL PENSTOCKS UNDER INTERNAL WATER PRESSURE

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KEYWORDS
Steel penstock, tubular, steel fiber reinforced concrete, reinforced concrete, internal water pressure

ABSTRACT
Reinforced concrete enased steel penstocks (RC-ESPs), which are usually laid on the downstream surfaces of dams, have been applied in many large hydraulic projects including the famous “The Three Gorges Project” of the Chinese mainland. Compared to conventional steel penstocks, which are usually embedded in dams, RC-ESPs are easier to construct, cost-effective and more safe and durable. However, a difficulty encountered in engineering practice for such a composite structure is in the crack control of the exterior reinforced concrete (RC) wall. To solve such a problem, steel fiber reinforced concrete (SFRC) can be used to replace the conventional concrete of the external RC wall to form a new type of structure: steel fiber reinforced concrete enased steel penstocks (SFRC-ESPs). The objective of this paper is to develop generic analytical models for analyzing the SFRC enased tubular steel penstocks (SFRC-ETSPs) under internal water pressure. A tri-linear curve is assumed in the models to describe the behavior of SFRC under tension. Based upon the developed models the strains and stresses of all the structural components, including the enased steel penstock and the steel reinforcement and SFRC in the exterior RC wall, can be predicted under different levels of internal water pressure. The analytical models provide a sound basis for designing the serviceability and safety of the SFRC-ETSPs.

INTRODUCTION
Reinforced concrete enased steel penstocks (RC-ESPs), which are usually laid on the
downstream surfaces of dams, emerged many decades ago in the former Soviet Union. The RC-ESP is a structural form composed of an interior steel liner and an exterior reinforcing concrete wall. Compared to the steel penstocks, the RC-ESP has some advantages as follows: 1) The difficulties in processing and welding of the thick high strength steel plates, which are usually needed for penstocks with a large diameter and a high water head, can be eliminated; 2) The RC-ESP benefits a ductile failure mode, leading to an improvement of the structural safety; and 3) As the RC-ESPs are always laid on the downstream face of a dam, the integrity of the dam can be well kept. While this structural form has been used in many hydroelectric projects including the famous “Three Gorges Dam” of the Chinese mainland, design methods for the RC-ESP are still under development. One of the pending issues is how to efficiently control the crack width of the exterior RC wall, which influences greatly the durability of the structure. In many existing projects larger cracks have been observed in the exterior RC walls when the RC-ESPs are subjected to internal water pressure or/and the complex temperature loading action during the construction and service periods. Efforts have been made to use steel fiber reinforced concrete (SFRC) to replace the conventional concrete in the exterior RC walls of RC-ETSP (e.g. He and Huang 2006) for a better crack resistance, while the design theory for SFRC encased tubular steel penstocks (SFRC-ETSP) is not yet available.

The existing analytical approaches for RC-ESPs fall into the following two categories: (1) numerical modeling approach; (2) analytical modeling approach. The numerical modeling approach is usually involved nonlinear three-dimensional (3D) finite element (FE) modeling of the RC-ESP system, which exhibits a significant advantage in dealing with complex loading conditions and can provide more accurate predictions. Comparatively, the analytical modeling approach is more commonly applied in practical engineering due to its simplicity, in which the RC-ESPs are simplified as a two-dimensional (2D) plane strain model. In the analytical modeling of RC-ESPs, the steel reinforcements in the exterior RC walls usually are simplified as multi-layered steel tubes (Dong 1986; Ma et al. 1986). Before its cracking, the concrete is considered to be a homogeneously elastic material while an orthotropic material after its cracking, which only transfers radial internal water pressure to the exterior reinforcement (equivalent steel tube) and bears no circumferential tensile stresses. Obviously, such an assumption is inappropriate when SFRC is used instead of plain concrete in the exterior RC walls since the SFRC usually has a significant softening phenomenon. After the cracking of concrete, the SFRC continues sustaining the circumferential stresses. The objective of this paper is therefore to develop explicit models to predict the stresses/strains of the encased steel penstock and the steel reinforcement and SFRC in the exterior RC wall when the SFRC-ETSP is subjected to internal water pressure, to provide a sound basis for both the serviceability and safety design.

**COMPUTATIONAL MODEL**

**Equilibrium Equation**

Because of the axisymmetrical feature of tubular penstocks and the internal water pressure, a half of the SFRC-ETSP is selected as the computational unit. Figure 1 shows the sketch of the structural model of the half SFRC-ETSP under the internal water pressure. The force equilibrium between the internal water pressure and the resultant sectional force in the internal steel penstock and encasing RC wall for a unit longitudinal length can be written as follows:
where, $p$ = internal water pressure of the penstock (MPa); $R_1$ = inner radius of the steel penstock; $t$ = thickness of the interior steel penstocks; $h_0$ = thickness of the exterior SFRC wall; $E_s$ = elastic modulus of steel penstock; $\varepsilon_{si}$ = the circumferential tensile strain of steel penstocks; $\varepsilon_{sbi}$ = the circumferential tensile strain of the $i^{th}$ layer of steel reinforcement; $n_i$ = number of steel reinforcement in the exterior SFRC wall within the unit longitudinal length of penstock; $m$ = number of the layers of steel reinforcement in the exterior SFRC wall; $\sigma_c$, $\varepsilon_c$ = hoop stress and strain of SFRC; $E_{sbi}$ = elastic modulus of the $i^{th}$ layer of steel reinforcement; $A_{si}$ = sectional area of individual steel reinforcement in the $i^{th}$ layer. When the stress $\sigma_c$ is calculated, the elastic modulus of concrete should be taken as $E_c = E_c / (1 - \mu_t^2)$ in consideration of the plane strain condition.

Figure 1: Structural model for SFRC-ETSP

Basic Assumptions for Analysis

In order to find explicit solutions for the Eqn 1, some basic assumptions are given for the structural model as follows:

1. The radial displacement of the section at any location is equal, since the radial compressive stress and deformation of the SFRC-ETSP is negligible small.
2. The relative slips between interior penstocks and the SFRC wall and between the steel reinforcement and surrounding SFRC in the exterior wall are neglected.
3. The tensile stress-strain relationship of the SFRC is independent of its radial strain, since the hoop strain caused by the radial force due to Poisson Effect is very small compared to the hoop strain of cracked FRC.

In engineering practice, a gap may exist between the interior steel penstock and the exterior SFRC wall due to constructional imperfections or/and the thermal contraction of steel penstock when it is filled with water. Under such a circumstance, the interior steel penstock needs to be ar alone an initial water pressure until it is closely contacted the exterior SFRC wall. In that case, the internal water pressure, $p$, used for Eqn 1 should be the design value deducted by that initial value. Assumption (1) implies that, for any point in the exterior RC wall, the hoop strain $\varepsilon$ can be expressed as $\varepsilon R = u$, where $u$ is the radial displacement of this point. In other words, the hoop strain at a location is inversely proportional to the radius of that location.
STRESS AND STRAIN ANALYSIS

Tensile Stress-strain Relationship of Steel Fiber Reinforced Concrete (SFRC)

Many models are available to describe the tensile stress-strain relationship of SFRC (e.g., Zhao et al. 1999). In general, they consist of two parts: a linear ascending branch and a nonlinear descending branch. Figure 2 presents an idealized tensile stress-strain relationship of SFRC, which exhibits a strain softening phenomenon (Soranakom et al. 2008). This relationship will be employed for the analysis of SFRC-ETSPs.

A tri-linear model can be used to describe the above relationship as follows:

\[
\sigma = E \varepsilon = \frac{f_p}{\varepsilon_p} \varepsilon \quad (\varepsilon < \varepsilon_p)
\]

\[
\sigma = f_p + \left(\frac{\sigma_{p1} - f_p}{\varepsilon_{p1} - \varepsilon_p}\right) (\varepsilon_p < \varepsilon < \varepsilon_{p1})
\]

\[
\sigma = f_{p1} \quad (\varepsilon > \varepsilon_{p1})
\]

where \(\sigma\) = tensile stress in SFRC; \(f_p\) = tensile strength of SFRC; \(\varepsilon_p\) = strain of SFRC corresponding to its tensile strength; \(f_{p1}\) = constant residual tensile stress of SFRC; \(\varepsilon_{p1}\) = strain at which SFRC starts to enter into the constant residual stress stage. The value of \(f_{p1}\) and \(\varepsilon_{p1}\) can be empirically correlated with the fiber volume fraction and their bond characteristics with the concrete matrix.

Four Critical Internal Water Pressure

According to the above tri-linear tensile stress-strain relationship of SFRC, the SFRC-ETSPs can be assumed to exhibit five typical loading stages, which correspond to four critical internal water pressures:
a) $p_1$, at which the stress of SFRC at the inner edge of exterior SFRC wall starts to reach the tensile strength of SFRC, $f_{ft}$;

b) $p_2$, at which the stress of SFRC at the outer edge of exterior SFRC wall starts to reach the tensile strength of SFRC, $f_{ft}$;

c) $p_3$, at which the stress of SFRC at the inner edge of SFRC wall starts to reach the residual strength of SFRC, $f_{ft}'$; and

d) $p_4$, at which the stress of SFRC at the outer edge of SFRC wall starts to reach the residual strength of SFRC, $f_{ft}'$.

![Diagram showing stress and strain distribution](image)

Figure 3: Sectional stress and strain distribution in the SFRC wall

Figures 3.a to 3.d present the stress and strain distributions of the cross-section of SFRC wall that correspond to the above four stages, respectively. Under such conditions and based upon the force equilibrium shown in Eqn 1, the critical internal water pressure corresponding to the above four status can be expressed using the following equations:

$$p_1 = \left( E_{st} \varepsilon_p t + \sum_1^m \frac{R_i}{R_i} n_i E_{sh} \varepsilon_{p1} A_{sl} + \frac{f_{ft}'}{E_p} R_i \ln \frac{R_i + h_0}{R_i} \right) / R_i$$

$$p_2 = \left( E_{st} \varepsilon_p t + \frac{R_i + h_0}{R_i} + \frac{R_i + h_0}{R_i} \sum_1^m \frac{R_i}{R_i} n_i E_{sh} \varepsilon_{p1} A_{sl} + Qh_0 - k \varepsilon_p (R_i + h_0) \ln \frac{R_i + h_0}{R_i} \right) / R_i$$

$$p_3 = \left( E_{st} \varepsilon_p t + \sum_1^m \frac{R_i}{R_i} n_i E_{sh} \varepsilon_{p1} A_{sl} + Qh_0 - k \varepsilon_p R_i \ln \frac{R_i + h_0}{R_i} \right) / R_i$$

$$p_4 = \left( E_{st} \varepsilon_p t + \frac{h_0}{R_i} + \frac{h_0}{R_i} \sum_1^m \frac{R_i}{R_i} n_i E_{sh} \varepsilon_{p1} A_{sl} + f_{ft} h_0 \right) / R_i$$

where $k = (f_{ft}' - f_{ft}) / (\varepsilon_{p1} - \varepsilon_p)$; $Q = f_{ft} + k \varepsilon_p$. 

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Strain and Stress Analysis for Different Stages

Elastic stage \((p < p_1)\)

Before the occurrence of the first cracking in SFRC, the SFRC can be treated as a homogeneous elastic material. Consequently, the SFRC-ETSPs can be rigorously analyzed following elastic mechanics (Timoshenko 1979). However, the penstock is still analyzed based on the aforementioned basic assumptions for such a stage for simplicity and consistency. As mentioned above, Assumption (1) indicates that the radial strain of each location on the section is inversely proportional to its radius, namely \(u_R = \varepsilon\) and can be expressed as:

\[
\varepsilon = \varepsilon_s R_i/(R_i + h),
\]

in which \(h\) is the radial distance from the location to the inner edge of the SFRC wall. Therefore, with the substitution of Eqn 2.1, Eqn 1 can be rewritten as:

\[
pR_1 = E_s l\varepsilon_s t + \varepsilon_{sl} \sum_{i=1}^{m} n_i E_{sbi} A_{si} R_i / R_i + \int_{h_0}^{R_1} \frac{f_{fl}}{\varepsilon_p} \left( \frac{R_1}{R_1 + h} \varepsilon_{sl} \right) dh
\]

when \(\varepsilon_{sl} = \varepsilon_p\), Eqn 3 can be obtained. The hoop strain of the interior steel penstocks, \(\varepsilon_{sl}\), can be obtained from Eqn 7 as:

\[
\varepsilon_{sl} = \frac{pR_1}{\left( E_s l\varepsilon_s t + \sum_{i=1}^{m} n_i E_{sbi} A_{si} R_i / R_i + \int_{h_0}^{R_1} \frac{f_{fl}}{\varepsilon_p} R_i \ln R_i + R_1/h_0 \right)}
\]

Given an internal water pressure \(p\), once the \(\varepsilon_{sl}\) is known, the strain and stress in the steel reinforcement and SFRC can be determined.

Crack propagation stage I \((p_1 < p < p_2)\)

When the cracks propagate from the inner side to the outer side of the SFRC wall, the SFRC experiences both ascending and descending branches of the tensile stress-strain curve along the thickness of the SFRC wall. So there is a location where the SFRC has the peak tensile stress \(f_{fl}\) and from which to the inner steel penstocks the radial distance is \(h_1\). The force equilibrium for this stage hence can be written as:

\[
pR_1 = E_s l\varepsilon_s t + \sum_{i=1}^{m} n_i E_{sbi} A_{si} \varepsilon_{sl} + \int_{h_0}^{R_1} \left[ f_{fl} + \frac{f_{fl} - f_{fl}}{\varepsilon_p} \right] (\varepsilon - \varepsilon_p) dh + \int_{h_0}^{R_1} \frac{f_{fl}}{\varepsilon_p} \varepsilon dh
\]

\[
\varepsilon_{sl} = \frac{pR_1 - Qh_1}{E_s l + \sum_{i=1}^{m} n_i E_{sbi} A_{si} - kR_1 \ln R_1 + h_1 + \int_{h_0}^{R_1} \frac{f_{fl}}{\varepsilon_p} R_1 \ln R_1 + h_0}
\]

where \(h_1 = (\varepsilon_{sl} - \varepsilon_p) R_i/\varepsilon_p\) according to the strain compatibility on the section. With substitution of \(h_1\) into Eq.10, \(\varepsilon_{sl}\) can be obtained. If the descending branch has a large slope in Fig.2, the crack propagation Stage I will be shifted to the following Stage II very quickly, in which the inner side of the SFRC wall will exhibit the residual tensile strength.

Crack Propagation Stage II \((p_2 < p < p_4)\)

In this stage, the crack has propagated throughout the thickness of the exterior SFRC wall. According to the tensile stress-strain curve shown in Fig.2, there are two possible tensile stress distributions of SFRC along the thickness of the exterior SFRC wall:

(a) all the SFR has the tensile stress within the BC branch shown in Fig.2 \((p_2 < p < p_3)\),
the force equilibrium equation can be written as:

\[ pR_i = E_{sl} \varepsilon_{sl} t + \sum_{i=1}^{m} \frac{R_i}{R_i} E_{sh} \varepsilon_{sl} A_{si} + \int_{0}^{h_i} \left( f_{ji} - k \left( \frac{R_i + h}{R_i} \varepsilon_{sl} - \varepsilon_p \right) \right) dh \]  

(11)

\[ \varepsilon_{sl} = \frac{pR_i - Qh_0}{E_{sl} t + \sum_{i=1}^{m} R_i n_i E_{sh} \varepsilon_{sl} A_{si} - kR_i \ln \left( \frac{R_i + h_0}{R_i} \right) \varepsilon_{sl} - \varepsilon_p} \]  

(12)

(b) Some part of SFRC wall has the tensile stress within the BC branch shown in Fig. 2 while the remaining part has the tensile stress within the CD branch \((p_3 < p < p_4)\), the intersection point for two parts at the location from which to the interior steel penstock the radial distance is \(h_2 = (\varepsilon_{sl} - \varepsilon_{p1})/R_i/R_{p1}\). Therefore, the force equilibrium can be expressed as follows:

\[ pR_i = E_{sl} \varepsilon_{sl} t + \sum_{i=1}^{m} \frac{R_i}{R_i} n_i E_{sh} \varepsilon_{sl} A_{si} + \int_{h_2}^{h_i} \left( f_{ji} - k \left( \frac{R_i + h}{R_i} \varepsilon_{sl} - \varepsilon_p \right) \right) dh + f_{ji} h_2 \]  

(13)

The value of \(\varepsilon_{sl}\) can also be obtained through solving Eqn 13.

**Ultimate stage** \((p > p_4)\)

When the outer edge of the SFRC wall starts to reach its residual tensile strength, all the SFRC along the whole wall thickness must lie in the CD stage of the tensile stress-strain curve. As a result, the force equilibrium can be written as follows:

\[ pR_i = E_{sl} \varepsilon_{sl} t + \sum_{i=1}^{m} \frac{R_i}{R_i} n_i E_{sh} \varepsilon_{sl} A_{si} + f_{ji} h_0 \]  

(14)

The strain of interior steel penstocks can be obtained as:

\[ \varepsilon_{si} = \frac{pR_i - f_{ji} h_0}{E_{sl} t + \sum_{i=1}^{m} n_i E_{sh} A_{si}} \]  

(15)

when \(\varepsilon_{si}\) reaches the yield strain, the composite penstock starts to enter into its ultimate state. Further increase of the internal water pressure will lead to the yielding of all layers of steel reinforcement. The ultimate internal water pressure \(p_u\) can be written as:

\[ p_u = f_{yi} t + f_{yi} \sum_{i=1}^{m} n_i A_{si} + f_{ji} h_0 \]  

(16)

where \(f_{yi} = \) the yield strength of steel material. Obviously, the use of SFRC-ETSP also leads to a higher load-carrying capacity compared to RC-ETSPs.

**CONCLUDING REMARKS**

Steel fiber concrete provides an effective way to improve the crack resistance of exterior RC walls of reinforced concrete encased steel penstocks. However, due to the addition of steel fibers, the analysis and design of the RC-ESP's become more complicated. On the basis of a tri-linear uniaxial tensile stress-strain relationship of SFRC, in this paper a simplified analytical model has been developed for the design of SFRC encased tubular steel penstocks. Using this model, the strain and stress conditions in the SFRC and steel reinforcement in the exterior wall as well as in the interior steel penstocks can be explicitly predicted from the elastic stage, crack propagation stage till the ultimate stage. The proposal model is expected to be applicable for the practical engineering design of SFRC-ETSPs.
REFERENCES


DYNAMIC PERFORMANCE OF BEAM OF GANGUE CONCRETE-FILLED CIRCULAR STEEL TUBE

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KEYWORDS
Gangue concrete-filled circular steel tube, Beam, Dynamical behaviors, Hysteretic curve, Ductility coefficient, Rigidity degradation, Energy Dissipation.

ABSTRACT
Compared with common concrete, the density of coal gangue concrete is lower by 25%-40% at the same compression strength. As a result of its excellent structure behavior, the coal gangue concrete filled steel tube has good application prospect in bridge engineering etc. The dynamic performance of beam of coal gangue concrete filled circular steel tube was investigated experimentally. In this foundation, take the parameter of steel ratio into account and analyses the characteristics of lateral load versus lateral displacement hysteretic curve. At the same time, effects of these parameters on hysteretic behaviors, elastic rigidity and ductility are analyzed according to lateral load versus lateral displacement framework curve from the experiment. At last, this paper gives a preliminary discussion on the rule of displacement ductility coefficient, rigidity degeneration, and the dissipation of energy.

INTRODUCTION
Concrete filled steel tube (CFST) not only has the advantages of both the steel tube and concrete but also overcomes the defect of the steel tube which is susceptible to local plate buckling. Thus, both the load bearing capacity and ductility are greatly improved. In recent years, with the theoretical studies on this type of structure and the development of new construction technology, the concrete-filled steel tube has become widely used (Schneider [1], Ge and Usami [2]). Researchers have made a large quantity of theoretical analysis and experiments on the mechanical property of CFST component under cyclic loading and many rich and practical referential experiences and conclusions were proposed, such as Varma [3], Aval et al. [4], Elremaily and Azizinamini [5]. However, little research has been conducted on the circular steel tube filled with coal gangue concrete. In this paper, the anti-seismic performance of the compositional structure formed by steel tube filled with coal gangue concrete is studied and this will promote the application of this structure in engineering practices (Li and Liu [6]).
SYNOPSIS OF DYNAMIC

Component design

The steel tube studied here is a seamless steel tube, which is 1500mm in length and smooth at both ends. At one end of the hollow steel pipe, a thickness of 20mm with a 4 bolt hole steel plate is welded as a cover plate of the test piece. The other end will be welded in two weeks after the tube is filled with concrete. This is because the conjunction place between the cover plate and the test piece is expected to take a large shear force. Therefore, the center of the cover plate is strictly in line with the geometric center of the tube, which will ensure the intensity of the welding line. In order to study the influence of the steel ratio of the tube on its power performance, three kinds of steel tubes with different thickness are divided into two groups for experiment. The specific data of test specimen is recorded in Table1. Coal gangue concrete is mixed by mechanical equipments. During concreting, the first thing is to erect the steel tube and then to pour the concrete from the top into the tube and use a vibrator wand to vibrate it until it is solid and compact. Two weeks later, use the cement mortar and angle grinder to smooth the surface of the concrete on the cast side and the steel tube and then weld the appropriate cover plate to make sure that the steel tube and the concrete in the core will receive the same force in the primary stage of loading experiment. During the period of maintaining, a frequent watering is needed and the test piece can be experimented after 28 days (Li and Zhao [7]).

<table>
<thead>
<tr>
<th>Number</th>
<th>D×t×L (mm)</th>
<th>α</th>
<th>ξ</th>
<th>fy (MPa)</th>
<th>fck (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-5-1</td>
<td>159×5×1500</td>
<td>0.139</td>
<td>.16</td>
<td>1.8</td>
<td>9.6</td>
</tr>
<tr>
<td>C-5-2</td>
<td>159×5×1500</td>
<td>0.139</td>
<td>.97</td>
<td>1.8</td>
<td>1.5</td>
</tr>
<tr>
<td>C-6-1</td>
<td>159×6×1500</td>
<td>0.170</td>
<td>.67</td>
<td>6.8</td>
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<tr>
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<td>.43</td>
<td>6.8</td>
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<tr>
<td>C-8-1</td>
<td>159×8×1500</td>
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<td>.76</td>
<td>5.5</td>
<td>9.6</td>
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<tr>
<td>C-8-2</td>
<td>159×8×1500</td>
<td>0.236</td>
<td>.42</td>
<td>5.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Where: \( \alpha = \frac{A_s}{A_c} \), \( A_s \): area of steel tube, \( A_c \): area of concrete. \( \xi = \frac{f_y}{f_{ck}} \), \( f_y \): steel yield stress, \( f_{ck} \): compressive strength of concrete, \( \xi \): confining factor. D: diameter of component. t: wall thickness of steel tube. L: length of component.

Device and method of experiment

This experiment is conducted in the Structural Engineering Laboratory of Shenyang Jianzhu University. The experimental facilities are demonstrated in Figure 1. There are two steel beams on the reaction pressure pier. The two steel beams are connected with a trench through the bolt on the ground. The tensile force on the bolt and the friction force between the two beams will keep the system from shifting vertically and horizontally. The specimen is horizontally placed with two ends hinged. A self-designed flat plate ream is connected with the other end of a reaction wall which is connected with a reaction block. The reciprocal force is implemented by a vertical MTS servo actuator placed in the mid-span. The allocation beam is functioned as a roller on the test piece, through which the applied force acted as pure bending moment is realized. As the experiment involves many complicated loading devices and structures, and in order to prevent the instability resulted from the MTS servo actuator and the axis of the test piece not being on the same layer, a lateral strut is designed, which is a triangle holder welded by I-beam plate on a solid chuck pulley. The bottom of the device is
solidly connected with a bolt on the ground to keep the test piece from shifting vertically and horizontally when loaded.

A displacement meter is placed at the one-third place of the test piece and the data from the displacement meter of the MTS system actuator is collected at the same time. In order to make sure the displacement meter does not slide from the circular section, strain foils are pasted at the middle section of the test piece. The method suggested in reference [8], named loading-displacement double controlling method, is adopted to add loading. Before reaching the yield point of the specimen, several load levels are applied to it, such as $0.25P_u$, $0.5P_u$ and $0.7P_u$ ($P_u$ is the ultimate load), and totally two circles at different load level have been conducted. $0P_u$ is estimated as the maximum receiving force vertically. On the verge of the yielding, displacement control method is adopted to control the load increment, for example, $1.0\Delta_y$, $1.5\Delta_y$, $2.0\Delta_y$, $3.0\Delta_y$, $5.0\Delta_y$, $7.0\Delta_y$, $8.0\Delta_y$, in which $\Delta_y$ is considered as the yielding displacement of the test piece. The secant stiffness of load-deformation curve is $\Delta_y=P_u/K_{0.7}$ where $K_{0.7}$ is loaded at $0.7P_u$. Circulate the load three times in the first three load level and the rest level circulate twice. Load until load-carrying capacity of the component decrease to 50% of the maximum carrying capacity or the actuator reaching the maximum displacement.

RESULTS AND ANALYSIS

Failure modes

Through the observation of the whole process of the test (in Figure 2), it was found that the six test pieces failed almost in the same way. When the applied load did not exceed the yield loading, the load-displacement relationship of the test piece was basically elastic without obviously residual deformation. However, when the applied load exceeded the yield loading, the vertical displacement of the actuator increased fast and a small bulge was occurred at the joint between the solid chuck and the test piece as well as the arch loading holder. In the following unloading and loading in reverse direction, the small bulge was evened and another small bulge was occurred on the other side where the pressure was received maximum. Thereafter, at the joint between the test piece and the solid chuck, the extent of the bending bulge increases and develops in a circumferential way. When the test piece was on the verge...
being destroyed, this bulge developed dramatically. At this moment, the load-carrying capacity decreased dramatically down to below 50% and the experiment was completed.

Figure 2: Typical failure modes

Load (P)-displacement(Δ) hysteresis curve

Figure 3 is a load-displacement hysteresis curve obtained from the experiment. It shows that the hysteresis curve of the test piece is in general full displaying a spindle shape without obvious pinching phenomenon. However, the load-carrying capacity of the test piece increases anyway. There is no part of declining in the curve. This indicates that there is a strengthening stage in the later part of the loading, which proves that the component has good ductility and energy consuming performances.
The Influence of steel ratio on the Skeleton Curve

The curve of the skeleton curve is an envelope curve connected by a series of peak points reached by the first circulation of every grade of loading of the load-deformation hysteresis curve. The envelope curve can reflect a general change of intensity and ductility of the test piece in the whole loading process. It can be seen from Figure 4 that steel ratio has a significant influence on the load-carrying capacity and ductility of the test piece, while the influence on the rigidity in the elastic stage is relatively small.

Rigidity degradation curve

The phenomenon of the rigidity of the component decreases with the increase of loading and the times of circulation is called rigidity degeneration. Figure 5 shows the degeneration of the rigidity of the pure bending component. For the convenience of comparison, the y-coordinate in above graph will use the non-dimensional parameter $\frac{EI}{(EI)_{\text{first}}}$, where $(EI)_{\text{first}}$ is the corresponding rigidity of the first load level. Figure 5 shows that with the development of deformation, the rigidity of the component drops significantly. In the primary state, the rigidity degenerates slowly. After 2 $\Delta y$, the rigidity degenerates significantly and then it becomes more gentle. In general, the ratio of the rigidity of the component $EI$ and the primary rigidity $(EI)_{\text{first}}$ is between 1~0.2 and the average value is 0.6. The most obvious degeneration happens to be the 5mm thick test piece with the...
lowest steel proportion, when the loading is added to the displacement of \( \Delta_y \) and the rigidity decreases to the point of primary rigidity 19%.

![Figure 5: Rigidity degradation curve](image.png)

**Ductility Coefficient**

The ductility of the component refers to the capacity of deformation under external force in the non-elastic stage of later period, which happens after the elastic stage and there is no significant degradation of carrying capacity. This is also true to certain part of the component of the structure. This paper adopts the displacement factor \( \mu \) to study the ductility of the component as below.

\[
\mu = \frac{\Delta_u}{\Delta_y}\quad (1)
\]

in which, \( \Delta_y \) is yield displacement and \( \Delta_u \) is ultimate displacement.

The load-displacement curve of coal gangue steel tube concrete component does not show a clear yield point. The measurement of the yielding displacement is to measure displacement of the intersection point of the spur line of elastic part of the skeleton line and the horizontal line passed the peak point. The measurement of the maximum displacement is to measure the corresponding displacement of the carrying capacity decreases to the point of 85% of the maximum carrying capacity [8]. According to the calculating methodology described above, the maximum displacement of the yielding displacement and the displacement ductility factor as demonstrated in Table 2. Since the anti-bending capacity of the 8mm thick steel tube coal gangue concrete test piece is strong that the loading does not drop to the point of 85% of the maximum carrying capacity until the end of this experiment. Therefore, there is no way to measure the ductility factor according to the definition of Eq. (1). From Table 2, it can be concluded that even the high steel proportion test piece reaches its maximum load capacity and the maximum displacement increases, the ductility factor decreases with the increase of the steel proportion.
TABLE 2

DUCTILITY COEFFICIENT

<table>
<thead>
<tr>
<th>Number</th>
<th>Loading direction</th>
<th>Yield point</th>
<th>Maximum load</th>
<th>Ultimate load</th>
<th>Ductility coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$P_y$ /kN</td>
<td>$\Delta_y$/mm</td>
<td>$P_u$ /kN</td>
<td>$\Delta_u$/mm</td>
</tr>
<tr>
<td>C-5-1</td>
<td>positive</td>
<td>149.91</td>
<td>13.21</td>
<td>209.62</td>
<td>106.09</td>
</tr>
<tr>
<td></td>
<td>negative 15</td>
<td>1.37</td>
<td>12.89</td>
<td>195.23</td>
<td>107.55</td>
</tr>
<tr>
<td>C-5-2</td>
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<td>13.44</td>
<td>231</td>
<td>99.85</td>
</tr>
<tr>
<td></td>
<td>negative 16</td>
<td>0.1</td>
<td>14.12</td>
<td>219.09</td>
<td>106.61</td>
</tr>
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<td>15.62</td>
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<tr>
<td></td>
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<td>15.97</td>
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<tr>
<td></td>
<td>negative</td>
<td>-</td>
<td>-</td>
<td>353.95</td>
<td>122.18</td>
</tr>
</tbody>
</table>

Where: $P_y$—yield load. $\Delta_y$—yield displacement. $P_u$—ultimate load. $\Delta_u$—maximum displacement. $\mu$—Ductility coefficient.

ENERGY CONSUMPTION

Energy consumption reflects the capacity of the test piece to absorb energy in the process of reciprocate loading. This capacity is a significant referent in evaluate the shock resistance capacity of the component. The more energy the structure absorbs from the earthquake, the more capacity to resist collapsing the structure will have. Figure 6 shows the typical load-displacement hysteresis loop of the test piece under reciprocal load.

![Figure 6: Influence of steel ratio on the $E-\Delta/\Delta_y$ curve](image)

CONCLUSION

The following is the primary conclusions of the paper based on the experiment:

1. The load-displacement hysteresis loop of the circular steel tube coal gangue concrete pure bending component is full without obvious pinch effect, which indicates a good ductility.

2. With the increase of the loading and displacement, the rigidity of the test piece degenerates significantly. When the test piece is on the verge of being damaged, the carrying...
capacity curve does not drop while the energy consuming capacity increases gradually.

(3) With the increase of the steel ratio, the carrying capacity and the maximum displacement of the test piece increases significantly, while the displacement ductility factor of the test piece decreases.

REFERENCES

ULTIMATE MOMENT OF SHEAR CONNECTIONS

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KEYWORDS

Simple connections, single-angle shear connections, double-angle shear connections, connection moment, yieldline analysis

ABSTRACT

Instead of establishing the mathematical model for the semi-rigid behavior of shear connections, the focus of this paper is to determine the bending moment of single-angle and double angle shear connections corresponding to the ultimate limit state of their supported beam. The connection moment, which is in the plane of the supported beam web, is caused by the end rotation of the supported beam and its nonzero rotational stiffness. Thus, the magnitude of the moment is much dependent on the amount of beam end rotation and the sources of connection rotation. Based on the observations from several series of tested full-scale connections at the strength limit of the supported beam, this study adopts a yieldline method to derive semi-analytical formulae to compute the connection moments. The theoretical moments are then compared with test data to demonstrate the applicability of the moment equations. The proposed moment formulae can be used to determine the eccentricity of bolt group which connecting a connection angle to the supported beam as well as can be used to estimate the tensile forces applied to the connectors which connect the connection to the supporting column.

INTRODUCTION

Shear connections develop a moment in the plane of beam web due to its rotational stiffness when the supported beam deflects under gravity loading. This moment is a function of connection rotation. As pointed out by Astaneh [1], in general, an end rotation of 0.02 radians is large enough to accommodate the curvature of the beam when the mid-span cross section reaches first-yield moment and a rotation of 0.03 radians is sufficient for the beam to reach its full plastic moment. Thus, for this study, a connection rotation at the level of 0.03 radians is taken as that corresponds to the ultimate limit state of the supported beam.
In a study on tee shear connections, Thornton [2] derived Eqn. 1 to calculate the maximum moment that a tee connection could develop. Figure 1 shows the yield line mechanism adopted by Thornton in which the second yield line occurred along the toe of the stem fillet. Thornton assumed that the tee connection was to rotate about its very lower end (i.e., the compression zone had a zero depth) and the force in tension zone was uniformly distributed \((T\) in Figure 1). Thornton’s equation was

\[
M_{th} = \frac{2m_p L^2}{b} \left[ 2 + \left( \frac{b}{L} \right)^2 \right]
\]

where: \(b\) is the distance between the two yieldlines at the upper end of the connection; \(L\) is the length of connection; and \(m_p\) is equal to \((F_y t^2/4)\), i.e., the plastic moment of flange plate per unit length \((F_y\) is yield strength of steel, and \(t\) is thickness of tee flange). Note that Eqn. 1 exists whether the tee flange is bolted or welded to the supporting member (Figures 1a and 1b, where \(k_1\) is fillet radius). Although Eqn. 1 is easy to use, the past study [3] also showed the equation often dramatically over-predicted the connection moment. It was not explained what caused this discrepancy.

Recently, the writer had conducted several experimental tests on both single and double-angle shear connections [4, 5, 6]. The yieldline mechanisms observed in these tests were not exactly the same as those described in Thornton’s works [1, 3]. Therefore, for this study, the writer will first derive the equations for the computation of the ultimate moment of single-angle shear connections based on the observations from the experiments. Then the equations will be extended to include double-angle shear connections. The theoretical analysis results will be compared with various test data to demonstrate the accuracy of the moment equations.

Note that the ultimate moment of a connection is defined hereafter as the moment corresponding to a connection rotation at the level of 0.03 radians. The supporting member of the supported beam is assumed to be rigid, allowing the connection to develop moment.

**SINGLE-ANGLE SHEAR CONNECTIONS**

Figure 2 illustrates the yieldline pattern of single-angle connections tested by Gong [4]. The inclined first yield line is on the outstanding leg. However, unlike Thornton’s model, the second yieldline occurred on the framing leg along the end of the beam web.
It is assumed that the neutral axis of connection \((x-axis \text{ in Figure 2a})\) is located at a distance of \(L_c\) above the lower end of the connection. The external couple, \(M_p\), is the moment imposed to the connection through the web of the supported beam. The first yieldline is inclined from the upper end to the \(x\)-axis in order to be compatible with the connection rotation. Corresponding to moment \(M_p\), \(T_y\) is the horizontal tension force per unit length applied to the angle from the beam web, and \(C_y\) is the compression force per unit length in the compression zone between the connection and the supporting column. Therefore, the equilibrium condition of the connection gives

\[
M_p = \int_0^{L_t} T_y y dy + \int_0^{L_c} C_y y dy = \sum T_y (\xi L_i) + \int_0^{L_c} C_y y dy
\]  

(2)

where: \(y\) is the coordinate of angle (Figure 2a); \(L_t\) is the depth of the tension zone; \(\sum T_y\) is the tensile force resultant, i.e., \(\sum T_y = \int_0^{L_t} T_y dy\); \(\xi L_i\) is the distance from the force resultant \(\sum T_y\) to the neutral axis; \(L_c\) is the depth of the compression zone; and the equilibrium of forces in horizontal direction requires \(\sum T_y = \int_0^{L_c} C_y dy\).

By assuming that the compression zone is a triangular area having a height of \(L_c\) and base of \(\lambda t\) (product of coefficient \(\lambda\) and angle thickness \(t\)) and that the compressive bearing stress \(\sigma_c\) is uniformly distributed over the compression zone, we obtain

\[
\int_0^{L_c} C_y y dy = L_c \lambda t \sigma_c / 2 \quad \text{and} \quad \int_0^{L_c} C_y y dy = \lambda t \sigma_c L_c^2 / 3
\]  

(3)

The average value of \(\lambda\) measured from the tested connections was 2.7, while \(L_c\) value was from 6% to 10% of length \(L\). The bearing stress \(\sigma_c\) must have exceeded \(F_y\) as the permanent plastic deformation in compression zone was evident in the tests. An upper bound value for \(\sigma_c\), which is approximately taken as 1.5 \(F_y\), is determined by considering local bearing of the compression zone according to Canadian standard CAN/CSA-S16 [7].

Denoting \(b_0\) as the distance from the first yieldline at the upper end to the angle heel (Figure 2a) and \(\Delta t\) the horizontal deflection of the angle heel at the upper end (Figure 2c), the work of \(T_y\) is then written as

\[
W_c = \int_0^{L_t} (T_y dy) \frac{\Delta_t}{L_t} y = \frac{\Delta_t}{L_t} \int_0^{L_t} T_y y dy = \frac{\Delta_t}{L_t} \left( \sum T_y \right) \xi L_i
\]  

(4)

The internal work of the first yieldline is

\[
W_{i,1} = m_p \sqrt{L_i^2 + b_0^2} \left[ \Delta_t \sqrt{1 + \left( b_0 / L_i \right)^2 / b_0} \right]
\]

(5)

where: \(m_p\) is the plastic moment per unit length of outstand leg plate; \(\sqrt{L_i^2 + b_0^2}\) is the length of the first yieldline; \(\Delta_t \sqrt{1 + \left( b_0 / L_i \right)^2 / b_0}\) is the rotational angle of the yieldline. The internal work of the second yield line is
\[ W_{i,2} = m_p L \Delta_i / b_0 = m_p L_i \Delta_i / b_0 \]  

(6)

where the rotation \( \theta = \Delta_i / b_0 \) is constant along the second yieldline. Since \( L_i \) is approximately 90% of \( L \) as observed from the test, \( L \) is replaced by \( L_i \) in Eqn. 6. Then, according to the virtual work principal, \( W_e = W_{i,1} + W_{i,2} \), we have

\[
\sum T_j = m_p L_i \left[ 2 + \left( \frac{b_0}{L_i} \right)^2 \right]
\]  

(7)

Substituting Eqn. 7 into Eqn. 3 gives

\[
L_c = \frac{2 m_p L_i}{\xi b_0} \left[ 2 + \left( \frac{b_0}{L_i} \right)^2 \right] \frac{1}{J \sigma_c}
\]  

(8)

Then from \( L = L_c + L_i \) we have

\[
j = \frac{L_i}{L} = \frac{1}{1 + \frac{2 m_p}{\xi b_0 J \sigma_c} \left[ 2 + \left( \frac{b_0}{L_i} \right)^2 \right]}
\]  

(9)

Figure 2: Yieldline mechanism for single angle connections

Substituting Eqn. 7 into Eqn. 3 gives

\[
L_c = \frac{2 m_p L_i}{\xi b_0} \left[ 2 + \left( \frac{b_0}{L_i} \right)^2 \right] \frac{1}{J \sigma_c}
\]  

(8)

Then from \( L = L_c + L_i \) we have

\[
j = \frac{L_i}{L} = \frac{1}{1 + \frac{2 m_p}{\xi b_0 J \sigma_c} \left[ 2 + \left( \frac{b_0}{L_i} \right)^2 \right]}
\]  

(9)
From Eqns. 3 and 7, the connection moment Eqn. 2 becomes

$$M_p = \frac{m_p L_i^2}{b_0} \left[ 2 + \left( \frac{b_0}{L_i} \right)^2 \right] + \frac{\lambda t \sigma_c L_c^2}{3} \tag{10}$$

It is readily shown that some error in the determination of $\sigma_c$ has negligible influence on the total moment. Hence, Eqn. 10 can be rewritten as

$$M_p = \frac{m_p L_i^2}{b_0} \left[ 2 + \left( \frac{b_0}{L_i} \right)^2 \right] + tL_c^2 F_y \tag{11}$$

where $\sigma_c$ is taken approximately as $1.1 F_y$ and $\lambda$ is taken as 2.7 based on the test results. For regular single-angle connections such as the tested [4], the value of $j$ in Eqn. 9 can be estimated using $(b_0/L_i)^2 \approx 0$ due to $b_0 \ll L_i$ for the connections having more than 3 bolts. Let $\sigma_c = \zeta F_y$, Eqn. 9 is reduced to

$$j = \frac{1}{1 + t/\zeta \lambda b_0} \tag{12}$$

Eqn. 12 is used to estimate the depth of tension zone. In order to estimate the values of $\xi$ and $\zeta$, the relationships between $j$ and $\xi$ and $\zeta$ are illustrated in Figure 3 for $0.4 \leq \xi \leq 0.65$ and $\zeta = 1.0$ and 1.5. Figure 3 tells that the $j$ value is not sensitive to the variation of $\xi$ and $\zeta$ values. Thus, in this study, the values of $\xi$ and $\zeta$ are arbitrarily taken as 0.55 and 1.1, respectively, for computing the $j$ value.

If the compression zone is assumed to be at the very lower end of the angle, i.e., $L_c = 0$ and thus $L_c = L$, the connection moment equation $M_p$ is then further simplified as

$$M_c = \frac{m_p L_i^2}{b_0} \left[ 2 + \left( \frac{b_0}{L} \right)^2 \right] \tag{13}$$

For single-angle connections, Thornton Eqn. 1 should be rewritten as

$$M_{th} = \frac{m_p L_i^2}{b} \left[ 2 + \left( \frac{b}{L} \right)^2 \right] \tag{14}$$

Note that Eqn. 13 is virtually the same as Eqn. 14, except Eqn. 13 uses dimension $b_0$ instead of $b$.

In summary, the procedure for computing the ultimate moment of a connection is

1. assume $\lambda = 2.7$, $\zeta = 1.1$ and $\xi = 0.55$, calculate $j$ by Eqn. 12, then $L_c = jL$ and $L_c = L - L_i$;
2. calculate $M_p$ through Eqn. 11, or through Eq. 13 if compression zone is assumed to be zero depth.
Gong [4] reported test of 19 single-angle all-bolted connections (Table 1). All connection angles had $t=9.5$ mm, $b_0=65$ mm and $F_y=445$ MPa. The high-strength bolts were snug-tightened. The test moments $M_t$ given in Table 1 were corresponding to the connection rotations greater than 0.03 radians. The angle heel fillet size $k$ is taken as 19 mm as per AISC Manual [8] for Thornton Eqn. 14 (i.e., $b=46$ mm). The observations from Table 1 are: 1) the ratio of Thornton moment $M_{th1}$ to Eqn. 13 moment $M_c$ is approximately equal to $b_0/b$ ($b_0/b$ is 1.4 for the connections); 2) the moment predicted by Eqn. 13 is about 7% greater than that by Eqn. 11, indicating that the compression zone contribution is insignificant for the connections; and 3) the predicted moments are all greater than the test moments, primarily due to the fact that bolt slip contributed about 0.01 to 0.02 radians to the connection rotations which reduced the strain level in the connections.

### TABLE 1

<table>
<thead>
<tr>
<th>Specimen ID#</th>
<th>Angle length $L$ (mm)</th>
<th>Test moment $M_t$ (kN-m)</th>
<th>Rotation corresponding to $M_t$ (rad.)</th>
<th>$M_p$ by Eqn.11 (kN-m)</th>
<th>$M_c$ by Eqn.13 (kN-m)</th>
<th>$M_{th1}$ by Eqn.14 (kN-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X2A 1</td>
<td>40</td>
<td>4.9</td>
<td>0.045</td>
<td>6.3</td>
<td>6.7</td>
<td>9.0</td>
</tr>
<tr>
<td>X2B 1</td>
<td>40</td>
<td>5.2</td>
<td>0.054</td>
<td>6.3</td>
<td>6.7</td>
<td>9.0</td>
</tr>
<tr>
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<td>0.046</td>
<td>6.3</td>
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<td>9.0</td>
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<td>5.0</td>
<td>0.046</td>
<td>6.3</td>
<td>6.7</td>
<td>9.0</td>
</tr>
<tr>
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<td>14.1</td>
<td>15.1</td>
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<td>15.1</td>
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<tr>
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<tr>
<td>N4C 29</td>
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<td>0.044</td>
<td>25.3</td>
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<td>37.7</td>
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<td>42.5</td>
<td>59.6</td>
</tr>
<tr>
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<td>29.3</td>
<td>0.041</td>
<td>39.8</td>
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<td>0.036</td>
<td>57.6</td>
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<td>86.6</td>
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<td>84.2</td>
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</tr>
<tr>
<td>X7B 52</td>
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<td>61.0</td>
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<td>X8B 59</td>
<td>6</td>
<td>71.2</td>
<td>0.037</td>
<td>103.3</td>
<td>110.4</td>
<td>155.7</td>
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</tbody>
</table>
DOUBLE-ANGLE SHEAR CONNECTIONS

Double-angle shear connections have one angle on each side of the beam web. Due to two angles, the moment Eqs. 11 and 13 are modified accordingly into Eqs. 15 and 16 respectively in the following.

\[
M_p = \frac{2m_cL_i^2}{b_0} \left[ 2 + \left( \frac{b_0}{L_i} \right)^2 \right] + 2tL_cF_y
\]  
(15)

And, if the compression zone is assumed to be zero depth, we have

\[
M_c = \frac{2m_cL_i^2}{b_0} \left[ 2 + \left( \frac{b_0}{L} \right)^2 \right]
\]  
(16)

Table 2 records the test data of four double-angle shear connections conducted by McMullin and Astaneh [9]. For each connection, the outstanding legs were welded to the supporting column while the framing legs were bolted to the test beam. The bolts were pretensioned and no obvious bolt-slip plateau was observed in the early stage of the reported moment-rotation curves. The test moments \(M_t\) were read directly from the moment-rotation curves at a rotation of 0.03 radians (except for connection 6 since its maximum rotation was 0.017 radians). The comparisons in Table 2 shows: 1) the compression zone effect is insignificant since the moments predicted by Eqn. 15 are only slightly less than the moments predicted by Eqn. 16; and 2) while Eqn. 16 makes excellent moment prediction for connections 4 and 5, Thornton Eqn. 1 makes better prediction for connections 6 and 9.

<table>
<thead>
<tr>
<th>Connection No.</th>
<th>Angle length (L) (mm)</th>
<th>Test moment (M_t) (kN-m)</th>
<th>Rotation corresponding to (M_t) (rad.)</th>
<th>(M_p) by Eq.15 (kN-m)</th>
<th>(M_c) by Eq.16 (kN-m)</th>
<th>(M_{th}) by Eq.1 (kN-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 520</td>
<td>562</td>
<td>76.5</td>
<td>0.030</td>
<td>70.8</td>
<td>73.9</td>
<td>90.3</td>
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<tr>
<td>5 368</td>
<td>368</td>
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<td>0.030</td>
<td>36.2</td>
<td>37.7</td>
<td>45.8</td>
</tr>
<tr>
<td>6 216</td>
<td>216</td>
<td>19.0</td>
<td>0.017</td>
<td>13.4</td>
<td>13.9</td>
<td>16.5</td>
</tr>
<tr>
<td>9 368</td>
<td>368</td>
<td>53.6</td>
<td>0.030</td>
<td>36.2</td>
<td>37.7</td>
<td>45.8</td>
</tr>
</tbody>
</table>

Note: All connection angles had \(F_y=303\) MPa, \(b_0=102\) mm, and thickness \(t=9.5\) mm.

SUMMARY AND CONCLUSIONS

The basis for the moment equations derived in this paper is the yieldline mechanism depicted in Fig. 2, in which the second yieldline must be on the framing leg of the connection angle. For the connections of this case, Thornton equation does not apply. The reasons for the second yieldline occurring on the framing leg include: 1) the outstanding leg of the angle(s) deflect vertically under the shear load from the supported beam, which leads the framing leg(s) to flex about the end of the beam; 2) the setback of the beam web is usually much greater than the thickness of angle plate, which allows the formation of the second yieldline on the framing leg; and 3) the moment \(M_c\) predicted by Eqn. 13 is less than the moment \(M_{th}\)
predicted by Thornton equation. The ratio of \( M_{th} \) to \( M_c \) is approximately equal to \( b_0/\beta \). Therefore, as long as actual conditions permit, the connection will form the second yield line on the framing leg instead of on the outstanding leg. The second yieldline may even form within the heel radius, as long as the increased yielding moment at the hinge does not result in \( M_c \) being greater than \( M_{th} \).

Equations 11 and 15 are most general and applicable to any design situations of angle connections. However, for regular connections with thickness less than 9.5 mm, by assuming a zero depth compression zone, the connection moment can be estimated easily and conservatively by Eqns. 13 or 16.

The equations from this study provide a means for directly computing the connection moment corresponding to the strength limit state of the supported beam. When applying these equations to connection design [4], the actual steel yield strength should be used instead of the specified yield strength. This actual material strength can be obtained from mill test or using the expected strength of the material. Note that the actual yield strength is used to compute moment only and thus will not compromise the safety of the designed structure.

The examples of applying moment equations in design practice include: 1) to determine the eccentricity of the bolt group which connecting a connection angle to a supported beam [4]; 2) to estimate the tensile forces applied to the connectors which connect the connection angle to the supporting column using Eqn. 7.

REFERENCES

AN EXPERIMENTAL STUDY OF STRENGTHENING OF DEEP CONCRETE COUPLING BEAMS WITH BOLTED STEEL PLATE

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KEYWORDS

Deep coupling beams, plate buckling, buckling restrained, seismic retrofitting, steel plate,

ABSTRACT

Existing deep reinforced concrete (RC) coupling beams with low shear ratios and conventionally reinforced shear stirrups tend to fail in a brittle way with limited ductility and deformability under reversed cyclic loading. In order to improve their performance under earthquake actions, this paper aims to develop a new retrofitting method for existing deep RC coupling beams which can enhance the deformability, energy dissipation ability but not the flexural stiffness. Experimental study was conducted to test three full-scale deep RC coupling beams with the same span-to-depth ratio (1.1) and the same geometry and reinforcement layout, of which one was acted as a control specimen without retrofitting, one was retrofitted by bolted steel plate on the side face of the beam and the other was retrofitted by bolted external steel plate with a buckling restraining device added. This paper presents the experimental results and compares the overall performance of these specimens. This study reveals that the deformation and energy dissipation ability of the deep RC coupling beams retrofitted with restrained steel plates were all improved while the flexural stiffness was not increased. Moreover, by using lateral restrained steel plates, the specimens had better post-peak behaviour, more ductile failure mode and better rotation deformability at beam-wall joints.

INTRODUCTION

Reinforced concrete (RC) coupled shear walls and core walls are widely employed as a lateral load resisting system for high-rise buildings to resist earthquake and wind loads. In this system, a number of individual wall piers are coupled together by coupling beams to increase the lateral strength and stiffness of the buildings. To ensure the desired behaviour of coupled core walls, seismic resistant coupling beams should be sufficiently strong, deformable and have good energy dissipation ability[1]. To increase the deformability and energy dissipation capacity of coupling beams, many researchers proposed various alternative design methods such as using diagonally reinforced coupling beams[2], steel coupling
beams[3], rectangular steel tube coupling beams with concrete infill[4] and embedded steel plate coupling beam[5]. However, little research had been conducted aiming at improving the deformability and reducing the strength degrading of existing heavily reinforced concrete coupling beams. Harries et al. [6] studied a shear strengthening method for coupling beams with a span-to-depth ratio of 3.0. In their study the retrofitting measures involved a number of different attachment ways to fix a thin steel plate to one side of the coupling beams. They found that the composite method of bolting with epoxy bonding to attach the steel plates both in the span and at the ends performed better. Su and Zhu[7] studied a shear strengthening method for RC coupling beams with a span-to-depth ratio of 2.5. They proposed to strengthen the coupling beams by bolting the steel plate to two ends of wall panels without adhesive bonding. They conducted experimental and numerical study to prove that this retrofitting method could greatly increase the shear capacity and deformability of coupling beams. In all their experiments, minor buckling of steel plate was observed and the influence of local buckling on the behaviour of composite coupling beams was not investigated. However, most of the previous studies focused on the coupling beams with span-to-depth ratios larger than 2.0. Deformation enhancement study of coupling beams, in particular, with small span-to-depth ratios (≤1.5), which behave quite differently from medium span coupling beams, has not been conducted. Deep RC coupling beams with low shear ratios and conventional shear stirrups tend to fail in a very brittle shear failure under lateral reversed cyclic loading. In order to improve their performance under wind or earthquake actions, this paper aims to develop a new strengthening method for existing deep RC coupling beams. In this study, we investigated using external restrained steel plates to retrofit existing deep RC coupling beams with small span-to-depth ratio of 1.1. The advantage of using lateral restraints instead of adding stiffeners to the steel plates for controlling plate buckling is that the flexural strength of the coupling beams will not be increased by the stiffeners, and the deformability instead of the flexural and shear strength capacities can be increased as much as possible. As the latter can cause over-coupling and the walls may fail prior to the failure of coupling beams and that could lead to undesirable brittle failure mechanism. The present method is suitable for seismic retrofitting of deep RC coupling beams in high seismicity areas.

**EXPERIMENTAL PROGRAMME**

*Description of Test Specimens*

Three specimens with the same dimensions and reinforcement details (see Fig 1), but with different retrofitting schemes were fabricated and tested. The sizes of coupling beams were 450mm deep by 120mm wide with a clear span of 500mm and with a span-to-depth ratio of 1.1. The top and bottom longitudinal reinforcements in the coupling beams were of four 12mm diameter high yield deformed reinforcement bars and the side bars were of four 8mm diameter mild steel round bars. Shear reinforcements in the coupling beams consisted of four 8mm diameter hoops with 125mm pitch. Sufficient longitudinal steel has been provided to ensure the control specimen to fail in a brittle fashion.

The first specimen DCB1 with plain RC arrangement was used for control purposes. Specimen DCB2 was retrofitted with 3mm grade 50 steel plate, while Specimen DCB3 were retrofitted with 3mm steel plates with a buckling restraining device added as shown in Fig 1. By providing two steel angles along the top and bottom edges of the steel plate, the possible lateral buckling of the steel plate in the span at the edges was suppressed. To avoid adding extra strength and stiffness to the composite coupling beam, the lateral stiffeners were
connected to steel plate by four bolt connections with slotted holes, which allowed the two lateral stiffeners to freely rotate and move in the longitudinal direction.

![Figure 1. Details of test specimens (all units in mm)](image)

The anchorage at the ends of the steel plates was designed to be strong enough to transfer all the forces from the steel plates to the wall anchors. In DCB3 with the restrained steel plates, the buckling strength of steel plate in the span may be higher than that in the anchorage regions. To avoid buckling phenomenon occurred in the anchorage regions, thicker steel plates 6mm were adopted by DCB3 at anchorage regions. As all the steel plates were attached to coupling beams solely by bolt connections, the details of the connections could have great influence on the behaviour of the entire retrofitting system. In this study, 20mm diameter mild steel bolts were used to fix the external steel plate. To minimize any possible slippage amongst various components at the connections, dynamic set washers developed by HILTI Corporation were used. The advantage of these washers is that bolt-slip can be minimized by injecting adhesive to fill up all the gaps between the bolt shank and surrounding concrete (see Fig 1). The material properties of the concrete, steel plate and reinforcement are given in Table 1.

**Test Setup and Loading Program**

The load frame as shown in Fig 2 designed by Kwan and Zhao[8] was employed in the tests. Reversed cyclic loading was applied by a 500kN servo-controlled hydraulic actuator through a rigid arm to the top end of the specimen with the line of action of the applied shear force passing through the centerline of the beam. In this way, the coupling beam was loaded with a
constant shear force along the span and a linearly varying bending moment with the point of inflection located at the mid-span, which simulated the real situation.

TABLE 1

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Concrete compressive strengths</th>
<th>Steel plate</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen</td>
<td>$f_{cu}$ (MPa)</td>
<td>$f_r$ (MPa)</td>
<td>$f_y$ (MPa)</td>
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<td>DCB3</td>
<td>33</td>
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</tbody>
</table>

The specimens were tested under reversed cyclic loading to simulate earthquake or wind applied forces. The first loading phase was load-controlled and the second phase was displacement-controlled. The reversed cyclic loading was applied to each specimen up to 75% of the calculated theoretical ultimate shear capacity ($V_u^*$) (see Table 2). The subsequent cycles were displacement-controlled, in which the specimen was displaced to nominal ductility factor ($\mu_n=1$) for one cycle, then to each successive nominal ductility factor for two cycles as illustrated in Fig 2. $\mu_n = \Theta / \Theta_{yn}$. Beam rotations ($\Theta$), defined as the differential displacement between the two beam ends ($\Delta$) in the loading direction divided by the clear span ($l$), were calculated using the displacements measured by linear variable displacement transducers (LVDTs) D3 and D4 as idealized in Fig 2. The nominal yield rotation ($\Theta_{yn}$) at $\mu_n = 1$ was obtained using the average of the $\Theta$ values corresponding to the positive and negative loads at 0.75 $V_u^*$ in the first cycle following the 4/3 rule. The test was terminated when the peak load reached in the first cycle of a nominal ductility level fell below the lesser of 0.8 $V_u^*$ and 0.8 $V_{max}$ and the test specimen was considered to have failed.

RESULTS AND DISCUSSIONS

Strength, Deformation and Ductility

Table 2 shows that the attached steel plates increased both the ultimate capacity and deformability of the coupling beams. Comparing the results of DCB2 with DCB3, the effects of restrained plates can be revealed. The shear strength $V_{max}$ of DCB2 and DCB3 was increased by 44% and 41% respectively while the ultimate rotation $\theta_{max}$ was increased by 64% and 82%. The results show that by adding a buckling restrained steel plate, the increase in the rotation deformability is much higher than that in the strength.
In terms of displacement ductility, DCB3 was increased by 30%. While for DCB2 (without a buckling control restraint), the ductility was slightly reduced due to the increase in the yield rotation when compared with that of the control specimen DCB1.

Secant stiffness $K_0$ was obtained from the test results by dividing the ultimate load by the yield displacement. As shown in Table 2, adding bolted steel plates would not increase the secant stiffness. It is because the increase in yield displacements (or yield chord rotations) is higher than that of the strength. Comparing the test results of the three specimens, it can be concluded that this new seismic retrofitting method can increase the deformation and ductility of deep RC coupling beams while avoiding increase in their flexural stiffness. As the total amount of base shear induced in a building under an earthquake excitation is dependent on the lateral stiffness of the structure, this method would not generate extra forces to the building after seismic retrofitting.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Brittleness</th>
<th>$V_u$ (kN)</th>
<th>$V_{max}$ (kN)</th>
<th>$V_{max}$% increased</th>
<th>$\theta_y$ (rad)</th>
<th>$\theta_{max}$ (rad)</th>
<th>$\theta_{max}$% increased</th>
<th>$\mu$</th>
<th>$\mu$% increased</th>
<th>$K_0$ (kN/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCB1</td>
<td>serious</td>
<td>230</td>
<td>238</td>
<td>N/A</td>
<td>0.0043</td>
<td>0.011</td>
<td>N/A</td>
<td>2.56</td>
<td>N/A</td>
<td>92</td>
</tr>
<tr>
<td>DCB2</td>
<td>serious</td>
<td>380</td>
<td>344</td>
<td>44%</td>
<td>0.0095</td>
<td>0.018</td>
<td>64%</td>
<td>1.89</td>
<td>N/A</td>
<td>60</td>
</tr>
<tr>
<td>DCB3</td>
<td>slight</td>
<td>400</td>
<td>335</td>
<td>41%</td>
<td>0.0060</td>
<td>0.02</td>
<td>82%</td>
<td>3.3</td>
<td>30%</td>
<td>93</td>
</tr>
</tbody>
</table>

**Crack Patterns and Failure Behaviours**

The crack patterns of concrete of the test specimens were similar. Figure 3 shows the crack patterns of DCB1 after the test. The extensive diagonal cracks indicate that the beams had insufficient shear capacity. This is in line with the anticipated design brittle shear failure mode as sufficient longitudinal steel has been provided.

The initial cracks of the test specimens occurred at the beam-wall joints and inclined at about 45°. As the applied load increases, major diagonal cracks were formed across the beam span. For Specimen DCB1, diagonal cracks occurred when the applied loads reached approximately ±148 kN. For the steel plate retrofitted specimens, diagonal cracks initiated at a later stage when the applied load went up to 172 kN and 231 kN, respectively, for DCB2 and DCB3.

For control specimen DCB1, shear links started to yield when the load went up to approximate 230 kN. Sooner after the yielding, no more shear force can be resisted and the specimen has reached its peak capacity (also at 230 kN). The results demonstrated that insufficient shear reinforcement is the primary reason for causing the failure of the control specimen. The present of high tensile strains in the diagonal compressive struts led to eventually crushing of concrete and failure of the specimen. When yielding of shear links occurred, the strength could not be increased much and yielding of shear link could accelerate the crack width increasing. Concrete cracking could have been necessary for activating the steel plate to resist shear force and the present of steel plate could bridge the cracks and help transferring the shear force across the diagonal cracks. When almost the entire beam was cracked mainly in shear, concrete crushing started to occur at the beam-wall joints and the concrete spalled off at the compression corners. Then local buckling of steel plate started to occur at these locations.
For DCB2 without control of buckling, unilateral local buckling of steel plate began at compressive zones near the beam-wall joints. Plate buckling initiated at the beam span near the beam-wall joints when the load reached +315kN (rotation 0.01 rad) and after peak strength, plate buckling became very serious, the buckled plate form a small pocket near the beam-wall joints holding the debris from spalling concrete. Because of that, the plate could not deform back to the original flat shape in the reversed load cycles. Shear transfer across the beam-wall joints through the steel plate was thus inactivated. Furthermore, the debris jammed in the gap between the steel plate and concrete and left some space for further spalling which led to deterioration of concrete in the subsequent load cycles. This can explain why the failure mode of DCB2 is brittle although steel plate was added. While for Specimens DCB3 with control of buckling, plate buckling initiated at a later stage when the load reached +342 kN (rotation 0.015 rad). Apparently, the buckling controlled device can delay the occurring of steel buckling. As the plate buckling at the beam-wall joints was suppressed, it provided a continuous shear transfer medium across the joints, the steel plate could continue to take up a larger share of the loading at the post-peak region and crushing of concrete at the compression region could be alleviated. This can explain why the failure mode of DCB3 was more ductile than DCB2.

![DCB1](image1)

![DCB2](image2)

![DCB3](image3)

Figure 3. Concrete failure pattern and buckling modes after the tests

**Load-Rotation Curves**

Fig 4 shows the load-rotation hysteretic loop of all the specimens. The load-rotation curve of the control specimen DCB1 exhibited substantial pinching after reaching the peak load. Such pinching associated with rapid stiffness degradation and less energy dissipation of the specimen in the post-peak regime. For DCB2 with retrofitting plate, pinching effect after peak load was also serious due to the reason that the steel plate could not be effectively activated when serious buckling occurred at beam-wall joints. While for DCB3 with control of buckling, the pinching became less serious. The load-rotation envelopes illustrate that the load-rotation curve of DCB3 with restrained plates was more ductile after the peak loads. Fig 4 also reveals that the load-rotation curve under positive and negative load cycles were very different, in particular for Specimens DCB1 and DCB2. As the earthquake loads are, by their nature, a type of reversed cyclic load, very un-symmetric or highly varied responses in the positive and negative cycles for the coupling beams are undesirable. For DCB2, buckling occurred in the span. The mechanical properties of steel plate at buckling in one loading direction were different from that in the opposite loading direction. Therefore, the hysteretic loops in the positive and negative loading direction are not symmetrical. While for plate restrained Specimens DCB3, the variations in positive and negative cycles are relatively small. The stable and repeatable response causes by the reliable shear deformation of steel plate make them best suit for seismic retrofitting.
Energy Dissipation and Strength Degradation

In order to compare the energy dissipation abilities of the specimens, the energy dissipated $W_d$ in each half-cycle is evaluated. The values of $W_d$ were equivalent to the area bounded by the load-displacement curves. Fig 5 shows the variations of $W_d$ with the nominal ductility level in the first cycles. It can be seen that adding steel plate could significantly enhance the energy dissipation ability. In the early cycles, all the specimens were able to dissipate an increasing amount of energy, and after $\mu_n=2$, the dissipated energy decreased for DCB1 and DCB2. While for DCB3 with restrained steel plates, the dissipated energy could be increased after $\mu_n=2$.

The strength degradations in repeated cycles were determined by comparing the peak loads of each specimen at a nominal ductility level ($V_{peak1}$) and in the repeated loading cycles ($V_{peak2}$). The retentions of load capacities in the first repeated cycles ($V_{peak1}/V_{peak2}$) at each nominal ductility level are presented in Fig 6. The results demonstrated that the strength retention ability for the plated beams, in particular DCB3 with control of buckling, is generally higher than DCB1. This is mainly due to the fact that the steel plate in Specimens DCB3 dissipated the energy through the stable shear deformation of steel plate. The results reveal that restrained steel plates can significantly improve the inelastic behaviour in terms of higher energy dissipation and lower strength degradation of the coupling beams.
CONCLUSIONS

1. The retrofitting method of bolting external steel plate to one side of concrete deep coupling beams with control of buckling can effectively increase the deformability and energy dissipation ability, and slow down the strength degradation. Failure of the beams was changed to a less brittle manner.

2. Buckling of steel plate has adverse effects on the structural performances of coupling beams. The strength cannot be increased anymore when buckling begin to occur. Without restraint the plate buckling, buckling can result in very brittle shear failure mode and poor energy dissipation ability.

3. Using lateral restrained steel plates, the deformation, ductility and energy dissipation ability of coupling beams can be greatly increased while the stiffness would not be changed so much. This new retrofitting method is suitable for seismic applications.

ACKNOWLEDGEMENTS

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REFERENCES

BEARING FAILURE OF BOLTED CONNECTIONS IN STAINLESS STEEL

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KEYWORDS

ABSTRACT
Using previously developed and validated finite element (FE) models of bolted connections in austenitic and ferritic stainless steel, an investigation into the bearing behaviour of stainless steel bolted connections has been carried out. The investigation showed that the deformation behaviour of stainless steel connections is different from that of carbon steel connections, with stainless steel exhibiting pronounced strain hardening. Both the bearing resistance and the locations of fracture initiation obtained from the numerical models match those observed during experimental studies of carbon steel and stainless steel connections. A parametric study to investigate the key parameters has been performed and the results utilised as the basis for design provisions for bearing failure in stainless steel bolted connections.

INTRODUCTION
The favourable properties of stainless steel, such as corrosion resistance, aesthetics and fire resistance are bringing about increasing usage in construction (Gardner [1]). Unlike carbon steel, the stress-strain relationship of stainless steel is rounded without a well-defined yield stress and the material exhibits significant strain hardening. Despite these differences, design provisions for bolted connections in stainless steel in current standards (Eurocode 3: Part 1.4 [2]; EuroInox [3]) are essentially based on the rules for carbon steel with some limited modifications. This is attributed to the lack of a comprehensive investigation on these structural components. Few experimental studies have been conducted to investigate the behaviour of stainless steel bolted connections and most have concentrated on cold-formed thin-gauge material (Errera et al. [4], Van Der Merwe [5]). The experimental study by Ryan [6] was the first investigation into stainless steel bolted connections composed of thicker hot-rolled plates. Recently, some numerical studies have been carried out to examine the behaviour of stainless steel connections (Kim et al. [7,8], Bouchair et al. [9]), but further investigation is still required. In this paper, a parametric study has been conducted to
investigate bearing failure in stainless steel bolted connections and design equations have been proposed.

**FINITE ELEMENT (FE) MODELS**

The finite element analysis software ABAQUS 6.6.1 [10] was used to develop numerical models for austenitic and ferritic stainless steel bolted connections to examine their response under static shear loading. The configurations of these specimens are presented in Figure 1. In order to reduce the size of the model and, consequently, the computational cost, only one quarter of the connection was modelled by applying appropriate boundary conditions. The 3D solid (brick) element with full integration – C3D8 – which has proved to be suitable to simulate lap bolted connections in many previous investigations (Chung and Ip [11], Ju et al. [12]), was employed in this study to model both the plates and bolts. Loading was applied by means of uniform displacement-control at the end of the central plate. Figure 1 illustrates the boundary conditions applied to the FE models.

The compound Ramberg-Osgood stress-strain model developed by Mirambell and Real [13] and Rasmussen [14] was adopted to represent the non-linear material response of stainless steel. Numerical analyses of bolted connections are expected to involve large inelastic strains, therefore, nominal stresses and strains were converted to the corresponding true values, which take into account the change in geometry under load. Contact between all components that are expected to interact with each other was defined using the surface-to-surface contact feature in ABAQUS. Frictional effects were taken into account by using the classical isotropic Coulomb friction model. Bolts were located centrally into the holes with a uniform clearance of 1.0 mm. Different levels of bolt preload were applied. Loads were carried initially by friction until the occurrence of slippage after which direct bearing was the primary means of load transfer.

Figure 1: FE models, (a) 2 bolts longitudinally (b) 2 bolts transversely (c) 3 bolts (d) 4 bolts

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VALIDATION OF THE FE MODELS

The overall deformation behaviour of the connections was validated by comparing the numerically obtained load-deformation curves with those obtained from 24 previously reported laboratory tests [6]. It is clear from the typical curves shown in Figure 2 that the predicted load-deformation curves are in good agreement with the tests. Note that the extension shown is the total deformation of the connection, as illustrated in Figure 3. In addition, the ultimate capacities obtained from the FE models closely matched those achieved experimentally, based on proposed numerical failure criteria (Salih et al. [15]). The deformed shape of a test specimen and the corresponding FE model simulation is shown in Figure 3.

BEARING CAPACITY OF LAP CONNECTIONS

The bearing resistance of shear bolted connections has been determined in previous studies either on the basis of strength or deformation criterion. These two criteria are discussed in this section.

Strength criterion

For this criterion, the bearing capacity of a bolted connection is taken as the maximum load attained in the test regardless of the associated deformation. Many researchers (Winter [16], Dhalla et al. [17], Errera [4], Rogers and Hancock [18], Puthli and Fleischer [19] and Kuwamura et al. [7]) who conducted experimental studies adopted this criterion to develop bearing design equations even though large deformations were observed at the ultimate load. For instance, Rogers and Hancock [18] developed a bearing design equation for cold-formed
carbon steel bolted connections by adopting the maximum loads from their tests, although in many specimens a level of deformation as large as 15 mm was reached. In the above-mentioned studies and in Kim’s [20] experimental work, the researchers observed that the failure was initiated by fracture at the edge of the elongated bolt hole at two symmetrical locations oriented at approximately $\theta = 45^\circ$ and $135^\circ$ (see Figure 4) which indicates the occurrence of peak strains at these locations.

**Deformation criterion**

The bearing resistance of a connection according to this criterion is taken as the applied load measured at a pre-specified acceptable deformation depending on the usage of the connections. This limit does not correspond to the maximum load attained in the test and hence, no rupture takes place in the material. The determination of bearing failure of bolted shear connections by limiting deformations was proposed by many researchers, but, there is no consensus about whether to limit the permanent or the total elongation and what is a suitable deformation limit. Perry [21] investigated carbon steel bolted connections, and recommended that the failure load be the load corresponding to a deformation of 6.35 mm, since beyond this level, the load-deflection curves of typical connections becomes virtually flat. Perry’s definition has been adopted in developing design guidance for carbon steel connections in the AISC [22]. The EuroInox [2] design provisions for stainless steel connections were developed on the basis of a 3.0 mm deformation limit at ultimate loading conditions. By imposing this limit, it was suggested that the deformation at the service loads would be of the order of 1.0 mm. Eurocode 3 Part 1.4 adopted the same design provisions.

**PARAMETRIC STUDY**

The developed FE models have been used to generate further results in order to examine the bearing behaviour of stainless steel connections. In the parametric study, lap connections with bolts in double shear were investigated as shown in Figure 4. Two types of thick hot-rolled stainless steel plates were investigated: austenitic and ferritic with plate thicknesses of 8.0 and 10.0 mm for each type. The two main parameters that were investigated are the end distance ratio $e_1/d_0$, varying from 0.8 to 4.0, and the edge distance ratio $e_2/d_0$ with four values: 1.5, 2.0, 3.0 and 4.0.

The distribution of plastic strain in the plate in front of the bolt, which is shown in Figure 5, indicates that the strains are very high at two symmetrical locations at about $\theta = 45^\circ$ and $135^\circ$. This strain distribution agrees with the observations during testing and confirms that bearing fracture occurs at these locations. This conclusion will be adopted to determine the bearing capacity using the strength criterion: when the peak plastic strain in the plate material in front of the bolt reaches the localized fracture strain of the material (Salih et al. [15]), fracture occurs and the maximum load is said to have been reached.

To illustrate the difference in the deformation behaviour of carbon steel and stainless steel bolted connections, an FE model was developed with the material behaviour of austenitic stainless steel and the geometry of a carbon steel connection tested by Kim [20]. Figure 6 shows that, for the carbon steel connection, the load-deflection response attained a relatively flat state once significant bearing deformations had occurred, and therefore, the exact deformation limit has relatively little effect on the “failure load”. However, for the stainless steel connection, owing to its rounded stress strain relationship and the pronounced strain hardening with increased deformation, a rising relationship, without significant flattening off
Figure 7 shows the stiffness at three stages of loading of a stainless steel connection from the parametric study. The stiffness at the point of bearing fracture is almost equal to the stiffness at 6.35 mm deformation and about 50% of the stiffness at 3.0 mm deformation; this loss of stiffness may be considered to be relatively modest. It may be concluded that defining bearing failure on the basis of deformation limits (3.0 mm or 6.35 mm), underestimates the true bearing resistance of stainless steel connections.

Figure 8: Stiffness of stainless steel connection

![Stiffness of stainless steel connection](image)

Figure 8: Stiffness of stainless steel connection

Figure 9: Comparison between carbon and stainless steel connection behaviour

![Comparison between carbon and stainless steel connection behaviour](image)

Figure 9: Comparison between carbon and stainless steel connection behaviour
EXISTING DESIGN PROVISIONS IN CODES

The ultimate bearing resistance of carbon steel connections in Eurocode 3 Part 1.8 [23] is given by the following equation:

\[ F_{b,Rd} = k_1 \alpha_b f_{udt} / \gamma_{M2} \]  

(1)

where \( \alpha_b \) is the smallest of \( \alpha_d ; f_{ub}/f_u \) or 1.0, \( \gamma_{M2} \) is partial safety factor =1.25. In the direction of load transfer, \( \alpha_d = e_1/3d_0 \) for end bolts, \( \alpha_d = e_1/3d_0 - 1/4 \) for inner bolts. In the direction perpendicular to load transfer, \( k_1 \) is the smaller of \((2.8e_2/d_0 - 1.7)\) or 2.5 for edge bolts and \((1.4p_2/d_0 - 1.7)\) or 2.5 for inner bolts. \( t \) is the plate thickness, \( d \) is the nominal bolt diameter, \( d_0 \) is the bolt hole diameter, \( e_1 \) is the end distance, \( e_2 \) is the edge distance and \( f_u \) and \( f_{ub} \) are the ultimate tensile strengths of the plate and bolt material respectively.

EuroInox and Eurocode 3 Part 1.4 adopt Eqn. 1 for stainless steel connections with a slight modification: a reduced ultimate strength of the plate material \( f_{u,\text{red}} \) obtained from Eqn. 2 is used in Eqn. 1 in place of \( f_u \). This modification was proposed to limit bearing deformations at the ultimate and service loads to acceptable levels, while maintaining the format of the resistance equation and the bearing coefficients for carbon steel.

\[ f_{u,\text{red}} = 0.5 f_y + 0.6 f_u \leq f_u \]  

(2)

PROPOSED DESIGN EQUATIONS

In order to exploit the high ductility and strain hardening characteristics of stainless steel, the concept in the AISC Standard [22] for bearing design will be adopted. Two bearing design equations will be proposed according to the requirement for a deformation limit under service loads. The first bearing design equation is for bolted connections where the deformation under service loads is not a design consideration. This equation will be developed by adopting the strength criterion to define the ultimate bearing capacity, as controlled by fracture. The second equation is for connections where the deformation under service loads is a design consideration, and therefore the equation will be developed by considering the deformation criterion to define the service load and consequently the corresponding ultimate bearing capacity. The ultimate strength of the stainless steel material will be adopted in the proposed design equations with a format similar to that of Eqn. 1.

Bearing capacity when deformation under service loads is not a design consideration

In order to establish a suitable design equation, the bearing coefficients \( \beta_{\text{FE}} \), defined by Eqn. 3, obtained from the parametric study are plotted in Figure 8 where deformation under service loads is not a design consideration, \( P_{b,\text{FE}} \) is the bearing fracture load; \( \sigma_{b,\text{FE}} \) is the bearing stress = \( P_{b,\text{FE}} / t d \). It can be seen that for \( e_2/d_0 > 1.5 \) the bearing stress factor for all FE models is greater than \( \beta_1 \) as defined by Eqn. 4.

\[ \beta_{\text{FE}} = P_{b,\text{FE}} / t d f_u = \sigma_{b,\text{FE}} / f_u \]  

(3)

\[ \beta_1 = 2.5(e_1 / 3d_0) \leq 2.5 \]  

(4)
Thus for \( \frac{e_2}{d_0} > 1.5 \), the proposed bearing design equation is given by Eqn. 5. Values of \( \frac{e_2}{d_0} \leq 1.5 \) requires further investigation.

\[
F_{b,Rd} = 2.5\beta_1 f_{udt} / \gamma_{M2}
\]  

(5)

**Bearing capacity when deformation under service loads is a design consideration**

In this case, the ultimate bearing capacity of the bolted connection should be defined such that the deformation at the serviceability limit state is kept within an acceptable limit. Examining the load-deformation curves for all FE models, it was found that at 1.0 mm deformation the connections remain elastic. Therefore, a service load corresponding to 1.0 mm deformation was adopted, and the corresponding ultimate bearing capacity \( P_{b,FE} \) (here controlled by deformation) was then obtained by assuming an average ratio of ultimate to service loads of 1.45.

Figure 9 shows the resulting bearing coefficient \( \beta_{FE} \) from Eqn. 3. A factor \( \beta_2 \) given by Eqn. 6 is proposed to provide a lower bound to the finite element data and plotted in Figure 9. For comparison, the bearing coefficient of Eqn. 4 is also plotted.

\[
\beta_2 = 1.25\left(\frac{e_1}{2d_0}\right) \leq 1.25
\]  

(6)

The ultimate bearing capacity for this category is therefore given by:

\[
F_{b,Rd} = 2.5\beta_2 f_{udt} / \gamma_{M2}
\]  

(7)

Eqn. 7 will ensure that the deformation at serviceability will be acceptable; there is, therefore no need to conduct a separate check.

![Figure 8: Bearing coefficient from parametric study by adopting strength criterion](image)
CONCLUSIONS

The bearing behaviour of stainless steel connections is investigated herein by means of FE parametric studies. The fundamental difference in the response of stainless steel and carbon steel connections is that, while the load-deformation curve for carbon steel connections flattens off after the initiation and spreading of yielding, for stainless steel connections this curve continues to rise significantly owing to strain hardening. For this reason, the limiting deformations used to define the bearing capacity of carbon steel connections were found to be unsuitable for stainless steel connections. Different failure definitions have therefore been devised for stainless steel connections, and bearing design equations that cover two cases – one restricting and one ignoring serviceability deformations – have been proposed. These equations define the bearing capacity in terms of the material ultimate strength $f_u$ instead of the so-called reduced ultimate strength $f_{u,red}$ and therefore, are consistent with the provisions for carbon steel connections. Statistical evaluation of the proposal is underway.

Figure 9: Bearing coefficient from parametric study by adopting deformation criterion
REFERENCES

FATIGUE STUDY OF PARTIALLY OVERLAPPED CIRCULAR HOLLOW SECTION K-JOINTS

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KEYWORDS
Fatigue assessment, Numerical modelling, Stress concentration factor, Stress intensity factors, Residual fatigue life, Partially overlapped circular hollow section K-joint

ABSTRACT
In this paper, a set of consistent geometrical models for partially overlapped circular hollow section (CHS) K-joint is developed for fatigue assessment. The geometrical models developed include realistic details of the weld profile and surface crack. To accomplish the fatigue assessment procedure, an accompanying set of automatic finite element (FE) mesh generation procedures is also developed. These FE mesh generation procedures are able to discretize the joints into different types of FE meshes including pure surface meshes, solid meshes either with or without weld profile, and solid mesh with both weld profile and surface crack details. The FE models obtained from these mesh generation procedures could then be used for the fatigue assessments of uncracked and cracked joints. The reliability of the geometrical models and the mesh generation procedure are validated by comparing the modelling results with full scale static and fatigue test results. The static tests involving basic and combined loadings were first applied to determine the stress concentration factors (SCF) along the intersection curves of the joints. Fatigue tests involving cyclic loading were then applied in order to study the stress intensity factor (SIF) and the fatigue life of the cracked joints. From the results obtained from the experimental study, it was found that the uncracked joint model could lead to reliable SCF and hotspot stress estimations while the cracked joint models could lead to overall conservative SIF predictions close to the measured results. Eventually, the proposed models could lead to satisfactory residual fatigue life prediction for the two cracked joints.

INTRODUCTION
In heavily loaded offshore and bridge frames, partially overlapped CHS K-joints are often used due to their high residual capacity. In fact, previous studies [1-2] shown that a well designed partially overlapped CHS K-joint could outperform its gap ped joint counterpart in both ultimate strength capacity and cost effectiveness of construction. However, up to now, relatively less research efforts had been spent on the study of fatigue performance of partially overlapped CHS K-joints. The only detail numerical study completed was reported by Efthymiou and Durkin [3]. Recently, full scale experimental tests conducted by Sopha et al. [4] and Lee et al. [5] found that the SCF formula suggested in reference [3] are not always conservative. Up to now, no large scale parametric study result on the fatigue performance of this joint type is available. One main reason for the lack of
numerical modelling results is that partially overlapped K-joint is the most complicated planar joint configuration constructed in practice [6]. The problem is further complicated by the fact that reliable hot spot stress and SCF values can only be obtained from 3D solid FE model. Together with the fact that detailed crack models for the crack surface and front are essential for the fatigue life estimation of a cracked joint, geometrical model and FE meshes generations are the two main obstacles that hinder large scale parametric study of partially overlapped CHS K-joints.

The main objective of this paper is to present a solution showing how the above obstacles that prevent researchers to carry out large scale parametric study for this joint type could be overcome. Two essential tools, namely, (i) a set of consistent and realistic geometrical models and (ii) a set of automatic mesh generation procedures are developed. The validity of the geometrical models and the mesh generation procedures are verified by comparing the modelling results with those obtained from full scale fatigue tests. In addition, efforts are focused to identify those parameters that may critically affect the accuracy of the modelling results. Finally, conclusions for the present study and potential areas of future research works are presented.

GEOMETRICAL MODELLING

The main method adopted in this study to obtain realistic geometrical models of partial overlapped CHS K-joint is to base on a hierarchical modelling approach [6] which follows the fabrication sequence of real joints. The geometrical models are constructed in the following steps:

1. Modelling of the intersection curves and intersection points (Fig. 1),
2. Modelling of the weld profiles (Fig. 2), and
3. Modelling of the surface crack shape and crack front details (Fig. 3).

AUTOMATIC MESH GENERATION

Similar to the derivation of the geometrical models, the mesh generation steps were carried out in a hierarchical manner. The starting point is the generation of a surface mesh, which is then converted into a solid mesh using an extrusion algorithm [7]. Finally, additional details such as weld profile and surface crack are added to the
mesh if they are needed. This method of mesh generation is flexible in such a way that at different generation stages, meshes with different levels of details can be included in the model and employed for different applications. In particular, the surface mesh (Fig. 4) could be employed for estimating the ultimate strength of the joint. While in case that a quick estimation of stress concentration factor is needed, the hybrid model (Fig. 5) which contains mainly surface elements and a small number of solid elements could be used. For a more detailed study on the distribution of hot spot stress, full solid meshes without (Fig. 6) or with welding details (Fig. 7) will be created by using the extrusion algorithm [7] and a weld profile meshing procedure. Eventually, if a full fatigue study for a crack joint is needed, a solid mesh with weld and crack details (Fig. 8) is generated by employing a specially designed procedure that inserts the crack details at appreciate location along the weld toe [6].

**APPLICATION RANGE OF THE MODEL GENERATION PROCEDURE**

The proposed mesh generation scheme is applicable to the special case of identical chord and braces diameter even when the joint has a high overlap ratio of 80%. Note that such special cases were not covered by the mesh generators done in the well cited works by Cao et al. [8] and Lie et al. [9]. An example of the mesh with same chord and braces dimensions and with overlap ratio of 80% is given in Fig. 9. Furthermore, as shown in Fig. 10, the current mesh generation scheme could also be employed to generate meshes for partially overlapped N-joints.

![Figure 5: Hybrid mesh](image)

![Figure 6: Solid mesh without welding details](image)
Figure 7: Solid mesh with welding details

Figure 8: Solid mesh with surface crack details

Figure 9: A FE mesh with identical braces and chord size and 80% overlap ratio
TEST SET UP AND SPECIMENS

The dimensions of the two full scale partially overlapped CHS K-joints named as Specimen S1 and Specimen S2 fabricated for the experimental study are shown in Fig. 11. The sizes of the CHS sections are deliberately selected to be almost identical in order to study the effects of the different loadings on the distribution of the SCF of the joints. The CHS employed to construct the joints are fully complied with the design code [10] with all welding fully complied with the American Welding Society specification [11]. During testing, the specimens were fixed at the two ends of the chord and the overlap brace and loadings were applied at the end of the through brace by three mutually perpendicular actuators.

FULL SCALE TESTS CONDUCTED

Both static and fatigue tests were conducted for the two specimens. For static test, the three basic loading cases, namely axial loading (AX) and in-plane bending (IPB) and out-of-plane bending were applied in turn for the study of the SCF and the hot spot stress distributions along the weld toes of the intersection curves. For the fatigue test, combined AX (200kN) and IPB (+45kN) sinusoidal constant amplitude cyclic loadings of 0.2Hz were applied. It should be stressed that in the fatigue test, the loading direction of the IPB loading applied to Specimen S1 is exactly opposite to that applied to Specimen S2. Such loadings were arranged so that for Specimen S1, a crack was eventually induced along the intersection curve between the chord and the through brace (Curve 1 and Weld 1 in Fig. 11). While for Specimen S2, the joint was eventually failed by a crack formed along the intersection curve between the through and the overlap brace (Curve 3 and Weld 3 in Fig. 11). In the fatigue test, cyclic loading was applied until the joint were finally broken with the formation of through thickness cracks were detected.

VALIDATION OF WELD PROFILE MODEL FOR UNCRACKED JOINTS

As it was expected that the peak hot spot stress was located along the crown of Weld 1 on the through brace side for Specimen S1 and along the crown of the through brace side of Weld 3 for Specimen S2, Fig. 12 and 13 plots the modelled and the actual weld thickness ($T_w$) along the through brace side of Weld 1 for Specimen S1 and the through brace side of Weld 3 for Specimen S2, respectively. It can be observed that the modelled weld thickness resembled the actual weld shapes and both of them satisfied the American Welding Society [11] with the actual weld thickness is larger (and hence conservative in SCF and hot spot stress prediction) than the modelled thickness (especially at crown heels).

VALIDATION OF SCF PREDICTIONS

Figs. 14 and 15 shows the hot spot stress distribution when the Specimen S1 and Specimen S2 are subjected to IPB loading, respectively. In order to check the accuracy of the SCF prediction, a detailed investigation was
carried out to compare the experimental results against the FE stress distributions obtained from the following models:

(i) Models based on the measured weld thickness before the full scale tests were carried out
(ii) Models based on the proposed weld thickness [6, 12]
(iii) Models without weld details

It was found that the SCF results obtained from the measured thickness are most accurate while the proposed weld model produced reasonable and conservative SCF predictions for the specimens tested. However, results from models without weld details always overestimated the SCF by a considerable large margin and are deemed to be too conservative and will underestimate the fatigue life of the joint if they are used in fatigue life estimation.

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Dimensional parameters (mm, degree)</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>D=275.0 T=250.0 d=244.5 t=10.1 65°-65° p=204.42</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>D=275.0 T=250.0 d=244.5 t=200.0 65°-65° p=201.90</td>
<td></td>
</tr>
</tbody>
</table>

\(D=\) diameter of chord, \(T=\) thickness of chord, \(d=\) diameter of braces, \(t=\) thickness of braces, \(\theta=\) angle between through brace and chord, \(\phi=\) angle between overlap brace and chord, \(p=\) overlapping ratio of the joint

Figure 11: Dimension of Specimens S1 and S2

Figure 12: Weld thickness for Specimen S1
VALIDATION OF STRESS INTENSITY FACTOR PREDICTION

In order to obtain a complete picture of the SIF as the crack developed, a number of models corresponding to different crack depths and crack lengths subjected to cyclic loading were created [13]. As the mesh generation procedure employed could be employed to generate solid FE meshes with crack surface details corresponding to any crack surface angle $\omega$ within the range [-20°, 20°], several sets of meshes corresponding to different values of $\omega$ were generated and analyzed. Fig. 16 and 17 compare the SIF at the deepest point of the surface crack obtained from the experiments with the proposed models with different values of $\omega$ for the Specimens S1 and S2, respectively. Note that in Figs. 16 and 17, $a'$ and $t$ are, respectively, the depth of the surface crack and the thickness of the section. For Specimen S1, it can be seen that the models with smaller absolute value of $\omega$ ($\omega = 0°$, $-5°$) gave SIF values closer to the experimental results during the initiation phase of the crack propagation. On the other hand, the FE models with larger absolute value of $\omega$ ($\omega = -10°$, $-15°$) gave SIF values closer to the experimental values at the end of the crack propagation. For Specimen S2, all models underestimated the SIF values especially when $a'/t \leq 0.60$. However, in general, for both specimens, good correlations with test results were obtained for all models, including the model with $\omega = 0°$ which was able to achieve closest approximations.
for the experimental results. Hence, it can be concluded that if the exact geometry of the crack surface angle is unknown, a value of $\omega = 0^\circ$ is recommended.

VALIDATION OF RESIDUAL FATIGUE LIFE PREDICTION

With the value of SIF at the deepest point of a surface crack is known, it is possible to estimate the residual life of a cracked joint from fracture mechanics using the Paris Law [14]. The residual fatigue lives for Specimens S1 and S2 were estimated by integrating the data shown in Figs. 16 and 17 with appropriate values of material parameters which are corresponding to the API-5L pipes tested in ambient temperature conditions [15]. Fig. 18 compares the residual fatigue life predictions obtained by the proposed numerical models with crack details with different crack surface angles (measured values, $\omega \in [0^\circ, -15^\circ]$ for Specimen S1 and $\omega \in [0^\circ, 15^\circ]$ for Specimen S2) against the actual life recorded. For both specimens, as expected, the numerical models based on the measured geometry gave the most accurate predictions. For Specimen S1, all the models gave conservative residual fatigue life predictions. In addition, the FE model with crack surface angle $\omega = 0^\circ$ gave prediction slightly more conservative than the model based on the measured crack shape. For Specimen S2, as the SIF predictions from all models are not conservative in the initiation phase of the crack propagation, the prediction of the residual life in the range $a'/t \leq 0.6$ are not conservative. However, it could be concluded that the model with crack surface angle $\omega = 0^\circ$ could be used in practice as it produces conservative prediction when $a > 6$ mm.

![Figure 16: SIF predictions at the deepest point of surface crack for Specimen S1](image1)

![Figure 17: SIF predictions at the deepest point of surface crack for Specimen S2](image2)
CONCLUSIONS

In this study, a set of consistent and reliable geometrical models and a corresponding set of hierarchical mesh generation procedure were developed. The final model of a partially overlapped CHS K-joint is first created in the form of a surface mesh, which is then converted into a solid mesh. Weld and crack details are then added subsequently using a series of mapping procedures. The mesh generator developed in this study is able to handle a wide range of parameters including the case of identical chord and braces dimensions with a large overlap ratio. In addition, by comparing the experimental results obtained from full scale tests with the numerical modelling results, it is shown that the geometrical models and the mesh generation procedures adopted could lead to consistent and reliable modelling results for partially overlapped CHS K-joints with and without crack. It is shown that the proposed finite element models could able to give reasonable conservative predictions for the SCF distributions along all intersection curves of the uncracked joint and the SIF values at the deepest point of the surface crack. Hence, the models are also able to estimate the residual fatigue life for a cracked joint with reasonable accuracy.

REFERENCE


EXPERIMENT AND ANALYTICAL STUDY ON CONNECTIONS BETWEEN STEEL PLATE SHEAR WALL AND CFTS

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KEY WORDS
Steel plate shear wall, concrete filled tubes, connection plate, experiment

ABSTRACT

Steel plate shear walls (SPSW) have been used in rehabilitation and new projects for more than 30 years for the attractions in weight, strength, construction and ductility. In recent years, some building projects use SPSW as the primary lateral force resisting system in China, such as the 75-storey building of Jinta Tower. The vertical boundary elements of SPSW in Jinta Tower are concrete-filled steel tubes (CFT), which play the roles both in anchoring the tension field developed in the infill steel panels and carrying most of the gravity load. To fully exert the post-buckling strength of the steel plate, the infill panel connections to the CFT should provide the anchor age equal to the yielding strength of the plate at an angle of web yielding. The tension in the panel tears the steel wall of CFT, which may also weaken the confinement effect on the concrete and subsequently decrease the axial loading capacity. In this project, the anchor plate embedded in the CFT is designed to alleviate the hoop stress in the steel wall. Based on the finite element analysis and a series of specimen tests, the force-transferring path of this connection is studied. The results show that, during the yielding or even the hardening process of the steel panel, the tension force can be transferred to the core concrete effectively. Also, the steel wall of CFT will not yield in the hoop direction and the effect on the loading capacity of CFT is small.

INTRODUCTION

The popularity of using Steel Plate Shear Walls (SPSWs) as the primary lateral force resisting system in tall buildings is rapidly increasing in recent years (FEMA 450[1], AISC 341-05[2], Abolhassan[3]). The SPSW system is being recognized for the benefits on ductility under earthquake and the high elastic stiffness under wind load, and is firstly used in China by Jinta Tower located in Tianjin. A series of SPSWs with Concrete Filled Tubular (CFT) columns are
arranged around the elevator shafts. The CFT columns perform the dual functions of resisting both lateral loads (anchorage of the tension field of the infill steel plate) as well as the vertical loads. The tension field of the infill plate relies on the anchorage to the CFT columns, so the connections between steel panels and CFTs should be designed as full-strength (Schumacher et al.[4], Sabelli et al.[5]). Also, the connections have the importance for maximizing the economy by facilitating the erection of the steel plate. Attaching the steel plate directly to the skin of the tubes without components embedded in the concrete core is convenient for construction. But this kind of connection will generate high local stress or distortion in the tube skin, which may cause fracture of the plate welds or the tube wall. Another concern during the designing is that the loading capacity of columns may be deteriorated by tearing of the steel skins of CFTs. The main objective of this research was to study the overall behavior of the connections between the steel panels and the CFTs. The results of a series of model tests are reported in this paper, and the validation through the finite element analysis is also presented.

EXPERIMENTAL STUDY

Test scheme

As the steel panels in Jinta Tower are continuous through the interior columns, which are more favorable in structural performance than the exterior ones, so the connections between steel plate and exterior CFT columns are chosen as the prototype to design the test specimens. By considering the main goal of the experiment and the convenience of the loading, the test setup shown in Figure 1 was designed to study the force transfer path in the connections between steel plate and CFT columns. The specimen consisted of two exterior CFT column segment and two steel plates, which represented the steel panels in the SPSWs. The specimens were approximately one thirds of the size of the prototype structures. The CFT tube had a circular hollow section with 520mm in diameter and 10mm in thickness, and the steel plates were 8mm in thickness and 500mm in width. The two CFT columns in each specimen had the same cross-sections, which were connected to the steel plate by high-strength bolts or weld at each side. For specimen CSPSW1, each steel plate was connected to the CFT columns by 15 pairs of M16-10.9 bolts on one end, and was connected by butt weld in the other end. For specimen CSPSW1a and specimen CSPSW2, both ends were connected to the columns by welding. In this test program, the strength of bolt connection was checked according to the design code of steel structures (GB 50013-2003[6]) with a reduction factor of 0.7 for the slot holes adopted both in the prototype and specimens. The connection plate was embedded in the concrete with the depth of one third of the tube diameter, which was also strengthened with a T plate at the end. For each specimen, eight shear studs with the diameter of 12mm and spacing of 250mm were welded on the upper connection plate at each end. Three specimens were fabricated and tested in this investigation. The test setup and the varied parameters are shown in Figure 1. Grade Q235B steel with the average yield strength of 283.5MPa and ultimate strength of 416.5MPa was used for the steel panels, and grade Q345B steel was used for the tubes of CFT columns. The average cubic compressive strength of concrete filled in the tubes were 86.3MPa for CSPSW1 and CSPSW1a, and 86.6MPa for CSPSW2.
Strain gauges were used on specimens to measure strain distributions at critical points (Strain A~G shown in Figure 1). Displacement transducers were also applied to measure the horizontal elongation of the steel plate between the left and right CFT columns.

In each loading process, the axial compression load of 4000kN was applied to each column through two hydraulic jacks at first, which was kept constant during the test. Then a hydraulic actuator with 5000kN capacity was used to apply horizontal tension load to the two steel plates. The tests were firstly performed by load control at five equal steps at the elastic stage. At the end of each loading step, the load was kept constant and the specimen was visually inspected. After the yielding of the steel plate, the tests were continued by displacement control until the failure of the steel plates occurred.

Test Results

For Specimen CSPSW1, initiation of slippage at the bolt connection of the lower steel plate was observed at the load of 1000kN. At the load of 1500kN, visible slippage occurred at the upper steel plate, which was also indicated by the small drop in the load curves. After the clearance of the holes had been taken up, the connections behaved in the way of bearing-type bolt connections. When the load reached 2000kN, the connection failed for the net section rupture. At the failure load, the mid-section of the steel plate was lower than the yielding stress, and no cracking was observed on the welding at right end. After the test, there was a notable elongation of the holes in the connection plate.

The load versus strain curves at each monitoring point for CSPSW1 is shown in Figure 2. By the comparison of strains from different strain gauges (e.g., location A, B and E), it can be found that a large portion of the tension force from the steel plate directly transferred into the infill concrete via the anchor plate. The remaining unbalanced force between location A and point E were carried by the skin of the steel tubes, which generated hoop forces. The strains in the steel along the anchor plate decreased with the increasing of the depth. The strains at point G and E were about 20% and 70% of that of location A, which showed that the tension force were effectively transferred to the concrete by the bond force on the interface. The hoop strain in the skin was decreasing from location B to location D, with the strain in location B and location D being about 30% and 15% of that of location A. Effected by the tearing of tube wall, the hoop strain at location B increased from $120 \times 10^{-6}$ imposed by axial force alone to $518 \times 10^{-6}$. 
For specimen CSPSW1a, the steel plate yielded and began necking under the load of 2100kN and 3100kN respectively. The test was terminated when the load reached 3300kN with the maximum horizontal tension displacement of 100mm, and no failure on the connections was observed. Similar failure modes were observed in the tests on CSPSW2. The steel plate yielded under the load of 2150kN, and formed necking under the load of 3100kN. According to the measured horizontal displacement between the two CFT columns, the maximum strain at failure was about 8% to 9%, with the stress of 410MPa approximately.

The development of the strains in specimen CSPSW1a and CSPSW2 are plotted in Figure 3 and Figure 4. At the elastic stage, the developments of the strain for these two specimens were similar with CSPSW 1. The measured strains show that the anchor plate embedded in the concrete played a very important role in the connections between steel panels and CFTs. When the load reached 3000kN (CSPSW1a) and 2700kN (CSPSW2) with the tension stress of 375MPa (CSPSW1a) and 338MPa (CSPSW2) at location A, the steel anchor plate yielded at point E and kept in elastic at point F and point G. During the whole loading process, the tube wall at location B generated the maximum hoop strains of $901 \times 10^{-6}$ to $1176 \times 10^{-6}$, which were far less than the yield strain. Specimen CSPSW2 had a wider T plate, so more tension force was undertaken by the anchor plate with smaller hoop strain at location B at the ultimate limit state.

Headed studs were added to the upper steel plate in every specimen to distribute tension forces to the concrete core. Comparison of the strains between the upper and lower connections shows that the effect of shear studs on the strain distribution can be ignored. The reason is that the stresses between steel anchor plate and concrete were lower than the bond strength, but the addition of shear studs will be helpful for improving the anchoring capacity under reversed loading.

Figure 3: Load vs. strains curves of CSPSW1a
Figure 4: Load vs. strains curves of CSPSW2

FINITE ELEMENT ANALYSIS

The FE Model

A finite element analysis was proposed to study the strains and displacement which cannot be observed during the test. The finite element model was validated by the comparison between the strains obtained from the analysis and the strains measured in the test specimens. The specimens were modeled using the commercial three-dimensional finite element analysis package of ANSYS. The 4-node shell element SHELL181 was used to model both the infill steel plates and the steel tubes, and the 8-node solid element SOLID65 was used to model the concrete filled in the tubes. A tri-linear constitutive relationship was used for all steel elements with parameters calibrated by material tests. The effect of confinement of the concrete was modeled using the constitutive relationship proposed by Zhong[7], which is expressed as:

\[
\sigma_e = \begin{cases} 
\sigma_u [A \frac{E}{E_0} - A(\frac{E}{E_0})^2], & \varepsilon < \varepsilon_0 \\
\sigma_u (1-q) + \sigma_c q(\frac{E}{E_0})^{(0.2+\alpha)}, & \varepsilon > \varepsilon_0 
\end{cases}
\]

(1)

where \( \sigma_u = f_{ck} [1 + (30/f_{cu} )^{0.4}(-0.0626\varepsilon^2 + 0.4848\varepsilon)] \); \( \varepsilon_0 = \varepsilon_c + 0.0036\sqrt{\alpha} \); \( \varepsilon_c = 0.0013 + 10^{-5}f_{cu} \); \( A = 2 - K \); \( B = 1 - K \); \( K = (-5\alpha^2 + 3\alpha)(50 - f_{cu})/50 + (-2\alpha^2 + 2.15\alpha)(f_{cu} - 30)/30 \); \( q = K/(0.2 + \alpha) \); \( \xi = \alpha f_y/f_{ck} \); \( \alpha = A_s/A_c \). \( f_{cu} \) is the compressive strength of cubic concrete block; \( f_{ck} = 0.8f_{cu} \) is the characteristic cylinder strength of concrete; \( A_s \) and \( A_c \) are the cross-section area of steel tube and concrete respectively.

For the symmetry of the specimen and the boundary conditions, only 1/4 of the model with symmetric boundary conditions on the symmetric planes was developed. Typical meshes of the steel element in the specimen are shown in Figure 5. FE model 1 and model 2 corresponded to CSPSW 1a and CSPSW 2 respectively. The details of the bolting or welding connections of the specimens were omitted. The only differences of the two models were the width of the T end plate.
Analytical Results

The overall force vs. displacement behavior of the specimens obtained from the FE analysis is compared with experiment results in Figure 6. The failure modes of the two FE models were all tension rupture of the steel plate with the ultimate load of 3370kN.

At the ultimate limit states, the bending deformation of the CFT column was small, and the tension strain of the steel plate reached hardening stage. Both the anchor plate and the steel tubes were found to be involved in the tension force transfer. The strain distribution of FE model 1 was similar with that of FE model 2. As model 2 had a wider end T plate than model 1, which may carry more tension forces, the hoping strain in the steel tube near the connection was smaller.

The calculated and measured strains of model 2 are shown in Figure 7 to study the force transfer mechanism. Comparing the strains measured in location A and location B or point E, it can be concluded that the internal part of the connections (the anchor plate) participated in the tension force transfer by more than 80%. Most of the tension force is transferred from the external part of connection plate directly into the embedded part, so very low local distortions of the steel tube wall near the connection plate were observed.
DISCUSSION

For CFT columns under axial compression, the loading process can be divided into three stages: the elastic stage, the elastic-plastic stage, and the plastic hardening stage as shown in Figure 8(a). In the initial stage of loading (the 0-a segment in Figure 8(a)), the Poisson’s ratio of concrete is lower than that of steel. Therefore, a trend of separation occurs between the concrete core and the steel skin, and the allocation of the force in the composite section is determined by the proportion of stiffness for the two materials. The CFT columns of the test specimens were just in this stage when the axial loading was proposed. As the load increases, the longitudinal strains of the steel tubes begin to yield, which indicates the elastic-plastic stage shown as the a-b segment, and force redistribution occurs in the composite cross-section with the decreasing of modulus of steel. The Poisson’s ratio of concrete is keeping increasing and gradually higher than that of steel, and the difference of lateral deformation generates hoop stress in the steel skin. Therefore, the confinement effect between steel tube wall and concrete core takes place. When the stress of the CFT tube reaches the yield surface, the steel tube becomes perfectly plastic and consequently can not resist more loads. But as the load increases further, the hoop stress developed in the steel tube makes the concrete core under tri-axial compression. This well known confined effect results in the increase of axial compression loading capacity for the CFT column. At point b shown in Figure 8(a), the steel tube reaches strain hardening range with the axial compression strain of about 0.02 to 0.03. After point b, the loading capacity of the CFT column can still increases in some degree due to the strain hardening effect of steel tubes and the confinement effect. At point c, the CFT column reaches the ultimate load and eventually fractures. For the vertical boundary element of CFT column, it will follow the above loading process if without the steel shear plate (Sakino et al.[8]).

By the influence of the steel plate, the stress of the steel tube goes through a different path, especially for steel at location B. The stress path of location B is shown in Figure 8(c). When the steel plate reaches yielding, hoop stress of about 130MPa is generated in point B. The tension stress in the hoop direction will reduce the yield strength to 260MPa in the axial direction, and causes the steel tube enter plastic stage earlier. As the load increases further, the confinement effect of the steel tube also increases, which improves the loading capacity...
of the CFT columns. At point c shown in Figure 8(c), the CFT column reaches the maximum loading capacity with the fracture of steel tubes. For the effectiveness of the anchor plate of transferring tension load from the steel plate to the concrete core, the tearing or distortion of the tube wall at location B and nearby regions are greatly relieved, and most parts in the steel tubes follow the similar stress path as shown in Figure 8(b).

According to above analysis, the tension of steel plate can cause local yielding of steel tube combined with the axial compression in early stages. If the connections can transfer most of the tension load into the concrete core, the early yielding of steel tube will limits in a small region near the connections. Thus, the confinement effect generated by the steel tubes won’t be weakened significantly by the local yielding, and the loading capacity of the CFT columns won’t be deteriorated. But this is depended on the effectiveness of connections by transferring tension forces between the steel infill panels and concrete core.

CONCLUSIONS

Based on both experimental and analytical results on the connections between steel panels and CFT columns in the steel plate shear wall system, the following conclusions can be drawn:

(1) The failure mode is the fracture of connection plate for bolt connections, and fracture of the steel plate for weld connections. For the latter type of connection, the steel plate shear wall can fully exert the post-buckling strength.

(2) The tension force introduced from steel panels can be effectively transferred to the boundary columns through the anchor plate, and the deterioration on the axial loading capacity of CFT due to the tearing of the steel skin can be neglected in design.

ACKNOWLEDGEMENT

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REFERENCES

FATIGUE DESIGN OF SQUARE HOLLOW SECTION TUBULAR T-JOINTS WITH CONCRETE-FILLED CHORDS UNDER IN-PLANE BENDING

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KEYWORDS
Tubular joints, concrete-filled tubes, high cycle fatigue, stress concentration factors (SCFs), failure modes

ABSTRACT
Previous research on fatigue of tubular joints has focussed mainly on empty welded joints subjected to various types of loading. Recent bridges have been constructed with welded tubular joints where the main chord is concrete filled to improve rigidity of the structure. This research investigates the behaviour of welded tubular T-joints made up of thin-walled square hollow section chord and braces with concrete-filled chords. The main chords are concrete-filled to simulate recent bridge constructions. High cycle fatigue tests of the T-joints are carried out under cyclic in-plane bending load in the brace. Stress concentration factors, failure modes and design data of the T-joints with concrete-filled chord are compared to those of empty tubular joints from previous research. Fatigue design is from an analysis of test data.

INTRODUCTION
Significant research has been carried out in the past on fatigue behaviour of empty tubular joints. Research on empty tubular joints has been carried out on square hollow section (SHS) T- and X-joints (van Wingerde 1992), circular hollow section (CHS) T- and Y-joints as well as overlap and gap K-joints (Efthimiou and Durkin 1985), multi-planar CHS and RHS tubular joints (Romeijn 1994, Panje-Shahi 1994). Based on this research, design guidelines have been formulated in national and international standards (IIW 2000, Zhao et al 2000, EC3 1992, SAA 1998). Recent bridge constructions in China, has seen an introduction of concrete filled tubular joints (Zhou and Chen 2003). In some of these bridge constructions, the tubular joints have a concrete filled chord member and empty brace members. The strength of these joints under fatigue loading is therefore in question, since little research has been carried out on concrete filled tubular joints under fatigue loading. Among some of the first work on concrete filled CHS tubular joints is that reported by (Udomworarat et al 2000). Preliminary work on fatigue behaviour of SHS T-joints with concrete filled chords was
reported by the authors in 2004, based on results of 9 fatigue tests (Mashiri and Zhao 2004). Research on tubular joints with concrete filled chords has been reported for K-joints with gap (Tong et al 2009) and CHS T-joints (Gu et al 2009).

This paper looks at the design of SHS T-joints with concrete filled chords based on a total of 17 tests. The main chords were filled with concrete having two mean compressive strengths of 25MPa and 50MPa. For each connection series, SCFs were measured in connections with chords filled by concrete with two different mean compressive strengths for comparison. Fatigue tests for these connections were carried out for the loading “in-plane bending in the brace”. The validity range for the fatigue design curves is defined based on the test series that has been investigated. One of the important observations in fatigue tests is the crack patterns that are observed during testing. These crack patterns depend on the levels of hot spot stresses around the tubular joints at the weld toes in the chord and brace. These crack patterns are significant in that they inform maintenance engineers in site inspections of joints in service. Analysis of the fatigue test data is carried out to compare the relative fatigue strength between SHS T-joints with concrete filled chords and empty SHS-T-joints. Fatigue design curves are also recommended.

TEST SPECIMENS

The specimens tested in this investigation are thin-walled square hollow section (SHS) T-joints with concrete filled chord as shown in Figure 1. The chords members were 3mm thick and the brace members 3mm and 1.6mm thick. The square hollow sections are high strength cold-formed tubes of grade C350LO. They have a minimum nominal yield stress of 350MPa and a minimum nominal ultimate tensile strength of 430MPa. The validity range of the specimens tested, are defined by non-dimensional parameters as follows: $0.35 \leq \beta \leq 0.67$; $25 \leq 2\gamma \leq 33$; and $0.5 \leq \tau \leq 1.0$ (Mashiri and Zhao 2004). The non-dimensional parameters are defined as follows: $\beta=b_1/b_0$, $\gamma=t_1/t_0$ and $2\gamma=d_0/t_0$, see Figure 1(a). Two grades of concrete, aimed at having a mean compressive strength of 25MPa and 50MPa, were used for filling the chords. The concrete mix design was estimated based on the British method (Neville and Brooks 1994). Details of the concrete constituents and cylinder strength at 28 days are given in Table 1.

<table>
<thead>
<tr>
<th>Mean compressive strength (MPa)</th>
<th>Concrete constituents based on British Method</th>
<th>Cylinder Strength (MPa)</th>
<th>Mean strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cement (kg/m³)</td>
<td>Fine Aggregate (kg/m³)</td>
<td>Coarse Aggregate (kg/m³)</td>
</tr>
<tr>
<td>25MPa</td>
<td>343</td>
<td>840</td>
<td>987</td>
</tr>
<tr>
<td>50MPa</td>
<td>490</td>
<td>689</td>
<td>991</td>
</tr>
</tbody>
</table>

STRESS CONCENTRATION FACTORS (SCFs)

Stress concentration factors were determined from experimental hot spot stresses determined from strains in strip strain gauges. The strip strain gauges were located in the extrapolation region next to the weld toes as recommended by fatigue design guidelines for the hot spot stress method (IIW 2000, Zhao et al 2000). The quadratic extrapolation method is recommended by fatigue design guidelines for tubular joints made up of square or rectangular hollow sections and was therefore adopted in determination of hot spot stresses in this investigation, see Figure 1(b). The stress concentration factors were defined as the ratio between the hot spot stress and the nominal stress in the member due to the basic member which causes that hot spot stress (IIW 2000).

For each connection series, stress concentration factors were determined in specimens with their chords filled by concrete with a mean compressive strength of 25MPa and 50MPa. For each of the specimens, SCFs were determined along the lines B, C and D in the main chord, see Figure 1(a).
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Lines B, C and D are the hot spot locations recommended by the standards (IIW 2000). For tubular T-joints made up of square or rectangular hollow sections these are the locations where maximum hot spot stresses occur. Fatigue crack initiation was predominantly observed to occur in this region of weld toes in the chord (Mashiri and Zhao 2004).

Figure 1(a): SHS T-joints with concrete filled chord (Mashiri and Zhao 2004)

A comparison of the SCFs for the SHS T-joints with concrete-filled chords and empty SHS T-joints is shown in Figure 2 for the 6 connection series tested in this investigation. Figure 2 shows that for the load case “in-plane bending in the brace”, there is a relatively significant decrease in SCF for SHS T-joints with concrete filled chord along lines C and D compared to empty SHS T-joints for connection series S3S1Con, S3S2Con, S3S4Con and S3S5Con. These connection series have relatively low to medium values of \( \beta \) ranging from 0.35 to 0.5. For connection series S6S1con and S6S2Con with relatively higher beta values of 0.67, although there is a decrease in SCF along line C, the SCFs along lines B and D are not distinctly affected by concrete filling of the chord. From the comparison in Figure 2, it can be concluded that for the load “in-plane in the brace” the concrete grade in the chord does not provide any trend to suggest an influence on the magnitude of SCFs at the hot spots. Therefore an average SCF was determined based on the SCFs measured in SHS T-joints with chords filled by 25MPa and 50MPa concrete for a given connection series.

A summary of the average SCFs along the lines B, C and D for each connection series is shown in Table 2, together with the SCFs for empty T-joints from previous research (Mashiri et al 2002) for comparison. The ratio of the maximum SCF in SHS T-joints with concrete filled chords to that in empty joints is also given in Table 2. This ratio shows that concrete filling of the chord in SHS-joints results in a reduction in the maximum SCF when compared to maximum SCFs in empty SHS T-joints. The maximum SCFs in each connection series will be used for analysing fatigue test data for the hot spot stress method.

### Table 2

**SCFS IN SHS T-JOINTS WITH CONCRETE FILLED CHORD AND EMPTY SHS T-JOINTS**

<table>
<thead>
<tr>
<th>Series Name</th>
<th>Experimental SCF (Average Quadratic)</th>
<th>Ratio of Maximum SCFs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T-joints: chord with concrete</td>
<td>Empty SHS T-joints</td>
</tr>
<tr>
<td></td>
<td>Line B</td>
<td>Line C</td>
</tr>
<tr>
<td>S3S1Con</td>
<td>9.0</td>
<td>7.9</td>
</tr>
<tr>
<td>S3S2Con</td>
<td>6.1</td>
<td>5.1</td>
</tr>
<tr>
<td>S3S4Con</td>
<td>5.7</td>
<td>6.8</td>
</tr>
<tr>
<td>S3S5Con</td>
<td>3.1</td>
<td>4.5</td>
</tr>
<tr>
<td>S6S1Con</td>
<td>3.1</td>
<td>7.2</td>
</tr>
<tr>
<td>S6S2Con</td>
<td>0.8</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Figure 1(b): Extrapolation for hot spot stresses, Line D, S3S4Con1-1
FATIGUE CRACKING IN SHS T-JOINTS WITH CONCRETE FILLED CHORD

Maintenance management of welded structures under service can be greatly enhanced by inspection regimes that are well informed about possible locations for crack initiation and subsequent crack propagation patterns in welded connections. This ensures that no unexpected failure of key structural components occurs, a scenario that can lead to catastrophic collapse of the structure. Typical locations for crack initiation and possible crack growth patterns for SHS T-joints with concrete filled chords under in-plane bending in the brace are therefore an important integral part of site inspections and are shown in Figure 3. Three types of failure were observed as follows (Mashiri and Zhao 2004):

(a) Chord-Tension-Side Failure: Crack initiation and propagation occurs in the main chord. This failure mode is predominant in specimens with small to medium values of $\beta$ ranging from 0.35 to 0.5, see Figure 3(a) and (b).

Figure 2: Comparison of SCFs in composite SHS T-joints and empty SHS T-joints
(b) Chord-and-Brace-Tension-Side Failure: Crack initiation occurs in the chord. Subsequent propagation leads to crack initiation and propagation in the brace. This failure occurred in some of the specimens with moderately higher $\beta$ values of 0.67, see Figure 3(c).

(c) Brace-Tension-Side Failure: Crack initiation and propagation occurs in the brace. This failure occurred in one specimen with a moderately higher $\beta$ value of 0.67, see Figure 3(d).

**FATIGUE STRENGTH COMPARISON: EMPTY VS SHS T-JOINTS WITH CONCRETE FILLED CHORD**

Seventeen (17) SHS T-joints with concrete filled chord were tested under cyclic in-plane bending in the brace. Fatigue test data was obtained relating stress range to the number of cycles to failure. The fatigue test data for the SHS T-joints with concrete filled chords is compared to that for 59 empty SHS T-joints from previous research (Mashiri et al 2002). Figure 5(a) shows that fatigue test data of SHS T-joints with concrete filled chords lies in the region that defines the upper bound or higher of empty SHS T-joints data. To enable fatigue strength comparison of the joints, the resultant S-N data has been analysed using the least squares method (ASTM 1980) to determine mean-minus two standards-deviations S-N curves. Mean minus two standards deviations curve define the design curves from a given set of fatigue test data. The statistical analysis was carried out for test data expressed in the classification. The classification method relates the nominal stress range derived from beam theory to the number of cycles to failure. The S-N data from the empty SHS T-joints and SHS T-joints with concrete filled chords are shown in Figure 5(a). The resultant mean minus two standards deviation S-N curve from the two sets of data are also shown in Figure 5(a). Results of the statistical analyses are shown in Table 3. The number of cycles defined by the mean minus two standards deviation curves show that the number of cycles to failure in SHS T-joints with concrete filled chords is about 1.7 times that of empty SHS T-joints for a given nominal stress range.

![Fatigue Strength Comparison](image_url)

Figure 3: Crack growth patterns in SHS T-joints with concrete filled chord under IPB
TABLE 3.
PARAMETERS, STANDARD DEVIATION AND LOWER BOUND DESIGN S-N CURVE EQUATIONS

<table>
<thead>
<tr>
<th>Test Data</th>
<th>Method: ( \log N ), dependent variable</th>
<th>Parameters ( A ) and ( B )</th>
<th>Standard Deviation, ( \sigma \log N )</th>
<th>Design S-N Curve Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty SHS T-joints</td>
<td>(i) ( A ) determined, ( B = -3 )</td>
<td>( A = 10.03 ) ( B = -3 )</td>
<td>( \sigma \log N = 0.63 )</td>
<td>( \log N = 8.78 - 3 \log S )</td>
</tr>
<tr>
<td>SHS T-joints with concrete filled joints</td>
<td>(ii) ( A ) determined, ( B = -3 )</td>
<td>( A = 10.66 ) ( B = -3 )</td>
<td>( \sigma \log N = 0.83 )</td>
<td>( \log N = 9.00 - 3 \log S )</td>
</tr>
</tbody>
</table>

FATIGUE DESIGN

There are two methods of fatigue design that can be used for welded connections. One is the traditional classification method which was used in the previous section to compare the fatigue strength between the SHS T-joints with concrete filled chords and empty T-joints. The classification method relates nominal stress range to the number of cycles to failure. Using the classification method, different fatigue design curves can be determined and assigned to different constructional details in different thickness ranges. The hot spot stress method on the other hand is obtained by analysis of fatigue test data relating the hot spot stress range to the number of cycles to failure. The hot spot stress range is the product of the nominal stress range and the highest stress concentration factor at a welded joint. One of the advantages of the hot spot stress method is that fatigue design curves are dependent on the wall thickness of the member in which fatigue cracks initiate and propagate causing failure. This means that regardless of the type of connection, uniplanar or multiplanar, joints can be designed using a unique design curve for a given wall thickness provided that the stress concentration factors can be determined for the evaluation of hot spot stress. The nominal stress ranges of the SHS T-joints with concrete filled chord shown in Figure 5(a) were converted into hot spot stress ranges using the maximum stress concentration factors in a connection series highlighted in Table 2. The resultant hot spot stress ranges versus number of cycles to failure for the SHS T-joints with concrete filled chords are shown in Figure 5(b). Crack initiation and propagation leading to failure occurred mostly in 3mm thick main chords similar to the failure that occurred in empty SHS T-joints. From the characteristics of fatigue data in the hot spot stress method, it is therefore expected that the fatigue data for SHS T-joints with concrete filled T-joints would be comparable to that for empty SHS-joints. The design S-N curves for empty SHS T-joints from previous research were defined by the following curves:

\[
\log(N) = 12.9502 - 3.4517 \cdot \log(S_{r,hs}), \quad \text{for } 10^3 \leq N \leq 5 \times 10^6 \tag{1}
\]

\[
\log(N) = 15.7543 - 5 \cdot \log(S_{r,hs}), \quad \text{for } 5 \times 10^6 \leq N \leq 10^8 \tag{2}
\]

The design curve for the empty SHS T-joints derived from equations 1 and 2 is shown in Figure 5(b). All the fatigue test data for the SHS T-joints with concrete filled chords in Figure 5(b) lie above the design curve for the empty SHS T-joints expressed in the hot spot stress method. Therefore the design S-N curve defined by equations 1 and 2 can be adopted for the SHS T-joints with concrete filled chords.

CONCLUSIONS

Seventeen (17) SHS T-joints with concrete filled chords were tested under cyclic in-plane bending in the brace and compared to empty SHS T-joints from previous research. The following observations were made:
1. There is a reduction of about 15% to 80% in the highest stress concentration factor in the SHS T-joints with concrete filled chord compared to empty SHS T-joints. This reduction in maximum stress concentration factor is attributed to the reduction in chord face deformation caused by the concrete filling.

2. Fatigue crack growth patterns observed in SHS T-joints with concrete filled chords are similar to those observed in empty SHS T-joints despite the reduction in SCFs. Three failure modes were observed. Chord-tension-side failure was the predominant mode of failure, followed by chord-and-brace-tension side failure and then brace-tension-side failure.

3. Fatigue life in SHS T-joints with concrete filled chords was found to be about 1.7 times that in empty T-joints because of the reduction in SCFs. It was shown that when using the hot spot stress method of design the same fatigue design S-N curve for empty SHS T-joints can be used for SHS T-joints with concrete filled chords.

Figure 5: (a) Comparison of fatigue test data, (b) Fatigue S-N data in the Hot Spot Stress Method

REFERENCES


INFLUENCE OF LOCAL DEFECTS ON BUCKLING BEHAVIORS OF PRESSURE STEEL PIPE

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KEYWORDS
Local defect, Buckling, Pressure pipe, Finite element, Compression, Local deformation.

ABSTRACT
An optimum design of pressure steel pipe requires knowledge of the local buckling behaviors under different local defects. In this study, numerical finite element model has been used to study the influence of local defects on buckling behaviors of pressure steel pipe. By introducing simple but reasonable local defect of somewhat rectangle pattern, three parameters are defined to characterize the local defects and used for parametric studies. A compression experiment was conducted first to verify the numerical model before systematic numerical simulations. The comparison between the results of numerical analysis and experiment shows a good agreement. It is found in this paper that, while presence of internal pressure decrease limit load of steel pipe, presence of local defect can further decrease it significantly. It decreases at a constant non-zero rate when the local defect develops in circumferential and thickness direction, but zero-rate or no apparently further decrease when develops in axial direction. Circumferential local defect was found to have the most influence on the decrease of load limit. Local defect also leads to a very short strain hardening region and earlier entering of softening especially for pipes of high pressure, which would cause local deformation develop rapidly. Once local defect occurs, the hardening ability of the pipe is reduced to a relatively constant value that mainly depends on internal pressure.

INTRODUCTION
Pipe systems of steel have being used in many industries around the world. Many of these pipelines either contain pressure gas or fluid, referred to hereafter as pressure pipe. The manufacture and installation of pressure pipe are regulated by code in each country, and to ensure safe operation of the system, they must meet stringent quality standards and performance levels under designed working environment. However, there are cases in which the working environment can change and become unpredictable after operation for some time.
It is not uncommon for occurrence of localized defects in operation process, for example, corrosion [1,2]. This poses a great threat to pressure pipe system as it causes a tendency for the pipeline to local deformation. Often the local deformation of the pipe wall results in local buckling of the pipe, and its post-buckling range of response, local buckles in the pipe wall develop rapidly and can thus quickly become significant in magnitude [3].

In example of gas pressure pipe system, it is reported in [4] that natural gas contributes to 20% to 50% of all earthquake related fire ignitions. If the pipe is damaged, higher pressures can further increase the amount of gas that can leak. Pressure pipe system is one kind of “lifeline” systems, of which safety under natural disaster should be considered seriously. While influences of imperfection, boundary, loading etc. on safety of pipes are studied by many researchers (see [5,6] for examples), few studies on influences of local defect can be found in literature, and majority of relative design codes do not clearly reflect these influences, for example, Seismic Design Guideline of High-pressure Gas Pipeline Japan [7], although an optimum design of pressure steel gas pipe requires knowledge of the local buckling behaviors under different local defects. To this end, the present study is directed.

NUMERICAL MODELS

Material, Local Defect and Internal Pressure

The carbon steel pipe, specified in Japanese Industrial Standard as SGP, was chosen in this study since it is wildly used by gas suppliers in Japan. The material behavior of SGP studied was got from common tension test of five samples. It should be noted that these samples are directly cut from finished products of SGP. So they are curved in nature, which is contrast to processed samples of flat plates used in [1]. A typical test result of stress-strain curve is shown in Figure 1(a) along with test result from [1] for a processed flat plate sample. It can be seen from Figure 1(a) that sample cut naturally from pipe has higher load ability than sample after flat plate processing, although they show no big difference and both good performance in ductility behavior. Table 1 summarized the test results, in which, refer to Figure 1(b), $\sigma_y$ and $\varepsilon_y$ denote the yielding stress and strain respectively, $E$ denotes the modulus of elasticity, $\nu$ denotes the Poisson’s ratio, $\sigma_u$ and $\varepsilon_u$ denote the ultimate strength and corresponding strain respectively, $E_{st}$ and $\varepsilon_{st}$ denote the tangent modulus and strain respectively.

![Stress-strain curves](image1.png)

Figure 1: (a) Typical stress-strain curves for curved sample tested and flat plate sample from [1]; (b) Linearized test result
TABLE 1

<table>
<thead>
<tr>
<th>Material</th>
<th>( \sigma_y (\text{MPa}) )</th>
<th>( E (\text{GPa}) )</th>
<th>( v )</th>
<th>( \varepsilon_y )</th>
<th>( \varepsilon_{sl} )</th>
<th>( E_s (\text{GPa}) )</th>
<th>( \sigma_u (\text{MPa}) )</th>
<th>( \varepsilon_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SGP</td>
<td>358.4</td>
<td>205.1</td>
<td>0.287</td>
<td>0.00175</td>
<td>0.0128</td>
<td>1.82</td>
<td>396.6</td>
<td>0.290</td>
</tr>
</tbody>
</table>

Effect of local defect has been modeled keeping in view the phenomenon of local defect. The most common result of local defect such as corrosion is decrease of effective volume. In this paper, this decrease in volume has been modeled by cutting and removing a block of somewhat rectangular form from middle part of steel pipe as shown in Figure 2. Thus, it is desirable to introduce the following three non-dimensional parameters to characterize the local defects:

\[
\bar{L}_c = \frac{c_{\text{loss}}}{2\pi r} \times 100\% \quad \bar{L}_a = \frac{a_{\text{loss}}}{L} \times 100\% \quad \bar{L}_d = \frac{d_{\text{loss}}}{t} \times 100\%
\]

where \( c_{\text{loss}} \), \( a_{\text{loss}} \), and \( d_{\text{loss}} \) are the length of the local defect block in circumferential, axial, and thickness direction respectively; \( t \) is the thickness of the pipe; \( L \) is the axial length of pipe; \( r \) is the inner radius of the pipe’s cross-section defined as \((D-t)/2\), in which \( D \) is the outer diameter of the pipe. Such definitions facilitate parametric study of their effects on buckling behaviors.

![Figure 2: Schematic of local defect](image)

It is well known for pressure pipe that fracture is governed by the hoop stress in the absence of other external loads since it is the largest principal stress, so internal pressure is introduced in form of hoop stress defined as

\[
\sigma_{\text{hoop}} = p\left(D-2t\right)/2t
\]

where \( p \) is the internal pressure. The hoop stress is further non-dimensionalized by yielding stress \( \sigma_y \) in this paper. An increment of \( \sigma_{\text{hoop}}/\sigma_y = 0.05 \), is equal to a pressure increment of 1MPa.

**Finite Element Model**

Commercial ABAQUS finite element computer program is used to simulate the buckling behaviors of pressure pipes of different localized defects. The element used is S4R which is 4-node, general-purpose, reduced integration with hourglass control and finite membrane strains shell element. Material properties of bilinear material law, suggested by Japan Gas Association, are based on sample test results in section 2.1. The modeled pipe is 500mm in axial length, with 216.3mm in outer diameter and 5.8mm in thickness as shown in Figure 3.
The global imperfection in the original geometry is considered. The first mode of linear eigenvalue buckling analysis of corresponding perfect structure is used to construct the imperfection. The maximum magnitude of the imperfection used is specified in the ECCS (1976) as:

\[
\delta_{d,max} = 0.04 \sqrt{l(D-2\pi)/2}
\]  

As successfully employed in similar previous study [2], the symmetry boundary condition is assigned to ends of pipe. This boundary condition was proved a good representation of working conditions of pipe in [2]. Two single nodes located at the center axial line as denoted in Fig. 3 are used to represent axial degree of freedom of shell nodes at the both ends of the pipe respectively, and shell nodes of both ends are attached to them by using equation constraint and specifying that axial degree of freedom at the shell nodes are constrained to the motion of the single node. Thus, in addition to internal pressure, displacement load can be applied on top single node directly while keep bottom single node fixed in axial direction. Moreover, to prevent rigid body motion, one node of bottom end nodes is fixed. This constraint is in addition to the symmetry end boundary conditions mentioned above and will not introduce an over constraint into the problem. The displacement load \( \delta \) history of the top single node and reaction history \( P \) of the bottom single node are recorded. In this paper, \( P-\delta \) response curves are used thoroughly for evaluation of behaviors of pressure pipe.

Stress singularities exist at the sharp corners of the local defect block of rectangle pattern. These singularities occur because of the idealizations of rectangle pattern block used for the local defect. But they are omitted in this study since our interest is not on the small details around the corners, and as numerical experiments have shown, these singularities have a negligible effect on the overall response of the model. Thus, for convenience, a uniform mesh distribution along both the circumference and length of the models are used. Mesh size \( 80 \times 80 \) is chosen in this paper, which is decided simply by series convergence studies based on load and displacement history curves. The same mesh size is deemed suitable in modeling applications for all cases studied.

VERIFICATION OF NUMERICAL MODEL BY EXPERIMENT

It is important for a numerical model to correctly reproduce physical behaviors of the corresponding physical model. In order to check this correctness, a compression experiment was carried out first for comparison purpose before some systematic numerical simulations. The tested specimen as can be seen in Figure 4 used the same SGP as in all the numerical
models. It has 500 mm long and 216.3 mm outside diameter by 5.8 mm wall thickness. There are two flange plates in the ends of pipe, which are used to facilitate introducing internal pressure as well as boundary conditions. Data on the local defect and internal pressure are summarized in Table 2.

The boundaries in the both ends of pipe are fixed in space except for the top end in the axial force direction. Displacement loads were incrementally applied on the top end of pipe specimen at a very low strain-rate. The relationship between load and the displacement are obtained. More detail description of the experiment can be found in [2]. Figure 4 shows the typical course of the experiment after occurrence of local buckling. It should be noted that local buckling not necessarily occurs at the middle part of the simulated local defect since of processing errors of local defect.

![Figure 4: Experiment setup and a typical course of the experiment after occurrence of local buckling.](image)

| Simulated local defect | Occurrence of local buckling | Introducing the internal pressure |

TABLE 2: LOCAL DEFECT AND INTERNAL PRESSURE OF THE SPECIMEN

<table>
<thead>
<tr>
<th>$L_c$</th>
<th>$L_d$</th>
<th>$L_{ld}$</th>
<th>$\sigma_{hoop}/\sigma_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>60%</td>
<td>51.7%</td>
<td>2.66%</td>
</tr>
</tbody>
</table>

Non-dimensionalized results from both experiment and the corresponding numerical model are given together in Figure 5, in which $P_y = \sigma_y A$, $\delta_y = \varepsilon_y A$, where $A$ is the cross-section area of pipe. Due to the inevitable errors, the two curves shown in Figure 5 are considered good agreement between each other. Thus, analysis result from numerical model in section 2 can be safely used to study the influence of local defects on buckling behaviors of pressure steel pipes.

![Figure 5: Comparison of load-displacement results between experiment and numerical simulation.](image)
RESULTS AND DISCUSSION

In this section, influences of local defects on pressure pipe are studied using the verified numerical model. The internal pressures are taken to have a range of \( \sigma_{\text{hoop}} / \sigma_y \) from 0.00 to 0.50, while varying parameters of local defect \( L_c \), \( L_a \) and \( L_d \) from 0.0% to 100% respectively. It should be noted here that in cases of \( L_c \), \( L_a \) or \( L_d = 0 \%), they correspond to structures of no local defect. From all of analysis results, two main parameters \( P_{\text{max}} \) and \( \delta_{\text{max}} \) representing characteristics of buckling behaviors are further extracted. Here, \( P_{\text{max}} \) and \( \delta_{\text{max}} \) are defined as the load maximum and its corresponding displacement in \( P - \delta \) history record respectively. As the present study is first one in a series study, \( P_{\text{max}} \) and \( \delta_{\text{max}} \) results are mainly report and compared here.

**Parametric Study of Influence of Circumferential Local Defect**

It can be seen from \( P_{\text{max}} \) results shown in Figure 6(a) that \( P_{\text{max}} \) decrease as the internal pressure and \( L_c \) increase, but following initial drops of \( P_{\text{max}} \), the decrease rates of \( P_{\text{max}} \) become nearly constant and moreover, all curves have almost the same decrease rate. It seems internal pressure has no influence on decrease rate of limit load of local defected pipe. Also, circumferential length of local defect appears to have no effect on decrease rate of limit load, as can be seen in Figure 6(a) that \( P_{\text{max}} \) keep a constant bands between each other for different values of \( L_c \). But attention should be paid to pipes of high internal pressure, as seen in Figure 6(a) that the reduction of \( P_{\text{max}} \) for the pipe of highest internal pressure studied are almost 2 times of those three of lowest pressure.

Figure 6: (a) \( P_{\text{max}} / \sigma_y \) versus \( L_c \) and (b) \( \delta_{\text{max}} / \delta_y \) versus \( L_c \) for different values of \( \sigma_{\text{hoop}} / \sigma_y \).

As for \( \delta_{\text{max}} \) results shown in Figure 6(b), all cases show significant drops with presence of local defect and internal pressure. Although it have relative low drops in case of \( \sigma_{\text{hoop}} / \sigma_y = 0 \) and \( L_c < 70\% \), the reduction values are still quite large and need attention. Moreover, as the internal pressure and \( L_c \) increase, \( \delta_{\text{max}} \) values converged to \( \delta_y \). In the presence of both local defect and internal pressure, the convergence rates are extremely faster, or in other words, the pipe enter the softening region and produce localized deformation rapidly, and large value of \( L_c \) and internal pressure tend to have no effect on strain-hardening behavior of pipe structure once the local defect occurs.
**Parametric Study of Influence of Thickness Local Defect**

$P_{\text{max}}$ in Figure 7(a) have similar trends as those in Figure 6(a): $P_{\text{max}}$ has linear relationship with $L_d$. After the initial drops from $P_{\text{max}}$ in case of no local defect; presence of high pressure causes a half-maximal reduction of $P_{\text{max}}$ in cases studied, and both $L_d$ and internal pressure have no significant effect on decrease rate of $P_{\text{max}}$. But local defect develops in thickness direction has less influence on the decrease of $P_{\text{max}}$ than that in circumferential direction, as can be seen that the slope of linear $P_{\text{max}}$ vs. $L_d$ relationship is apparently smaller than that slope of linear $P_{\text{max}}$ vs. $L_a$ relationship.

Also, some similar trends in Figure 6(b) can be seen in Figure 7(b) for pipes of locale defects: $\delta_{\text{max}}$ reduces and converges to $\delta_y$ with internal pressure increasing, and varying of $L_d$ makes no significant different except in two cases ($20 < L_d < 70$ ). But, in figure 6(b), high value of $L_a$ cause high reduction of $\delta_{\text{max}}$ to $\delta_y$ even in case of $\sigma_{\text{hoop}} / \sigma_y = 0$, and this correlations are not true for $L_d$, of which even large value the local defect has, the pipe can still have quite large value of $\delta_{\text{max}} / \delta_y > 1$ depending on internal pressure.

**Parametric Study of Influence of Axial Local Defect**

$P_{\text{max}}$ results in Figure 8(a) show different relationships as those in Figure 6(a) and Figure 7(a). It can be seen that, increasing of internal pressure cause reduction of $P_{\text{max}}$ as it is true for all cases studied in this paper, but development of defects by increasing $L_a$ no more changes $P_{\text{max}}$ significantly.
Figure 8(b) shows that, variations of $L_o$ do not change $\delta_{\text{max}}$ significantly, this similar trends can also be seen in Figure 6(b) and Figure 7(b) with exceptions of few cases. Also shown in Figure 8(b) is that, even presence of low internal pressure can cause high reductions of $\delta_{\text{max}}$ to $\delta_y$. Figure 6(b) have this similar effect but only when $L_o > 40\%$.

**CONCLUSIONS**

All cases studied and described in last section show a consistent result that, presence of internal pressure decrease limit capacity and reduce hardening of steel pipe as can be expected. But presence of local defect can further decrease the limit load significantly. Furthermore, the limit load decreases at a constant non-zero rate when the local defect develops in circumferential and thickness direction, but zero-rate or no apparently further decrease when local defect develops in axial direction. This result is reasonable since compression capacity is controlled by the weakest cross-section of the pipe and development of local defect in axial direction do not contribute to further weaker or decrease of effective area in that cross-section. Among all the results, circumferential local defect was found to have the most influence on the decrease of load limit.

Local defect leads to a very short strain hardening region and earlier entering of softening, thus local deformation would develop rapidly, especially for pipes of high pressure. This kind of local deformation is studied in detail by Murray [8]. It is suggested that, if one defines failure in the spirit of limit state design, the behavior of the pipe in terms of the initiation and development of local deformation should becomes a rational type of behavior upon which to base such limit state criteria. We also found in this study that dimensional variations of local defect do not have significant effects on strain-hardening. Thus, once local defect occurs, the hardening ability of the pipe is reduced to a relatively constant value that mainly depends on internal pressure.

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**REFERENCES**

DESIGN OF ECCENTRICALLY CONNECTED CLEAT PLATES IN COMPRESSION

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KEYWORDS
Eccentric cleat connection, CHS member, design method

ABSTRACT
The paper presents experimental results from 12 full scale test samples using different CHS member lengths and eccentric cleats combinations. The test results show that the governing connection failure is a sway collapse mechanism. Based on this evidence, a simple design method for eccentric cleat connection in compression is proposed. Results from the proposed design methods are in good agreement with the obtained experimental results and with FE predictions.

INTRODUCTION
Unlike open structural sections where it is possible to connect the structural member directly to a gusset plate, the connection of structural steel hollow sections require certain fabrication such as slotted-end plate, welded-tee and flattened-end connections. This paper deals with slotted-end plate connection type. The end plate is commonly bolted through a cleat or a gusset plate to a supporting member. This will result in an eccentric connection with unavoidable bending stresses that adversely affect the connection capacity under compression. The slotted-end plate remains the most common type of connection used in practice. The design of eccentric cleats is covered by a number of specifications [1-3] with a wide spectrum of design capacities determined according to these specifications. In Australia, the design of eccentric cleats is covered by the ASI guide [1]. The recommendations in this guide were based on experimental and analytical work presented by Kitipornchai et al [4]. The work of Kitipornchai et al identified three possible failure modes for a compression member with eccentrically connected cleat plates at both ends, these are: overall member buckling, connection failure with sway and connection failure without sway. They had shown that analysis based on the first-yield approach resulted in an overly conservative result while ignoring the eccentric moment generated in the cleat plates would result in unconservative
design. Furthermore, they concluded that for most practical applications the relative bending  

stiffness between the cleat plate and the member to which the cleat is attached will  

approximate a fully fixed connection. Following this conclusion, they derived six possible  

plastic collapse mechanisms for the connection that were confirmed by experimental results  

conducted on isolated cleat plates. Based on this, a design method was proposed that was  

adopted by the Australian Steel Institute, ASI [1]. This design approach will overestimate the  

connection capacity if the connection fails in a sway rather than fully fixed mode. This was  

first recognized during the development of steel connection design software Limcon V3 [5].  

In response to this, the ASI issued an advisory note [6] recommending using the design  

procedures outlined in the Australian steel structures code [7] taking into account the  

combined bending and compression actions for designing eccentric cleat connections rather  

than using the ASI design guide [1]. More recently, New Zealand Heavy Engineering  

Research Association (HERA) recommended a new step-by-step design method for eccentric  

cleats [8]. This method suggests that the dominant failure mode of this type of connection is  
a sway mode. It assumes that at least one end of the overlapping cleat plates is effectively fully  

fixed while the connection to the end of the supported member can be free to rotate. Particular  

attention is given to identifying the exact type of support conditions at both ends of the  
cleat plate. A procedure to determine whether a cleat is effectively fixed to the supported  
member is also outlined in this method. In this paper the results obtained from full-scale  

experimental program using 12 samples of different member length and eccentric cleats  
combinations will be presented. The experimental results are compared with available design  
methods for eccentric cleat connections and with FE predictions and a new design procedure  
is proposed.

![Figure 1: Geometric details of eccentric cleat connection](image)

**EXPERIMENTAL RESULTS**

A total of twelve test specimens were used, three samples of each of the following member  

lengths (Ls); 3m, 4m, 5m, 6.5m. All the members were Grade 350 hot rolled circular hollow  

sections CHS with an outside diameter of 139.7mm and a wall thickness of 3.5mm. Different  

member lengths were used in order to assess the effect of the relative stiffness of the  

overlapping cleats and the member on the design capacity of the connection. A constant cleat  

length Lc= 170mm (Figure 1) was used for the 3m and 6.5m members. For the 4m and 5m  

length members, three different cleat lengths, Lc, were used. These were; 170mm, 220mm  
and 270mm. This was done in order to assess the effect of cleat plates’ slenderness on the  
connection capacity while the member slenderness is kept constant. All cleat plates used were  
180x10mm Grade 300 steel. Two coupon tests were conducted for the CHS sections and four  
for cleat plates. These tests indicated an average yield stress of 345MPa for the CHS and  
320MPa for the cleats. Cleat connections usually require two weld locations, the first is
between the slotted end plate and the supported member (the CHS member) and the second is between the gusset plate and the supporting member. The slotted end plate is usually welded along its entire slot length. For the test specimens a 6mm SP fillet weld with a slotted length, \( L_s \), of 260mm is used (Figure 1). This weld length is adequate for even distribution of shear transfer across the slot length. Bracing members are usually connected to stiffer beams or columns. In the experimental setup used, this is achieved by welding the gusset plate to a thicker square plate (180mmx20mm). The weld used here is a full penetration butt weld. The square plate is attached to a rigid support as shown in Figure 2. Good design of bolted connections requires minimizing the length of the cleat assemblies. For cleat assemblies that require two rows of bolts (Figure 1), the minimum length is comprised of the pitch distance, \( S_p \) (70mm used in our samples), edge distance, \( A_e \) (35mm is used) and clearance to the support (minimum of 15mm is used). The pitch and edge distances are governed by the size of bolt used to connect the plate. All the cleats were assembled using four M20 snug tight bolts (two rows). These length values gave a minimum cleat length \( L_c \) of 170mm. Table 1 summarizes the details for the 12 samples used in the experiment. The CHS member was placed horizontally in the test rig with the cleat plates oriented in the vertical plane. Four displacement measurements using LVDTs were recorded for each test. Two of these were placed at the interface between the slotted end plates and the CHS at each end to measure the lateral displacement and the other two at the mid-span of the CHS to measure the lateral and the vertical displacement. Similarly, strain measurements using strain gauges were recorded for four of the test samples (one for each of the 3m, 4m, 5m and 6.5m length samples). Strain gauges were placed at opposite sides of the CHS at mid-span and the other two on the opposite sides of the slotted end plate at the supported end of the samples. Tables 2 summarizes the test results. Nine test samples (sample A, B and C with 3, 4 and 5m lengths respectively) experienced a sway collapse mechanism at the cleat connection at the supported end of the members. Figure 3 (a, b and c) shows the resulting sway collapse mechanism for \( L_c \) 170, 220 and 270mm respectively. In contrast, samples D1-D3 (6.5m) failed by elastic buckling of the CHS member. When taking the cleat length \( L_c \) into account, samples D1-D3 have actual length of 6.84m. Using the experimental failure loads (Table 2) for D1-D3 and a 6.84m length, an effective length factor, \( k \), of 0.98, 1.06 and 0.98 is obtained for D1, D2 and D3 respectively. Figures 4a and 4b show some of the obtained experimental load-lateral deflection curves at mid-span for Samples B1-B3 (Figure 4a) and Samples A1, B1 and C1 (Figure 4b). From these Figures, it is clear that the connection capacity is much more sensitive to the slenderness of the cleat plates (Figure 4a) rather than to the member (CHS) slenderness (Figure 4b).
Figure 3: Sway failure of the cleat connection, a) 170mm cleat, b) 220mm cleat, c) 270mm cleat

Table 1: Details of test samples

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Brace CHS 139.7x3.5 Length Lₚ (mm)</th>
<th>Cleat plates (mm)</th>
</tr>
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<tr>
<td></td>
<td>Lc</td>
<td>t</td>
</tr>
<tr>
<td>A-1</td>
<td>3000</td>
<td>170</td>
</tr>
<tr>
<td>A-2</td>
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<td>170</td>
</tr>
<tr>
<td>A-3</td>
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<td>170</td>
</tr>
<tr>
<td>B-1</td>
<td>4000</td>
<td>170</td>
</tr>
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<td>B-2</td>
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<td>220</td>
</tr>
<tr>
<td>B-3</td>
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<td>270</td>
</tr>
<tr>
<td>C-1</td>
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<tr>
<td>C-2</td>
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<tr>
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<td>D-3</td>
<td>6500</td>
<td>170</td>
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</table>
Table 2: Experimental failure loads and predictions

<table>
<thead>
<tr>
<th>Test specimen</th>
<th>Cleat slenderness ratio L_c/r</th>
<th>Brace slenderness ratio</th>
<th>Experimental failure Load (kN)</th>
<th>Proposed design method Pu (kN)</th>
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<tr>
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<td>62</td>
<td>158.5</td>
<td>151</td>
</tr>
<tr>
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<td>62</td>
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<tr>
<td>A-3</td>
<td>59</td>
<td>62</td>
<td>159.8</td>
<td>151</td>
</tr>
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<td>59</td>
<td>83</td>
<td>175.1</td>
<td>151</td>
</tr>
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<td>B-2</td>
<td>76</td>
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<td>155.4</td>
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<td>94</td>
<td>83</td>
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<td>C-1</td>
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</table>

**PROPOSED DESIGN METHOD**

Based on the experimental observations presented earlier, the sway collapse mechanism (Figure 3) is the governing mechanism that determines the connection capacity. A two steps design method for eccentric cleat connection in compression is proposed in this section. A schematic of the sway mechanism with two plastic hinges is shown in Figure 5, the external work V (ignoring second-order effects) is

\[ V = P \varepsilon \theta \]  

(1)

Where \( P \) is the applied load, \( \varepsilon \) is the load eccentricity which equals the average thickness of the supported and supporting cleat plates and \( \theta \) is the rotation as shown in Figure 5. The internal energy, U, is

\[ U = (M_{p1} + M_{p2}) \theta \]  

(2)

Where \( M_{p1} \) and \( M_{p2} \) are the plastic moments of the supported and supporting cleat plates respectively (for identical cleat plates with \( \sigma_y \) yield stress, \( M_{p1} = M_{p2} = M_p = \sigma_y wt^2/4 \)).

Equating the external and internal energies (eqs 1 and 2) and assuming identical supporting and supported cleat plates, the collapse load, \( P \), for the connection can be obtained

\[ P = 2M_p/t \]  

(3)

A nondimensional collapse load, \( \eta \), is obtained by normalizing the load \( P \) from eq. 3 by \( P_y \) (cleat’s squash load, \( P_y = \sigma_y wt \))

\[ \eta = P / P_y \]  

(4)

The elastic buckling load, \( P_{EC} \) of the cleat, assuming sway mode, is calculated

\[ P_{EC} = \frac{\pi^2 EI_c}{(1.2L_c)^2} \]  

(5)

Where \( E \) is the elasticity modulus, \( I_c \) is the cleat plate moment of inertia \((wt^3/12)\) and \( L_c \) is the cleat plate length as shown in Figure 1. Depending on slenderness ratio, the critical buckling load \( P_C \) is given by [4]

\[ \Lambda > \sqrt{2} \rightarrow P_C = P_{EC} \]  

\[ \Lambda \leq \sqrt{2} \rightarrow P_C = P_y (1 - \frac{P_y}{4P_{EC}}) \]  

(6)
Where
\[
\Lambda = \left( \frac{P_y}{P_{EC}} \right)^{0.5} (7)
\]
The ultimate load \( P_U \) of the connection can be obtained from
\[
P_U = \frac{P_C}{1 + \frac{P_C}{\eta P_y}} (8)
\]
Once the ultimate load is calculated from eq 8, a second (and last) step starts by modifying the plastic moment to account for the effect of the axial load \( P_U \) in the connection
\[
M_p = M_p [1 - \left( \frac{P_U}{P_C} \right)^2 ] (9)
\]
The revised plastic moment \( M_p \) from eq. 9 is used to obtain a new collapse load \( P \) from eq 3, a new factor \( \eta \) from eq 4 and a revised ultimate load using eq.8 (in the 2\textsuperscript{nd} step, \( P_y, P_{EC}, P_C \) and \( \Lambda \) are remain unchanged). The ultimate loads predicted by the proposed approach for the test samples used in the current experiment are listed in Table 2. The proposed method gives a good estimate of the connection capacity compared to test results.

Figure 4: Experimental load-Deflection response, a) effect of cleat slenderness, b) effect of member slenderness

Figure 5: Sway collapse mechanism
PARAMETRIC STUDY OF CLEAT CONNECTIONS

A parametric study for cleat connections using various combinations of cleat length L_c, thickness, t, and width, w, was conducted using three approaches. These are: nonlinear finite element analysis of the cleat using Strand7 software [9], modeling using connection design software LIMCON and the proposed design method. In the nonlinear FE analysis, both the cleat plates and the CHS member are modeled using an assembly of beam elements and an elastic-perfectly plastic material representation is used. Figure 6 shows the results from the parametric study. The failure load is normalized by the cleat squash load P_y and presented as a function of the cleat slenderness ratio L_c/r. The three points shown in this Figure are the experimental results, each point represents the average of the experimental results from specimens A, B and C. Generally a good agreement is obtained between the proposed design method, the experimental results, the numerical (FE) analysis and LIMCON results.

CONCLUSION

The results from 12 full scale test samples using different member lengths and eccentric cleats combinations were presented in this paper. These results were compared with available design methods and FE results. Based on the experimental observations, the governing connection failure is a sway collapse mechanism. A simple two steps design method for cleat connection design is proposed. A good agreement between results from the proposed design method, experiments and FE predictions is obtained.
REFERENCES

REINFORCEMENT OF BOX-SECTION BEAM-TO-COLUMN CONNECTION IN STEEL BRIDGE PIER

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Tobata, Kitakyushu, Japan

ABSTRACT

Since large resultant forces and stress concentration can take place, the design of beam-to-column connections is important for a steel moment frame. To date, in practice, the analytical study of stress concentration due to shear lag has been a mainstay for the design of these connections exclusively. However, the validity of such a design approach in large earthquake has not been clarified. In recent years, many existing connections have been found to suffer from fatigue cracks. To overcome the problems, the reinforcement by fillets (the M-fillets) has been investigated and the effectiveness has been confirmed. In the present study, a smaller fillet (the C-fillet) is proposed. The stiffness of the C-fillet is smaller than that of the M-fillet, and the stress reduction is insufficient. The observation of the stress distribution in the C-fillet indicates the necessity of the reinforcement along the free edge. The attachment of a rib along the free edge indeed improves the stress in the beam-to-column connection: the state of stress in the beam-to-column connection can be as good as that with the M-fillet. The fatigue resistance therefore can be considered sufficient. The performance of the beam-to-column connection with the C-fillets under seismic loading (large horizontal cyclic loading) is then investigated and turns out to be even better as the displacement at the peak load is larger than that of the connection with the M-fillet. The present study concludes that the beam-to-column connection of a thin plate with a smaller fillet, the C-fillet, can be yet another choice of a connection detail.

KEYWORDS

beam-to-column connection, box-section member, stress concentration, fatigue resistance, seismic resistance, fillet

INTRODUCTION

Many steel bridge piers of moment-frame type have been built for urban expressways in Japan. In this type of steel structure, a beam-to-column connection must be designed carefully, since large resultant forces act and stress concentration due to shear lag takes place as well. Although the load-carrying capacities of the beam-to-column connections have been studied by several researchers [1-2], their mechanical behaviors under large cyclic loading...
have not been investigated much, so that the seismic design has not been established well. Moreover, in recent years fatigue cracks have been found in the beam-to-column connections of a steel bridge pier. To reduce the possibility of fatigue crack, the introduction of fillets to a beam-to-column connection has been tried by Metropolitan Expressway Co., Ltd. [3].

The authors have investigated the mechanical behaviors of the beam-to-column connections numerically with an emphasis on the effects of haunches and fillets [4, 5]. In the proposed design by Metropolitan Expressway Co., Ltd., not only shall the fillets be attached, the beam-to-column connection shall be designed including the effect of shear so that the plate thickness is much larger than the remaining part of the beam and the column. However, according to the previous study of the authors, there is a chance that the attachment of the fillets would eliminate the consideration of the stress concentration due to the shear lag: the plate thickness of the beam-to-column connection then needs not be so large. The large plate thickness would inevitably make not only erection but also fabrication difficult. The difficulty in fabrication could lead to initial defect, which could be a source of fatigue crack initiation.

Against the background of the above information, the authors have explored the possibility of using thin plates with the type of fillet of Metropolitan Expressway Co., Ltd. It has been found feasible even though a rib may be necessary along the free edge of the fillet [6].

The fillet of Metropolitan Expressway Co. is basically a right triangle with the free edge curved in the vicinities of the connections to the beam and column members. On the other hand, the haunch used in a moment frame for a railway has a circular free edge. The latter tends to be smaller, which may be important in some cases to provide more space under the beam of a moment frame.

The present study explores the possibility of reducing the size of a fillet. In particular, the effect of a fillet with a circular free edge is investigated from the viewpoints of both fatigue and seismic resistances. For the fatigue resistance, stress distribution in the beam-to-column connection under ordinary loads is studied, for which linear analysis will do. On the other hand, for seismic design, large cyclic loads must be applied, requiring geometrical and material nonlinear analysis. In this study, the linear analysis is conducted by NASTRAN [7] and the nonlinear analysis by ABAQUS [8].
ANALYSIS MODEL

Beam-to-Column Model

The same beam-to-column models as those used in the previous study [4-6] are employed for the present numerical study. Specifically, two models, G-1 and G-3, serve as the bases of the analysis models. The two models are similar to each other, and G-1 is illustrated schematically in Figure 1. Both beam and column are thin-walled box members of the same dimensions. The plate thickness in a beam-to-column connection of G-1 is 16 mm while the remaining portion is 12 mm thick. The portion having the plate thickness of 16 mm is referred to as a beam-to-column connection in the present study, which is the shaded zone in Figure 1. The difference between the two models lies only in the plate thickness in the beam-to-column connection: the plate thickness of this portion in G-3 is 22 mm. The difference comes from the fact that shear-lag effect is taken into account in the design of G-3 while it is neglected in G-1. The width of a flange plate is 410 mm while that of a web plate is 380 mm in both models.

G-1 and G-3 have a haunch, which can be seen in Figure 1. Removing the haunch from these basic models, the analysis models of G10 and G30 are constructed. G30 is the conventional standard beam-to-column connection of a bridge pier in Japan.

In the present study, for the reinforcement of the beam-to-column connection, a fillet with a circular free edge, which is called the C-fillet, is attached to G10. This beam-to-column model with the C-fillet is then called G1C. The plate thickness of the C-fillet is made equal to that of a beam-to-column connection, i.e. 16 mm.

The reinforcement practiced by Metropolitan Expressway Co., Ltd. corresponds to the attachment of the fillet of a right-triangular shape, which is called the M-fillet herein, to G30. This beam-to-column connection model with the M-fillet attached is called G3M. The M-fillet is somewhat bigger than the C-fillet.

The stiffness of the beam-to-column connection in G1C is smaller than that in G3M, suggesting that additional reinforcement might be necessary for G1C. The attachment of a rib to the free edge of the fillet is therefore considered. The model with the rib is called G1CR. The sizes of the fillet and the rib are presented in Figure 2. The thickness of the rib is made equal to that of the fillet, i.e. 16 mm. Further to see the effect of the rib, the thickness of the rib is increased to 32 mm. This model is called G1CR2.
Loading

As shown in Figure 1, the analysis models are hinged at both ends. As for the translational movement, the column end is fixed while the beam end is movable in the horizontal direction. The horizontal load is applied at the beam end.

For fatigue resistance, it is sufficient to study the behavior under ordinary load. Therefore, the deformation is small so that linear analysis will do. To be specific, horizontal load is applied in the negative direction, and the stress state in the beam-to-column connection is investigated.

Larger load is applied in the investigation of seismic resistance. It is common for this type of study to give load by controlling displacement. In the present study, the displacement path in Figure 3 is employed. This is a cyclic loading and the amplitude is increased each time one loading cycle is completed. Note that $\delta_y$ is the horizontal displacement at the loading point when the nominal normal stress in the axial direction at the intersection between the column and the beam reaches the yield stress.

Constitutive Model

The Young Modulus, the Poisson ratio and the yield stress are 206 GPa, 0.3 and 355 MPa, respectively. The nonlinear material behavior is assumed to be described by Von Mises type of elasto-plasticity with the kinematic hardening. The skeleton curve of stress-strain relationship is of bilinear type with the second slope beyond the yield stress being 2.06 GPa.

RESULTS

Stress State under Ordinary Loading

Metropolitan Expressway Co., Ltd. usually measures the flange stress along the line 50mm away from the intersection between the column and the beam to avoid the influence of stress disturbance due to the welding bead. Considering the size of the analysis model herein, the maximum principal stress distribution along the line 10mm away from the intersection between the column and the beam (the dotted line in Figure 4) is presented in Figure 5. Since this is a linear analysis, the results are normalized by the maximum stress value obtained in
G10. For the sake of brevity, the maximum principal stress is called simply the stress and the largest value of the maximum principal stress in each model is the maximum stress in what follows, unless stated otherwise.

The comparison between G10 and G30 in Figure 5(a) indicates that the thickening the plate of the beam-to-column connection reduces the stress but that the reduction is only about 18%. On the other hand, the M-fillet decreases the stress significantly. The reduction of the stress near the web is especially noticeable: the stress at the intersection between the flange and the web has become 0.35 in G3M. Since the flange-web intersection is a region of welding where fatigue crack is initiated if it ever occurs, the reinforcement by the fillets can thus improve the fatigue resistance very effectively.

On the other hand, the C-fillet decreases the stress, but the reduction is not that good, as the stress distribution of G1C in Figure 5(b) indicates: the stress at the flange-web intersection is still as large as 0.55.

![Figure 5: Principal stress distribution across flange](image)

(a) Effect of plate thickness and M-fillet  
(b) Effect of C-fillet

![Figure 6. Normal stress distribution across fillet](image)
Figure 6 presents the distribution of the normal stress $\sigma_{xx}$ along the dotted line AB shown in the figure of the C-fillet. It is observed from this result that the stress tends to increase toward the free edge, implying that the reinforcement along the free edge can reduce the stress further. The results of G1CR in Figures 5 (b) and 6 indeed confirm the effectiveness of the rib along the free edge: both stresses in the beam flange and in the fillet are reduced considerably: the stress at the flange-web intersection in G1CR is 0.38, which is only about 9% larger than that in G3M. To reduce the stress further, the thickness of the rib plate is doubled: the stress at the flange-web intersection in G1CR2 is 0.31, even smaller than that of G3M.

**Overall Behavior under Cyclic Loading**

Hysteretic load-displacement relationship is obtained under cyclic loading. Since the behaviors in the positive and negative displacements are very similar to each other, only the envelopes of the horizontal load-displacement curves on the positive side at the loading point are presented in Figure 7.

In Figure 7(a), the influence of the plate thickness of the beam-to-column connection can be observed. The displacement at the maximum loading is largest in the case of G10 while the load-carrying capacity is not much influenced by the plate thickness, as the difference between the maximum loads that G10 and G30 carry is small. The performances of G30 and G3M are almost the same, which leads to the conclusion that the fillets do not play a role when the beam-to-column connection is thick. The reason behind it is revealed by the observation of the deformation states: the out-of-plane displacement takes place only outside the beam-to-column connection in G30 and G3M, while in G10 the out-of-plane displacement is observed also in the beam-to-column connection. Since the plate thickness of the beam-to-column connection in G10 is 16 mm, and the plate thickness outside the beam-to-column connection in G30 and G3M is 12 mm, plastic deformation tends to prevail more in G10 before the maximum strength is reached, which results in more ductile behavior. It is noted that for seismic resistance, larger displacement before the peak load is favorable because it absorbs more seismic energy while a structural behavior is stable.
The attachment of the C-fillet appears to increase the stiffness of the beam-to-column connection, thus suppressing the penetration of the out-of-plane deformation into the beam-to-column. This results in less ductile behavior, the decrease of the displacement at the point of the maximum load-carrying capacity (Figure 7(b)). Yet the displacement at the maximum load-carrying capacity that G1C has achieved is much larger than that of G3M.

The rib on the C-fillet further enhances the tendency of less ductile behavior. Nevertheless, to be quite interesting, the difference in the overall behavior between G1CR and G1CR2 is very small although the doubling of the rib thickness changes the stress state in the beam-to-column connection.

CONCLUDING REMARKS

Metropolitan Expressway Co., Ltd. requires the fillet reinforcement (the M-fillet) in addition to the consideration of shear-lag effect in designing the plate thickness of a box-section beam-to-column connection. This detail is becoming the de facto standard of the beam-to-column connection of a steel bridge pier for highway bridges in Japan these days. In the previous study, the authors challenged the detail and showed that the attachment of the rib made it possible to design the connection solely based on the nominal stress. The research has been extended to explore the usage of a compact fillet, the C-fillet. The results are presented in this paper, based on which it can be concluded that the C-fillet with a rib can increase the fatigue resistance as much as the M-fillet and improves the seismic resistance even better than the M-fillet, if the thickness of the rib plate is designed appropriately. Therefore, the beam-to-column connection of a thin plate with the C-fillet can be considered yet another alternative to the beam-to-column connection detail proposed by Metropolitan Expressway Co., Ltd.
REFERENCES


BOLT PRYING IN HOLLOW SECTION BASE PLATE CONNECTIONS

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KEYWORDS
Steel connections, cold-formed hollow sections, RHS, SHS, SSHS, plates, bolts, tension, prying, experimental methods.

ABSTRACT
This paper presents test of cold-formed hollow section column baseplates under uplift with different bolt patterns with an aim to assess the magnitude of bolt prying forces. Individual bolt loads were monitored during the experiments. The results indicate that under service loads prying is not a significant factor, and prying forces developed only after significant deformations, which would probably not occur at service loads. At ultimate conditions, additional prying forces of the order 40% were observed.

INTRODUCTION

The T-stub analogy is a well established and recognized model for the phenomenon of prying. Kulak et al (2001) provides a comprehensive summary. Figure 1 shows a T-stub plate with an applied load \( N_t \). Due to the flexibility of the plate, contact or prying forces, \( N_q \), develop between the two plates, and consequently the total bolt force is greater than the applied \( N_t \).

The magnitude of the prying is a function of the plate flexural stiffness, bolt size and geometry. In a rigid plate model, the deformations of the plate are relatively small, and the prying forces are small. For intermediate, more flexible plates, the amount of prying is maximized. For very thin plates, a plastic yield line mechanism will normally form in the plate before substantial prying forces can develop. Figure 2 shows these different types of behaviour.
Theoretical calculations of bolt prying can be complex (Kulak et al 2001), and for simple design office calculations approximations are often considered. Hogan (ASI, 2007) suggests that an increase of 0 - 10% may be appropriate for thick plates, and 20 – 40 % for thin plates.

Most experimental investigations that have incorporated prying have been based on simple T-stub tests, or I-section connections. Less information is available on prying of hollow section connections. Wheeler (2007) examined moment connections between bolted moment endplates for square hollow sections. Willibald et al (2002) had hollow section test results with prying ratios of up to 30%.

This paper outlines the results of tests on axial tension uplift of hollow section baseplate connections which were specifically instrumented to determine prying forces. The tests were performed as part of the undergraduate honours theses of Williams (2005) and Edwards (2005) at The University of Sydney.

**TEST METHOD**

Figure 3 shows the two different bolt patterns considered. Figures 4 and 5 show a typical test setup. A steel seat was fixed to the base of the testing machine. The seat was a thick steel plate, welded to steel channels. The steel channels were bolted securely to the testing machine and provided a gap under the plate to allow bolt fixing. Concrete blocks were cast with oversized vertical holes. These holes matched the bolt pattern on the base plate, and hole in the steel seat. Long threaded rods were secured under the base of the seat by bolts. These rods were made from the same Grade 4.6 material as standard baseplate anchor bolts.
A 3-part epoxy grout known as Chockfast Red, made by Epirez, was used to fill the concrete voids. Each batch consisted (by weight) of 21 parts of aggregate, 0.75 parts of hardener and 1.85 parts of resin. This grout was chosen because of its 2 hour work time. A 300 mm depth of pour was required to fill the bolt holes in the concrete and another 20 mm was poured between the base plate and concrete. Prior to pouring a levelling nut was placed on each rod just above the concrete surface and all were aligned. After pouring the steel plate was pushed down on top to the nuts and grout began to seep out along the edges of the formwork. A complete contact between the plate and the grout was assumed to have occurred.

Figure 3: Typical base plate dimensions

Figure 4: Diagramatic representation of test set up
Foil strain gauges (10 mm), made by Showa Measuring Devices, measured the strain in the threaded rods. Spatial limitations of the steel-on-steel connections prevented strain gauges being attached to the rods. They were placed 105 mm, as shown in Figure 5, below the top of the bolt head, so that they would lie just below the concrete surface. The position of each strain gauge is shown in Figure 6.

The column was loaded in tension at a constant strain rate in stroke control mode. Specimens were tested until failure.
TEST OBSERVATIONS

Typically five stages of behaviour were observed:

1. Initial “sticking” and separation caused by the adhesive contact and suction between the underside of the baseplate and the grout.
2. Normal linear elastic behaviour where the centre of plate lifted, the plate started to bend.
3. Plate plastic mechanism where yield lines and a distinct drop in stiffness were observed.
4. Plate arching and membrane action in which the plate yield lines further deformed, plate prying was clearly visible and the bolts were bent significantly.
5. Final failure which was caused by either the bolts snapping in tension, or the plate tearing adjacent to the weld between the RHS and plate.

A typical load-deflection curve is shown in Figure 7 and a typical plate yield line pattern in Figure 8.

![Figure 7: Typical Load deflection behaviour](image1)

![Figure 8 : Typical plate failure mode](image2)
BOLT PRYING RESULTS

Several bolts were instrumented with strain gauges to determine bolt loads. By comparing the DARTEC applied load with the combined bolt tension it was possible to monitor the effect of prying during each test. Figure 9 shows a typical relationship between bolt load and total load. Three typical stages can be seen. In the initial elastic zone the bolt load and total load are approximately equal. It is expected that prying would be minimal in this region. Once the plastic yielding started to form in the plate, prying naturally starts to occur. Figure 6 shows that the bolt load also drops. This is potentially not necessarily correct, as by this stage the bolts have undergone large bending strains and it is possible that the readings are inaccurate.

Table 1 highlights the difference between the load in the bolt group and column at 3 points of interest:

1. Yield load comparison; the amount of prying corresponding to the estimated plate yield load.
2. Maximum prying; the prying magnitude, relative to the column load, was greatest between the plate yield load and ultimate load.
3. Ultimate load comparison; signifies the amount of prying when the plate had reached its maximum load.

Figure 9: Typical bolt force behaviour
TABLE 1

COMPARISON BETWEEN LOADS ON THE BOLT GROUP AND THE COLUMN

<table>
<thead>
<tr>
<th>Bolt Pattern</th>
<th>RHS Size</th>
<th>Yield load Comparison</th>
<th>Maximum Prying</th>
<th>Ultimate load Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Narrow Pitch</td>
<td>150×100×4</td>
<td>1.22</td>
<td>1.40</td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td>150×50×4</td>
<td>0.94</td>
<td>1.26</td>
<td>1.24</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>1.08</td>
<td>1.33</td>
<td>1.28</td>
</tr>
<tr>
<td>St. Deviation</td>
<td></td>
<td>0.20</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Crucifix</td>
<td>150×100×4</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>150×50×4</td>
<td>1.09</td>
<td>1.25</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Table 1 shows the increase in bolt load due to prying. It can be seen that the loads attributed to this action are significant, increasing the load on the bolts by 40%. An inspection of Figure 9 (and other test results from Williams (2005) and Edwards (2005)) shows that after the bolts hit a peak load, relative to the column, the stiffness of the bolts drop and, hence, prying is reduced. This corresponds to the point in time when the bolt sections become plastic and undergo large deformation. The results for the Narrow Pitch pattern show only 1 drop in the stiffness and this can be explained by the connection symmetry; all the bolts have the same load. The results of the Crucifix pattern show 2 drops, which, correspond to the successive yielding of the bolts next to the flange followed by the bolts next to the web.

The results indicate that under service loads prying is not a significant factor. As the plate yield mechanism occurs, prying factors of up to 1.40 may apply. The AISC (2002) design model did not include prying in the bolt design for the column base plate, but suggested that the recommendations of Packer (1996), which included a prying factor of 1.35, be followed. The results from this set of tests indicate that the prying factor recommended by Packer 1996 (1.35), and the tests of Willebald et al (2002) (1.30-1.40) is confirmed.

CONCLUSIONS

This paper has experimentally examined bolt prying forces in hollow section base plate connections in tension. The results indicate that under service loads prying is not a significant factor, and prying forces developed only after significant deformations, which would probably not occur at service loads. At ultimate conditions, additional prying forces of the order 40% were observed.

ACKNOWLEDGEMENTS

The results presented in this paper were obtained by authors 3 and 4, as part of their undergraduate thesis in The School of Civil Engineering at The University of Sydney. Contributions from Smorgon Steel Tube Mills Ltd (now Australian Tube Mills, part of OneSteel) to support part of the experimental work are acknowledged. The experiments were carried out in the J. W. Roderick Laboratory for Materials and Structures. Thanks to the Civil Engineering workshop and laboratory staff for the preparation of the specimens and assistance in testing.
REFERENCES


CAPACITY OF SCREWED CONNECTIONS BETWEEN FABRICATED FITTINGS AND COLD-FORMED HOLLOW SECTIONS

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KEYWORDS
Steel connections, cold – formed hollow sections, plates, screws, shear.

ABSTRACT
An easy to assemble domestic floor system has been designed in Australian for domestic housing applications. A key design feature is the production of fabricated fittings to be used in simple joints connected by Tek screws. This paper describes an investigation in the axial capacity of baseplate fittings. Parameters included the number and layout of the screws, and the thickness of the joined members. The main conclusion is that a screwed connection reduction factor is required.

INTRODUCTION
The Australian steel manufacturer, OneSteel, has a range of pre-engineered products for the domestic housing market. The DuraGal Flooring System comprises of a set of cold-formed hollow sections, and manufactured joint fittings designed for simple in-situ fabrication, predominantly using screwed connections. A diagrammatic representation of the system is shown in Figure 1.

Two baseplate options have been designed as shown in Figure 2. There is a short internal fitting designed to be a pinned connection, and a more substantial external sleeve fitting intended for potential bending moment transfer. The designs allow for some potential tolerance in length. The external moment sleeve connection is tight fitting for a fixed set of external dimensions (but possible variable thickness). The internal sleeve pin connection will have a varying degree of gap depending on the thickness within the same set of external dimension posts.
The aim of the project is to assess the axial tension and compression capacity of these fittings. Parameters that were varied include:

- Thickness of the hollow section post
- Number and location of screws
- Tolerance and miss-fitting dimensions by varying possible gaps or misclous between the components.

The tests were performed as part of the undergraduate theses of A Yang (2008), A Yang (2008) and Ning (2008) at The University of Sydney.
TEST METHOD

Both compression tests and tension tests were carried out using the 2000 kN DARTEC hydraulic testing machine as shown in Figure 3. The SHS tubes and base plates were assembled at ground level. Power tools were used to install screws. The assembled specimen was then fixed on the testing machine. The bases of specimens were locked to the machine by high strength bolts. In tension tests, the top part of specimen was clamped to the loading ram via a testing hook. The loading ram directly applied forces to specimens in compression tests. Specimens were then loaded and aligned vertically in the DARTEC machine. 31 tests were performed in total, including 5 compression tests and 26 tension tests.

Buildex self-drilling screws (commonly known as ‘Tek screw’) were used. The screw size was 14-20×22 mm with hexagon head. The nominal shear capacity for a single screw is 11.2 kN. The post sizes were 89×89×2.0, 90×90×2.5 and 89×89×3.5 all in Grade C450 to AS 1163.

Different screw arrangements were included in the project. The internal pinned baseplate allowed for a maximum of 4 screws, and combinations of 2, 3 or 4 screws were trialed. The moment sleeve connection could accommodate up to 16 screws. Combinations of 8 screws on either opposite of adjacent faces, or all 16 screws were tested. These are shown in Figure 4. Figure 5 shows the type of tolerance constructions gaps that were planned. However, the nature of the installation process of the screws made it difficult to maintain these gaps, and the resulting type of gap is also shown.

The column was loaded in tension at a constant strain rate in stroke control mode. Specimens were tested until failure.
RESULTS

The experimental results revealed that all specimens ultimately failed in screw shear. The failure load increased when more screws were used in the connection. During the testing most of specimens behaved similarly. Besides screw shearing, specimens also experienced hole bearing and member deformation. Figure 6 shows some typical failed specimens.

The typical behavior of specimens in the testing is shown in Figure 7. The diagram plots load versus displacement of the 90×90×2.5 SHS connecting a normal pinned base plate by four screws. There are two friction slips at the start of the curve. They represent screw slipping at the beginning of the test. When the testing machine started to load the axial force in the specimen was low. Friction between two connecting faces originally resisted the axial force rather than screws since the screws were installed by power tools.
Table 1 summarises the maximum load in each test in comparison to the prediction of AS/NZS 4600 (2005). Each specimen was named according to its characteristics. The specimen ‘2.0-NT-4-1’ can be interpreted as a SHS with 2.0 thickness connecting a normal base plate by 4 screws in tension test #1. Similarly, ‘M’ stands for moment plates, ‘C’ stands for compression test, and ‘A’ indicates that screws were installed on two adjacent faces of the tube.

Figure 6: Typical load deformation response

Figure 7: Failure modes
TABLE 1
SUMMARY OF RESULTS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Testing Failure Load (kN)</th>
<th>AS/NZS 4600 Prediction (kN)</th>
<th>Specimen</th>
<th>Testing Failure Load (kN)</th>
<th>AS/NZS 4600 Prediction (kN)</th>
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</thead>
<tbody>
<tr>
<td>2.0-NT-2-1</td>
<td>21.43</td>
<td>17.01</td>
<td>2.5-NT-3-1</td>
<td>29.18</td>
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<td>17.01</td>
<td>2.5-NT-3-2</td>
<td>38.23</td>
<td>31.89</td>
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<td>2.0-NT-3-1</td>
<td>36.20</td>
<td>25.52</td>
<td>2.5-NC-4-1</td>
<td>57.62</td>
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<td>2.0-NT-3-2</td>
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<td>25.52</td>
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<td>36.14</td>
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<td>50.00</td>
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<td>2.0-NT-4-1</td>
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<td>85.05</td>
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<td>152.01</td>
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<td>2.0-MT-16-1</td>
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<td>27.43</td>
<td>21.26</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Effect of number of screws**

AS/NZS 4600 assumes that as the number of screws increases the connection capacity is proportional to the number of screws in the connection. However Laboube and Sokol (2002) suggested a reduction factor of the form:

\[ R = \left( 0.535 + \frac{0.467}{\sqrt{n}} \right) \leq 1.0 \]

However, this type of factor based on the number of screws, n, is more appropriate for a lap connection.

![Figure 8: Effect of number of screws](image)
Figure 8 plots the relationship between connection capacity (associated with screw failure) versus the number of screws. A simple linear degradation of capacity has been initially proposed. A Yang (2008), B Yang (2008) and X Ning (2008) have performed some initial reliability analyses on the possible equation. The preliminary results suggest that the mean line of $R = 1 - 0.006n$ is not suitably reliable, and some modifications are still required to create a suitable reduction factor.

Effect of gap

Gaps between the fitting and the SHS connection can reduce the connection strength.

Figure 9: Reliability Analysis for Reduction Factors

Bambach and Rasmussen (2006 & 2007) concluded that for same type of screws, the connection strength is linearly reduced as the gap increase due to increased bending in each of the screws.

Specimens were originally designed to have gaps of up to 3 mm as shown in Figure 5. However, in the process of assembling specimens the gap was impossible to control. Gaps only varied between 1 mm to 1.5 mm. The presence of gaps in connections and its ability to reduce connection strength are acknowledged. Test results did not show any clear pattern of relationship between connection strength and gaps.

Asymmetric screw patterns

Figure 4 shows some of the different screw patterns considered. This was to investigate the effect of incomplete installation. A small number of specimens were used to study the eccentricity affecting connection strength. Connections with 8 screws were installed in two ways. One way was to install them on two opposite face of the SHS only, and the other way was to install 8 screws on two adjacent faces (Figure 4). Test results are given in Table 2.

No clear difference is observed between two groups of specimens. Hence the eccentricity did not affect the strength of DFS screwed connections.
TABLE 2
RESULTS FOR DIFFERENT SCREW PATTERNS

<table>
<thead>
<tr>
<th>Specimen (opposite screws)</th>
<th>Failure Load (kN)</th>
<th>Specimen (adjacent screws)</th>
<th>Failure Load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0-MT-8-1</td>
<td>80.52</td>
<td>2.0-MT-8A-1</td>
<td>81.31</td>
</tr>
<tr>
<td>2.0-MT-8-1</td>
<td>84.97</td>
<td>2.0-MT-8A-2</td>
<td>85.09</td>
</tr>
<tr>
<td>2.5-MT-8-1</td>
<td>86.93</td>
<td>2.5-MT-8A-1</td>
<td>84.30</td>
</tr>
<tr>
<td>2.5-MT-8-2</td>
<td>88.67</td>
<td>2.5-MT-8A-2</td>
<td>88.39</td>
</tr>
</tbody>
</table>

CONCLUSIONS

This paper has presented results of a small project examining the axial capacity of prefabricated screwed joints in cold formed steel. The base joints failed in screw shear and showed a non-linear relationship between capacity and screw numbers. More analysis is required to obtain a reliable reduction factor. It was found there was no significant difference between connections with screws on varying faces of the post members.

ACKNOWLEDGEMENTS

The results presented in this paper were obtained by authors 2, 3 and 4, as part of their undergraduate thesis in The School of Civil Engineering at The University of Sydney. Contributions from One Steel Australian Tube Mills to support the experimental work are acknowledged. The experiments were carried out in the J. W. Roderick Laboratory for Materials and Structures. Thanks to the Civil Engineering workshop and laboratory staff for the preparation of the specimens and assistance in testing.

REFERENCES

EXPERIMENTAL RESEARCH ON THE BEHAVIOR OF SPATIAL INTERSECTING CONNECTIONS OF A DIAGRID STRUCTURE SUBJECTED TO AXIAL LOADING

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KEYWORDS
Diagrid structure, high-rise building, concrete filled steel tube (CFST), spatial intersecting connection, axial loading, compressive test, finite element analysis (FEA), failure modes, bearing capacity

ABSTRACT
In an innovative structural system known as the diaphragm structure, connection commonly consists of four obliquely intersecting columns. On the basis of a newly developed concrete filled steel tubular (CFST) planar intersecting connection, this paper presents a type of spatial intersecting connection with both in-plane and out-of-plane angles. Two specimens were tested under monotonic axial compressive loading, with the purpose of investigating the failure mode, performance, and bearing capacity of the connections. The parameter in this study was the thickness of elliptical plate. The lateral constraints of the connection were simulated by three steel bars, and their diameters were determined by an elastic finite element analysis (FEA) before the test was conducted. As a result, the connection exhibited strain-hardening characteristics, and the failure was due to the tube bulging in the column zone. The ratios of the bearing capacity to the maximum design load of the connections were 4.41 and 4.47 for specimen 1 and 2, respectively. The thickness of elliptical plate has no significant influence on the behavior of the connection. The results show that both connections can fulfill the seismic design criteria “stronger connection, weaker components” defined by current Chinese codes. FEA result is substantiated to be accurate by comparing to the test results. In addition, an adequate lateral constraint is verified to be essential for the connection in practical use.

INTRODUCTION
Recently, an innovative structural system, with the term of ‘diaphragm structures’ or ‘diagonal frame structures’, has become widely used in the world. Diaphragm structural system is characterized by vertical components which are not conventional columns but columns that obliquely intersecting each other at a certain angle. This system has emerged as both structurally efficient and architecturally aesthetic for application in high rise buildings. Due to the significant lateral stiffness, diaphragm structures are commonly used as the outer tube of high rise buildings with tube-in-tube structural
systems. For instance, the Swiss Re Building in London, the Hearst Headquarters in New York, the Lotte Super Tower in Korea, the Mode Gakuen Cocoon Tower in Tokyo, the Tornado Tower and the building for Ministry of Foreign Affairs of Qatar in Doha, the new Guangzhou TV tower and the Guangzhou West Tower in China (with height of 432m, as depicted in Figure 1) have applied diagrid structural system as their outer tubes. These buildings are appreciated by the public and engineers for their noticeable appearances and outstanding structural behavior.

The use of diagrid structures has attracted great interests from scholars and structural engineers. Moon [1] presented research on the characteristics and methodology for the preliminary design of all-steel diagrid structures. Moon [2] also proposed guidelines for determination of the bending and shear deformations for optimal design, which use the least amount of structural material to meet the stiffness requirements. Fu [3] conducted experimental and analytical research on the structural design of a reinforced concrete diagrid structure at Qatar. The focus is on the behavior of diagrid structures.

The obliquely intersecting columns in diagrid structures usually carry gravity loads as well as lateral forces owing to their triangulated configuration, thus they resist greater axial loads than the columns of common frames. Concrete-filled steel tubular (CFST) columns can provide excellent structural properties for seismic resistance, such as high strength, high ductility and large energy absorption capacity. Furthermore, experimental and analytical studies on CFST components have been on-going for many decades [4-8], while the CFST structures have been widely used in practical buildings in China, Japan and U.S. etc. Therefore, CFST columns are ideal alternatives for the obliquely intersecting components due to their high strength under axial loading.

CFST diagrid structures usually have spatial intersecting connections comprised of four obliquely intersecting CFST columns which exist in two or more planes. Figure 2 shows that the cross-sectional area at the connection is reduced to that of each CFST column, so the connection details become the key issue in the structural design. Han [9] and Huang [10] have presented two types of planar intersecting connection that consists of four obliquely intersecting CFST columns in one plane. They have also studied the behavior of the connection by experiments and finite element analyses. However, the practical connection (e.g. the connection used in the Guangzhou West Tower) may consist of columns that intersecting with both in-plane and out-of-plane angles. This paper proposes a type of connection detail and two specimens were tested under monotonic axial loading. The deflection, stress, failure modes and capacity of the specimens are experimentally obtained and analyzed. As a result, the connections have been verified to satisfy the requirement of design codes.

**EXPERIMENTAL INVESTIGATION**

Two specimens of a spatial intersecting connection were tested under monotonic axial loading. The purposes of the study were (1) to observe the failure modes of the connections, (2) to investigate the ultimate bearing capacity of the connection subjected to axial loading, and (3) to obtain the strain...
distribution, the axial deformation and the lateral deflection of the specimens during the entire test procedure, (4) to assess the effectiveness of the connection in practical use.

**Test Specimens and Parameters**

According to the architecture view of Guangzhou West Tower (Figure 1), the floor area varies with the height. The 30th floor has the largest floor area of the building. Therefore, the four columns of a connection obliquely intersect with two different directions: an angle of 35° in the vertical plane and an out-of-plane angle of 1°, as shown in Figure 3. Four columns are first welded to an elliptic plate, then one ring reinforcing plate and two circumferential plates are welded outside the center of the connection. In addition, several stiffening ribs are welded between them to inhibit a local buckling. It is worth noting that the elliptical plate and ring reinforcing plate are employed to enhance the confinement effect to the concrete core, while the circumferential plates are used to link the structural beams with the connection. This type of connection is convenient to be manufactured in the factories and mounted on sites. The application can significantly facilitate the constructional process.

Two 273 mm diameter hot finished circular hollow sections (CHS) with 9 mm and 7 mm thickness were used for the steel tube in the connection and column zone, respectively. Steel tubes and plates were manufactured using mild structural steel with a nominal yield stress of 345 MPa. The material properties of the steel tubes and plates were obtained from tensile tests of coupons taken from steel tube and plate before manufacturing. The concrete grade in the connection and column zone of all specimens was C90 and C70 respectively, according to the Chinese code for design of concrete structures [11]. Compression tests were carried out on three cube specimens (150 mm in lengths) to determine the compressive strength of the concrete. As a result, the measured 28-day target concrete strength was 70.2 MPa.

The test parameter was the thickness of the elliptical plate: 17 mm for specimen 1 and 10 mm for specimen 2. Furthermore, before the test, two steel sleeve barrels with 10 mm thickness were welded on the loading end while a 20 mm thick plate was welded on the fixed end.

**Test procedure**

Two sets of loading systems were used to apply an axial load on each column separately. Each of the loading systems consists of a 10000 kN hydraulic jack and a diagonal supporting bracket. Each bracket consisted of two steel plates welded on four rib stiffeners of different lengths. A steel self-equilibrating rigid reaction frame with a loading capacity of 20000 kN in the horizontal direction and 2000 kN in the vertical direction was designed to resist the reaction generated by the jacks and steel bars, as shown in Fig. 4. Three steel bars were mounted to simulate the lateral constraint provided by structural beams and floors in the prototype structure.
Detailed instrumentation was used to measure the strains of the steel tube and the deformations of the connection. The steel tube in the connection and column zone was instrumented with 64 strain rosettes and four strain gauges on nine cross-sections. Each strain rosette comprised of three strain gauges with an intersecting angle of 45°. Meanwhile, 22 Linear Variable Displacement Transducers (LVDTs) were placed at different locations on the specimen to measure the displacements. These strain rosettes and LVDTs were connected to a computer acquisition system to record the data at each loading interval.

The two columns were first loaded simultaneously at 500kN increments until the loading reached 5000 kN, and then with 250kN increments until the connection failed. Any one of three conditions concluded the test: (1) The loading decreased to 85% of the maximum, (2) the maximum deformation reached 200mm, or (3) obvious features of failure occurred on the specimens (e.g. the steel tube bulged noticeably or a welding crack occurred).

A FINITE ELEMENT ANALYSIS SIMULATION OF THE TEST SETUP

As mentioned above, the steel bars provided a significant lateral constraint to the connection, so their diameters are essential factors in the test. In addition, the bending moment might generate deflection on the vertical reaction frame, further led to an influence on the constraint to the connection. Therefore, a simulation using finite element analysis (FEA) of the entire test setup was performed to determine the diameters before the test was conducted (Figure 5). Due to the complexity of the test setup, the convergence and accuracy of a nonlinear finite element analysis could be unconvincing. Therefore, an elastic finite element analysis using SAP2000 [12] was conducted to determine the diameters of three steel bars.

In this FEA model, the Young’s modulus of the concrete C70, C90, and the steel were 37.0 GPa, 39.0 GPa and 206.0 GPa respectively, on the basis of Chinese code for design of concrete structures [11]. One of the steel bars was vertical, and the others had a horizontal angle of 30°. The reaction frame was fixed at four corners. The diameters of steel bars were obtained by comparing the internal forces of the connection to that of the prototype structures (provided by the design institute), shown in Table 1. As a result, the eccentric ratio \( e \) of the prototype structure and FEA model was the same, which indicated that the lateral constraint provided by the steel bars was consistent with that in practical use.

EXPERIMENTAL RESULTS

Failure modes

It can be observed that the failure was due to the tube bulging in the column zone, as depicted in Figure 6a. It produced some Lüders slip lines along the surface of the tubes near the loading ends as shown in Figure 6b. In addition, bending in-plane occurred in the column zone of the specimens.
### TABLE 1

**COMPARISON OF INTERNAL FORCES OF THE PROTOTYPE STRUCTURE AND FEA RESULT**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Diameter of a column $D$ (m)</th>
<th>Axial compressive force $P$ (kN)</th>
<th>Bending moment $M$ (kN·m)</th>
<th>Eccentricity ratio $M/(P·D)$</th>
<th>Vertical steel bars</th>
<th>Diagonal steel bars</th>
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<td>Prototype structure</td>
<td>1.6</td>
<td>80292.3</td>
<td>5729.0</td>
<td>4.46%</td>
<td>1 $\phi$ 42</td>
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<tr>
<td>FEA model</td>
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<td>5000.3</td>
<td>60.7</td>
<td>4.45%</td>
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</table>

### Load-steel strain and load-deflection

The axial load-strain relationships of different locations of two specimens are illustrated in Figures 7a ~ 7c. These curves show the point of steel yielding, the maximum load and the ductility for each specimen. It can be seen that the trend of all the curves is similar to that of the planar intersecting connection [9]. The five significant events for each specimen during testing are as follows: (1) the steel tube was compressed in the axial direction while being tensioned in the hoop direction, and the axial strain was larger than the hoop strain, (2) the axial and hoop strains increased linearly until the axial strain in connection zone reached 2000$\mu$e, while the load on single column was approximately 5500 kN, (3) subsequently, the strains in both column and connection zone increased remarkably with a non-linear trend, which indicated that the steel tube yielded under a combination of axial compression and hoop tension, (4) although the tube yielded, the load kept increasing until it reached 7000kN ~ 7200kN due to the stress redistribution between the concrete and the tube, (5) when the load reached the ultimate value, steel bulge occurred in the loading or fixed end of the column zone as described in section 4.1. Meanwhile several Lüders slide lines occurred along the surface of the tubes.
The curves of Figure 7d illustrate that the axial deformation increased linearly until the load reached 5500kN, so that the axial stiffness of the connection is significant and thus applicable to high-rise buildings. Furthermore, the connection exhibited strain-hardening characteristics after the tube yielded, due to the increasing strength of concrete core under triaxial compression. Local wall buckling did not occur prior to yield of the CFST column for both specimens. The lateral deflection also increased with a nonlinear trend after the tube yielded.

**Comparison of experimental and design values of bearing capacity**

Table 2 shows the loads applied on each column at different stages and the design load of the connection. The yield load was calculated as the load at which the axial strain reached the steel tube yield strain (i.e. 1675 με in this test). The bearing capacity was the maximum value recorded in the test for each single column of the connections. The design load, which was derived from the structural design data, was the maximum load of the prototype connection under combinations of different load, such as dead load, live load, wind load and the seismic effect. The corresponding maximum load was obtained by the design load divided by the square of the scale factor (i.e. 5.86 in this test), in order to compare with the test results. The safe coefficient, defined as the ratio of axial bearing capacity to the corresponding maximum load of the connections, was calculated as 4.41 and 4.47 for specimen 1 and 2, respectively. According to the test results, the design strengths of the connection were substantiated to be conservative for practical use.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield load (kN)</td>
<td>5250</td>
<td>5750</td>
</tr>
<tr>
<td>Bearing capacity (kN)</td>
<td>7000</td>
<td>7100</td>
</tr>
<tr>
<td>Design load of the prototype structure (kN)</td>
<td>54556</td>
<td></td>
</tr>
<tr>
<td>Corresponding maximum load (kN)</td>
<td>1589</td>
<td></td>
</tr>
<tr>
<td>Safe coefficient</td>
<td>4.41</td>
<td>4.47</td>
</tr>
</tbody>
</table>

Table 7 and Table 2 show that the capacity of two connections are more or less the same. Specimen 2 offers a slightly larger yield load and bearing capacity than specimen 1. Figure 7c demonstrates that elliptical plates with different thickness have almost the same load-strain relationship. Hence it can be concluded that the thickness of elliptical plate has only slight influence on the behavior of the connection under axial loading.
Verification of design principle and the FEA results

In Chinese code for seismic design of buildings [13], the seismic design criteria known as “stronger connection, weaker components” stipulate that the capacity of the connection or joint must exceed that of the adjacent components. In this test, the proposed connection can satisfy this principle because the connection zone offers a larger bearing capacity than the column zone, according to the failure modes and load-strain relationships mentioned above.

It is also seen that the lateral deflection obtained from FEA is consistent with the test results in the elastic state, as depicted in Figure 7d. Hereby, the accuracy of FEA and simulated lateral constraint provided by steel bars is verified. Compared to the test of planar intersecting connections [10], the spatial intersecting connection has a higher bearing capacity due to the lateral constraint. Therefore, an adequate lateral constraint is essential for the connection when it is applied to a practical structure.

CONCLUSIONS

This paper has presented a type of connection for CFST diagrid structures. Each connection connected four CFST columns in two planes. The behavior of the connections with two different elliptical plate thicknesses was studied under monotonic compressive load. The failure modes and bearing capacity were obtained from experiments, which offered a basis for the structural design. The failure was due to the steel tube bulging within the column zone for both specimens. Their bearing capacity can satisfy the requirement of structural design. In fact, the ratios of the bearing capacity to the maximum design load of the connections were 4.41 and 4.47 for specimen 1 and 2, respectively.

The thickness of elliptical plate has no significant influence on the behavior of the connection. Both connections can fulfil the seismic design criteria “stronger connection, weaker components” defined by current Chinese codes. FEA result is substantiated to be accurate by comparison with the test results. Finally, an adequate lateral constraint is verified to be essential for the practical connection.

The experiments and FEA was conducted on the connection with a specific detail and two specific elliptical plates, hence the conclusions may or may not be valid for other connections. Further research on CFST obliquely intersecting connections with different details is still needed.

ACKNOWLEDGE

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REFERENCES


INFLUENCE OF BOLT PRELOADING AND FLEXURAL EFFECTS ON THE ULTIMATE BEHAVIOUR OF BOLTED T-STUBS

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KEYWORDS

Preloading, Mechanism Type-2, T-stub, Ductility, Modelling, Experimental, Component method.

ABSTRACT

The paper deals with a refinement of a model for predicting the ultimate behaviour and the deformation capacity of bolted T-stubs. The model, in its original formulation, generally gives very satisfactory results for T-stubs failing according to type-1 mechanism; conversely, it can be poor in predicting the plastic deformation capacity of T-stubs failing according to type-2 mechanism (i.e. flange yielding with bolt failure). The unsatisfactory prediction in this case is generally due to: 1) the simplified modelling regarding the bolts, assumed to be in tension without any bending moment and 2) the neglecting of the influence of bolt preloading. Therefore, in this paper some improvements to the original model are presented by introducing a simple approach to account for the influence of bolt preloading and the effects of bending on the bolts. The accuracy of the improved model is validated by comparison with new experimental tests carried out at the Material and Structure Laboratory of the Department of Civil Engineering of Salerno University.

INTRODUCTION

The prediction of the behaviour of beam-to-column connections can be obtained by means of the so-called “component method”.

The most important components of bolted beam-to-column joints can be modelled by means of equivalent T-stubs, i.e. two equal T-shaped elements connected through the flanges by means of one or more bolt rows.

The prediction of the ultimate behaviour of bolted T-stubs (force vs. displacement curve) has been carried out by the authors in previous works (Faella et al., 1997; Piluso et al., 2001a). In
this paper the effects of bolt preloading and bolt bending occurring in type-2 mechanism is discussed. The main goal of this work is the improvement of the original model for type-2 collapse mechanism. The refinement of the model will be validated by means of the comparison with the results of four experimental tests carried out at the Material and Structure Laboratory of the Department of Civil Engineering of Salerno University.

PREDICTION OF ULTIMATE BEHAVIOUR OF BOLTED T-STUBS

T-stub collapse mechanisms are generally classified by considering the number and the location of plastic hinges at collapse. Three collapse mechanisms are usually considered. In this paper, the attention is focused on type-2 failure mechanism (Fig. 1), where failure is due to the attainment of the ultimate bolt strength $B_u$ and to the formation of two plastic hinges located at the flange-to-web connection.

![Figure 1: T-Stub failure modes: a) Specimens' geometry definition](image)

The original model (Piluso et al., 2001a) for simulating the complete force versus displacement curve of bolted T-stubs, by means of a quadrilinear law was based on the evaluation of the T-stub displacements occurring for increasing bending moment of the T-stub flange at the web-to-flange connection section depending on the plastic rotation of the hinges involved in the collapse mechanism as shown in Figures 2 and 3. Displacements are obtained by considering the kinematics of each collapse mechanism and by evaluating plastic rotations of involved hinges by integration of the curvature diagram, directly obtained by the bending moment diagram.

The model has been validated (Piluso et al., 2001b) by experimental tests carried out at the Material and Structure Laboratory of the Department of Civil Engineering of Salerno University. Experimental tests have shown a very good agreement between model predictions and experimental curves in case of specimens failing according to type-1 mechanism; conversely, in case of type-2 mechanism further investigations are needed.

The unsatisfactory prediction obtained in this last case is due: 1) the model adopted for bolts, assumed to be in tension without any bending moment; 2) the simplified assumption regarding the loading process, stating that the point of zero moment along the T-stub flange remains unchanged during the loading process; 3) the influence of bolt preloading, because neglected.

In this work some improvements for type-2 mechanisms are proposed, dealing with bolt modelling and preloading effects.
INFLUENCE OF BOLT PRELOADING

As already shown in Figure 1, type-2 mechanism is characterized by the attainment of the ultimate moment $M_u$ only at the flange-to-web connection. Conversely, at the bolt axis, where the plastic hinge does not attain its ultimate resistance or does not develop, bending moment is expressed as a part of $M_u$:

$$ M = \xi M_u $$

(1)

The parameter $\xi < 1$ depends on the mechanical and geometrical properties of the T-stub:

$$ \xi = \frac{\lambda(2 - \beta_u)}{\beta_u(1 + \lambda)} $$

(2)

where $\beta_u$ is the parameter governing the expected collapse mechanism provided by the following relationship:

$$ \beta_u = \frac{2M_u}{B_u m} $$

(3)

where $M_u$ is the ultimate bending moment of the flange, $m$ is the distance between the bolt axis and the web-to-flange connection, $n$ is the distance between the bolt axis and the flange edge of the T-stub and, finally, $\lambda = n/m$.

In case of type-1 mechanism (Figures 1 and 2) there isn’t any detachment between the T-stub flanges in the region between the edge and the bolt axis. Conversely, in case of type-2 mechanism, a detachment between the flanges occurs due to the plastic engagement of the bolts.

With reference to type-2 collapse mechanism, it has been recognized that the original model does not account for the influence of bolt preloading on the loading process. This means that the detachment between the flanges is assumed during the whole loading process. On the contrary, during an initial part of the loading process, the bolt preloading prevents such detachment so that, from a kinematic point of view, the T-stub is initially “forced” to behave like a type-1 mechanism (Piluso et al., 2008). In particular, this occurs until the force acting on the bolt $B$ is less than the bolt preloading $N_s$. Of course, the T-stub kinematically behaves like a type-2 mechanism as soon as the value of $N_s$ is exceeded.

In order to account for the influence of bolt preloading, an improvement of the model is herein proposed. The refinement consists of the introduction of a new value of the parameter $\xi$, that will depend on the development of the previously described loading process. For a type-1 mechanism, the force $B$ acting on the bolt, for an assigned moment $M$ at the bolt axis, is derived from equilibrium equations:
The condition $B=N_s$, corresponds to the onset of the flange detachment The corresponding bending moment $M_s$ acting at the bolt axis is given by:

$$M_s = N_s \cdot \frac{\lambda}{2\lambda+1} m$$

Hence, the development of the loading process will consist of two phases:

An initial phase (Figure 4), for $B<N_s$, according to a type-1 mechanism, where the moment acting at the bolt axis is equal to the one acting at the flange-to-web connection;

The phase following detachment (Figure 5), for $B>N_s$ starting from the previous attained value $M_s$, where the moment at the bolt axis is equal to $\zeta$ times the incremental moment arising at the flange-to-web connection.

$$\Delta M_{hinge} = M_u - M_s$$

As a consequence, the corresponding increment of the moment at the bolt axis, $\Delta M_{bolt}$, is expressed by the following relationship:

$$\Delta M_{bolt} = \zeta \cdot \Delta M_{hinge}$$

By considering the moment attained at the bolt axis at the end of the loading process, a new equivalent value $\xi_{eq}$ of the parameter originally given by Eq. (2) can be obtained:

$$\xi_{eq} = \frac{M}{M_u} = \frac{M_s + \xi \cdot \Delta M_{hinge}}{M_u} = \xi_s + \xi \cdot (1 - \xi_s)$$

where $\xi = M_s/M_u$. Because of the parameter $\xi_{eq}$, the influence of the bolt preloading is taken into account, adopting the original model provided that the coefficient $\xi$ is substituted by $\xi_{eq}$.
It is important to stress that the adoption of $\xi_{eq}$ since the initial phase of the loading process still corresponds to maintain the point of zero moment unchanged during the loading process, but in a new location, with respect to the original model, which is more close to the point of zero moment occurring for type-1 mechanism.

**BENDING OF THE BOLTS**

During the loading process, because of the significant plastic deformation of the flanges, bolts are subjected to bending moment as a result of the rotation $\phi_b$ occurring at their ends due to the compatibility requirement with the flange deformation as depicted in Figures 6 and 7.

In this paper, a simple procedure to account for the influence of bolt bending moment is proposed.

The proposed procedure neglects the bending moment transmitted to the T-stub flanges by the bolt head, accounting for bolt bending only in the definition of the bolt deformation capacity.

This approach leads to the introduction in the original T-stub model of a new criterion for checking bolt plastic deformation.

Due to the bending moment acting on the bolts, it can be assumed that bolt collapse occurs due to the attainment, in the extreme fiber of the bolt section, of a strain $\varepsilon$ exceeding the ultimate bolt strain $\varepsilon_{ub}$.

The rotation $\phi_b$ of the bolt ends can be derived by considering, for type-2 mechanism, the kinematics of the T-stub (Fig. 7):

$$\phi_b = \theta_{p1} - \frac{\theta_{p2}}{2}, \text{ with } \theta_{p1} \geq \frac{\theta_{p2}}{2} \quad (9)$$

As the deformed shape of the bolts is characterized by equal end moments, it means that bolts are subjected to a constant bending moment, so that the deformation of bolts is characterized by a constant curvature $\chi_b$.

The relationship between the curvature $\chi_b$ and the rotation on the bolt end $\phi_b$ can be derived as:

$$\phi_b = \int_0^{L_b/2} \chi_b dz = \frac{\chi_b L_b}{2} \quad (10)$$

so that, as the rotation $\phi_b$ is known from compatibility requirements, the constant curvature of the bolt is equal to:
\[ \chi_b = 2 \frac{\phi_b}{L_b} \] (11)

where \( L_b \) is the bolt length.

As depicted in Figure 3, the plastic displacement occurring at the bolt axis location, due to the deformed shape of flanges is given by:

\[ \delta_{pb} = 2n \cdot (\theta_{p1} - \theta_{p2}) \] (12)

Due to compatibility requirements, the above displacement provides the bolt elongation. Such displacement gives rise to a plastic strain \( \varepsilon_0 \), whose computation is a paramount aspect from a theoretical point of view, because the bolt length subjected to plastic engagement is not easy to be defined.

Aiming at this evaluation, experimental tests carried out on bolts showed that almost all the plastic displacement is concentrated in the nut zone, so that \( \varepsilon_0 \) can be computed as:

\[ \varepsilon_0 = \frac{\delta_{pb}}{t_n} \] (13)

where \( t_n \) is the nut thickness.

Starting from the knowledge of \( \varepsilon_0 \) and \( \chi_b \), the maximum strain occurring in the extreme fiber of the bolt, in the nut region, can be computed as:

\[ \varepsilon_{max} = \varepsilon_0 + \chi_b \cdot \frac{d_{pb}}{2} \] (14)

where \( d_{pb} \) is the resistant bolt diameter.

Therefore, during the loading process, increasing values of \( \theta_{p1} \) and \( \theta_{p2} \) are obtained, leading to increasing values \( \delta_{pb} \). Bolt failure occurs when the condition:

\[ \varepsilon_{max} \geq \varepsilon_{ub} \] (15)

is satisfied, being \( \varepsilon_{ub} \) the ultimate bolt strain.

The ultimate bolt strain \( \varepsilon_{ub} \) has been obtained, in terms of true strain, from technical data of an Italian bolt manufacturer (Bulloneria Fontana, 2004) providing the necking ratio of the bolt material.

Therefore, the T-stub force vs. displacement curve is determined by means of the procedure described in Piluso et al. (2001a) by adopting the \( \xi_{eq} \) parameter to account for bolt preloading and by assuming that the ultimate displacement occurs when the ultimate strain develops either in the flanges or in the bolts.

**PRELIMINARY VALIDATION OF THE MODEL**

A preliminary validation of the refined model has been carried out by means of six experimental tests carried out at the Material and Structure Laboratory of the Department of Civil Engineering of Salerno University. Six specimens (Figure 8), made out by commercial rolled profiles, have been tested (Figures 15-18). Geometrical and mechanical properties of specimens are summarized in Table 1.
Figure 8: Geometrical parameters of tested specimens

### Table 1

<table>
<thead>
<tr>
<th>Geometrical and Mechanical Properties of Tested Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specimen</strong></td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Bolt nominal diameter ([d_b]) [mm]</td>
</tr>
<tr>
<td>Bolt head diameter ([d_{bh}]) [mm]</td>
</tr>
<tr>
<td>Washer diameter ([d_{wh}]) [mm]</td>
</tr>
<tr>
<td>Bolt head thickness ([t_{bh}]) [mm]</td>
</tr>
<tr>
<td>Nut thickness ([t_{bn}]) [mm]</td>
</tr>
<tr>
<td>Bolt resistant area ([A_{rb}]) [mm²]</td>
</tr>
<tr>
<td>Flange thickness ([t]) [mm]</td>
</tr>
<tr>
<td>Distance between plastic hinge and bolt axis ([m]) [mm]</td>
</tr>
<tr>
<td>Distance between prying force and bolt axis ([n]) [mm]</td>
</tr>
<tr>
<td>Flange-to-web radius ([r]) [mm]</td>
</tr>
<tr>
<td>Effective width ([b_{eff}]) [mm]</td>
</tr>
<tr>
<td>Flange yielding stress ([f_y]) [MPa]</td>
</tr>
<tr>
<td>Flange ultimate stress ([f_u]) [MPa]</td>
</tr>
<tr>
<td>Hardening modulus ([E_h]) [MPa]</td>
</tr>
<tr>
<td>Post-necking modulus ([E_{un}]) [MPa]</td>
</tr>
<tr>
<td>Hardening strain ([\varepsilon_h]) [%]</td>
</tr>
<tr>
<td>Fracture strain ([\varepsilon_u]) [%]</td>
</tr>
<tr>
<td>Ultimate bolt strength ([f_{ub}]) [MPa]</td>
</tr>
<tr>
<td>Ultimate bolt strain ([\varepsilon_{ub}]) [%]</td>
</tr>
</tbody>
</table>

Experimental curves and model predictions (for the original and the refined model) are depicted in Figures 9 to 14.

It can be observed that the proposed refined model leads to a significant improvement of the prediction of the force vs. displacement curve for all the four experimental tests.

Regarding the prediction of the ultimate displacement, the model prematurely leads to bolt collapse when compared with the experimental results. However this is an expected result, because it can be explained considering that the ultimate strain of bolt material \(\varepsilon_{ub}\) adopted in the model is just the lower bound stated by the manufacturer. The actual values of \(\varepsilon_{ub}\) are probably greater than those adopted in the analyses, due to random material variability, so that the actual ultimate displacement is greater than the one provided by the numerical model.
Notwithstanding, displacements computed by means of the proposed procedure \( d_{u,th} \) better agree with experimental data \( d_{u,exp} \) in comparison with the displacements obtained by applying the original model. The obtained ultimate displacements are summarized in Table 2 where, in particular, the ratio \( d_{u,th}/d_{u,exp} \) has been highlighted.

Figure 9: Force vs displacement curve of TS HEB 240

Figure 10: Force vs displacement curve of TS HEB 300

Figure 11: Force vs displacement curve of TS HEA 400

Figure 12: Force vs displacement curve of TS HEB 400

Figure 13: Force vs displacement curve of TS HEA 500

Figure 14: Force vs displacement curve of TS HEB 500
TABLE 2
EXPERIMENTAL AND THEORETICAL DISPLACEMENTES OF TESTED SPECIMENS

<table>
<thead>
<tr>
<th></th>
<th>HEB 240</th>
<th>HEB 300</th>
<th>HEB 400</th>
<th>HEB 400</th>
<th>HEB 500</th>
<th>HEB 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{u,\text{exp}}$ [mm]</td>
<td>25.66</td>
<td>25.10</td>
<td>44.50</td>
<td>29.70</td>
<td>43.37</td>
<td>15.70</td>
</tr>
<tr>
<td>$d_{u,\text{th}}$ [mm]</td>
<td>25.70</td>
<td>14.05</td>
<td>34.44</td>
<td>16.23</td>
<td>21.20</td>
<td>17.58</td>
</tr>
<tr>
<td>$d_{u,\text{th}}/d_{u,\text{exp}}$ [-]</td>
<td>1.00</td>
<td>0.56</td>
<td>0.77</td>
<td>0.55</td>
<td>0.49</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Figure 15: Specimens TS HEA500 at collapse
Figure 16: Specimens TS HEB500 at collapse
Figure 17: Specimens TS HEA400 at collapse
Figure 18: Detail of bolt at collapse (specimen TS HEA500)

CONCLUSIONS

In this work, the influence of bolt preloading and bolt flexural deformation effects on the behaviour of T-stubs failing according to type-2 mechanism have been analysed. Aiming to account for these effects, a refinement of a model for predicting the ultimate T-stub behaviour has been proposed. Its accuracy has been verified by means of a comparison between experimental results and model predictions.
REFERENCES


A TESTING MODEL STUDY ON DYNAMIC PROCESS OF TRUSS STRUCTURE INTRODUCED BY LOCAL MEMBER FAILURE

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KEYWORDS

Progressive collapse, model experiment, testing device, planar beam-truss structure

ABSTRACT

Experiment on planar beam-truss structure model which is supposed to undergo a progressive collapse process is carried out. The purpose of the research includes the following aspects: to make a testing model to be easily assembled and reused after part of elements fail; to invent a manual controlled initial failure apparatus with repeatability and portability; to develop a measuring system which is able to catch dynamic strains and dynamic displacements in failure process. The results show that the laboratory testing system to simulate progressive collapse failure caused by local member failure is possible. By the measured data and observed phenomena, the collapse mechanism is also discussed.

INTRODUCTION

The progressive collapse is defined by ASCE/SEI (SEI [1]) as the spread of an initial local failure from element to element, eventually resulting in collapse of an entire structure of a disproportionately large part of it. Since the event of Ronan Point Apartment collapse due to blast happened, the design method for preventing progressive collapse of building structure has been much attention, and previous research achievements have been gradually adopted by general building codes and standards (JSSC [2], DOD [3] and GSA [4]). The rational design shall depend on the thorough understanding of the failure mechanism. So the researchers and engineers have been developing the numerical analysis programs to simulate the process of structural collapse. It is much helpful and efficient to reveal the characters of the progressive collapse, compared with the costly and time-consuming experiment in laboratory. By means of numerical analysis, engineers have been aware of many key points, for example, the importance of redundancy, the effect of load redistribution routine, and the criteria to judge the key elements by which the structures suffered from accident action or...
terror attack can survive of complete collapse. However, when the numerical simulation is carried out, few or less, assumptions are always needed, including simplicity of structural model, mechanical properties of structural members, constrained condition between members and members with basement, and some factors are often neglected because of the complication, such as impulse caused by sudden break of members or connections, dynamic process in a continuously changed structure which members lost one by one. For a precise simulation, the model parameters usually need to be calibrated by physical experiments. Furthermore, experiment shall bring to view the unnoticed phenomenon, and the validity of the countermeasure to prevent progressive collapse need to be checked by experiment.

By overcoming many difficulties, experiments on collapse or dynamic failure of structures have been carried out recently. Progressive collapse resistance of an actual six-story reinforced concrete frame structure (the nearly century old structure) was evaluated following predefined initial damage which was caused by the simultaneous explosion of two adjacent exterior column (Sasani [5]). It is a full-scale experiment expensive in cost. A pseudo-static experiment was carried out on a small-scale plane reinforced concrete frame, estimating the ultimate bearing capacity of the catenary action (Yi [6]), where the middle column of bottom floor was replaced by mechanical jacks to simulate its initial failure. However, it is impossible in this experiment to follow the tracks of dynamic history of the structure in progressive collapse due to the limitation that only static response of structure was performed following initial local failure. Experiential study of the impact effect of sudden fracture of beam-to-column connections in a one storey one span moment resisting steel frame was executed (Chen [7]) by authors to investigate the effect of impulse load caused by crack or break of structural members or connection during severe earthquake or other imaged extreme loading event. One more case is an experiment on dynamic buckling of a single-layer reticulated shell model under drop hammer impact was carried out (Li [8]). The last two experiments mentioned above reveal some failure mechanism in dynamic procedure under a specific external force.

To the research on progressive collapse, a non-expensive experimental system is significant. The following features are expected for the system: (1) Small-scale model, thus an amount of parametric researches can be performed in a relatively economic way; (2) Controlled initial failure apparatus with repeatability and portability, by which sudden failure of local member shall be realized, and impulse effect as in actual structures can be demonstrated; (3) Smart and reliable measurement device, which shall all record the dynamic strain and dynamic displacement identical with the dynamic behavior in progressive collapse following predefined initial failure. In this paper, the authors shall establish a laboratory experimental system. There are two main components of the system. One is the structural model with the initial failure inducing apparatus. Another one is data record device which includes high frequency data processing device, and digital camera and its data transform program changing image data to digital data.

As we know from the available literatures, including design guidance, numerical analysis and experiment reports, most of the consideration till now is concerned with the frame structures in multi-storey or tall buildings. However, large span truss structure is also a common used system, and possesses different redundancy features from frame structures. In this paper, the truss model is taken as the research objective.
PLANE TRUSS-BEAM MODEL STRUCTURAL DESIGN

Planar truss-beam model setup

Truss-beam structure is characteristic by upper chords and lower chords being continuous bars. All members in the testing model are aluminum pipes possessing lower strength and stiffness, with diameter of 10 mm and thickness of 1 mm. Fig. 1 shows planar truss-beam model structure. Interval between nodes of upper chord is 500 mm and overall length of upper chord is 2000 mm. Interval between nodes of lower chord also is 500 mm and overall length of lower chord is 2500 mm. The height of the truss is 300 mm. Connections between upper/lower chord and diagonal web members are made from two steel blocks. Upper or lower chord is clamped by means of tightening bolts through two steel blocks and steel pin with diameter of 3 mm connecting two steel blocks and upper/lower chord (Fig. 2). Supports mainly includes connections between one steel block and the lower chord and between the steel block and one diagonal web member through steel pin with diameter of 3 mm respectively (Fig. 3). The components of general displacement UX, UY, UZ, RX, RY at support are constrained, UZ, RX, RY at nodes of upper chords are constrained by lateral constraint apparatus fixed on base apparatus and made from plexiglass which can be benefit for picking up image of displacement observation spot in the course of progressive collapse, where UX, UY, UZ is translational displacement along X, Y, Z coordinate, RX, RY is rotation about X, Y coordinate, Z direction can be obtained by right-hand rule. The steel pin is used for all web member to chord member connection, thus the advantage for easily fitting and reusing in the model experiment comes true. Loading apparatus include suspenders (thread steel pole with diameter of 6 mm), and loading block weights. One end of suspender is linked to nodes of upper chords and the other end of that to a padding which can be put block weights. Weight of one block is 19.6N. The loading apparatus can satisfy need of different vertical loading cases and masses of the entire structure following deformation in the course of progressive collapse.

![Figure 1: Planar truss-beam model structure](image-url)
Initial failure inducing apparatus

How to induce initial local failure which is expected to cause possible progressive collapse is much crucial. In this experiment the location of initial failure apparatus is selected at diagonal web member for easy assembly. The bearing capacity of the diagonal web member predefined shall be soon lost through triggering initial failure inducing apparatus. In addition, the apparatus can substitute any diagonal web member which may bear pressure or tensile force and shall not change mechanical properties of the entire model structure significantly. Based on such an idea, an initial failure inducing apparatus which is made up of two identical aluminum pipes with wedgy steel toe (Fig. 4) and a steel clip (Fig. 5, 6) is designed. Fig. 5 and Fig. 6 show service mechanism of initial failure inducing apparatus under pressure. Where, P₁ is internal force from the external loading, P₂ is force of reaction from steel clip and P₃ is force of reaction from the other aluminum pipe. While steel clip closes, P₁, P₂ and P₃ are in equilibrium making initial failure inducing apparatus normal working. While steel clip opens, denoting that P₂ becomes zero, wedgy steel toe starts to slide instantly due to unbalance forces, subsequently P₁ and P₃ become zero, and then initial failure of the member comes true. So does initial failure inducing apparatus if the member is under tensile force.

MEASURE TECHNOLOGIES

Due to predefined initial failure of the model structure the complex dynamic procedure will take place, therefore dynamic strains and dynamic displacements in the process of progressive collapse of the model structure are chosen as the main measure objects.
Dynamic strain gauges in progressive collapse can be recorded by using DH5922 dynamic signal measurement and analysis system. The measure indicators, such as the highest sample frequency of 50 kHz and the highest analysis frequency of 19.53 kHz, can satisfy experiment requirement because basic frequency of the entire model structure is less than 20 Hz obtained from previous numerical analysis and follow-up experiment verification. The reliability of strain gauge in failure process had been verified (Chen [7]). Strain gauges were stuck to the upper surface of upper chord portions 1 to 4 and lower chord portions 1 to 15, to the left surface of diagonal web members 5, 7, 10 and to the right surface of diagonal web members 6, 8, 9 (Fig. 1).

It is of rather importance for acquisition of dynamic displacements of the model structure. However, the common displacement transducer can’t record dynamic displacement in progressive collapse and only apply to displacement measurement in static or pseudo static experiments. Hardware integration method is unfit because experiment instrument is easily damaged in progressive collapse. Therefore a non-contact measure technique for dynamic displacement, digital image identification method, is developed. The main measuring procedure for dynamic displacement may be described as follows: first the relucant tags are stuck to corresponding observation spot before testing, then the visible photos tracking observation spots can be taken by high-speed camera, and the photos may be transferred into pieces of photos at a fixed time interval. Finally dynamic displacements at observation spots can be obtained by Matlab software program which can read data recorded in photo files and analyze position change of observation tags. In addition, the static loading experiment verified that the measurement error is limited within 0.2 mm, which is satisfied enough for explaining failure mechanism.

EXPERIMENT ON PLANAR TRUSS-BEAM STRUCTURE

Experimental process and testing phenomena

Suspender were first fixed on all nodes of upper chords, then block weights were added to suspender by one in each step, and there were 7 steps in the entire loading process. That means the model structure was loaded with 7 block weights as gravity load on each node of upper chords. After that, initial failure inducing apparatus at the indicated diagonal web member was triggered. Subsequently, the model structure abruptly dropped off at a high speed and collapse took place in very short time. Suspender finally touched the floor one after another when structure dropped off at certain height. During the loading and collapse processes, strain data and displacements of observation spots were recorded. In failure scenes, the entire model structure had considerably large damage and was disproportionate to original initial local failure. Two main types of failure phenomena were displayed (Fig. 9e): (1) Buckling of members 2, 3 and 5. (2) Break at left end of members 11 and 15.

Analysis of experimental results

According to stress versus strain curve obtained by tensile experiment of aluminum pipe (Fig. 7), yielding strain of aluminum pipe was adopted by criterion of 0.2% residual strain as \( \varepsilon_y = 1800 \mu \varepsilon \). It can be seen from strain history of all strain gauges in the course of experiment that maximum absolute value at strain gauges 2, 3, 5, 11 and 15 (Fig. 8a-8e) were greater than yielding strain, namely these members were in elastic-plastic stage; yet maximum absolute value at other strain stations were less than yielding strain all along, that is, these members was still in elastic stage. Thus the main discussion is focused on member 2, 3, 5, 11 and 15.
Figure 7: Stress-strain curve

Figure 8: Strain history of the main objects

Two special moments $T_0$ and $T_1$ are marked in Fig. 8a to 8e:

1. At moment $T_0$, initial failure inducing apparatus of one diagonal web member was triggered (Fig. 9b), thus apparent change of strain took place and displacements of observation spots (Fig. 10) began to increase. Internal force redistribution of the model structure with local failure took place for the first time. At certain moment after $T_0$, member 2 buckled (Fig. 9c).

2. At moment $T_1$, one suspender at node A (referring to Fig. 9a) touched the floor due to the large falling move and thus was supported by the floor. Apparently, there was additional impact acting on node A and the internal force of model structure changed significantly again. This can be seen from dramatic change with reverse sign of strain values at strain gauges shortly after moment $T_1$ in Fig. 8a to 8e. Actually, after moment $T_1$ other suspenders dropped to floor in succession. Consequently, $T_1$ can be treated as the demarcation moment to distinguish the first and second process of internal force redistribution.
Strain gauge data display that moment $T_0$ is 133.31s and $T_1$ 133.62s, and interval between $T_0$ and $T_1$ is 0.31s, which almost keeps consistent with the time needed for reaching the first maximum displacements of observation spot A in Fig.10. In addition, the maximum displacement of spot A is also nearly keeps consistent with the distance between bottom of suspenders and floor.

In the second process of internal force redistribution, the absolute values of strain gauge 2, 3 and 5 rapidly exceeded the valid range of strain measurement (-9708.8 $\mu\varepsilon$ ~ 9708.8 $\mu\varepsilon$), while the absolute values at strain gauge 11 and 15 kept at a low level after the sharp fluctuation. Those two kinds of strain record can be explained by two test phenomena at the end of experiment (Fig.9d): (1) the great compression plastic strain indicates the occurrence of great deformation at the corresponding member 2, 3 and 5, that is, buckling came into being in these members; (2) The small strain of strain gauge 11 and 15 indicates the break of the corresponding members. Thus the final collapse was accompanied by both overall buckling of compressive members and break of tensile members.

**SUMMARY**

(1) A model structure easily fitting and reusing was designed. The key apparatus, manual controlled initial failure inducing apparatus with repeatability and portability was realized.

(2) Failure process of the entire model structure following predefined initial local failure can
be traced by dynamic strain measure technology and dynamic displacement obtained by using
digital image identification method.

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INTEGRATED OPTIMAL PLACEMENT OF DISPLACEMENT TRANSUDERS AND STRAIN GAUGES

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KEYWORDS
Hybrid sensor system, Optimal placement, Strain gauges, Displacement transducers, Estimation error

ABSTRACT
This paper focuses on the problem of optimal design of hybrid sensor systems with both displacement transducers and strain gauges. Unlike traditional sensor placement approaches in which these two types of sensors are often plated separately to monitor structural deformations and displacements respectively, the integrated design procedure presented in this study treats the sensor system as a whole. The number and locations of strain gauges and displacement transducers will be optimized simultaneously, and their measurement data will be fused together to better predict the unobserved structural response. The theoretical criterion for the optimization procedure is first formulated based on the strain and displacement mode shapes extracted from finite element models. Then the initial candidate sensor locations are reduced to a smaller optimal set with minimized prediction error of structural response. A two-dimensional cantilever beam is then analyzed as a numerical example to investigate the effectiveness and accuracy of the presented optimal sensor placement approach. The results indicate that the hybrid sensor system provides better estimation of structural response than single-type sensor system.

INTRODUCTION
Structural health monitoring system (SHMS) is one of the cutting-edge technologies to ensure the safety of structures during their long service lives. Among SHMS, sensor system always plays an important role, and its spatial allocation and the quality of collected data may affect the functionality of the entire SHMS. Considering the number of sensors is often limited, especially for those large-scale structures with great spatial complexity, the optimal sensor placement has attracted increasing interest in the past two decades. Numerous techniques
have been developed for solving the optimal sensor placement problems [1-6]. For example, Kammer [1,2] proposed a method, termed Effective Independence (EfI) method, to optimize sensor locations based on the contribution of each sensor location to the linear independence of the identified modes. Such a method was further extended that the number of sensors is determined during the course of sensor placement to maintain a desired level of signal-to-noise ratio[7]. Nevertheless, little research has been carried out on the design of sensor system with multiple types of sensors. Therefore, this paper presents an integrated design approach of sensor systems with both strain gauges and displacement transducers. The integrated approach for optimal sensor placement of hybrid sensor systems is an extension of the EfI method for single-type sensor systems. The locations of strain gauges and displacement transducers are optimized simultaneously so that we can best estimate structural response of interest from limited measurements, while their total number are determined in the optimization in order to reduce estimation errors to a desired level. In particular, the normalization using noise variance is adopted to solve the problem caused by combining displacement measurements and strain measurements together. A case study of a cantilever beam is presented as an example. The results of the numerical analysis indicate that the hybrid sensor system, composed of strain gauges and displacement transducers, can provide better estimation of the entire structure response than single-type sensor system.

STRAIN-DISPLACEMENT RELATIONSHIP

In order to integrate the design of sensor system with both strain gauges and displacement transducers, the strain-displacement relationship is first presented in this section in the context of finite element method (FEM). It is noteworthy that the strain refers to normal strains in this study since strain gauges measure only normal strains in a certain direction. A strain vector can be expressed as

$$\varepsilon_t = (\varepsilon_1, \ldots, \varepsilon_j, \ldots, \varepsilon_n)^T = (B_1 T S_1, \ldots, B_j T S_j, \ldots B_n T S_n)^T d_t = C d_t$$

(1)

where subscript t indicates that the vector corresponds to the complete set of DOFs, $d_t$ is the displacement vector; $S$ is a selection matrix; $T$ is a transformation matrix; $B$ is a strain-displacement matrix; $\varepsilon_i$ is the strain at a location in element $i$; $C$ is transformation matrix between the displacement vector and the strain vector.

A modal expansion of the displacement vector yields

$$d_t \approx \sum_{r=1}^{k} \phi_r q_r = \Phi q$$

(2)

Subsequently, the strain vector can be expressed with the modal coordinates:

$$\varepsilon_t = \sum_{r=1}^{k} \psi_r q_r = \Psi q$$

(3)

where $\Phi_t$ is the displacement modal matrix; $q$ is the modal coordinates; $\Psi_t$ is strain modal matrix. Thus the strain modal matrix satisfies the following relationship with the displacement modal matrix:

$$\psi_r = C \phi_r \quad \Psi_t = C \Phi_t$$

(4)

Optimization Strategy

The optimal sensor placement presented in this section aims to minimize the error in the estimation of unmeasured displacements and strains. Particularly, the locations of strain gauges and displacement transducers are optimized simultaneously, unlike the conventional design methodology in which they were usually carried out separately. The total number of
both sensors is determined within the optimization procedure based on the target error levels of the estimation instead of being prescribed.

Assume the response vector $y$ includes the displacements and strains at the locations of interest for a structure. For a linear structural system, the response can be expressed as

$$y = \begin{bmatrix} \varepsilon \\ d \end{bmatrix} = \begin{bmatrix} \Psi \\ \Phi \end{bmatrix} q = \Gamma q$$  \hspace{1cm} (5)$$

where $q$ is the vector of modal coordinates; $\Gamma$ is the general modal matrix which includes both strain mode shapes $\Psi$ and displacement mode shapes $\Phi$. Here, $\Psi$ and $\Phi$ are only sub-matrices of aforementioned modal matrices $\Psi_t$ and $\Phi_t$. From Eqn. 5, the measured response can be expressed as

$$y_m = \begin{bmatrix} \Psi_m \\ \Phi_m \end{bmatrix} q + w = \Gamma_m q + w$$  \hspace{1cm} (6)$$

where $\Gamma_m$, $\Psi_m$ and $\Phi_m$ are partitioned model matrices corresponding to the positions with sensors, and $w$ represents the zero-mean stationary Gaussian noise, which is assumed uncorrelated with each other, and sensors of same type are of equal variance. The covariance matrix of the measurement noise yields

$$E(w w^T) = \begin{bmatrix} \sigma_{\varepsilon}^2 I \\ \sigma_d^2 I \end{bmatrix} = \Sigma_m$$  \hspace{1cm} (7)$$

where $\sigma_{\varepsilon}^2$ and $\sigma_d^2$ is the noise variance for strain gauges and the displacement transducers respectively, $I$ is the identity matrix. To estimate the response at unmeasured DOFs, O’Callahan [10] introduced the System Equivalent Reduction-Expansion Process (SEREP) method. Similar approach is adopted here to estimate the response vector $y$,

$$y_e = \Gamma \Gamma_m^+ y_m$$  \hspace{1cm} (8)$$

where the subscript $e$ denotes the estimation; $\Gamma_m$ is of full column rank.

Kammer [7] extended the Efl method to include the effects of measurement noise, where the number of sensors is determined based on a predetermined level of signal-to-noise ratio in modal coordinates. This approach is further extended to achieve the optimal placement of sensor system with both displacement transducers and strain gauges. The estimation error $\delta$, i.e. the difference between the estimated response and the real response, can be obtained from Eqn. 5, Eqn. 6 and Eqn. 8:

$$\delta = y_e - y = \Gamma q + \Gamma_m^+ w = \Gamma \Gamma_m^+ w$$  \hspace{1cm} (9)$$

However, the magnitudes of strain and displacement responses are of different orders, and as such their absolute estimation errors. The optimization procedure may considerably bias one type of sensors. In view of this, the relative estimation error, that is the ratio of the estimation error to the measurement noise, is used in this study

$$\tilde{\delta} = \begin{bmatrix} \varepsilon_e - \varepsilon \\ \sigma_{\varepsilon} \\ d_e - d \\ \sigma_d \end{bmatrix} = \Sigma^{-1} \delta$$  \hspace{1cm} (10)$$

Similarly, the noise-normalized mode shape matrices are defined as

$$\tilde{\Gamma} = \begin{bmatrix} \frac{1}{\sigma_{\varepsilon}} \Psi \\ \frac{1}{\sigma_d} \Phi \end{bmatrix}^T = \begin{bmatrix} \Psi \\ \Phi \end{bmatrix}^T = \Sigma^{-1} \Gamma$$  \hspace{1cm} (11)$$
\[
\hat{\bm{\Gamma}}_m = \left[ \frac{1}{\sigma_x} \Psi_m \frac{1}{\sigma_y} \Phi_m \right]^T = \left[ \hat{\Psi}_m \hat{\Phi}_m \right]^T = \Sigma_m^{-1} \Gamma_m
\]

where ‘\(\sim\)’ denotes the noise-normalized vectors or matrices, \(\Sigma\) is a matrix that has similar format but different dimensions with \(\Sigma_m\). The estimation of the structural response can be computed by

\[
\hat{\bm{y}}_e = \Sigma \hat{\bm{\Gamma}}_m \hat{\bm{y}}_m = \Gamma \hat{\bm{m}} \hat{\bm{y}}_m
\]

The covariance matrix of the normalized estimation error vector can be expressed as

\[
\hat{\Delta} = E\left(\hat{\Delta} \hat{\Delta}^T\right) = E\left(\Sigma^{-1} \hat{\Delta} \hat{\Delta}^T \Sigma^{-1}\right) = \hat{\bm{\Gamma}}_m (\hat{\bm{\Gamma}}_m)^{-1} \hat{\bm{\Gamma}}^T
\]

The diagonal elements in the covariance matrix of the normalized estimation error \(\hat{\Delta}\) represent the variance of the normalized estimation error for corresponding response (strain or displacement), and they are of the same order of magnitude. Therefore, such normalization enables the simultaneous selection of the optimal locations of displacement transducers and strain gauges. The maximum diagonal element denotes the maximum normalized estimation error, while the trace of the matrix represents the sum of normalized estimation errors at all the locations of interest.

\[
diag(\hat{\Delta}) = \left[ \hat{\sigma}_1^2 \hat{\sigma}_2^2 \ldots \hat{\sigma}_n^2 \right]
\]

The maximum and average estimation errors at all the locations can be computed by

\[
\hat{\sigma}_{\text{max}}^2 = \max(\text{diag}(\hat{\Delta}))
\]

\[
\hat{\sigma}_{\text{avg}}^2 = \frac{\text{tr}(\hat{\Delta})}{n} = \frac{\text{tr}\left[\Gamma (\hat{\Gamma}_m^T \hat{\Gamma}_m)^{-1} \hat{\Gamma}^T\right]}{n} = \frac{\text{tr}\left[(\hat{\Gamma}^T \hat{\Gamma})(\hat{\Gamma}_m^T \hat{\Gamma}_m)^{-1}\right]}{n}
\]

Consequently, the sensor optimal placement can be done by minimizing the maximum normalized estimation error, average normalized estimation error or both. The optimization objectives in this study are

\[
\begin{align*}
&\text{minimize } \hat{\sigma}_{\text{max}}^2 \quad \Rightarrow \quad \text{minimize } \max(\text{diag}(\hat{\Delta})) \\
&\text{minimize } \hat{\sigma}_{\text{avg}}^2 \quad \Rightarrow \quad \text{minimize } tr\left[(\hat{\Gamma}^T \hat{\Gamma})(\hat{\Gamma}_m^T \hat{\Gamma}_m)^{-1}\right]
\end{align*}
\]

The maximum and average estimation errors will be reduced with the increase of the number of sensors. The total number of strain gauges and displacement transducers can be thus determined to achieve the prescribed criterion for estimation errors. A simple procedure is to delete the candidate sensor positions which contribute most to the trace of the matrix \((\hat{\Gamma}^T \hat{\Gamma})(\hat{\Gamma}_m^T \hat{\Gamma}_m)^{-1}\) one by one until the target error level is reached.

**Case Study**

A cantilever beam with a length of about 2.0 m and a cross section of 50.8mm×50.8mm is investigated in the analytical study. The beam is modeled by two-dimensional beam element, and the finite-element model consists of 21 nodes and 20 equal-length elements, shown in Figure 1(a). A random excitation is applied vertically at the end of the cantilever beam, and it induces the flexural vibration of the beam. Considering that it is not easy to measure the rotations at nodes in practice, rotational DOFs are eliminated in the concerned mode shapes. The stain gauges are attached to the upper face at the middle of the elements to measure the flexural deformation of the beam. As a result, 20 element strains, and 20 vertical nodal displacements are identified as the response of interest, and they are also taken as the candidate locations for strain gauges and displacements transducers.
The following three cases are studied and compared:

**Case 1:** The noise variances for strain gauges and displacement transducers are taken as $\sigma_\varepsilon = 25 \mu \varepsilon$ and $\sigma_d = 0.7 \text{mm}$, and they are equivalent to noise-to-signal ratio of 15% and 10% respectively.

**Case 2:** Only strain gauges are installed. The noise variance are taken as $\sigma_\varepsilon = 25 \mu \varepsilon$.

**Case 3:** Only displacement transducers are installed. The noise variance are taken as $\sigma_d = 0.7 \text{mm}$.

The first five mode shapes are considered in the case study. In the first case, the proposed approach is adopted to optimize the locations of strain gauges and displacements transducers simultaneously, and the total number of sensors is determined based on the objective of $\tilde{\sigma}_{\max}^2 \leq 1.0$ and $\tilde{\sigma}_{\text{avg}}^2 \leq 0.5$. Finally, totally 9 sensors are selected in case 1, composed of 7 strain gauges and 2 displacement transducers in y-direction, shown in Figure 1(b). In the last two cases, only one type of sensors is installed on the beam, and their performance is compared with the counterpart-case 1. The single-type sensor systems are placed as using the conventional Kammer method. The total number of sensors is set equal to that in case 1, seen in Figure 1(c) and Figure 1(d).

Figure 2 depicts the variation of the average and maximum normalized estimation error variance in the optimization procedure in case 1. The sensor location which contributes most to the trace of the error variance matrix is removed from the candidate locations in each step. Both curves ascend with the decrease of the sensor number. Figure 3 illustrate the comparison of the estimated response and the real response at element 3 and node 18 for all three cases. As illustrated by Figure 3, both the estimated strain and displacement can match the real response fairly well in case 1, but the time history of displacement response in case 2 becomes slightly worse in comparison with case 1. Moreover, a huge discrepancy is witnessed by the strain time history in case 3. It is because that the displacement responses generally contain less high-frequency components than strain responses.
In particular, the time histories of measurement noise and estimation error for the displacement at node 19 and the strain in element 4 are shown in Figure 4. The comparison indicates that the estimation errors are even smaller than the measurement noise for both displacement and strain response.

Figure 5 and Figure 6 present the comparison between the theoretical and actual estimation errors, all being normalized by sensor noise variances. It can be seen that the theoretical values derived using the presented approach can well predict the actual errors in this case study. Slight discrepancy that can be observed is caused by truncated higher mode shapes in
theoretical formulation. As shown in Figure 5(a) and Figure 6 (a), the values of the average and maximum normalized estimation errors in case 1 are $\tilde{\sigma}_{\text{avg}}^2 = 0.483$ and $\tilde{\sigma}_{\text{max}}^2 = 0.915$ for strains, and $\tilde{\sigma}_{\text{avg}}^2 = 0.123$ and $\tilde{\sigma}_{\text{max}}^2 = 0.410$ for displacements. The criteria for the maximum and average normalized estimation error are also shown in Figure 5(a) and Figure 6 (a) by two dashed lines, and both are satisfied by actual estimated responses. Moreover, $\tilde{\sigma}_{\text{max}}^2$ is less than 1 in both figures, which means that the estimation errors are smaller than the measurement noise levels at all the locations. In this perspective, we can conclude that it is beneficial to calculate the responses at the points even with sensor measurements.

![Figure 5: Comparison of Normalized theoretical and actual estimation errors for strains](image)

![Figure 6: Comparison of Normalized theoretical and actual estimation errors for displacements](image)

Figure 5(b) and Figure 6(b) show the distribution of normalized estimation errors for case 2. Compared with case 1, case 2 has similar error levels for strain estimations, ($\tilde{\sigma}_{\text{avg}}^2 = 0.517$ and $\tilde{\sigma}_{\text{max}}^2 = 0.718$), whereas the displacement estimation error levels become larger ($\tilde{\sigma}_{\text{avg}}^2 = 0.246$ and $\tilde{\sigma}_{\text{max}}^2 = 0.973$). Similar observations can also be made for case 3 in figure 5(c) and figure 6(c). If only the displacement transducers are installed, the estimation errors in strain response become considerable ($\tilde{\sigma}_{\text{avg}}^2 = 168.182$ and $\tilde{\sigma}_{\text{max}}^2 = 346.539$), although the estimation of displacements is still acceptable. Through the comparison of cases 1, 2 and 3, it is seen that the optimized hybrid sensor system, a combination of displacement transducers and strain gauges, contains more information of the entire structural response than single-type sensor system, provided that the data from multi-type sensors can be appropriately fused.

**CONCLUSIONS**

An optimal sensor placement approach is presented in this study for monitoring systems with both strain gauges and displacement transducers, both of which are optimized simultaneously. The optimization objective is to best estimate structural strains and displacements of interest.
using limited measurements from sensors, and the total number of sensors is determined to achieve the desired error levels in structural response estimations. A numerical analysis of a cantilever beam indicates that this approach offers an effective way to design such a hybrid sensor system as a whole and reduce a relatively large initial candidate location set to a much smaller optimum set. The actual estimation error levels induced by measurement noises can be fairly well quantified by the theoretical formulation, and the target estimation error levels can be successfully achieved through the optimization procedure. The results of a comparative study also demonstrate that the hybrid sensor network with multiple types of sensors can provide more complete and accurate information about structural response, compared with single-type sensor systems.

Reference


STRUCTURAL HEALTH MONITORING SYSTEM FOR STEEL ANTENNA MAST OF GUANGZHOU TELEVISION AND SIGHTSEEING TOWER

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KEYWORDS
supertall tower, high-rise structure, steel spatial structure, antenna mast, structural health monitoring (SHM) system.

ABSTRACT

The Guangzhou Television and Sightseeing Tower (GTST) located in Guangzhou, China, is a supertall structure with a total height of 610 m. One of the outstanding features of GTST is its 156 m high antenna mast founded on the top of the main tower which is a 454 m high tube-in-tube structure. The lower part of the antenna mast is a steel lattice structure and the upper part is a steel box structure. In view of the slenderness of the antenna mast as well as its extreme high erection location, a long-term structural health monitoring (SHM) system is being implemented for the antenna mast to watch its structural condition, especially the stability and whipping effect. This SHM system comprises anemometers, tiltmeters, accelerometers, and fiber optic sensors, totaling nearly 90 sensors in number. In addition, GTST is also designed with a hybrid control system with the intention of mitigating wind-induced vibration of the main tower. As operation of this control system requires the structural conditions as input, a data acquisition system is being implemented by the authors to feed the acceleration, velocity and displacement information to the control system. This paper briefs the SHM system for the antenna mast and the data acquisition system for the hybrid vibration control of the main tower.

INTRODUCTION

The Guangzhou Television and Sightseeing Tower (GTST) located in Guangzhou, China, as shown in Figure 1, is a supertall structure with a total height of 610 m. It comprises a 454 m high main tower and a 156 m high antenna mast. The main tower is a tube-in-tube structure consisting of a
reinforced concrete inner structure and a steel lattice outer structure. The inner structure has a constant ellipse cross-section of 14 m × 17 m throughout the height. The outer structure has a hyperboloid form, which is generated by the rotation of two ellipses, one at the ground level and the other at an imaginary horizontal plan 454 m above the ground. The tightening caused by the rotation between the two ellipses forms the characterizing “waist-line” of the tower. The cross-section of the outer structure is 50 m × 80 m at the ground, 20.65 m × 27.5 m (minimum) at the waist level (280 m high), and 41 m × 55 m at the top (454 m high). The antenna mast founded on the top of the main tower is a steel spatial structure. The lower part of the antenna mast is a steel lattice structure, and the upper part is a steel box structure. Both the section form and sectional dimension of the antenna mast vary with height. The diagonal length of the octagonal cross section at the bottom is 14 m and the side length of the square cross section on the top is 0.75 m. The tower serves for various functions including television and radio transmission facilities, observatory decks, Ferris wheels, exhibition spaces, revolving restaurants, computer gaming, conference rooms, shops, and 4D cinemas.

A sophisticated long-term structural health monitoring (SHM) system has been designed and implemented by The Hong Kong Polytechnic University to monitor GTST at both in-construction and in-service stages [1, 2]. It consists of more than 700 sensors, including a weather station, anemometers, wind pressure sensors, temperature sensors, corrosion sensors, seismographs, GPSs, digital video cameras, tiltmeters, accelerometers, vibrating-wire strain and temperature gauges, and FBG fiber optic strain and temperature sensors. This SHM system is developed mainly for the main tower while few sensors have been deployed on the antenna mast. Nevertheless, same importance should be attached to the 156 m high antenna mast founded on the top of the main tower (454 m high), especially its stability. In early 2009, The Hong Kong Polytechnic University was commissioned by the owner of GTST to develop a SHM system for the antenna mast. In the meantime, the authors have also been commissioned to develop a data acquisition system for the hybrid vibration control system of the main tower, which consists of two tuned mass dampers (TMDs) coupled with two active mass dampers (AMDs). The data acquisition system monitors the acceleration, velocity and
displacement of GTST, and feed them to the control system as input. This paper describes the SHM system for the antenna mast and the data acquisition system for the hybrid vibration control of the main tower.

ANTENNA MAST OF GTST

The antenna mast of GTST is a steel spatial structure, as shown in Figure 2. The lower part of the antenna mast is a steel lattice structure which is composed of columns, diagonal bracings and horizontal bars. The typical cross section of the steel lattice part is octagon, as shown in Figure 2(b). The diagonal length of the octagon is 14 m at the bottom and decreases as the height rises. The upper part of the antenna mast is a steel box structure. The form of the cross section varies with height, being square, hexagon and square again. The side length of the square cross section on the top of the antenna mast is 0.75 m only. The antenna mast has a total weight about 1,800 tons.

To suppress wind-induced vibration of the antenna mast, two TMDs with a total weight of 4 tons are suspended at the heights of 571 m and 575 m, respectively. Accordingly, the SHM system for the antenna mast is designed with a special function of verifying the effectiveness of this control system. In addition, a hybrid control system consisting of two TMDs coupled with two AMDs is installed at the height of 438 m for mitigating wind-induced vibration of the main tower. The monitoring data acquired by the SHM system for the antenna mast will be provided to this hybrid control system as references in decision making on its activation or locking.

SHM SYSTEM FOR ANTENNA MAST

Making use of the modular design concept, the SHM system for the antenna mast is so designed as to be able to integrate into the existing SHM system for the main tower. In this system, four types of sensors, i.e., anemometers, tiltmeters, accelerometers, and fiber optic strain and temperature sensors, are deployed on the antenna mast, as summarized in Table 1. Hereafter, the system will use the existing data acquisition and transmission system, data processing and control system, structural
health data management system, and structural health evaluation system to fulfill the functions of data acquisition, transmission, processing, visualization, management, analysis, etc.

### TABLE 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Sensor type</th>
<th>Monitoring items</th>
<th>Number of sensors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Anemometer</td>
<td>Wind speed and direction</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Tiltmeter</td>
<td>Inclination</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Accelerometer</td>
<td>Acceleration</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>Fiber optic sensor</td>
<td>Strain and temperature</td>
<td>80</td>
</tr>
</tbody>
</table>

**Wind Monitoring**

Wind effects are the major concern in the design of supertall structures located in the coastal cities of China, such as Hong Kong, Shenzhen, Guangzhou, and Shanghai. In the current Chinese “Load code for the Design of Building Structures”, the maximum gradient wind level is 450 m [3]. Thus, the wind-resistant design parameters obtained from the specification may be unable to provide an accurate description of the fluctuating wind loads acting on the antenna mast. For the purpose of design verification, one propeller anemometer is mounted on the antenna mast at the height of 578 m, which is the world’s highest installation position of anemometers. In addition, there is one more anemometer on the top of the main tower. The wind monitoring will provide useful information about the wind condition, turbulence characteristics and its effect on fluctuating surface pressures, and wind loading distribution at the height exceeding the gradient wind level.

**Inclination Monitoring**

As the 156 m and 1,800 tons antenna mast is founded on the top of the main tower at the height of 454 m, not only the antenna mast itself may incline but also the inclination of the main tower will contribute to the inclination of the antenna mast. Therefore, the inclination of the antenna mast is of great concern. Recalling the aforementioned two TMDs with a total weight of 4 tons that are suspended to the antenna mast for wind-induced vibration mitigation, the effect of the inclination of the antenna mast on the appropriate operation of TMDs is another concern. To address the concerns, one bi-axial tiltmeter is placed on the top of the main tower, and another tiltmeter is mounted on the antenna mast at the height of 529 m for the measurement of the inclination of both main tower and antenna mast.

**Acceleration Monitoring**

In view of the slenderness of the antenna mast as well as its extreme high erection location, it is of great importance to monitor the wind-induced whipping effect of the antenna mast. To this end, two accelerometers are mounted on the antenna mast at the height of 529 m to monitor its dynamic response and modal properties. A special type of accelerometers with a wide measurement range is selected for the measurement of the acceleration of the antenna mast because of the large vibration amplitude expected for the antenna mast.

**Strain and Temperature Monitoring**

A fiber optic sensing system based on fiber Bragg gratings (FBGs) is implemented on the antenna mast to fulfill the strain and temperature monitoring. The antenna mast is welded into the steel-tube columns of the inner structure. Two levels of the steel-tube columns are selected for instrumentation with fiber optic strain sensors. One is below the welding line of the steel-tube columns between the...
inner structure and an antenna mast, and the other is above the welding line. At each level, 32 fiber optic strain sensors are attached to the surfaces of eight steel-tube columns with four sensors each. On each column, two fiber optic strain sensors are aligned longitudinally at two opposite sides of the steel-tube column. Similarly, two sensors are tangent to the surface. At the upper level, 16 fiber optic temperature sensors are also attached to the surfaces of the eight steel-tube columns with one sensor at each opposite side. In total, 64 fiber optic strain sensors and 16 fiber optic temperature sensors are mounted on the antenna mast.

**DATA ACQUISITION SYSTEM FOR HYBRID VIBRATION CONTROL**

The vibration control system for mitigating wind-induced vibration of the main tower is a hybrid system, which consists of two TMDs coupled with two AMDs as shown in Figure 3. The TMDs are two water tanks located at the floor of 438 m height for firefighting purpose. The two water tanks have a weight of 600 tons each totaling 1,200 tons, which is about 0.6% of the total weight of the main tower. On the top of each water tank, one AMD with a weight of 100 tons is seated. Because the operation of the AMDs requires the structural conditions as its input, a data acquisition system is developed to feed the acceleration, velocity and displacement information to this control system.

![Figure 3: Hybrid vibration control system of main tower](image)

The data acquisition system for hybrid vibration control of the main tower has a total of 20 sensors. 12 accelerometers are installed on the main tower, water tanks, and antenna mast to measure the acceleration responses: four accelerometers are respectively deployed at 1/4, 1/2, 3/4 and 438 m height level of the main tower with one sensor at each level; four sensors are placed at the centroids of the water tanks with two on each water tank; and four sensors are mounted on the antenna mast at two levels of 529 m and 578 m high with two at each level. Four velocity meters are used to monitor the velocity of the main tower with one sensor at 1/4, 1/2, 3/4 and 438 m height level, respectively. Four displacement transducers are employed for measurement of the displacements of the water tanks with two sensors for each water tank. The sensors deployed for this data acquisition system are summarized in Table 2. The selected sensor locations are expected to suffer large structural responses, which coincide with the sensor locations of the SHM system for the main tower. As a redundancy, the accelerometers in the SHM system for the main tower will be used as the backup sensors when the accelerometers in the data acquisition system for the hybrid vibration control of the main tower are malfunctioned. The redundancy of sensors also enables a cross validation of the monitoring data. In addition, the hybrid vibration control system for the main tower will make references to the monitoring data collected by the SHM systems for the main tower and the antenna mast, such as the wind and seismic input and response data, in decision making on its activation or locking.
TABLE 2
SENSORS DEPLOYED FOR HYBRID VIBRATION CONTROL OF MAIN TOWER

<table>
<thead>
<tr>
<th>No.</th>
<th>Sensor type</th>
<th>Monitoring items</th>
<th>Number of sensors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Accelerometer</td>
<td>Acceleration</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>Velocity meter</td>
<td>Velocity</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Displacement transducer</td>
<td>Displacement</td>
<td>4</td>
</tr>
</tbody>
</table>

SUMMARY

This paper outlines the SHM system for the antenna mast of GTST and the data acquisition system for hybrid vibration control of the main tower of GTST. The SHM system for the antenna mast comprises nearly 90 sensors in four types for measurement of wind speed and direction, inclination, acceleration, and strain and temperature. The data acquisition system for hybrid vibration control of the main tower measures the acceleration, velocity and displacement responses by making use of a total of 20 sensors. The two systems make the existing SHM for GTST (mainly the main tower) more complete and effective. Analysis of the monitoring data from the two systems will be carried out in future work.

ACKNOWLEDGEMENTS

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REFERENCES

CRITICAL DISTANCE METHOD TO PREDICT THE FATIGUE STRENGTH FOR WELDED STEEL STRUCTURES

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KEYWORDS
welded steel structure, fatigue strength, fatigue notch factor, effective stress concentration factor, residual stress, area method, critical distance method

ABSTRACT
This paper is concerned with the basis of the area method which is taking the mechanical parameters of the semi-circular region near the weld to control the fatigue fracture. The numerical simulation was made to residual stress field by finite element analysis, which used the APDL to write the mobile heat-load program and "element of kill and birth" to simulate the welding heat input and weld formation. Then established the critical distance method which considered the structure's geometric style, load type, stress ratio, residual stress field to predict the fatigue strength of welded steel structures, and the typical types of welded steel joints were theoretically predicted and found that the predictions are in good agreement with the experimental values. The application of the critical distance method to predict the fatigue strength of welded steel has a significant reduction in fatigue experiments, so it has a certain reference value to the application of engineering.

INTRODUCTION
Due to many advantages of welded steel structures, it has been widely applied into industry, marine engineering, architecture and bridge construction. Fatigue fracture, a major form of steel structure failure under the alternating load, makes the fatigue assess of welded joint be the main issue we paid attention to in the project. However, the impact of fatigue strength has close relations with many factors such as geometric style, load type, stress ratio and residual stress etc. thus, it makes the research on fatigue strength become more complicated. The experimental method, which exerts a special load on the steel structure, is the main method to obtain fatigue strength. Using the least square method, the S-N curve has been obtained based on the experimental result[1]. Whereas, the reason of the experiment needs great effort and takes more time, while makes the prediction of fatigue strength miscellaneous with trifles, brings much restrictions on applying this method into the project.
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The international institute of welding has been paying a great importance to the local approach in recent years. French scholar Janosch etc. put forward by optimizing the geometry of fatigue loaded fillet welds to analyze fatigue strength[2-3]. Owing to it takes the local weld mechanical parameter into consideration, this method, which can reflect the actual situation of fatigue fracture, has been becoming a new research hotspot at present. Taylor etc. advanced critical distance method which includes point method, line method, area method and volume method[4-7], on the basis of field intensity analysis of infinity plate with center through-thickness crack under tensile load. Later, it was extended to predicting fatigue strength by eliminating residual stress in welded joints[8]. Adib etc. established the method of volumetric approach [9-10] for fatigue life prediction in notch components, and applied it to welded steel structure. Chen Jun-mei etc. used local approach to predict fatigue strength of cross joint in steel Q235B[11], the result shows that this method is fully suitable for steel Q235B. Above-mentioned proves the validity of using local approach. Nevertheless, due to the distributing of residual stress field and the mechanism of affecting fatigue strength are too complicated, it makes a tough thing to quantitatively describe the influence to the fatigue strength. A great deal of experiment shows that residual stress field has a relatively big influence to the fatigue strength. Therefore, it’s significant meaningful to analyze critical distance method for predicting fatigue life with the consideration of residual stress field.

THE AREA METHOD

As shown in Figure 1 there is an infinite plate with a through crack was loaded by uniform tensile stress $\sigma$.

![Figure 1: One point stress around crack tip in an infinite plate](image)

According to the analysis of linear elastic fracture mechanics theory, any points’ stress near the crack tip can be written by Eqn.1.

$$
\begin{align*}
\sigma_x &= \sigma \left( \frac{a}{2r} \cos \frac{\theta}{2} (1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) \right) \\
\sigma_y &= \sigma \left( \frac{a}{2r} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) \right) \\
\tau_{xy} &= \sigma \left( \frac{a}{2r} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right)
\end{align*}
$$

Crack stress intensity factor $\Delta K$ can be defined as:

$$
\Delta K = M \sigma (\pi a)^{1/2}
$$

where $M$ is geometric modified factor, because the cracking will be terminated when the crack is effected below the threshold Stress value. Therefore, we may obtain the crack sample’s fatigue limit:

709
\[ \sigma_{oc} = \Delta K_{th} / M(\pi a_c)^{1/2} \]  \hfill (3)

where \( \Delta K_{th} \) is the threshold stress intensity factor. Through the introduction of material constants of \( l_0 \), Topper amended Eqn. 3 as:

\[ \sigma_{oc} = \Delta K_{th} / M[\pi(a_c + l_0)]^{1/2} \]  \hfill (4)

here \( l_0 \) is a constant related with the material which is determined by:

\[ l_0 = \frac{1}{\pi} \left( \frac{\Delta K_{th}}{M\sigma_{-1}} \right)^2 \]  \hfill (5)

where \( \sigma_{-1} \) is the fatigue limit of smooth material specimens as shown in Figure 1, the infinite plate with a through crack is loaded by a uniform tensile stress \( \sigma \), the elastic stress \( \sigma(r) \) along the loading direction in crack polar coordinates plane can be concluded as equation with parameter deciding by the distance from the crack tip.

\[ \sigma(r) = \sigma/[1 - (a_c/(a_c + r))^2]^{1/2} \]  \hfill (6)

when \( r \ll a_c \), the Eqn. 6 can be changed to:

\[ \sigma(r) = \sigma(a_c/2r)^{1/2} \]  \hfill (7)

expanding the stress function on the style in any directions to get the expression of \( \sigma_y \):

\[ \sigma(r) = \sigma(a_c/2r)^{1/2} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) \]  \hfill (8)

We take the average stress on the direction of \( \sigma_y \) in semi-circular region near the crack tip as shown in Figure 2.

![Figure 2: Semi-circular region ahead of the crack tip](image)

So:

\[ \sigma_{av} = \frac{2}{\pi l_0^2} \iint \sigma_y dA = \frac{2}{\pi l_0^2} \iint \sigma(\frac{a_c}{2r})^{1/2} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}) dA = 1.1\sigma_{-1} \]  \hfill (9)

neglected the factor of 1.1 then it changes to Eqn. 10:

\[ \sigma_{av} = \sigma_{-1} \]  \hfill (10)
It can be learned that the average stress of the semi-circular region with the radius of \( l_0 \) along the loading direction near the crack tip is equivalent to the fatigue limit of smooth specimen. And applies this conclusion to the notch specimen, the fatigue notch factor can be obtained, because the welded steel structure forms a geometric notch near the weld, so the above conclusions also apply to welded steel.

**THE ANALYSIS TO RESIDUAL STRESS OF WELDED STEEL JOINTS**

Due to the inhomogeneity of residual stress and thermal stress' field which are generated by welding, excessive deformation and crack initiation of welding specimen, which is extremely detrimental to the structure, have been appeared in the welding joint. However, the general analysis of the residual stress is very difficult. The finite element analysis is convenient, fast, accurate etc. So use it to analyze the residual stress field of the steel components is very appropriate.

Welding simulation is a plate with V-shaped groove, thickness 4mm, materials for low-alloy steel, the yield strength for 441MPA, the middle solder for aluminum, the yield strength for 550MPA, selected the 13th two-dimensional element to establish the solid model. When the mesh to be improved the accuracy of calculation in the weld department with smaller meshing size, at both ends of it with the larger meshing size. In order to facilitate the element's birth, restart of the analysis and re-load operation, at both ends of the plane use mapped meshing but the free meshing in the middle.

Assuming no convection and radiation, structural model of the left, right and bottom surface are constrained. The left and the right side are given the 30°C of temperature constraints. The reference temperature of low-alloy steel is 30°C and 1500°C of the solder. Ignore the structural analysis of the transient effect, only consider the transient effects of thermal analysis. Through the APDL to write subroutine of mobile thermal loading and the "element of kill and birth" to simulate the heat input and formation of welding, the middle solder is divided into a total of 147 elements, welding process is composed with packing of solder and cooling. A total of 735 seconds of welding time, cooling time is 50000 seconds. When welding is finished, the maximum residual stress occurred in the connections of steel and the solder, the maximum value of residual stress is 346MPA. Deformation occurred in the welding seam, this is because of the compression plastic deformation while the heat is loading and the contraction after cooling. This is also the same with the actual welding deformation.

**CRITICAL DISTANCE THEORY OF WELDED STEEL STRUCTURES**

The conclusions of Area Method to predict fatigue strength of welding steel structure is applied to notch specimen, the average stress at the region with the radius of \( l_0 \) near the notch can be obtained through numerical calculation. Calculate the ratio between average stress and stress, the fatigue notch factor can be obtained, and then get the fatigue limit. Residual stress was generated at jointing process will affect the fatigue strength. Thus, limited fatigue notch factor is not able to measure fatigue strength. The fatigue strength of welding steel structure was affected by both limited fatigue notch factor and residual stress, and then take affective stress concentration factor to describe fatigue strength of welded steel structure. The affective stress concentration factor of \( \beta_R \) under unsymmetrical cyclic can be defined as:
\[ \beta_R = \frac{\sigma_R}{\sigma_{RW}} \]  

(11)

Here, \( \sigma_R \) is the fatigue limit of smooth materials specimen when the stress ratio is \( R \); \( \sigma_{RW} \) is the fatigue limit of welding steel when the stress ratio is \( R \). \( \beta_R \) has a relationship with materials, geometrical type, loading type, Stress gradient, stress ratio and residual stress, etc. It is an eigenvalue that reflects comprehensively the fatigue strength of welding steel. Usually, \( \beta_R \) can be determined by experiment. Through an amended Basquin equation, the fatigue strength of smooth materials specimen when the average stress \( \sigma_m \) is existent can be denoted as:

\[ \sigma_{Ra} = (\sigma'_f - \sigma_m)(2N)^b \]  

(12)

Here, \( \sigma_{Ra} \) is the stress range when the stress ratio is \( R \); \( N \) is the number of stress cycles; \( \sigma'_f \) is the fatigue strength factor; \( b \) is the fatigue strength index. The relationship between average stress and stress range is shown in Eqn. 13:

\[ \sigma_m = \frac{1+R}{1-R} \sigma_{Ra} \]  

(13)

Thus, Eqn. 13 in Eqn. 12 and let stress range when \( N = N_L \) be the fatigue limit, the equation for the fatigue limit \( \sigma_R \) when the stress ratio is \( R \) can be concluded as:

\[ \sigma_R = \sigma'_f (2N_L)/[1+\frac{1+R}{1-R}(2N_L)^b] \]  

(14)

Because of the existence of notch effect and residual stress in weld edge, the equation of average stress which considers the notch through Morrow can be obtained:

\[ K_f \sigma_{RWa} = (\sigma'_f - \sigma_r - \sigma_m)(2N_L)^b \]  

(15)

Here, \( K_f \) is the fatigue notch factor and \( \sigma_{RWa} \) is the stress range of welded steel when the stress ratio is \( R \). Let the stress range when \( N = N_L \) be the fatigue limit \( N_L = 2\times10^6 \) in the paper. Considering Eqn. 15, \( \sigma_m \) and \( \sigma_{RWa} \) meet the relationship of Eqn. 13, so the fatigue limit of welded steel when the stress ratio is \( R \) can be obtained:

\[ \sigma_{RW} = \frac{(\sigma'_f - \sigma_r)(2N_L)^b}{K_f + \frac{1+R}{1-R}(2N_L)^b} \]  

(16)

Thus, through Eqns. 11, 14 and 16, the equation to predict affective stress concentration factor when welded steel is under unsymmetrical cyclic load can be obtained:

\[ \beta_R = \frac{K_f + \frac{1+R}{1-R}(2N_L)^b}{1+\frac{1+R}{1-R}(2N_L)^b} \times \frac{\sigma'_f}{\sigma'_f - \sigma_r} \]  

(17)

When stress ratio \( R = -1 \):
\[ \beta_1 = K_f \frac{\sigma'_f}{\sigma'_f - \sigma_r} \]  

(18)

Therefore, based on the finite element analysis, using area method Eqn. 10, the fatigue notch factor is obtained. The effective stress concentration factor can be acquired through the Eqn. 17 and Eqn.18. So the prediction of the fatigue strength for welded steel is also obtained. We would name this theory as the critical distance method to predict the fatigue strength for welded steel structure.

EXPERIMENTAL VERIFICATION OF CRITICAL DISTANCE METHOD

In order to verify the accuracy of the critical distance method which established above, we compared theoretical calculation with the experimental values. The material constants which are used in calculating are the same with the experimental data. In accordance with the critical distance method, through the finite element analysis to establish respectively model of Figure 3 – Figure 5 shown in butt joint, T-shape joint and cruciform joint.

FIGURE 3: butt joint

FIGURE 4: T-shape joint

FIGURE 5: cruciform joint

Adopting with Plane82 of two-dimensional and 8-nodes structural solid element in ANSYS software to mesh the model, the butt joint was divided into 2691 elements and 8006 nodes, 2297 elements, 6108 nodes of T-shape joint, and 2506 elements, 7607 nodes of the cruciform joint. Each joints’ surface is loaded by the tensile load of 100N/m². Then obtained its structure stress field, semi-circular radius \( l_0 \) is determined by Eqn. 5, thus, where \( \Delta K_{th} = 294 N/mm^{3/2} \) and \( \sigma_{th} = 548 MPa \), correction coefficient \( M = 1.12 \). Use Eqn. 10 to calculate the average stress of the semi-circular area, which is the fatigue limit of the
structure. In the state of unsymmetrical cycle load (that is \( R = 0.2 \)), the following equation is used for calculating the constants:

\[
b = -\frac{1}{6} \log \left( \frac{2(1 + \frac{344}{\sigma_{-1}})}{\sigma_f} \right)
\]

\[
\sigma'_f = 345 + \sigma_{-1}
\]

(19)

where \( \sigma'_f \) is the fatigue strength coefficient. By Eqn.19 we may obtain: \( b = -0.0854 \), \( \sigma'_f = 894 \text{MPa} \), \( N_L = 2 \times 10^6 \). The maximum residual stress is 346MPA through the finite element analysis. Therefore, we can work out the fatigue notch factor, and then through the Eqns. 17 and 18 to calculate the effective stress concentration factor of structure. The results were listed in Table 1 and compared with the experimental value of the reference [13].

### TABLE 1

<table>
<thead>
<tr>
<th>Structure shape</th>
<th>( K_f ) (Calculation)</th>
<th>( \beta_{AA} ) (Calculation)</th>
<th>( \beta_{-1} ) (Experiment)</th>
<th>( \beta_{0,2AA} ) (Calculation)</th>
<th>( \beta_{0,2} ) (Experiment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butt joint</td>
<td>1.27</td>
<td>2.07</td>
<td>2.37</td>
<td>1.95</td>
<td>1.97</td>
</tr>
<tr>
<td>T-shape joint</td>
<td>1.39</td>
<td>2.27</td>
<td>2.55</td>
<td>2.09</td>
<td>2.11</td>
</tr>
<tr>
<td>Cruciform joint</td>
<td>1.48</td>
<td>2.41</td>
<td>2.91</td>
<td>2.21</td>
<td>2.35</td>
</tr>
</tbody>
</table>

It can be learned from the table 1 that the prediction of the effective stress concentration factors are in good agreement with the experimental value, which verify the effectiveness of predicting the fatigue strength of welded steel structures with critical distance method.

### CONCLUSIONS

1. This paper presented a method which considered the affect of geometric style, load type, stress ratio and residual stress field to the structure, take the mechanical parameters of semi-circular region near the weld to control the fatigue fracture and established the theory of critical distance method to predict the fatigue strength for welded steel structure.

2. The numerical simulation was made to welded steel structure by finite element analysis. And the residual stress field was obtained. The technology of "element of kill and birth" can be effectively simulated the heat input and weld formation and comparatively reflect the actual conditions of thermal effects of welding. So it is able to reduce the costs and improve the efficiency of research.

3. The typical types of welded steel were theoretically predicted with the critical distance method and found that the calculation results are in good agreement with the experimental value to verify the effectiveness of this method. Because it will reduce a lot of fatigue experiments. Therefore this method has economic value and is useful for practical engineering application.
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FAILURE CRITERIA FOR COMPOSITE SLABS SUBJECT TO EXTREME LOADING CONDITIONS

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KEYWORDS

Composite slabs, extreme loading, fire conditions, failure criteria, strain concentrations

ABSTRACT

This paper is concerned with the ultimate behaviour of composite steel/concrete floor slabs under extreme loading situations, particularly those that occur during severe building fires. The study focuses on the failure state associated with rupture of the reinforcement in composite slab members which become lightly reinforced in a fire situation due to the early loss of the steel deck. An account of a series of large scale ambient tests, undertaken on full slab members, is presented in the paper. The experimental arrangements are described together with the details of the specimens. Complementary analytical studies, carried out to assess the salient factors influencing the failure of composite slab members are also summarised. The assessments utilise detailed numerical models which adopt novel finite element formulations including geometric and material nonlinearities, as well as simplified analytical models for the prediction of failure deformations and associated load levels. The results of this investigation offer detailed insights into the key factors that govern the ultimate behaviour of composite floor systems under extreme loading conditions, and provide simplified tools which are suitable for implementation in performance based design procedures.

INTRODUCTION

The performance of buildings with composite steel/concrete floors under extreme loading conditions has been the subject of extensive research investigations in recent years, e.g. [1-6]. Particular attention has been given to structural fire response after it was observed during real fires (e.g. the Broadgate and Basingstoke fires) that buildings with composite floors had an inherent resistance against failure greater than that accounted for in design. Moreover, large-scale fire tests were conducted in the UK by the Building Research Establishment (BRE) and Corus (formerly British Steel) on the full-scale eight-storey building at Cardington [1, 7]. The findings of these tests, coupled with other numerical and experimental studies [e.g. 2-6], have verified the important role played by the composite floor slab in carrying the gravity loading
within the fire compartment, even after the loss of strength in the supporting secondary steel beams due to elevated temperature. Although the slab exhibits significantly lower bending capacity, the development of tensile membrane action coupled with several sources of over-design leads to considerable fire resistance capabilities. To this end, progress in the development of improved design approaches needs to be based on detailed assessment of the behaviour of floor slabs, using reliable and realistic modelling approaches coupled with the application of appropriate failure criteria.

One of the most important failure criteria for composite slabs is that related to rupture of the reinforcement. Under fire conditions, in addition to the possible failure of unprotected secondary steel beams, the thin steel deck within a composite slab incurs high temperature and becomes largely ineffective at an early stage. Thus, the slab behaves primarily as a concrete element with light mesh reinforcement which can experience large deflections thereby enabling the development of tensile membrane action. Prediction of the displacement and load levels corresponding to the fracture of the reinforcement is however a complex issue that necessitates a detailed treatment of the interaction between the concrete material and steel reinforcement, with due account of the appropriate loading and boundary conditions. Due to the uncertainties involved in various important material and response parameters, this problem also requires experimental validation and calibration.

This paper provides an overview of recent studies carried out to examine the performance of floor slabs. A series of idealised ambient slab tests are described, including the experimental set-up, specimen details and key results and observations. Particular emphasis is given to identifying and assessing the salient parameters with a view to investigating the appropriate failure criteria. Although the experiments described herein were conducted at ambient temperature, this is believed to be a significant step and essential precursor towards quantifying the elevated temperature behaviour, as well as the response under other extreme loading conditions. In addition, the main focus in this paper is on the response of simply-supported flat slabs, although the wider test programme has included tests on both one- and two-way spanning elements, with various restraint conditions and cross-sectional properties; the results of these can be found elsewhere [8].

After describing the experimental arrangement and discussing the main test results, the paper discusses the analytical procedures which have been developed to further investigate the response. These include a simplified analytical model which can represent the member response up to failure, and also more complex nonlinear finite element simulations. Importantly, the comparative assessments enable the calibration of realistic levels of idealised bond properties that can be used in analytical models for predicting the ultimate response. Although the work presented in this paper is restricted to simply-supported slabs without axial or rotational restraint, it accounts for the influence of key material and geometric parameters and can be readily adjusted to account for other structural and loading configurations.

EXPERIMENTAL PROGRAMME

A large number of ambient tests have been conducted on two-way spanning reinforced concrete slabs (out of which only eight tests are discussed herein) with a view to (i) gaining a greater understanding of the mechanisms dominating ultimate behaviour; (ii) assessing and quantifying the key parameters influencing behaviour; and (iii) establishing the appropriate
failure criteria. A description of the testing arrangement is included herein together with the details of the materials and specimens examined.

Testing arrangement

The experiments were conducted in a purpose-built testing arrangement consisting of four large steel sections which were of sufficient strength and stiffness to resist the applied loads. A schematic of the testing arrangement is presented in Fig. 1. The steel sections were positioned on four large concrete blocks at each corner and these were, in turn, fixed to the laboratory strong floor. The slabs were free to move both axially and rotationally at the edges and the arrangement could be readily modified to accommodate either rectangular or square members by adjusting two of the steel beams. Due to the nature of the behaviour, involving significant membrane action, a high-precision large-stroke actuator, operating in displacement-control, was utilised. Load was applied to the slab through 12 widely-spaced points, through a system of hinges attached to the actuator, in order to simulate distributed loading. Careful consideration was given to ensuring that an equal load was applied at each of the twelve points and also that the application remained vertical for the duration of the experiment. As a result, a system of square hollow sections, steel plates and ball joints was configured. The maximum vertical deflection in the centre of the slab was measured using a displacement transducer.

Figure 1: Experimental arrangement of slab tests

Specimen details

This paper discusses the results of eight selected slab tests which included specimens with various geometric and material properties. As mentioned previously, these tests formed part of a wider test programme, the details of which can be found elsewhere [8]. The ultimate behaviour is particularly sensitive to several reinforcement characteristics, such as those related to the reinforcement ratio and the development of bond strength; accordingly, several tests were designed to examine these effects. To this end, the bond strength was inherently varied within the test programme by utilising both typical deformed bars (D6) and also welded mesh reinforcement (M6), thus providing a realistic assessment of the prevalent characteristics. Both reinforcement types had similar values of yield and ultimate strength which were approximately 550 and 600 N/mm², respectively. On the other hand, the ultimate strain exhibited by the reinforcement at fracture varied from 2.5% for M6 to 4% for D6. Careful attention was given to using equipment that can provide the full stress-strain relationship of the reinforcement bars up to fracture, as the actual shape of the curve can have a significant influence on the behaviour. For the concrete, an average compressive strength of about 40 N/mm² was considered in each of the tests.
TABLE 1
SLAB TEST DETAILS

<table>
<thead>
<tr>
<th>Test</th>
<th>$L_1$ (mm)</th>
<th>$L_2$ (mm)</th>
<th>$h$ (mm)</th>
<th>Bar Type</th>
<th>$\rho_1$ (%)</th>
<th>$\rho_2$ (%)</th>
<th>$F_u$ (kN)</th>
<th>$F_{f,\text{test}}$ (kN)</th>
<th>$U_{f,\text{test}}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>2250</td>
<td>1500</td>
<td>40</td>
<td>D6</td>
<td>0.35</td>
<td>0.35</td>
<td>32.3</td>
<td>56.6</td>
<td>90</td>
</tr>
<tr>
<td>S2</td>
<td>2250</td>
<td>1500</td>
<td>60</td>
<td>D6</td>
<td>0.24</td>
<td>0.48</td>
<td>48.4</td>
<td>104.5</td>
<td>84</td>
</tr>
<tr>
<td>S3</td>
<td>2250</td>
<td>1500</td>
<td>60</td>
<td>D6</td>
<td>0.24</td>
<td>0.24</td>
<td>40.4</td>
<td>72.5</td>
<td>76</td>
</tr>
<tr>
<td>S4</td>
<td>1500</td>
<td>1500</td>
<td>60</td>
<td>D6</td>
<td>0.24</td>
<td>0.24</td>
<td>51.3</td>
<td>87.6</td>
<td>68</td>
</tr>
<tr>
<td>S5</td>
<td>1500</td>
<td>1500</td>
<td>60</td>
<td>D6</td>
<td>0.52</td>
<td>0.52</td>
<td>108.8</td>
<td>167.5</td>
<td>63</td>
</tr>
<tr>
<td>S6</td>
<td>2250</td>
<td>1500</td>
<td>60</td>
<td>M6</td>
<td>0.24</td>
<td>0.24</td>
<td>46.3</td>
<td>71.1</td>
<td>69</td>
</tr>
<tr>
<td>S7</td>
<td>2250</td>
<td>1500</td>
<td>60</td>
<td>M6</td>
<td>0.24</td>
<td>0.24</td>
<td>48.6</td>
<td>82.2</td>
<td>64</td>
</tr>
<tr>
<td>S8</td>
<td>1500</td>
<td>1500</td>
<td>60</td>
<td>M6</td>
<td>0.24</td>
<td>0.24</td>
<td>46.3</td>
<td>78.3</td>
<td>74</td>
</tr>
</tbody>
</table>

Table 1 provides the relevant geometric and material properties pertaining to each slab, including the length of the long and short spans ($L_1$ and $L_2$, respectively), the depth ($h$) and also the reinforcement ratio in the long and short spans ($\rho_1$ and $\rho_2$). In order to provide a realistic insight into the bond and cracking behaviour, priority was given to employing realistic values of $h$ and $\rho$, as well as bar diameter ($\varphi$), although this resulted in comparatively low span/depth ratios owing to experimental constraints. In all specimens, the reinforcement was positioned at mid-depth of the section, with the longer bars placed at a greater effective depth than those across the short span.

**Test Results**

A large amount of data was obtained through the measurement of displacements, loads and strains in the tests. However, emphasis is placed herein on the deformation at failure of the specimens, which corresponds to fracture of the reinforcing bars. The overall load-displacement response obtained for S1 to S8 are given in Figs 2a and 2b below. In addition to the figures, the key experimental data relating to the failure load attained ($F_{f,\text{test}}$) and the corresponding failure displacement ($U_{f,\text{test}}$) is provided in Table 1. Also included in the table are the theoretical ultimate loads ($F_u$) according to classical yield line theory. All tests failed by fracture of the reinforcement across a localised through-depth crack. This type of failure was typically accompanied by a loud and distinctive noise, as well as a sudden drop in load.

![Figure 2](image_url)

(a) (b)

Figure 2: Load-displacement response for specimens with (a) D6 and (b) M6 reinforcement

Each slab surpassed the theoretical ultimate load, confirming the development of membrane action in all cases. The ratio of $F_{f,\text{test}}$ to $F_u$ varied between around 1.5 (Slab S6) and 2.2 (Slab
S2). The scale of the load enhancement due to membrane action is directly related to the ductility of the reinforcement as low-ductility steel causes premature failure, thus preventing the large deflections necessary for significant tensile catenary action. The maximum load achieved was 167.5 kN in S5, which contained the greatest reinforcement ratio of all the tests whereas Slab S1 exhibited the lowest load-carrying capacity and failed at 56.6 kN. Analysis of the test results indicate that many factors influence the limiting levels of load and displacement that can be sustained by the slab. The load-carrying capacity and also the ability of the specimen to develop membrane forces is positively dependent on the type of reinforcement, the reinforcement ratio and the slab depth. On the other hand, the failure displacement is directly related to the crack pattern that develops as greater cracking results in a lower concentration of strain in the reinforcement, thereby delaying failure. In this respect, $U_{f,\text{test}}$ was shown to be directly proportional to the reinforcement ductility, the reinforcement ratio (as this effects the level of cracking) and also the aspect ratio. However, failure is expedited for relatively deeper slabs. The results demonstrate that there are a number of inter-related parameters which influence the ultimate behaviour of floor slabs, hence necessitating in the development of realistic analytical procedures that can capture the influence of these key factors.

**ANALYTICAL MODELLING**

The experimental results presented in the previous section furnished direct information on the relative influence of a number of geometric and material parameters on the ultimate response of simply supported floor slabs. In order to provide further insight into the behaviour, and to enable quantification of the response for the purpose of future design studies, there is a need for suitable analytical models that are validated and calibrated against experimental results and detailed numerical simulations. To this end, a simplified analytical model has been developed at Imperial College to predict the post-yield load-deflection response of floor slabs, as well as the level of deformation and load corresponding to failure, both at ambient and elevated temperature [9]. The approach provides a simple and realistic estimation of the ultimate conditions with due account of main geometric and material characteristics including the important influence of bond-slip. A detailed description of the procedure can be found elsewhere [9], but an outline of the key aspects and formulations are provided herein.

The model is developed building on an earlier one-dimensional simplified strip model [10, 11] and therefore assumes that the slab comprises a series of strip elements through the length and width of the element. The overall response is obtained by integrating the response of each strip (Fig. 3). It is assumed that cracks form in the locations predicted by conventional yield line theory and along the short span in the centre of the slab. The steel is assumed to have a rigid-hardening constitutive relationship defined by the yield strength $f_y$, ultimate strain $\varepsilon_u$ and hardening modulus $E_2$. On the other hand, the bond-slip relationship is idealised as a rigid-plastic relationship with a constant bond strength $\sigma_b$; this has been validated elsewhere [8]. Furthermore, the various parts of the slab, that are bounded by the full-depth cracks, are free to rotate both in-plane and out-of-plane in a rigid manner. Failure occurs by rupture of the reinforcement at mid-span in the short direction across the through-depth crack.
Figure 3: Quarter of the slab for formulation of simplified analytical model [9]

The width of each crack ($\Delta_x$) is calculated as a function of the vertical deflection ($U$) and then employed to obtain the total force in the steel across each crack ($T_x$). This is based on the bond strength that exists between the steel and the concrete and therefore accounts for strain concentration in the reinforcement. The total energy dissipated from the reinforcement as the slab deforms is determined as

$$\dot{D} = T_x \frac{d\Delta_x}{dU} \quad (1)$$

On the other hand, under a uniformly distributed load $q$, the total work done is equal to $q$ multiplied by the volume created by the slab as it deforms. Thus, the rate of external work performed by $q$ over the quarter slab is given by

$$\dot{E} = q \frac{L L_s (3 - 2\eta)}{24} \quad (2)$$

where $\eta$ is the yield line geometric parameter. Equilibrium is enforced by equating the rates of overall energy dissipation and the external work, enabling the determination of $q$ for a specific $U$. This leads to a load-deflection relationship, which accounts for the development of stress concentrations within the reinforcement at crack locations and depends on the bond strength, material response of steel reinforcement, and crack widths as influenced by the slab deflection. Moreover, the level of load and deflection corresponding to rupture of the reinforcement can be obtained by assuming that the bond-slip length, $x_d$, is bounded by half the distance between the crack and the intersection of the yield lines

$$x_d = \frac{1}{2} \left( \frac{L_s}{2} - \eta L_s \right) \quad (3)$$

Thus, the predicted failure displacement ($U_{f,p}$) is obtained from either Eq. 4(a or b) according to

$$U_{f,p} = \frac{L}{2 A_s E_2 \sigma_b} \left( T_u - T_y \right) \quad \text{if } x_d \geq \frac{T_u - T_y}{\sigma_b} \quad (4a)$$

$$U_{f,p} = \frac{L}{2 A_s E_2 \sigma_b} \left( (T_u T_y + \sigma_b x_d)^2 - 2\sigma_b^2 x_d^2 \right) \quad \text{if } x_d < \frac{T_u - T_y}{\sigma_b} \quad (4b)$$

where $A_s$ is the area of steel, and $T_u$ and $T_y$ are the ultimate and yield forces in the reinforcement.

Further verification of the load-displacement response was carried out through detailed finite element simulations using the advanced nonlinear analysis program ADAPTIC [12]. For brevity, a full description is not included herein but can be found elsewhere [8]. The program
employs 2D shell elements [13,14] which combines computational efficiency with numerical accuracy and accounts for both geometric and material non-linearities. However, since the conventional smeared crack representation is used in the finite element models, they do not account for bond-slip hence cannot capture the strain concentration that occurs in the reinforcement across cracks. However, the finite element simulations can still provide valuable information regarding the behaviour of the floor slabs under extreme loading conditions [13,14], particularly in terms of the overall load-displacement response.

COMPARITIVE ASSESSMENT

This section compares the experimental results to the analytical predictions of both the simplified analytical model (hereafter referred to as the SAM) and the finite element model (denoted as the FEM). Two experiments (S1 and S8) are selected for detailed analysis of the load-deflection performance; these slabs incorporate both reinforcement types. The FEM analysis employs a mesh comprising 30×20 uniform-thickness shell elements, based on a mesh sensitivity assessment. On the other hand, the SAM is based on rigid-plastic hardening behaviour and hence the elastic and elasto-plastic displacements are not included. This procedure terminates upon either fracture of the reinforcement or crushing of the concrete and hence the final point on each curve represents failure. In addition to the SAM and FEM predictions, simulations from another analytical model, which has been developed by BRE [2,3], are also provided. This method assumes a similar crack pattern to the SAM and also ignores the elastic and elasto-plastic stages of the response. Unlike the SAM which is based on an assumed kinematic mode, the BRE approach is based on an assumed internal stress distribution.

The experimental load-deflection responses for S1 and S8 are presented in Figs 4a and 4b, respectively, together with the corresponding analytical simulations. It is evident that the FEM predicts the elastic response and cracking load reasonably well in both cases. The experimental responses generally exhibit greater strain hardening properties in the plastic range than is depicted by the model as the smeared-crack solution procedure cannot reliably assess the strain concentrations across cracks. Significant cracking occurred in both tests and, as a consequence, the bond-slip length and the length of reinforcement undergoing plastic deformation was greater than if less cracks had developed. In spite of this, the overall load-deflection response simulated by the FEM compares reasonably well with the test results. On the other hand, the initial load resistance predicted by the SAM corresponds to a value very close to the yield line capacity ($F_u$ in Table 1). As the displacement increases further, the predicted behaviour correlates well with the experimental response in the plastic range. The prediction of maximum load capacity is almost identical to the actual behaviour. It is also evident that the BRE prediction provides a reasonable correlation with the experimental response. The SAM response is generally stiffer than the BRE proposal as it is based on an assumed kinematic mode rather than an internal stress distribution. In summary, the favourable comparisons provided through these examples demonstrate the reliability of the kinematic expressions and the corresponding load-deflection response characteristics of the proposed model.
One of the most important aspects of the proposed SAM model is establishing the appropriate levels of load and displacement corresponding to failure. To this end, the failure displacements and loads obtained from the simplified analytical model ($U_{f,\text{SAM}}$ and $F_{f,\text{SAM}}$, respectively) are included in Table 2, together with the corresponding test values. Also included are the predicted failure displacements according to the BRE model, $U_{f,BRE}$.

### TABLE 2

**FAILURE ANALYSIS**

<table>
<thead>
<tr>
<th>Test</th>
<th>Bar Type</th>
<th>$F_u$ (kN)</th>
<th>$F_{\text{foot}}$ (kN)</th>
<th>$F_{f,\text{SAM}}$ (kN)</th>
<th>$U_{\text{foot}}$ (mm)</th>
<th>$U_{f,\text{SAM}}$ (mm)</th>
<th>$U_{f,BRE}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>D6</td>
<td>32.3</td>
<td>56.6</td>
<td>56.4</td>
<td>89.8</td>
<td>89.2</td>
<td>51</td>
</tr>
<tr>
<td>S2</td>
<td>D6</td>
<td>48.4</td>
<td>104.5</td>
<td>88.5</td>
<td>83.6</td>
<td>82.8</td>
<td>51</td>
</tr>
<tr>
<td>S3</td>
<td>D6</td>
<td>40.4</td>
<td>72.5</td>
<td>75.5</td>
<td>76.2</td>
<td>75.1</td>
<td>51</td>
</tr>
<tr>
<td>S4</td>
<td>D6</td>
<td>51.3</td>
<td>87.6</td>
<td>87.2</td>
<td>68.4</td>
<td>66.8</td>
<td>34</td>
</tr>
<tr>
<td>S5</td>
<td>D6</td>
<td>108.8</td>
<td>167.5</td>
<td>167.8</td>
<td>62.9</td>
<td>63.2</td>
<td>34</td>
</tr>
<tr>
<td>S6</td>
<td>M6</td>
<td>46.3</td>
<td>71.7</td>
<td>67.7</td>
<td>68.5</td>
<td>66.6</td>
<td>50</td>
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<tr>
<td>S7</td>
<td>M6</td>
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<td>46.3</td>
<td>82.2</td>
<td>77.1</td>
<td>64.2</td>
<td>62.6</td>
<td>33</td>
</tr>
</tbody>
</table>

It is evident from Table 2 that the analytical predictions provide a reasonably accurate assessment of the experimental failure conditions. However, the results are clearly dependent on a realistic representation of the material properties, including the bond strength between steel and concrete. The test results therefore provide direct calibration of the idealized bond strength that needs to be employed in the model. As discussed previously, the bond-slip behaviour is idealised as a rigid-plastic relationship. On this basis, representative values of effective bond strength ($\sigma_b$) were determined to be in the range of 0.2-0.3 N/mm$^2$ for D6 and 0.3-0.4 N/mm$^2$ for M6. Clearly, due to the different loading and behavioural conditions, these values are considerably lower than those measured in conventional pull-out bond tests. It is also important to recall that minimum cracking is assumed in the analytical model; hence, the bond strength employed implicitly accounts for the resulting influence of any additional cracks that develop. In addition, whilst the BRE model is semi-empirical in its treatment of several key parameters, it seems to provide conservative predictions for typical ranges.
CONCLUDING REMARKS

This paper described recent studies into the ultimate behaviour of composite floor slabs, including large-scale experimental testing as well as the development of a simplified analytical model. Particular emphasis was given to the failure condition associated with reinforcement fracture. The results demonstrated the reliability and accuracy of the proposed model. Importantly, the model enables a reliable prediction of the limiting levels of load and displacement which can be sustained. It captures the influence the key geometric and material parameters including the crucial influence on bond-slip. The work described herein is part of a wider experimental and analytical study, which has investigated the effects of other material, geometric, boundary and loading conditions.

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EXPERIMENTAL TESTS ON STRUCTURAL MEMBERS FABRICATED FROM HIGH STRENGTH STEEL MATERIALS

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KEYWORDS

Design standard, flexural behaviour, experimental test, high strength steel, plate local instability, residual stress distribution, plate slenderness limit, section moment capacity

ABSTRACT

Typical high strength steels (HSS) have exceptional high strengths with improved weldability making the material attractive in modern steel constructions. However, due to lack of understanding, most of the current steel design standards are limited to conventional low strength steels (LSS, i.e. \(f_y \leq 450\) MPa). This paper presents the details of full-scale experimental tests on short beams fabricated from Bisplate80 HSS materials (nominal \(f_y = 690\) MPa). The various slenderness ratios of the plate elements in the test specimens were chosen in the range near the current yield limit (AS4100-1998, etc.). The experimental studies presented in this paper have produced a better understanding of the structural behaviour of HSS members subjected to local instabilities. Comparisons have also been presented in the paper regarding to the design predictions from the current steel standards (AS4100-1998). This study has enabled to provide a series of proposals for proper assessment of plate slenderness limits for structural members made of representative HSS materials. This research work also enables the inclusion of further versions in the steel design specifications for typical HSS materials to be used in buildings and bridges. This paper also presents a distribution model of residual stresses in the longitudinal direction for typical HSS I-sections.

INTRODUCTION

The growing demand from blooming markets led to the successful development of new construction materials, high strength steels (HSS). The new steel has excellent high strength and better weldability making it attractive for structural design in modern buildings and bridges. With their high strength, typically in the range of 500 to 900 MPa (yield stress \(f_y\)) along with reduced self-weight, it frees imaginations of contemporary architects and opens up new possibilities. The advantage of the intrinsic property of the HSS makes it feasible to achieve applications in a cost-effective manner (see Figure 1).
However, most of the current steel design specifications, including AS4100 [1], prohibit the use of HSS in design and limit to traditional low strength steel (LSS) materials (i.e. \( f_y \leq 450 \) MPa). Similar situations also exist in the UK (BS5950 [2]) and Europe (EN1993 [3]). In consequence, in Australia and around the world, structural members made of HSS are usually designed in accordance with overseas standards, such as AISC [4], which allowing the design for structures fabricated from HSS materials. On the other hand, the design provisions of AISC were mainly based on experimental and analytical studies on standard LSS (i.e., \( f_y \leq 450 \) MPa). HSS exhibit mechanical properties that are in some measure different from conventional LSS (i.e., higher yield ratio, lower ductility). Moreover, the engineers outside of the United States are unfamiliar with design procedure and approach of the AISC specifications, which explain why design engineers around the world are reluctant to use the American codes (AISC [4]) in the design of HSS members. Therefore research into the structural behaviour of HSS members is fundamental to address this shortcoming.

FULL-SCALE SHORT HSS BEAM TESTS

In order to explore the structural behaviours of HSS beams, in particular those influenced by HSS mechanical characteristics, a series of full-scale experimental tests on I-shaped beams fabricated from BISPLATE80 HSS materials (nominal \( f_y = 690 \) MPa) was conducted. The test beams adopted were relatively short and fully restrained in lateral direction. The following sections report the design and fabrication of the test specimens, test set-up and procedure, and the test results including section moment capacity and moment-deflection curves. Assessment and discussions of the structural behaviour of the fully-restrained HSS test beams are also included.

Test Specimens

As the objective of this research project was to investigate the section moment capacity of I-shaped cross-sections fabricated from typical HSS (i.e. BISPLATE80 materials), an experiment program was launched in the structural laboratory at the Queensland University of Technology (QUT) to study the structural behaviour of these sections in bending. BISALLOY STEEL is the only manufacturer in the Australasian region producing a wide range of high tensile and abrasion-resistant quenched and tempered (QT) steel plates. Previous researches (Kuhlmann [5]; Ricles, et al. [6]; Green, et al.[7]) indicated that flexural members without moment gradient are more critical than those with moment gradient Therefore the test specimen was designed as a four-point bending configuration with a constant moment
segment between the loading points as shown in Figure 2 that was provided with no twisting by the lateral restraints to the specimen at critical locations such as the supports and the load applications. The specimens include four, 3500 mm long, I-shaped short beams (see Figure 2 & Table 1) made of BISPLATE80 HSS (nominal \( f_y = 690 \text{ MPa} \)). With the testing capacities available in the structural laboratory at QUT, a nominal 8 mm thick plate of BISPLATE80 HSS for the flanges and a nominal 4 mm thick plate for the webs were selected to fabricate the test specimens. The chosen specimens comprised a combination of various slenderness ratios, which were close to the yield limits according to the current steel design specifications (namely, AS4100 [1], AISC [4], EN1993 [3], and BS5950 [2]), of plate elements within each cross-section. The nominal dimensions of the specimens are given in Table 1 (Tang [8]).

![Figure 2: Test Specimen of Short HSS Beam](image)

The design of flexural members in the conventional practice is based on a constant moment condition representing the “worst case” with reference to the occurrence of member and section stability. However, to avoid any kind of shear failure, the test specimens must be designed with sufficient shear capacity, so that flexural failure governed. Referring to the recommendation made by the SSRC (Galambos [9]), the spacing of lateral restraints on test specimens was chosen to sufficiently eliminate any lateral instability effects and to fully develop its section moment capacity within the cross-section, whereas the length of the shear segments should be greater than twice the depth of the short beam member (see Figure 2). In order to capture the desired failure modes, the section moment capacity of each specimen was also estimated performing calculations according to the current design codes (i.e. AS4100 [1], AISC [4], EN1993 [3], and BS5950 [2]), bearing in mind that some of these codes are not for the design of structural members fabricated from HSS materials. Table 1 also presents the design predictions of the section capacity for each test specimen.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( b )</th>
<th>( t_f )</th>
<th>( f_y )</th>
<th>( d_f )</th>
<th>( t_c )</th>
<th>( f_y )</th>
<th>( d_c )</th>
<th>Failure</th>
<th>( M )</th>
<th>Error</th>
<th>( M )</th>
<th>Error</th>
<th>( M )</th>
<th>Error</th>
<th>( M )</th>
<th>Error</th>
<th>( M )</th>
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</tr>
</thead>
<tbody>
<tr>
<td>NN-B1</td>
<td>130</td>
<td>260</td>
<td>765</td>
<td>401</td>
<td>281</td>
<td>385</td>
<td>331</td>
<td>219</td>
<td>29.3%</td>
<td>218</td>
<td>29.1%</td>
<td>219</td>
<td>28.8%</td>
<td>213</td>
<td>30.5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS-B2</td>
<td>160</td>
<td>8</td>
<td>765</td>
<td>401</td>
<td>281</td>
<td>385</td>
<td>331</td>
<td>295</td>
<td>26.4%</td>
<td>284</td>
<td>22.0%</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>NS-B3</td>
<td>130</td>
<td>340</td>
<td>700</td>
<td>365</td>
<td>278</td>
<td>318</td>
<td>310</td>
<td>284</td>
<td>22.0%</td>
<td>22.0%</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>SN-B4</td>
<td>160</td>
<td>260</td>
<td>700</td>
<td>311</td>
<td>213</td>
<td>261</td>
<td>242</td>
<td>220</td>
<td>29.3%</td>
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<td>28.5%</td>
<td>15.4%</td>
<td>20.8%</td>
<td>27.0%</td>
<td></td>
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</tbody>
</table>

During the fabrication process, there may be a certain amount of plate bending which is usually a side-effect of the welding process. Since HSS materials are very susceptible to heat, the traditional post-welding treatment, using heat to stretch and straighten the plate, cannot be used on the test specimens. However, to fabricate the test specimens, a steel strip
as the bottom flange was first clamped to the welding table preventing possible distortion of the strip, and then web strip was tack welded along the centreline of the flange strip at intervals of approximately 250 mm. Finally, top flange steel strip was clamped and tack welded at similar intervals. Furthermore, to avoid possible distortions of the top flange strip, a thick steel plate was clamped to the top flange strip as referred to as back-to-back welding technique. The standard procedures TN15-96 (WTIA [10]) and AS1554 (SA [11]) were closely adhered to the welding of test specimens as recommended by the manufacturer BISALLOY STEELS. More details of the experimental test can be found elsewhere (Tang [8]).

**Properties of Test Specimens**

A series of tension tests was performed on sample coupons cut from the virgin sheets of BISPLATE80 materials supplied by the steel distributor SMORGAN STEELS and also cut from the fabricated test specimens according to AS1391 (SA [12]). The mechanical properties of BISPLATE80 HSS have consequently been presented in Figure 3 and Table 1. Evaluating the tensile test results, BISPLATE80 steels represents typical HSS materials possessing the higher yield ratio $Y_R$ combined with the lower value of the $\varepsilon_u/\varepsilon_y$ ratio $\mu_c$. These material characteristics, combined with the lower initial strain-hardening $E_{st}$, can result in reduced ductility failure modes due to a less reserve of structural ductility for some tension failure modes, in particular in the inelastic range. Premature local buckling of the flange and/or web plate element may occur resulting in a loss of section capacity without reaching the necessary ductility capacity required by the current design specifications AS4100 (SA [1]).

Due to shrinkage along the weld lines during the cooling period, longitudinal residual stresses are present in built-up sections reducing their section capacities hence causing early non-linearity (Tebedge et al. [13]; Galambos [9]). Thus experimental measurements of longitudinal residual stresses in the test specimens were conducted in this research project (Tang, [8 & 14]). It has been confirmed that the residual stresses generally have relatively less influences on the sections made of HSS than those made of conventional LSS (Nishino et al. [15]; Kishima et al. [16]; Bjorhovde, [17]). The reduced ratios of compressive residual stresses to the yield stresses of steel materials used in the cross-sections benefit both strength and stability capacities of structural members in general.

![Sample Stress-Strain Curves](image1.png)

![Distribution of Residual Stresses](image2.png)

Figure 3: Experimental Measurements of BISPLATE80 Material Properties
Geometric imperfections have inevitably been induced during the process of section fabrication. The measurements of the initial geometric imperfections included those of local plate imperfections at the free edges of the flange plates and the web centreline, and also overall imperfections of the test specimens. They were measured in the longitudinal direction of all the test specimens within the constant moment segment at intervals of 50 mm using the measurement table set-up. Three separate runs were repetitively taken for each measurement. It has perceptible been observed that the repeatability of the measurement gives reasonably high-quality results with a maximum discrepancy between each individual run of approximately 0.75%, 0.83%, 1.13%, and 1.37% for specimens NN-B1, SS-B2, SN-B3, and NS-B4, respectively. The corresponding COV (coefficient of variations) is 0.004, 0.004, 0.006, and 0.001 for specimens NN-B1, SS-B2, SN-B3, and NS-B4, respectively. However, the magnitudes of the measured initial geometric imperfections were very small as if the plate elements of the test specimens were perfect flat (Tang [8]) since the back-to-back welding technique effectively eliminated any excessive initial geometric imperfections in the test specimens during the fabrication process. This “zero” initial geometrical imperfection can only be achieved in closely monitored laboratory situations.

**Test Set-Up and Instrumentations**

In order to conduct the experimental tests on the HSS beam specimens, a test rig (see Figure 4) containing a pair of 500 kN hydraulic jacks has been designed and set-up in the structural laboratory at QUT. Lateral restraints were arranged so that the test specimen would only deform and fail in-plane about the major axis thus preventing the undesired out-of-plane deformations. The surfaces between the lateral constraints and the test specimen were lubricated prior to the test eliminating possible frictions between them. Intentionally, the constant moment segment in the test specimens was also designed to install all the measuring instrumentations with sufficient length in the region, yet prohibiting out-of-plane deformations. The applied transverse loads were measured using the load cells located between each hydraulic jack and the test specimen. Figure 4 shows the test arrangement that produces a constant bending moment within a segment between the load application points.

**TEST RESULTS AND DISCUSSIONS**

This section presents the results from experimental tests on the four beam specimens fabricated from BISPLATE80 HSS materials (nominal \( f_y = 690 \) MPa). The test results are
evaluated and discussed in relation to the section moment capacity of the HSS flexural members in the following sections. Prior to the experimental tests, the section moment capacity of each test specimen was estimated by performing calculations in accordance with the current conventional design specifications (AS4100 [1], AISC [4], EN1993 [3], and BS5950 [2]) to compare them with the test results.

**Test Results**

The hydraulic jack loads were closely monitored and recorded at regular intervals using a computer data acquisition system, and the consequential failure moments of the beam specimens were measured as the applied jack load times the distance between the central end support and the load application point. The test specimens failed in bending as anticipated due to plate local buckling of either flange or web element, or combined of both (see Figures 5 and 6) at a section moment of 308, 401, 365 and 311 kNm (see Table 1), with a corresponding failure load level of 275, 358, 325 and 277 kN for each test specimen, respectively. The section moment capacity at failure was 21.2%, 6.7%, 3.1% and 6.6% higher than the design predictions of the section moment capacities based on the American specifications (AISC [1]). Figure 5 presents the test results of the short beams as the mid-span moment versus the in-plane vertical deflection.

![Sample Local Buckling of a Beam Specimen](image)

**Discussions and Comparisons**

From Table 1, it is obvious that most conventional design codes are under-predicting the section capacities in comparison with the test results. From a safety viewpoint, this must be considered conservative, however, from an economical viewpoint, it may be considered unacceptable. Furthermore, with the use of capacity reduction factors makes it worse with under-predictions by greater than 30%. In general, most of the current steel design specifications (namely, AS4100 [1], AISC [4], EN1993 [3], and BS5950 [2]) failed to produce decent section capacities for the test specimens presented in this project. This confirms that the current design codes cannot be used for the design of structural members fabricated from steel materials with the yield stress beyond 450 MPa (i.e. $f_y > 450$ MPa), unless adequate plate slenderness limits are defined for those made of HSS materials.

On the other hand, it was also noticed that although the AISC specification (AISC [4]), which was regarded as an appropriate design standard for the use of Bisplate80 HSS materials (nominal $f_y = 690$ MPa) by the manufacturer Bisalloy Steels, produces overall better predictions for the section moment capacities of the test specimens, it gives a fairly poor prediction for cross-sections with relatively stockier plate elements, such as the specimen...
NN-B1. It may imply that the yielding slenderness limits of plate elements, defined in the AISC-LRFD [4], may be considered to be incongruous for sections fabricated from steel materials other than conventional low strength carbon steel grades, or equivalent alloy steel grades, which have mainly been the bases of experimental and analytical studies conducted on these materials by the American specification.

A numerical investigation (Tang [8]) indicated that yield ratio ($Y_R = \frac{f_y}{f_u}$) in steel materials is the main influences of the ductility capacities of structural members in bending. Since the yield ratio $Y_R$ is inversely proportional to the ductility capacity for short beams, typical HSS materials, such as BISPLATE80 (nominal $f_y = 690$ MPa), thus possess a high level of the yield ratio $Y_R$. Hence, structural members fabricated from typical HSS materials, i.e. $Y_R > 0.7$, have relatively limited ductility capacities compared to those made of conventional LSS materials (i.e. $f_y \leq 450$ MPa).

CONCLUSIONS AND RECOMMENDATIONS

This paper has presented the details of full-scale experimental tests on four short beams fabricated from BISPLATE80 HSS (nominal $f_y = 690$ MPa). The test results clearly indicated that the section capacities of these short beams were all higher than the design predictions produced by the current codes implying that the current codes including the AISC specification [4], which is regarded by the manufacturer as a suitable design code for structural members made of HSS materials, are unable to predict accurate design solutions for structural members made of steel materials other than standard LSS (i.e. $f_y \leq 450$ MPa). The inaccuracy of design predictions may be attributed to unsuitable plate slenderness limits for cross-sections made of HSS materials that may lead to incorrect section classification, other effects such as the interactions of local instabilities within the cross-section, etc.

The BISPLATE80 HSS materials (nominal $f_y = 690$ MPa) have limited ductility as they have very short length of plastic plateau in its mechanical property relationship. These Australian HSS materials have high $Y_R = \frac{f_y}{f_u}$ and low $\mu_e = \frac{\varepsilon_u}{\varepsilon_y}$ as most typical HSS. As a result, the strength reserve and strain-hardening properties, $\gamma_e = \frac{E_{st}}{E}$, of the material are in that case relatively lower than conventional LSS, which in turn will adversely affect the inelastic stability criteria in the design codes. These are the major differences between typical HSS (representative $f_y = 500$–$700$ MPa) and commonly used LSS (nominal $f_y \leq 450$ MPa) as the current design specifications (i.e. AS4100 [1]) were mainly based on experimental and analytical studies conducted on structural members made of these LSS.

This research project has confirmed that the residual stresses generally have less influence on the cross-sections made of HSS materials than those made of conventional LSS. The reduced ratios of compressive residual stresses to the yield stresses of steel materials used in the flange elements generally benefit both strength and stability capacities of structural members. This research project has also found that the ECCS’ recommendation [18] produces unsuitable predictions of section moment capacities for cross-sections made of typical HSS materials (Tang [8 & 14]).

ACKNOWLEDGMENTS

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EXPERIMENTS ON THE RESIDUAL STRESS OF 420MPA STEEL EQUAL ANGLES

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KEYWORDS
Residual stress, sectioning method, steel equal angle, Q420, high strength

ABSTRACT

High strength steels have been applied more and more widely in steel structures in the world. In China, Q420 steel angle members with the nominal yield strength of 420MPa have been applied in many steel structures especially in transmission towers. However, there is lack of study on its residual stress, which is one of the most important imperfections and has significant effects on the buckling behavior of steel members. The previous research works were only based on ordinary strength steels, whose yield strength is around 200MPa, and the distribution models of the residual stress in any countries’ steel structures design codes are only applicable for ordinary strength steel members. Consequently, the study on the residual stress of high strength steel sections is quite necessary. Some measuring tests on high strength steel welded box and I sections have been conducted before, but no hot-rolled sections. In this paper, the residual stress measuring test for Q420 steel equal angle sections is conducted. Sectioning method is used, and there are 5 different sections with a total of 15 specimens were measured. Based on the test results, the characteristics of the residual stress distribution of Q420 high strength steel equal angles are analyzed, as well as the magnitudes of the compressive and tensile residual stresses. Besides, the test results were compared with the residual stress distribution of ordinary strength steels. Finally, the effects of the width-thickness ratio of the angle leg and the steel strength are discussed. The test results can be used to establish new distribution models of Q420 high strength steel equal angles and it can be applied in the buckling analysis of such kind of angle members.

INTRODUCTION

With the development of the steel material properties and the production methods, high strength steels have begun to be applied in many steel structures, including building and bridge structures in Japan, Europe, America, Australia, China and other countries and districts (IABSE [1], Pocock [2], Shi [3]). The construction of ultra-high-voltage transmission
lines with large section bundled conductors has also begun to apply high strength steels, and Q420 high strength steel angles have been used in transmission towers in China (Qin [4]).

The research on the buckling behavior of high strength steel structures has just begun. Some axial compression column tests and residual stress measuring tests for welded sections were conducted (Ban [5], Ban [6]), as well as the finite element analysis (FEA) on the buckling behaviour of high strength steel welded sections were implemented (Shi [7]). The design codes for steel structures in Europe (CEN: prEN 1993-1-1 [8], CEN: prEN 1993-1-12 [9]) and America (AISC [10]) added design methods for S460-S700 (\(f_y=460\text{MPa}-700\text{MPa}\)) and A514(\(f_y=690\text{MPa}\)) steels respectively, but they just simply apply the original design method of ordinary strength steel structures mechanically, and there is lack of the basic research (Shi [11]). It is the same for the Chinese design code of steel structures, in which Q420 (\(f_y=420\text{MPa}\)) high strength steels were added (GB50017-2003 [12], GBJ 17-88 [13]). Consequently, the experimental research and design method on the buckling behavior of high strength steel structures are both urgently needed. The research on the initial imperfections is one of the most important basic ones, especially the residual stress. In this paper, the residual stress measuring test of Q420 hot-rolled equal angles is conducted, which can provide the experimental basis for the further study on the buckling behavior of Q420 high strength steel angles.

The residual stress measuring tests of ordinary strength steel angles were conducted in America, which were the experimental basis for residual stress distributions of angles adopted in the Chinese code (CDSSC [14]). For equal angles, the distribution model in the Chinese code is shown in Figure 1(a). \(f_y\) means the steel yield strength, the symbols \(\beta_1\), \(\beta_2\) and \(\beta_3\) stand for the residual stress factor values, and ‘+’ and ‘-’ mean the residual stress tensile and compressive respectively. The residual stress factor values adopted in the Chinese steel structures design code are shown in Table 1.

```
+ \(\beta_1 f_y\)
+ \(\beta_2 f_y\)
+ \(\beta_3 f_y\)
- \(\beta_4 f_y\)
- \(\beta_5 f_y\)
- \(\beta_6 f_y\)
```

(a) (b)

Figure 1: The residual stress distribution model of hot-rolled equal angles adopted in the Chinese code and dimension symbols for the angle section

The residual stress distributions and magnitudes in Figure 1(a) and Table 1 are only based on the ordinary strength steel angle tests, which cannot provide the experimental basis for the research on the buckling behavior of high strength steel angle columns. Consequently, it is necessary to measure the residual stress of high strength steel angles. Q420 steel equal angles are tested in this paper.
TABLE 1
THE RESIDUAL STRESS FACTOR VALUES FOR EQUAL ANGLES ADOPTED IN THE STEEL STRUCTURES DESIGN CODE OF CHINA

<table>
<thead>
<tr>
<th>No.</th>
<th>Literature resources</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ref. [14]</td>
<td>-0.22</td>
<td>0.24</td>
<td>-0.25</td>
</tr>
<tr>
<td>2</td>
<td>-0.30</td>
<td></td>
<td>0.30</td>
<td>-0.30</td>
</tr>
<tr>
<td>3</td>
<td>-0.25</td>
<td>Ref. [15]</td>
<td>0.25</td>
<td>-0.25</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>-0.20</td>
<td>0.20</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

TEST PROGRAMME

Preparation of the Test Specimen

This experiment mainly studied on the longitudinal residual stress of Q420 equal high strength angles. 5 sections: L125×8, L140×10, L160×10, L180×12 and L200×14 were tested, and each section type has 3 specimens, that is 15 specimens totally. Every specimen has a label to differ from each other, shown in Table 2. “R” means the residual stress, and “L” means the angle section, and the number following “L” is the nominal width value of the specimen section, and the last number is the serial number for specimens with the same nominal section dimensions.

Sectioning method was used in this experiment. Specimens were sawed into several pieces along the longitudinal direction, and the residual strain of each piece can be calculated by measuring the deformation of each piece through two gage holes after sectioning. Finally the residual stress can be obtained by the steel elastic module times the residual strain. Sectioning method has been used for over 40 years to measure residual stresses in structural steel members, and now it is still adequate, accurate and economical if taking proper care in the specimen preparation and measurement procedure (Tebedge [16]).

Previous experimental researches have shown that, the test section must be far enough from the member end in order to reduce end effects, and a distance of 1.5 to 2.0 times the lateral dimensions is required (Tebedge [16]). Besides, the length of specimens cannot be less than 3 times the lateral dimensions (Wang [17]). The specimens tested in this study all have a length of 650mm (larger than 3 times the maximum lateral dimension which is 200mm), and a distance of 450mm from ends (larger than 2.0 times the maximum lateral dimension). The width of pieces $w_0$ is 10mm-11mm, shown in Table 2, and the length of each piece is 260mm, with two gage holes at ends, which are 254mm (10in) apart. The details of the specimen dimension are shown in Figure 2 (take specimen RL200-1 for example).

The details of gage holes of test specimens are shown in Figure 3. Electric drill was used to make the gage hole. Most of the holes are drilled thoroughly (shown in Figure 3(a)), but those close to the corner cannot because of the operating space (shown in Figure 3(b)).

Test Setup

According to the experimental research by Tebege, N. and Wang, G.Z. (Tebedge [16], Wang [17]), Whittemore strain gage was used to take strain measurements over a 254mm (10in) gage length in this study, shown in Figure 4. Electric spark cutting was used to slice the specimens. It has been proved that the heat input is minimal (Liu [18], SEMTFMMB [19]) so that there is little effect on the residual stress distribution.
TABLE 2
DIMENSIONS OF THE SPECIMENS AND TEST RESULTS

<table>
<thead>
<tr>
<th>Specimen label</th>
<th>(w_0/\text{mm})</th>
<th>(f_y/\text{MPa})</th>
<th>(\beta_{1,\text{min}})</th>
<th>(\beta_{2,\text{max}})</th>
<th>(b/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RL125-1</td>
<td>441.893</td>
<td>11</td>
<td>-0.013</td>
<td>0.037</td>
<td>12.875</td>
</tr>
<tr>
<td>RL125-2</td>
<td>435.514</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL125-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL140-1</td>
<td>451.787</td>
<td>10</td>
<td>-0.073</td>
<td>0.115</td>
<td>11.600</td>
</tr>
<tr>
<td>RL140-2</td>
<td>432.956</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL140-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL160-1</td>
<td>464.429</td>
<td>10</td>
<td>-0.000</td>
<td>0.046</td>
<td>13.400</td>
</tr>
<tr>
<td>RL160-2</td>
<td>457.727</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL160-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL180-1</td>
<td>460.247</td>
<td>10</td>
<td>-0.034</td>
<td>0.094</td>
<td>12.667</td>
</tr>
<tr>
<td>RL180-2</td>
<td>465.498</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL180-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL200-1</td>
<td>458.580</td>
<td>10</td>
<td>-0.037</td>
<td>0.050</td>
<td>12.000</td>
</tr>
<tr>
<td>RL200-2</td>
<td>453.821</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL200-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Dimensions of the specimen RL200-1 (Unit: mm)

Figure 3: Dimensions of the gage hole (Unit: mm)

Figure 4: Whittemore stain gage

Measuring Process and Accuracy of Measurements

In this study, using sectioning method to obtain the residual stress mainly includes the following steps:
(1) Prepare specimens with a length of 650 mm and a distance of 450 mm from raw steel angles ends, and drill gage holes at ends of pieces to be cut in step 3, shown in Figure 2 and 5(a). The gage holes should be as perpendicular to the plate surface as possible.

(2) Measure the distance between the two gage holes at each piece by using the Whittemore stain gage, and label the readings as \( r_1 \). To be noticed that readings of Whittemore strain gage can be considered as the increase value of distances.

(3) Cut a length of 260 mm section from the specimen by using the electric spark cutting, shown in Figure 2 and 5(b), and measure the gage holes distance and label the readings as \( r_2 \).

(4) Cut the specimens into pieces and measure the gage holes distance again, and label the readings as \( r_3 \).

Each reading is the average value of no less than 3 measurements. Temperature correction was also considered during measurements.

Figure 5: Measuring processing

The residual strain \( \varepsilon \) of each piece after sectioning can be calculated by using Eqn. 1.

\[
\varepsilon = \frac{(r_3 + \Delta r_3) - (r_1 + \Delta r_1)}{L_0 + r_1 + \Delta r_1}
\]

In Eqn. 1, \( r_1, r_2 \) and \( r_3 \) are the readings mentioned above. \( L_0 \) is the standard distance of Whittemore strain gage, which is 254 mm (10 in). \( \Delta r_1 \) and \( \Delta r_3 \) are temperature corrections for \( r_1 \) and \( r_3 \) respectively.

During the sectioning process steels keep elastic, so the residual stress \( \sigma_r \) can be calculated by using Eqn. 2.

\[
\sigma_r = -E \cdot \varepsilon
\]

In Eqn. 2, \( E \) is the steel elastic modulus, and 2.06E5 is adopted in this study. If the residual stress is tensile (positive) at some location, the corresponding piece will be shorter after sectioning (negative strain), otherwise the strain will be positive. So, there is a negative sign in Eqn. 2.

**TEST RESULTS**

The steel yield strengths \( f_y \) for each specimen is shown in Table 2, which were obtained from tensile coupon tests. In this study, the residual stresses \( \sigma_r \) calculated from test results are all divided by the steel yield strength, and labeled as \( \beta (\beta = \sigma_r / f_y) \), which is called residual stress.
factor in this paper. Figure 6(a)-(e) show test results of residual stress factors for each specimen. Most of the factors are the average of the inside and outside measuring values of the angle leg, except for the gage points at the inside corner because of the operating space, where only the outside values were obtained. Positive values mean tensile residual stresses and negative mean compressive stresses.
Figure 6: Residual stress test results and relationship between the residual stress and the width-thickness ratio of angle legs
TEST RESULTS ANALYSIS

Residual Stress Distribution

Based on test results, the residual stress distribution of Q420 hot-rolled equal angels has the following characteristics.

1. The residual stresses at the edge of legs are basically compressive, and those at the median of legs are basically tensile, which accord with the distribution model in the Chinese steel structures code. The residual stresses at the corner are mainly compressive, and the data are discrete because there are only test results at the outside surface.

2. The residual stress magnitudes at the leg edge are slightly less than those at the median.

3. The residual stress magnitudes differ significantly among specimens with different width-thickness ratios.

4. In general, test results are discrete. There are some possible reasons, such as breakage during transport and different cooling conditions after hot rolling.

Residual Stress Magnitudes

In the Chinese steel structures design code, the residual stress distribution models of hot-rolled angles depend on three values, which are the minimum residual compressive stress factor at the edge of legs $\beta_1$, the maximum residual tensile stress factor at the median of legs $\beta_2$ and the minimum residual compressive stress factor at the corner $\beta_3$, shown in Figure 1. In this experimental study, since the test values at the corner are more discrete relatively (shown in Figure 6(a)-(e)), the other two factors at the edge and median of legs are being focused on. In Table 2, $\beta_{1,\text{min}}$ is the minimum factor among the average values of 3 specimens with the same section at the leg edge, and $\beta_{2,\text{max}}$ is the maximum factor among the average values of 3 specimens with the same section at the leg median. Compared with the values in Table 1, we can find that the test results are much smaller than those according to the Chinese steel structures design code. The maximum magnitude of $\beta_{1,\text{min}}$ and $\beta_{2,\text{max}}$ is 0.115, while the minimum magnitude of the residual stress distributions adopted in the Chinese code is 0.20. From Figure 6(a)-(e) it can be also found that the distribution model with the minimum factor values (0.20) can envelope most of the test results.

Figure 6(f) shows the relationship between the residual stress factors and the width-thickness ratios $b/t$ (shown in Table 2). The width $b$ and thickness $t$ are shown in Figure 1(b). It can be obtained from Figure 6(f) that with the increase of $b/t$, the magnitudes of the residual stress factors become smaller, and the maximum value in this test is 0.115. Considering the safety and applicability for other angle sections with smaller width-thickness ratios, a magnitude value of 0.15 is suggested for Q420 equal angle sections as the residual stress factor, which is smaller than that of the Chinese steel structures design code.
CONCLUSIONS

A residual stress measuring test for Q420 steel equal angles was conducted. Based on the test results, the residual stress distribution and magnitudes were analyzed. It showed that there was no obvious difference between the distributions of Q420 and ordinary strength steel equal angles. However, the magnitude of the Q420 steel angle residual stress is significantly smaller than that of ordinary strength steel angles, and a factor of 0.15 is suggested in this paper. These conclusions are the same as that obtained from the previous experimental study on the welded high strength steel sections. This study can provide experimental basis for the research on proposing the residual stress distribution models of high strength steel angles, and also be helpful for the future study on the buckling behavior of high strength steel angle columns.

REFERENCES


FINITE ELEMENT STUDIES ON HORIZONTALLY CURVED COMPOSITE PLATE GIRDER

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KEYWORDS
Horizontally curved, composite, plate girder, finite element analysis, tension field action, ultimate shear strength, load-deflection, web openings, trapezoidal webs.

ABSTRACT
In bridge construction horizontally curved plate girders are used to support reinforced concrete slab acting compositely with the steel part. This paper is concerned with ultimate load behaviour of such girders curved in plan under shear loading. Finite element analysis using the computer package LUSAS was employed to investigate the behaviour and ultimate strength capacity of the girders. Parameters that affect the behaviour of these girders are slenderness of the web d/t, web panel aspect ratio b/d, type of web such as corrugated webs, openings in the web panels, size of the openings relative to depth of the web d/o, shape of the openings, shear force at the opening, bending moment at the centre line of the opening, stiffness of the flange and degree of curvature. Influence of these parameters on the collapse behaviour due to shear loading is investigated in this study. The ultimate loads, failure mechanism and load-deflection curves are obtained and some typical results are presented herein.

INTRODUCTION
Horizontally curved I-girders are often employed in the construction of modern highway bridges. At the design stage, in order to simplify the analysis, these girders are sometimes assumed to act independent of the deck slabs resting on them even though the deck slabs are connected to the girders by means of shear connectors. The advantage of composite action between the steel girders and concrete deck is not considered. The present study is concerned with such composite action in horizontally curved I-girders. Neither experimental nor analytical works are available currently in literature on the horizontally curved composite plate girders. However, an experimental investigation on full-scale horizontally curved steel plate girders has been carried out to study their overall behavior and to determine the shear strength by Zureick et al. [1]. Web plate slenderness (d/t) approximately equal to the largest value permitted for transversely stiffened members in AASHTO [2] and panel aspect ratio b/d = 1.5 and 3.0 were tested to failure. Shanmugam et al. [3] investigated the ultimate load behaviour and load carrying capacity of medium size plate girders curved in plan. Similarly, straight steel-concrete composite plate girders under combined shear and negative bending were
tested to failure by Allison et al. [4]. Shanmugam and Baskar [5,6,7] studied both experimentally and numerically the effects of combined shear and bending on straight composite plate girders. It has now become normal practice to locate openings in plate girder webs to carry the services through the beam openings instead of underneath the beams thus saving building heights considerably. Lian and Shanmugam [8] carried out experimental and finite element studies on horizontally curved plate girders containing centrally located circular web openings. Parametric studies have been carried out and simple design method proposed [9]. Application of corrugated webs in plate girders has been studied by a number of researchers. Elgaaly et al. [10] studied experimentally the ultimate load behavior of girders with corrugated webs subjected to shear loading or to uniaxial bending or to partial compressive edge loading. Elgaaly and Seshadri [11] employed nonlinear finite element modeling to analyze the girders under different types of loadings; the study showed the behavior up to failure, failure modes and ultimate capacity. Recent study on horizontally curved composite plate girders by Basher et al. [12] has indicated that considerable benefits in terms of ultimate load capacity could be derived by considering the composite action between the steel girder and concrete deck. Results from the analyses on the effect of web openings on such girders [13] show that web openings display significant effect on the behavior of these girders. The object of this paper is to highlight the behavior of horizontally curved composite plate girders with and without web openings and to illustrate the effect of corrugated webs in such girders.

FINITE ELEMENT ANALYSIS

Three-dimensional finite element models were developed by idealizing the flange, web, and stiffener plates in plate girders using thin shell elements in the LUSAS element library. Concrete slab was idealized by three-dimensional hexahedral isoparametric solid continuum elements with higher order models capable of modeling curved boundaries. The plate girders were modeled as simply supported girders. Steel plate girders were modeled using ungraded Mild Steel in LUSAS material library. Young’s Modulus and Poisson’s ratio were assumed as 205 kN/mm² and 0.3, respectively, for the material. The initial imperfection of the modeled girders was obtained from buckling analysis whereby the deformed mesh from the first eigen-value was used for the nonlinear analysis. Suitable restraints by means of truss elements were provided to the girder corresponding to points of lateral restraints. Concentrated loads were simulated in the analyses by means of displacement control. Nominal residual stresses and imperfections for the steel part of the girder have been assumed in the analyses. The finite element models were so chosen that they represent the behavior of girders with solid webs or perforated webs or corrugated webs. Typical mesh suitable to a particular structural model was chosen based on convergence studies, is shown in Figure 1. These models were applied to the respective girders and the analyses carried out.

ANALYSIS OF HORIZONTALLY CURVED COMPOSITE PLATE GIRDER

Girders with solid webs

Horizontally curved steel plate girders the cross section details of which are shown in Figure 2 were taken as the steel part of composite girders analyzed in this study. The chord length for all the girders was kept as 11.58 m. Transverse stiffeners along the girder length were positioned such that the panel aspect ratio was 3 in the case of S1 and 1.5 in the case of S1-S. The overall depth of the girders was around 1.22 m whilst the top and bottom flanges of around 22.9 mm thick varied in width from 546.4 mm as in the case of S1 to 546.9 mm for S1-S. The girders S1 and S1-S had a nominal radius of
63.63 m. These girders were tested by Zureick et al. [1] and analyzed by the finite element method by Jung and White [14]; the overall geometry of the test girders, the test set-up along with the applied loading may be found elsewhere [14,15]. To each of these two steel girders, concrete slab of 200 mm thick and 2400 mm wide was added at the top flange, as shown in Figure 3, to act compositely with the steel part. The dimensions of the slab were chosen as per AASHTO recommendations for a composite girder. Full interaction in the composite action was assumed. The resulting composite girders are identified herein in the text as C1 and C1-C corresponding to S1 and S1-S respectively.

The finite element analyses provided detailed output in terms of displacements, stresses, strains, etc. However, for brevity only the most relevant results are presented herein for discussion. Figure 4 shows the load-vertical deflection plots for the four girders. Deflection measured at the center of the supported span is plotted against the corresponding shear force (P). In these figures, results corresponding to steel girders [15] are also presented along with those for composite girders for comparison. An elastic behaviour at the initial stages is observed for all the girders and it becomes nonlinear so on after reaching the ultimate condition. The behaviour of the composite girders is similar to that of the steel girders and enhancement in stiffness and ultimate load-carrying capacity compared to steel girders can be witnessed in all the composite girders. Composite girders exhibit significant gain in the ultimate load capacity, ranging from 28% to 30% over the corresponding values for steel girders. The gain should be attributed to the composite action between steel girder and reinforced concrete slab.

**Girders with perforated webs**

Girders C1 and C1-C considered in the previous section were analyzed were web openings in order to assess the effect of such openings on the ultimate load behaviour and load carrying capacity. Openings of different proportions were positioned in each of the panels. The total scheme of the studies on girders with web cut outs consisted of two phases. In the first phase circular or square openings were placed in each of the web panels of the girders C1 and C1-C as shown in Figures 5(a) and 5(b), respectively. These girders are identified in the text as C1-Cr 0.1D, C1-Sq 0.1D, C1-C-Cr 0.1D and C1-C-Sq 0.1D. In the notations C1-Cr or C1-Sq indicates circular or square openings, respectively, in C1 and 0.1D refers to the size of the openings, D being the girder depth. Five different depths viz., 0.1D, 0.2D, 0.3D, 0.4D and 0.5D for the openings were considered in the analyses in order to assess the effects of opening size on the behaviour of the girders. In the second phase, analyses were carried out to determine the effects of opening location on the behavior of the girder. Only one size of the openings viz., 0.5D in the girder C1 was considered in this case. As shown in Figures 6(a) and 6(b), openings were positioned accordingly in the middle panel in which the shear was maximum or in the outer panels subjected to the least shear force.

![Figure 2: Cross-section of the steel part of the composite girders](image1)

![Figure 3: Cross-section of the composite girders](image2)
Figure 7 shows the load-deflection relationship for the girder C1 with circular openings of different size and, the corresponding plots for square openings are presented in Figure 8. As in the previous case deflection measured at the center of the supported span is plotted against the corresponding shear force (P). In each of these figures results for the girder with web openings of size varying from 0.1D to 0.5D are plotted. The effect of opening size on the behaviour and ultimate load of the girder is illustrated in these figures. Openings were placed in all the web panels of these girders. It can be seen from the figures that the web openings, circular or square, have significant influence on the behaviour and ultimate load capacity of horizontally curved composite plate girders. The effect becomes more pronounced with increase in size of the openings, irrespective of the shape of the openings. Drop in load carrying capacity due to the presence of openings can be noticed in the figures and, the drop gradually increases as the size of the cut outs gets larger. The curve in respect of the girder with web openings becomes more and more flexible and the ultimate load level drops gradually as the opening size is increased. The rate of decrease in ultimate strength is large, 11% when the opening size is increased from 0.3D to 0.4D and the rate decreases to 5% when the opening size is increased further to 0.5D. Also, the stiffness of the girder is reduced along with the size of the openings and, the drop in stiffness is more pronounced when the opening size is increased from 0.3D to 0.4D. The effect of increase in size of the openings, circular or square, is obvious from the figures.
For small size of openings viz. 0.1D the drop in load is only around 2% to 3%, not so significant. However, when the opening size is increased further it can be noticed that the drop is bigger and substantial, around 40% when the opening size is increased to 0.5D.

Figure 9 shows the effect of location and size of circular openings on load carrying capacity of the girder C1. Results are presented for girders with openings placed in all web panels, openings in the interior panel only and, openings placed in the exterior panels for comparison. Results for steel girder, S1 with openings in all panels are also shown in the figure to illustrate the effect of composite action between the steel girder and the corresponding composite girder. As noted earlier, the load carrying capacity in all girders is seen to drop with the increase in size of openings and, the drop and the rate of drop increases as the opening size gets larger. Up to opening size of around 0.2D the drop in ultimate load is small and hence the presence of openings can be ignored in the design. Similar recommendations are given in the codes for opening size up to 0.1D. Openings located in the interior panel only where the shear is larger resulted in higher reduction in ultimate load capacity compared to the openings placed in the exterior panels only in which the applied shear is smaller. Reduction in load carrying capacity in girders with openings in all panels or only in the interior panels is around 40% for large openings (0.5D) whereas the corresponding reduction in the girders with cut outs in the exterior panels only is around 26%. It is, therefore, recommended that the openings, if necessary, should always be located at sections with low shear value.

Figure 9: Comparison of results for different size of circular openings placed at various locations along the girder
Girders with trapezoidally corrugated webs

Analyses were carried out to study the effect of corrugated webs on the behavior of horizontally curved composite plate girders. Four horizontally curved steel plate girders, S1-T20, S1-T100, S1-T300 and S1-T500 with flat width b_w of the corrugated web equal to 20mm, 100mm, 300mm and 500mm, respectively, were considered in the analyses. The flange slenderness (b_f/2t_f), in accordance with the recommendations by AASHTO, was taken as 12.19. The overall cross-sectional dimensions of the steel girder were kept same as those for the girder S1 considered in the earlier sections. Plan view and elevation along with cross-sectional dimensions of a typical girder are shown in Figure 10. To each of these four steel girders, concrete slab of 200 mm thick and 2400mm wide was added at the top flange, as shown in the figure, to act compositely with the steel part. Full interaction in the composite action was assumed as in the previous case. The resulting composite girders are identified herein in the text as C1-T20, C1-T100, C1-T300 and C1-T500. All steel and composite girders were modeled as shown in Figure 1 using the finite element code LUSAS and analyzed with supported span of 8m and overhang equal to 4m as in the case of girders with non-corrugated webs. The girders were subjected to two concentrated loads, one equal to 3P applied at the centre of the supported span and the other equal to P at the free end of the overhang as shown in Figure 10. Also, selected girders were analyzed with circular openings located in the flat part of the corrugations as shown in Figure 10.

Typical results obtained from the finite element analyses are presented in Figures 11-13. Load-deflection plots for the composite girders C1-T20, C1-T100, C1-T200 and C1-T500 are given in Figure 11 in which deflection at the centre of the supported span is plotted against shear force (P). Load-deflection curve corresponding to the girder C1 is also included in the figure for comparison. All the girders with corrugated webs display better performance in terms of initial stiffness and ultimate load carrying capacity compared to the girder C1 with non-corrugated webs. The girder C1-T100 with corrugated webs having flat width of 100 mm appears to be stronger compared to all the girders. Lower load capacity in the case of the girders with corrugation flat width of 300 mm and 500 mm may be due to the fact that local buckling in the flat part of the web has caused premature failure in these girders. The effect of web openings is clearly shown in Figure 12 in which load–deflection curves for the girders C1-T500 with no web opening and C1-T500 Cr0.3D with web cut-out of 0.3D diameter are given. Once again it is apparent from the figure that load carrying capacity and initial stiffness are significantly reduced by the presence of openings in the web. All girders containing web openings show similar behaviour with drop in ultimate load increasing with increase in opening size.

Figure 10: Plan view, elevation and cross-section of a typical girder
Views after failure of typical girders are shown in Figure 13. In the girders without web openings the web panel in the zone of higher shear appears to have suffered buckling whereas no buckling is seen in other panels. On the other hand web panels containing openings irrespective of the opening size have been subjected to buckling. The buckling behaviour is similar to the tension field action observed in the girders with non-corrugated webs.

CONCLUSIONS

Finite element analyses of horizontally curved composite plate girders have been presented in this paper. Behaviour of girders with non-corrugated webs and those with corrugated webs were considered in the study. Also, the effect of web openings in the webs was also examined. It has been found from the results obtained in the study that composite action in horizontally curved girders results in higher shear strength, better initial strength and enhanced load carrying capacity. The influence of web panel size on the performance of these girders is so significant that designers should choose right spacing for transverse stiffeners. Presence of web openings affects the behaviour of these girders in the same way as in the case of straight plate girders. Girders with corrugated webs display significant improvement in the performance of these girders and could replace the transverse stiffeners.
REFERENCES


FINITE ELEMENT ANALYSIS OF THE SUBSTRUCTURE IN A SLIM FLOOR FRAME SUBJECTED TO ACCIDENTAL LOAD

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KEYWORDS

Slim floor, WQ-beam, Substructure, Progressive collapse,

ABSTRACT

In this paper, in order to study the anti-collapse behaviour of the slim floor frame with the WQ-beams and the composite circular steel hollow section columns, finite element analysis of the substructure in a slim floor frame is carried out under the condition of accidental load. The substructure is simulated by using the general purpose software Abaqus, a three dimensional (3-D) finite element program. The model has been calibrated while the finite element analysis of the shear capacity of the joint is being performed [2]. The local failure is simulated through nominal removing the internal column. Then the non-linear static analysis is selected during the analysis, and two-step loading system is performed to model the actual loading situation. Finally, the anti-collapse behaviour of the substructure is investigated through analyzing its load-bearing capacity and deformation capacity.

INTRODUCTION

Structural safety in building design is assumed implicitly through reliability-based load and resistance factors. However, such provisions do not account for extreme loading events that may lead to progressive collapse. The progressive collapse of a structure refers to the condition in which the failure of local components (or a local region) leads to global system failure.

The capacity of a structure to resist localized failure is an important aspect of progressive collapse analysis. Conventional structural design simply considers a building’s stability on the basis of the undamaged state of its components and by introducing a safety factor to consider uncertainties in actual structures. But accidental loads are difficult to quantify; they include gas explosions, construction errors, a vehicular collision, a bomb explosion etc. Therefore, in evaluating the response of a structure to progressive collapse, current methods consider the strength and deformation limits of structural components as the results of a predetermined localized failure, such as the loss of a column. If these limits are satisfied, then the structure is considered to be strong enough to resist progressive collapse.
MODEL DEVELOPMENT FOR COLLAPSE

In order to provide a comprehensive understanding of the behaviour of a slim floor frame with a beam-to-column joint under the extreme situation, a substructure instead of a joint is therefore simulated. As a general procedure to derive the robustness requirement, a numerical study is carried out in order to see what happens when part of the structure is destroyed, as well as how and how far redistribution takes place. In this study, the hollow-core slab is ignored during the progressive collapse analysis.

A first storey of the slim floor frame with two-bays and a nine-storey are simulated in this study. In order to model the condition of local damage to the inter-middle column, one internal column is nominally removed, and then two bay structures are changed into one long bay structure. In the actual design, the sections of all elements are designed on the basis of the two bay system, so that it can not satisfy the design requirement after one column has been destroyed. Major deformation will occur in the beams or in the joints, and may even lead to the collapse of part or the whole of the structure. In order to avoid this phenomenon, the progressive collapse of the structures should be examined under such situations as, for example, local failure in structures. For the purpose of avoiding the collapse of the structure after the nominal removal of the key element, the remaining structures should have enough capacity to bridge the force from the removed column to the other elements. Thus, robustness of design is an important issue in ensuring integrity of the structures.

To achieve the targets mentioned above, the response of the substructure should be as close as possible to that of the prototype frame. The model set-up is created as shown in Figure 1 according to the aims outlined above. In order to make the calculation faster, just half of the substructure is selected for simulation. After the nominal removal of the column, the second and above storey will move downward as a consequence of displacement. If each beam remains well connected with the column, then they will work together, that is to say, each storey will have the same downward displacement (Δ\text{remove}). If the beams in each storey are designed to undergo the plastic deformation under the influence of an accidental load, the substructure should just be designed to resist the corresponding storey accidental loading. But, in actual design, the sections of the beams and columns in each storey maybe designed differently for many reasons. Generally, the cross-section of the roof beam is smaller compared to that of the other beams, and the section of the column reduced with the number of the storey of the building. Hence, it is possible for one substructure in the frame to have not enough capacity to resist an accidental load in certain case. But the whole structure will not collapse immediately; the vertical load can be borne by the remaining substructures, therefore, it is necessary to check the capacity of the substructures subjected to the load from the above storey. The substructures should be studied on the basis of two loading steps, i.e. the accidental load applied to the WQ-beam and the load applied to the top of the removed column. In the set-up shown in Figure 1, half the height of the external column above and below the joint is selected, and a full one bay WQ-beam length is chosen. The internal column below the joint is cut out to simulate the nominal removal of the column; but the joint is retained there. The boundary conditions of the model set-up are demonstrated below:

- the top and bottom boundary conditions of the external column are assumed to be pinned;
- the top of the removed column is also pinned, but only displacement in the vertical direction is permitted in order to simulate the potential vertical load from the above storey.

The load path during the analysis is described as follows on the basis of two loading steps mentioned above, the accidental load and the removed load:
- a uniformly distributed pressure is first applied to the WQ-beam of the substructures according to EN1990: 2002(E) to simulate the accidental load in the actual building;
• after that, keeping the accidental load constant, the vertical loading is applied to the top of the removed column until the collapse of the structures occurred. This step is performed in order to study the capacity of the structure to resist the load from the above storey.

![Figure 1: Model set-up for progressive collapse analysis](image)

**FINITE ELEMENT MODELLING**

Three-dimensional models are created to simulate the joint. In total there are two types of elements to be adopted. All main components, such as structural steel and concrete filled in the column, eight-node, first-order solid continuum element (C3D8I) are selected for the modeling. The longitudinal reinforcing bars are modelled via a two-node, first-order truss element (T3D2). All the structural steel parts, such as the WQ-beam, hollow steel column, console and tension-bar are modelled as an elastic-plastic material with a strain hardening both in compression and in tension, using Von Mises plasticity. The modulus of elasticity, yielding stress, and ultimate stress are shown in Figure 2. “Damaged plasticity” is used to model the plastic properties of the concrete. This model provides a general capacity for the analysis of concrete structures under static or dynamic and cyclic loading. It is based on the damaged plasticity algorithm. Compared to the companion model (smear crack model), “damaged plasticity” models concrete behaviour more realistically but at a relatively high cost. The yield function is based on the Drucker-Prager plasticity model. In this model, the compressive stress-strain curve and tensile stress-strain curve are specified separately. The stress-strain curves are calculated on the basis of the EN 1992-1-1[8]. The material properties of the concrete are illustrated in Figure 3.

![Figure 2: Stress-strain curve of the structural steel and reinforcement bar](image)
In the model, all three types of interaction are used to model the contact between components. They are embedded constraints, tie constraints, and friction interaction.

Embedded constraints: The reinforcement is embedded in the solid element (the concrete part of the column), which means that no change is allowed in the spacing and geometry of the reinforcement.

Tie constraints: In this model, all the welded connections between components are modelled by using tie constraints as follows. Surface and surface contact are defined for the tie constraints.
- between the console and the steel part of the column;
- between the tension-bar and the steel part of the column;
- between the tension-bar and the WQ-beam.

Friction interactions: Besides the embedded and tie constraints between components, friction interaction is another important method to define the remainder of the contacts. The utilization of the friction interactions are listed below:
- between the tension-bar and the concrete part of the column;
- between the steel part of the column and the concrete part of the column;
- between the endplate of the WQ-beam and the console.

Penalty is selected to model the friction formulation and “hard-contact” is considered to simulate normal behaviour. The friction coefficient is 0.3 for the interaction between steel and concrete and 0.2 for the interaction between steel and steel.

In order to accurately simulate the model set-up developed in Figure 1. Three different boundary condition groups are created: at the top and bottom of the external column, a pinned supports are used; at the top of removed column, pinned support is also selected, but the displacement in the column direction is released. For the purpose of simulating the contact between the WQ-beam endplate and the gap concrete, both bottom edges of the WQ-beam endplate are prevented along the WQ-beam direction in the accidental load step. However, only the bottom edge of the WQ-beam endplate in the external joint is prevented along the WQ-beam direction during the removed load step. The removed load is applied to the top of the internal column; the load path is as follows:
- downward displacement of the WQ-beam is applied to the contact surface to make convergence easier; the downward displacement is only 0.001 mm;
• in the second step, a uniform pressure load is applied to both sides of the bottom flange of the WQ-beam in order to simulate the accidental load, which includes characteristic permanent load and quasi-permanent value of a variable action for long-term effect. The value of the pressure load for each side is 81 kN/m, it is calculated by \(6 \text{kN/m}^2 \times 12 \text{m} + 0.3 \times (2.5 \text{kN/m}^2 \times 12 \text{m}) = 81 \text{kN/m} \) when the slab is 12 m long, the remaining structures should be checked whether they can resist the loads between the midlines of the remaining columns in the slab direction from the two-bays. The width of the loading is 40 mm away from the external edge of the bottom flange of the WQ-beam.

![Figure 4: Overview of mesh, boundary condition and loading system of the substructures](image)

• in the third step, for the purpose of studying the capacity of the substructure to resist the load from the above storey, a vertical load is applied to the top of the removed column in order to simulate the loads from the above storey. All details can be seen in Figure 4.

**MODELING RESULTS**

**Accidental load step**

The deformation of the substructure and Von Mises stresses distribution in the joints regions are shown in Figures 5-6. The accidental load (A) is plotted in Figures 7-8 against different vertical and horizontal displacements. It can be seen from Figure 5 that the maximum vertical displacement appears in the WQ-beam where in the middle site. The maximum vertical displacement is almost two times higher than WQ-beam end in the internal joint, and it is especially higher than at WQ-beam end in external joint. The plasticity appears both in the tension-bars in the external and internal joint, especially obvious in the tension-bar in the external joint. As shown in Figures 7-8, the vertical and horizontal displacements increase smoothly with the accidental load. This indicates that the substructure resists the accidental load well. The accidental load is applied to the bottom flange of the WQ-beam, the value of which is 1230.06 kN through calculating \(2 \times 81 \text{kN/m} \times 7.56 \text{m} = 1230.06 \text{kN} \) when the lengths of the hollow-core slab and the WQ-beam are 12 m and 7.56 m respectively. The maximum vertical displacement in the substructure is 50.47 mm. The maximum displacement is almost two times of those at the top of the removed column and WQ-beam end in internal joint, the values of which are 27.89 mm and 28.38 mm, respectively. The change trends of the displacement at the top of the removed column are in accordance with that at WQ-beam end in internal joint all along the accidental load step. In addition, the maximum horizontal displacement of the external column gradually increases with the accidental load up to 5.74 mm because the tension-bar in the external joint is taken effect throughout, and it is so small that it cannot lead to the plasticity hinge on the column.
Figure 5 Deformation of the whole model in the accidental load step (in m)

Figure 6: Von Mises stresses distribution (a) internal joint, (b) external joint (in Pa)

Figure 7: Relationship between accidental load and vertical displacements
Figure 8: Relationship between accidental load and horizontal displacement

**Removed load step**

The deformation of the substructure and Von Mises stresses distribution of the joint regions are shown in Figures 9 -10. The vertical and horizontal displacements, rotation of the WQ-beam and the substructure measured in Abaqus are plotted in Figures 11-13 against the removed load ($V_{\text{remove}}$). The maximum vertical displacement appears in the internal joint. As shown in Figure 10, because the tension-bars have a lower flexure strength and stiffness compared to those of the WQ-beam and the composite column; catenary action develops and is concentrated in the tension-bar in the internal joint; full plastic deformation is formed under the catenary action as well, the full section is indicated in tension. In addition, a plastic deformation also develops in the tension-bar in the external joint, and the spread of the plasticity is basically concentrated on it. The removed load is plotted in Figure 5.11 against the vertical displacements at WQ-beam ends in the internal and external joint, and the top of the removed column. It shows the tendency that they follow the same change trend along the removed load step at WQ-beam end in the internal joint and the top of the removed column. It means that the joint in the external joint keep its integrity throughout. The maximum displacement at the top of the removed column is 258.96 mm; the maximum displacement at the WQ-beam end in the internal joint is 243.83 mm. Therefore, the maximum difference in displacement between the top of the removed column and WQ-beam end in the internal joint is 15.13 mm. The different in displacement begins to obviously increase at a load of approximately 1450 kN, before this point, it basically remains unchanged and the value is very small. Therefore, it can be concluded that the internal joint keeps
integrity well during this step again. The difference in displacement is entirely caused by the plastic deformation in the tension-bar in the internal joint due to the action of catenary action. Figure 12 shows the relationships between the removed load ($V_{\text{remove}}$) and the rotations of the substructure and the WQ-beam ($\theta_{\text{remove}}$, $\theta_B$). The rotations $\theta_{\text{remove}}$ can be calculated through the distances between the midlines of the columns divided by the vertical displacement of the top of removed column; $\theta_B$ can be calculated through distances between the WQ-beam end in the internal joint and midline of external joint divided by the vertical displacement of the WQ-beam end in internal joint. It indicates that they follow the similar change trend throughout. The plastic rotations of the substructure and the WQ-beam are 31.97 mrad and 30.10 mrad and are both higher than the minimum required values of 15 mrad in GSA and DoD for a semi-rigid joint. The maximum removed load is 1728.8 kN. The load is first to increase rapidly up to 1357 kN, and then, it slowly increases up to the maximum value of 1728.8 kN, meaning that the substructure shows a deformation in ductility, as shown in Figures 11-12. The maximum horizontal displacement of the external column remains 5.89 mm before the load reaches approximate 1429.6 kN and decreases during the catenary stage. After that, this horizontal displacement increases up to 6.84 mm due to the fact that removed load is transferred to the external joint. The horizontal displacement of 6.84 mm is so small that it cannot lead to plastic hinge on the column.

![Figure 9 Deformation of the whole model in the removed load step (in m)](image)

![Figure 10: Von Mises stresses distribution (a) internal joint, (b) external joint (in Pa)](image)

![Figure 11: Relationship between removed load and vertical displacements](image)

![Figure 12: Relationship between removed load and rotations](image)
CONCLUSIONS

The load-bearing capacities of this substructure are 1230.06 kN and 1728.8 kN for accidental load step and removed load step. When the accidental load is applied, the maximum deflection appeared close to the middle of the WQ beam, the substructure kept integrity well. At the edge of collapse of the substructure, the maximum deflection appeared in the joint where the column is removed, and the rotation of the substructure and the WQ-beam are 31.97 mrad and 30.10 mrad respectively, these rotations are higher than the required value of a semi-rigid joint by GSA and DoD. Furthermore, the load-bearing capacity kept steady after reaching the maximum load, the substructure has good ductility when the internal column is nominal removed.

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References
ANALYSIS OF THE SHEAR-LAG EFFECT IN STEEL-CONCRETE CABLE STAYED BRIDGES BY MEANS OF DECK FINITE ELEMENTS

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Cable stay ed bridges, effective width, finite element method, shear-lag, steel-concrete composite structures, decks.

ABSTRACT
This paper presents the derivation of a finite element formulation for the analysis of composite decks accounting for partial interaction theory and shear-lag effects. The particularity of the proposed element, referred to as the deck finite element, relies on its ability to capture the structural response of cable stayed bridges, while preserving the ease of use of a typical line element. For these particular bridges, stress concentrations are induced in the slab by the application of concentrated forces due to the anchorage of stays. The ease of use of the proposed deck finite element is outlined considering a case study for which the calculated results have been compared against those obtained using a more refined model implemented using shell elements in a commercial finite element software.

INTRODUCTION
Steel-concrete composite decks are used in many bridge typologies. In addition to the simple scheme of multi-span continuous beam, widely used in viaducts and flyovers, the composite decks are adopted also in combination with other elements in the bridges with complex static schemes such as arch bridges, bow-string bridges and cable stayed bridges. In these kinds of structures the usual assumption of bending theory, according to which the plane cross sections remain plane after loading, is not realistic and non uniform stress distributions usually arise in the slab (shear-lag effect) reducing its effective width. In addition to the shear-lag effect, proper due to vertical loads, other stress concentrations can arise in the slab due to the application of longitudinal forces and in the case of anchorage of prestressing cables or stays, and in the case of differential non-mechanical deformations (e.g., shrinkage, thermal effects). In these cases the method of the effective width proposed by the main codes of practice (e.g., EN 1994-2 [1]) cannot be applied and the stress concentration can be captured with refined finite element analyses by modelling the structure with shell or solid finite elements. Unfortunately, the results of such analyses are not synthetic and have to be post-processed in order to verify the structure.
The authors proposed a beam finite element for the time dependent analysis of composite steel concrete decks taking into account the beam-slab partial interaction and the shear-lag effect on the slab [3]. This element, which will be called hereafter deck element, permits to capture the concentration of the longitudinal normal stresses in the slab due to generic loads. Treating the whole deck as a beam, the model gives the results in terms of stress resultants which are very useful for the structure verification.

In this paper, the deck finite element previously defined is used in combination with other elements (truss and Bernoulli’s beams) to model cable stayed bridges. Problems encountered in modelling the spatial structure are discussed and the results obtained on a realistic bridge are compared with those given by a more refined shell finite element analysis.

MODEL OUTLINE

A typical composite steel-concrete twin girder bridge deck is considered (Fig.1). The cross section is considered to be rigid in its own plane whereas the deformability of the interface shear connection enables relative longitudinal displacement (slip) to occur between the steel beam and the slab while preventing their separation. The loss of planarity, which occurs in the slab subjected to member forces due to the composite action and localized longitudinal forces, is considered by means of a unique known warping function modulated along the deck axis by an intensity shear-lag function [4].

A local reference system is introduced for each element with the origin located at node \(i\). If \(\vec{x}_1\) is the longitudinal axis of the element, oriented from joint \(i\) to joint \(j\), axes \(\vec{x}_2\) and \(\vec{x}_3\) complete an orthonormal reference system. The element nodal displacements are defined according to the positive directions of the local reference axes. For each node, three displacement components and three rotation components are considered. Two additional degrees of freedom are introduced to define the beam-slab interface slip \(\Gamma\) and the shear-lag function \(\omega\) that measures the intensity of the slab warping (Fig.1a). These last two generalized displacements are defined as scalar quantities and thus do not depend on the adopted reference system.

In a dual manner generalized nodal forces which consist of the three force components and three moment components, the longitudinal shear force at beam slab interface and the slab bi-moment may be defined (Fig.1b). As for the displacements, the last two generalized forces are scalar quantities.

The generalized nodal displacements and forces of element \(e\) are grouped in the vectors

\[
\mathbf{s}_i^T = \begin{bmatrix} u_{1i} & u_{2i} & u_{3i} & \phi_{1i} & \phi_{2i} & \phi_{3i} & \Gamma_i & \omega_i \end{bmatrix}, \quad i \in [1, n]
\]

\[
\mathbf{f}_i^T = \begin{bmatrix} f_{1i} & f_{2i} & f_{3i} & m_{1i} & m_{2i} & m_{3i} & q_i & \beta_i \end{bmatrix}, \quad i \in [1, n]
\]

FIGURE 1: Deck finite element: (a) nodal displacements; (b) nodal forces
In the case of linear behaviour of all the materials and of the interface shear connection, the usual relationship \( \mathbf{f}_e = \mathbf{K}_e \mathbf{s}_e \) holds, where \( \mathbf{K}_e \) is the stiffness matrix of the element.

**Rotation of the element**

The deck element can be oriented freely in the three dimensional space in which an ortho-normal global reference system is introduced. If \( a_\alpha (\alpha = 1, \ldots, 3) \) are the unit vectors of the global system and \( \vec{a}_\alpha (\alpha = 1, \ldots, 3) \) are those of the local reference axes, the rotation operator \( R_e \) is defined by the matrices

\[
R_e = \begin{bmatrix}
R & 0 & 0 & 0 & 0 & 0 \\
0 & R & 0 & 0 & 0 & 0 \\
0 & 0 & I_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & R & 0 & 0 \\
0 & 0 & 0 & 0 & R & 0 \\
0 & 0 & 0 & 0 & 0 & I_{22}
\end{bmatrix}
\]

\[
R = \begin{bmatrix}
\vec{a}_1 \cdot a_1 & \vec{a}_2 \cdot a_1 & \vec{a}_3 \cdot a_1 \\
\vec{a}_1 \cdot a_2 & \vec{a}_2 \cdot a_2 & \vec{a}_3 \cdot a_2 \\
\vec{a}_1 \cdot a_3 & \vec{a}_2 \cdot a_3 & \vec{a}_3 \cdot a_3
\end{bmatrix}
\]

\[
I_{22} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

The nodal displacement vector and the nodal force vector can then be transformed from the local to the global reference system by the following relationships:

\[
\mathbf{s}_e = R_e s_e \\
\mathbf{f}_e = R_e f_e \quad (3a,b)
\]

Thus, transformations (3a,b) result in the rotation of the vectorial entities of the three-dimensional space (displacements and rotations) whereas the scalar degrees of freedom (interface slip and shear-lag function) and the relevant forces (longitudinal shear flow and bi-moment) are not modified. From (3a,b), standard considerations permit it to refer the element stiffness matrix to the global reference system as

\[
\mathbf{K}_e = R_e^T \mathbf{K}_e R_e \quad (4)
\]

**Element connectivity**

Although different choices of the nodal displacements may be done [3], that adopted in this paper, which considers the six usual displacement components (translations and rotations) of the steel part, and two scalar components for the slab, is particularly suitable for practical cases. The deck element can be connected with other kinds of finite elements such that usual beam elements and truss elements in a quite natural manner. Only the displacements of the steel part can be connected to those of the other kind of elements and no interaction can happen with the slab of the composite deck. Although this is a theoretical limitation it is not a real limitation considered the practical where connection between the steel elements is always privileged.
In modelling cable stayed bridges (Fig.3), where the stays are connected laterally to the deck, it is suitable to use rigid links to catch the real position of the connections and thus the eccentricity of interaction forces.

The choice done for the degrees of freedom of the deck element permits to establish a kind of hierarchy among the various elements (truss elements 3+3 degrees of freedom, beam elements 6+6 degrees of freedom, deck element 8+8 degrees of freedom). The admissible degrees of freedom associated to each joint are inherited from the element with the higher level of hierarchy among those connected to the joint. The stiffness matrix assembly is thus very simple and descends from the element connectivity on the basis of the structural topology by enforcing the consistency of the nodal displacements and can be performed as usual by a summation procedure suitably selecting the contributions due to the various elements [5].

Restraints and constraints

In addition to the usual external restraints which prevent the six displacement components, other two kinds of restraints, able to prevent the interface slip and the slab warping, may be introduced. The restraints on the slip can be used to model deck with rigid shear connections whereas the restraint on the slab warping can simulate the presence of rigid structural elements (e.g. end transverse beams). The case of flexible transverse beams limiting the slab warping can be caught by defining special generalized springs. Obviously, the degree of restraint imposed to the joint must agree with the relevant admissible degrees of freedom defined as previously described.

Using of deck finite element subjected to flexural action in the vertical plane

In the cases of decks subjected only to flexural actions in the vertical plane, the composite finite element suggested by Gara et al. [3] may be used. Being the composite deck element characterised by only 10 degrees of freedom, the relative 10x10 stiffness matrix needs to be expanded to dimension 16x16 by adding rows and columns of zeros to comply with the spatial problem. Obviously, this results in an ill-conditioned problem that may be solved by imposing special constraints to the deck so as to have rigid motions with respect to horizontal deflections and torsional rotations [6].

APPLICATION TO A REAL BRIDGE

In order to demonstrate the capability of the method proposed, the results obtained on a realistic cable stayed bridge are compared with those given by a refined model. The case study considered is shown in Fig.4. Fig.5(a) depicts the model prepared using the proposed deck finite element combined with the common truss and frame elements, while Fig.5(b) outlines the meshing adopted in SAP2000 [7] where both steel beams and slab have been modelled using shell elements. For the former model, rigid links have been specified at the ends of the deck element to ensure the correct positioning of the stays at the sides of the bridge.

![Figure 4: General layout and dimensions of the bridge studied](image-url)
The loading conditions adopted in the analyses are illustrated in Fig.6(a) which include both a symmetric component simulating the deck self-weight and a non-symmetric one representative of a possible traffic load applied over half of the internal span. At the beginning of the simulations the bridge stays are pretensioned to provide an initial condition of zero deflection throughout the length of the bridge when subjected to its self-weight only. The additional deflection of the bridge due to the non-symmetric loading is depicted in Fig.6(b) which illustrates how both the proposed deck finite element and the shell FE models produce equivalent results. Very good agreement is also shown for the values for the longitudinal stresses calculated in both the slab and the steel joists. Fig.6(c) plots the stresses calculated at mid-height of the slab along the bridge length at the location of the steel member axis and along the centre-line of the deck while the stresses obtained for the top and bottom steel flanges are illustrated in Fig.6(d). The variation of the effective width is determined based on the calculated stress distributions and is illustrated in Fig.6(e). It is worth highlighting that the localised minimum values observed for $B_{eff}$/B near the end supports and near mid-span are not relevant from a design viewpoint as related to bridge segments along which the stress levels are relatively low. This is also shown in Fig.7 which outlines the stress distributions across the deck width at mid-height of the concrete depth at four different locations along the bridge defined in Fig.6(a). Also in this case the results calculated using the proposed deck element well agree with those obtained using the more refined shell FE model. Similar considerations can be drawn for the concrete stresses calculated at the top and bottom of the slab and, for this reason, their graphical representation has been omitted. It is

<table>
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<tr>
<th>Stays</th>
<th>Proposed deck FE</th>
<th>Shell FE model Error</th>
<th>Stays</th>
<th>Proposed deck FE</th>
<th>Shell FE model Error</th>
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</table>

Figure 5: Finite element models: (a) proposed deck elements; (b) shell finite elements
worth highlighting how the maximum stresses in the concrete occur at both the centre-line of the deck and along the member axis of the steel beams depending on the location of the cross-section considered. In fact, for sections 1 and 3 defined in Fig.6(a), the peak values for the concrete stresses take place at the location of the steel joint (Fig.7) while for sections 2 and 4 these are observed in the middle of the deck width.

Comparisons have also been carried out for the axial forces induced in the bridge stays using the two finite element models (Fig.5) and the maximum observed error is very small, i.e. less than 0.4% as specified in Table 1. An overview of the deflected shape of the whole model is illustrated in Fig.8.

CONCLUSIONS

This paper presented a finite element formulation for the modelling of cable-stayed bridges where stress concentrations may be induced in the slab by the application of concentrated forces due to the anchorage of stays. The main advantage of the proposed formulation relies on its ability to retain the ease of use of a typical line element while being able to capture the complex response of the bridge deck along its length as well as across its width. This has been achieved as the proposed analytical and numerical model is derived based on partial interaction behaviour theory and is capable to capture shear-lag effects. To highlight the ease of use of the proposed finite element one case study has been considered. The results calculated using the proposed finite element have been compared against those obtained using a more refined model implemented using shell elements in a commercial finite element software and, in general, the results have been shown to be in good agreement. The limitation of the proposed formulation relies on its inability to account for torsion and transverse bending. The authors do not feel that this compromises the usefulness of the proposed element which intends to complement the analytical tools available to bridge designers. In fact, it could be used in preliminary design and costing, and as an efficient tool to determine effective width values for complex bridge systems.

REFERENCES

Figure 6: (a) loading conditions; (b) deck deflection; (c) longitudinal normal stresses at mid-height of the slab; (d) longitudinal normal stresses in the top and bottom flanges of the steel beam; (e) effective width over the actual deck width
Figure 7: Transverse distribution of the normal stresses of the concrete slab at different locations along the bridge length.

Figure 8: Deflected shape of the cable-stayed bridge obtained using the proposed Deck FE model.
A CONTRIBUTION TO NON-LINEAR ANALYSIS OF STEEL FRAME WITH FLEXIBLE AND ECCENTRIC CONNECTIONS

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KEY WORDS

Steel structure, Plane frame, Semi-rigid connection, Eccentric connection, Shear deformable beam

ABSTRACT

This paper deals with the effects of flexible and eccentric connections on the dynamic behavior of plane steel frames. Focusing on the effects of rotational spring and its eccentricity of flexible connections, this work extends the elastic analysis developed in Reference [1] and later developments [2] to shear deformable beams [3].

The eccentric flexible connection is idealized by a linear rotating spring together with a linear translational spring, normal to the beam axis, positioned at a certain distance from the axes node. The stiffness matrix is obtained using power series expansion of trigonometric functions and all finite element matrices have been developed in an explicit form. Using kinematic relationships all the internal degrees of freedoms are condensed out obtaining a standard stiffness matrix readily implementable on standard FEM codes. These kinematic relationships depend exclusively upon the eccentricity and flexibilities of the connections.

Linear and non-linear dynamic analyses are used to illustrate the efficiency and the accuracy of the present model and results are compared with values obtained by the known software package SAP 2000 [4].

INTRODUCTION

The conventional way of analysis and design of steel frames is to suppose the joint connection (beam-column) as a fully rigid or ideally pinned even if practically most of the connections are semi-rigid. Therefore this idealization is not adequate as all types of connections are more or less flexible or semi-rigid. Several authors have studied the effects of semi-rigid connections on steel structures regarding different aspects. Most of these models developed are, usually, a sophisticated numerical solution or just an approximation based upon information obtained by experimental tries. Among these we can recognize two main group: simplified or approximated methods, and analytical solution of complex system where this paper lies. In Reference [5] Lionberger et al. have been among the first authors who have studied the dynamic behavior of the structures with semi-rigid connections, while
Kawashima et al. [6] have defined the stiffness matrix for a beam model with springs and dampers at the extremities. Suarez et al. [7] have developed a linear dynamic analysis for frames with semi-rigid connections, considering also the effect of the eccentricity of the connections. From the elastic analysis developed by Goto et al. [8] several authors have studied dynamic and static analyses at second order for frame structures with semi-rigid connections [9-12] lately extended addressing to inelasticity property of steel material [2].

This paper deals with the static and dynamic behavior of plane frame with flexible and eccentric connections where the beam is modeled using the Timoshenko theory or including the shear deformability [13,14]. In particular this paper extend the elastic analysis developed in Reference [1] to shear deformable beams. The stiffness matrix is obtained using power series expansion of trigonometric functions.

As a practical comparison of the present model we can refer to the DWA connection type since is one of the most sensitive to rotational effects: the beam goes to the columns trough two web angle, that usually, during the construction stage, are smaller than the final beam element sections.

The paper is organized as follows: in the first Section we determine the stiffness matrix of the shear deformable beam element with fixed ends. In the second Section we obtain the stiffness equations for a shear deformable beam with flexible and eccentric connections: corrective matrix for the rigidity and the eccentricity effect of the connection are developed. The stiffness matrix is then obtained as a sum of stiffness matrices of each contribution. In this section the vector of generalized forces and the mass matrix will be derived. In the third Section some numerical example are given.

**Equilibrium equations for deformable shear beam with fixed ends**

The second order differential equations for shear deformable beams can be obtained from the principle of virtual works [1]. They can be expressed in the following form, with displacements $u_0$ and $v_0$ and rotation $\psi_0$ as primary unknowns:

$$\left[ EA \left( u_0' + \frac{1}{2} v_0'^2 \right) \right] = 0, \quad (1)$$

$$\left[ Nv_0' \right] + \left[ kG_oA \left( v_0' + \psi \right) \right] + q(x) = 0, \quad (2)$$

$$\left( EJ\psi' \right) - kG_oA \left( v_0' + \psi \right) = 0. \quad (3)$$

where $E$ is modulus of elasticity, $A$ and $I$ are cross-sectional area and moment of inertia, and $q(x)$ is lateral distributed load, while superscript denotes partial derivative with respect to $x$ and $k$ is:

$$k^2 = \frac{A}{J} \left( u_0' + \frac{1}{2} v_0'^2 \right), \quad (4)$$

These equations are nonlinear and coupled, and hence, an analytical solution cannot generally be obtained. The relations between internal forces and displacements are given as:

$$N = EA \left( u_0' + \frac{1}{2} v_0'^2 \right), \quad S = kG_oA \left( v_0' + \psi \right), \quad M = EJ\psi'. \quad (5)$$

Supposing that there are no distributed load along the axial direction, we can consider that $N=const$ and in case the Equation (2) can be solved respect to $v_0$ regardless the Equation (1) [3]. Supposing $N_i>0$, and setting $q(x)=0$, the solution of Equations (1-3), results to be:

$$\psi(x) = kx - \frac{\sin kx}{\left( 1 - \alpha k^2 \right)} \left[ \frac{M_i l^2}{EJ} \left( 1 - \cos kx \right) - \psi_i \sin kx \right], \quad (6)$$

$$v_0(x) = v_0 + \frac{S_i l}{EJ} \frac{1}{(kl)^3} \left[ \sin kx \left( 1 - \alpha k^2 \right) + \frac{M_i l^2}{EJ} \left( 1 - \cos kx \right) \left( kl \right)^2 \right], \quad (7)$$
where:

\[ k = \sqrt{\frac{N_i}{EJ}}, \alpha = \frac{EJ}{k_0 G_0 A}. \]  

If \( N_i > 0 \), the solutions are obtained, by replacing \( k_l = j_k l \) and using the relations \( \sinh (k_l) = -\sin (j_k l) \) and \( \cosh (k_l) = -\cos (j_k l) \). From (6) we can see that if we disregard the shear deformability, \( \alpha = 0 \), we obtain the equations shown in Reference [4].

![Figure 1: A shear deformable beam element with eccentric flexible connections](image)

The evaluation of the value of the axial force at the extremities is needed since different are the results if \( N_i \) is positive or negative. Moreover, if the axial force \( N_i \) inclines to zero, the coefficients of the equations of rigidity become indefinite, causing a numerical instability of the solution. In order to avoid these problems, we introduce this development in power series that is independent of the value of the axial force, and is not affected by the numerical instability when \( N_i \) inclines to zero:

\[
\begin{align*}
\sin k l = & \left[ k l + k l \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left( -\frac{N_i l^2}{EJ} \right)^n \right] \\
\sinh k l = & \left[ \sum_{n=1}^{\infty} \frac{1}{(2n)!} \left( \frac{N_i l^2}{EJ} \right)^n \right] \\
\cosh k l = & \left[ \sum_{n=1}^{\infty} \frac{1}{(2n)!} \left( \frac{N_i l^2}{EJ} \right)^n \right] \\
\cos k l = & \left[ -\sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left( \frac{N_i l^2}{EJ} \right)^n \right]
\end{align*}
\]  

where \( EJ \) shows the rigidity for unit length, \( l \) shows the length of the element. The stiffness matrix is expressible as it follows:

\[
\begin{bmatrix}
S_1 \\
M_1 \\
S_2 \\
M_2
\end{bmatrix} = \begin{bmatrix}
12\xi_1 & 6\xi_2 & -12\xi_1 & 6\xi_2 \\
6\xi_2 & 4\xi_3 & -6\xi_2 & 2\xi_4 \\
-12\xi_1 & -6\xi_2 & 12\xi_1 & -6\xi_2 \\
6\xi_2 & 2\xi_4 & -6\xi_2 & 4\xi_3
\end{bmatrix} \begin{bmatrix}
\psi_1 \\
\psi_2
\end{bmatrix}
\]

where:

\[
\begin{align*}
\xi_1 = & \frac{1}{12\xi_1} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left( \frac{1}{N_1} \right)^n \right] \\
\xi_2 = & \frac{1}{6\xi_1} \left[ -\sum_{n=1}^{\infty} \frac{1}{(2n)!} \left( \frac{1}{N_1} \right)^n \right] \\
\xi_3 = & -\frac{1}{4\xi_1 \cdot N_1} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \left( \frac{1}{N_1} \right)^n \right] \\
\xi_4 = & \frac{1}{2\xi_1 \cdot \left( 1 - k^2 \right) N_1} \left[ -k^2 \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left( \frac{1}{N_1} \right)^n \right] \\
\xi = & \frac{1}{N_1} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \left( \frac{1}{N_1} \right)^n \right] \left( 1 - k^2 \right) - \frac{1}{N_1} \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left( \frac{1}{N_1} \right)^n, \quad \text{and} \quad \tilde{N}_1 = \frac{N_i l^2}{EJ}.
\end{align*}
\]
The functions $\xi_i$, result to be trigonometric for a compression axial load ($\bar{N}_i > 0$) or hyperbolic if the axial load is a traction force ($\bar{N}_i < 0$). In compact form:

$$\mathbf{F}_s = \mathbf{k}_s \mathbf{q}_s,$$  (17)

where $\mathbf{F}_s$ is the vector of the nodal forces, $\mathbf{k}_s$ is the second order stiffness matrix for the Timoshenko’s beam and $\mathbf{q}_s$ is the vector of the nodal displacements

$$\mathbf{q}_s = [v_1, \psi_1, v_2, \psi_2].$$  (18)

Figure 1 shows a beam element with eccentric semi-rigid connections and relative degrees of freedom. The eccentricity is indicated using the length of two rigid links $e_1$ and $e_2$. The semi-rigid connection is modeled using a rotational spring with stiffness $R_{ki}$, and a translational spring with stiffness $R_{vi}$. Connection devices are considered massless and sizeless.

**Rotational and translational springs contribution**

Rotational and spring contribution could be defined using the following vector:

$$\delta_i^{T} = [\dot{\psi}_i, \dot{\psi}_i, -v_2, \psi_2].$$  (19)

where:

$$\dot{\psi}_i = \frac{M_i}{R_{ki}}, \dot{\psi}_i = \frac{S_{vi}}{R_{vi}}, (i = 1, 2)$$  (20)

Figure 2(c) shows it is possible to see that displacements and rotations are tied by these equations:

$$v_1 = v_1 + \bar{v}_1, \quad v_2 = v_2 - v_2, \quad \bar{\psi}_1 = \psi_1 + \dot{\psi}_1, \quad \bar{\psi}_2 = \psi_2 + \dot{\psi}_2.$$  (21)

or in compact form:

$$\mathbf{q}_s = \mathbf{q}_s + \delta.$$  (22)

where

$$\mathbf{q}_s = [v_1, \bar{\psi}_1, v_2, \bar{\psi}_2].$$  (23)

By replacing (22) in (20), for the (17) we obtain the following expression:

$$\mathbf{F}_s = \mathbf{F}_s + \mathbf{k}_s \delta.$$  (24)

Inverting this last Equation

$$\mathbf{F}_s = \mathbf{R}^{-1} \mathbf{k}_s \mathbf{q}_s$$  (25)

the detailed expression of the elements matrix of $\mathbf{R}$ is reported in [3]. Using the Equation (25) it is possible to rewrite the vector (19), by replacing the (20) and the (21) in the (19), obtaining:

$$\delta = \mathbf{S}_n \mathbf{q}_s,$$  (26)

The matrix $\mathbf{S}_n$ is responsible of the transformation in case of presence of both rotational and linear springs in a non-linear analysis. For the sake of brevity the detailed expression of the elements matrix of $\mathbf{S}_n$ is reported in [3]. By replacing the (26) in the (22) we obtain:

$$\mathbf{q}_s = \left[\mathbf{I} - \mathbf{S}_n\right] \mathbf{q}_s,$$  (27)

and

$$v = \mathbf{N}_v \mathbf{q}_s = \mathbf{N}_v \left[\mathbf{I} - \mathbf{S}_n\right] \mathbf{q}_s, \quad \psi = \mathbf{N}_\psi \mathbf{q}_s = \mathbf{N}_\psi \left[\mathbf{I} - \mathbf{S}_n\right] \mathbf{q}_s.$$  (28)

Detailed expressions of $\mathbf{N}_v$ and $\mathbf{N}_\psi$ are reported in the companion paper [3].

**Connections Eccentricity contribution**

Consider Figure 1(c) and the new degrees of freedom:

$$v_{e_1} = v_1 + \bar{v}_1 + e_1 \psi_1, \quad v_{e_2} = v_2 - v_2 - e_2 \psi_2.$$  (29)

and considering the (23), (29) and the (28), the relation between these dofs and the global dofs could be expressed as follows:

$$\bar{\mathbf{q}} = \left[\mathbf{I} + \mathbf{E}\right] \mathbf{q}_s,$$  (30)

where
and we made use of the shifter $E$:

$$E = \begin{bmatrix}
0 & -e_1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & e_2 \\
0 & 0 & 0 & 0 
\end{bmatrix}. \quad (32)$$

By replacing (30) in the (28), we obtain:

$$v = N_v (I + G_n) q, \quad \psi = N_\psi (I + G_n) q. \quad (33)$$

the corrective matrix $G_n$ is:

$$G_n = -S_n + E - S_n E. \quad (34)$$

where $S_n$ represents the bending and shear effect in the connection, $E$ expresses the effect of the eccentricity, while $S_n E$ introduces the combined. Setting $S_n$ and $E$ equal to zero we obtain that $G_n = 0$ and that the (33) becomes the well-known equations for the elements with rigid connections.

**Stiffness Matrix and equivalent nodal force vector**

The stiffness matrix for the beam element with flexible connections could be found through the total potential energy as follows:

$$U = \frac{1}{2} \int_0^l \left[ EA \left( u_0 + \frac{1}{2} v_0^2 \right)^2 + EJ \psi'^2 + kG_n A \left( v_0 + \psi \right)^2 \right] dx + \sum_{i=1}^2 R_{ii} \theta_i^2 + \sum_{i=1}^2 R_{ij} \theta_i \theta_j \quad (35)$$

or in compact form

$$U = U_a + U_f + U_s + U_i + U_m \quad (36)$$

where $U_a$ is the axial deformation energy, $U_f$ is the deformative flexional energy, $U_i$ is the shear deformation energy, and $U_s$ and $U_m$ the potential energy of the rotational and linear springs respectively. Using Equation (33) $v_0$ and $\psi_0$ becomes:

$$v = N_v (I + G_n) q = B_b (I + G_n) q, \quad \psi_0 + \psi = N_\psi (I + G_n) q + N_\psi (I + G_n) q = B_s (I + G_n) q. \quad (37)$$

where

$$B_b = N_v, \quad B_s = N_\psi, \quad B_s = N_v + N_\psi = B_b + N_\psi. \quad (38)$$

The elastic axial deformation energy and the shear deformation energy are connected by the parameter $k^2$. Setting $k^2 = cost$, we can express separately this two contributes as shown in Reference [7]:

$$U_f = \frac{1}{2} q^T (I + G_n)^T \left[ \int_0^l B_b^T E J B_b dx \right] (I + G_n) q \quad U_i = \frac{1}{2} q^T (I + G_n)^T \left[ \int_0^l B_s^T kG_n A B_s dx \right] (I + G_n) q$$

(1)(2)

or in compact form

$$U_f + U_i = \frac{1}{2} q^T (k_{\text{at}} + k_{\text{ref}}) q + \frac{1}{2} q^T (k_{\text{at}} + k_{\text{ref}}) q. \quad (39)$$

where matrices $k_{\text{at}}$, $k_{\text{ref}}$ and $k_{\text{at}}$, $k_{\text{ref}}$ are defined as:

$$k_{\text{at}} = \int_0^l B_b^T E J B_b dx, \quad k_{\text{ref}} = \int_0^l B_s^T kG_n A B_s dx, \quad (40)$$

$$k_{\text{ref}} = G_n^2 k_{\text{at}} + G_n^2 k_{\text{at}} + G_n^2 k_{\text{at}} G_n, \quad k_{\text{ref}} = G_n^2 k_{\text{at}} + k_{\text{at}} G_n + G_n^2 k_{\text{at}} G_n. \quad (41)$$

The potential energy of the springs can be expressed in the following matrix form:

$$U_s + U_m = \frac{1}{2} \delta^T C_m \delta, \quad (42)$$

where $C_m$ is a diagonal matrix containing on the diagonal the terms $R_{ii}$ and $R_{ij}$. Setting:

$$\mathbf{G}_n = S_n (I + E). \quad (43)$$

The Equation (42) becomes:
Neglecting the axial term, the potential energy of the beam could be expressed as:

\[ U = \frac{1}{2} \mathbf{q}^T \left[ \mathbf{k}_{ll} + \mathbf{k}_{el} \right] \mathbf{q} \]  

(45)

and the expression of the stiffness matrix of the whole system could be expressed in the following matrix form:

\[ \mathbf{k}_{ll} = \mathbf{k}_{el} + \mathbf{G}_{ll}^T \mathbf{k}_{el} + \mathbf{k}_{el} \mathbf{G}_{ll} + \mathbf{G}_{ll}^T \mathbf{k}_{el} \mathbf{G}_{ll} + \mathbf{k}_{el} \mathbf{G}_{ll} + \mathbf{G}_{ll}^T \mathbf{k}_{el} \mathbf{G}_{ll} + \mathbf{k}_{ll} \]  

(46)

Detailed expressions of the elements of stiffness matrices are reported in Reference [3].

The generalized forces vector \( \mathbf{Q}(t) \) could be obtained in the usual way considering a distributed force field \( q(x) \) acting on the deformable beam portion:

\[ \mathbf{Q} = \mathbf{Q}_1 + \mathbf{Q}_2 \]  

(47)

where \( \mathbf{Q}_1 \) and \( \mathbf{Q}_2 \) are the contribution of standard generalized force vector for a beam element and the contribution of eccentricity and rotational and linear flexibility of the connections

\[ \mathbf{Q}_1 (t) = \left[ \int_0^t \left( q(x,t) \mathbf{N}_{ei} + c(x,t) \mathbf{N}_{ei} \right) dx \right], \quad \mathbf{Q}_2 (t) = \left[ \int_0^t \left( q(x,t) \mathbf{N}_{ei} + c(x,t) \mathbf{N}_{ei} \right) dx \right] \mathbf{G}_n. \]  

(48)

**NUMERICAL EXAMPLES**

The ten floors frame shown in Figure 2 has been proposed as a benchmark to check the behavior of semi-rigid connections [10]. As shown by Figure 1 only the connections beam-column are considered semi-rigid and eccentric. Based on the above theoretical considerations, a computer program has been developed. For the sake of brevity, only some typical results are presented herein and are compared with the values obtained by the known software package SAP 2000 [4].

Figure 2 it shows the plane lateral displacement and the corresponding shear strains due by horizontal concentrated forces applied on the nodes as a function of the fixity factors defined as

\[ t_i = \frac{1}{1 + 3EJ/lR_{ki}}, \quad R_{ki} = \frac{3EJ}{l} \left( t_i - 1 \right) \]  

(49)

where \( R_{ki} \) is the initial rigidity of the connection. Three different values of the fixity parameters are used in this first analysis: \( t_i = 0.8, \ t_i = 0.9 \) and \( t_i = 1.0 \). Analysis in this range the relation between the horizontal displacement and the fixity factor is almost linear [3]. The solicitations shown in Figure 2 calculated for \( t_i = 1.0 \) overlap to the first order theory and SAP2000 solicitations.

![Figure 2: Ten story frame. Lateral displacements and shear values varying fixity factors: \( e_{1,2} = 0 \).](image-url)
Moreover the analysis reported in [3] shows that the behavior of averaged shear and the moment in the two columns at the base of the building. The fixity factor has a strong effect on the structure, but more pronounced in the displacement response than in the force responses.

Figure 8: Time history displacement at the left top floor node and at the 4th floor node of the frame with various values of the flexibility factor: (a) eccentricity = 0%; (b) eccentricity = 0.16%.

Two-steps ground acceleration

The frame is subjected to the sudden discontinuous two-steps uniform ground acceleration shown in Figure 3: here time histories of the lateral displacements at the left top of the frame with various connections are plotted. Gravitational loads considered as additional lumped masses at the beam nodes are included. Figure 3 shows as expected the frame, with flexible and eccentric connections, has longer amplitudes and period when compared with the rigid joint case. The displacement amplitudes and period increase with a decrease in joint stiffness. It is obvious that the displacement response of the frame decays with time for both flexible and rigid types of connections, but with different time. The influence of viscous damping at connections on displacement response of frame would be appreciable in this context, with particular regards to the flexible connections.

CONCLUSIONS

In this paper an analysis at second order for shear deformable beams for steel frame structures with semi-rigid connections has been developed. The finite element matrices incorporating the eccentricity and flexibility effect of the connections have been developed in an explicit form. Using kinematic relationships all the internal degrees of freedom are condensed out obtaining a standard 4x4 stiffness matrix. These kinematic relationships depend exclusively upon the eccentricity and flexibilities of the connections.

Numerical examples show that the structural responses of the frames with flexible connections and the frames with rigid connections are considerably different: an increase of the flexibility of the connection reduces the total rigidity of the frame as expected. This reduction is more marked when the rigidity of the connection decreases, rather than the eccentricities increase. Moreover it is observed that the connection flexibility has most effect on the lowest frequencies, and this particular is interesting regards to the seismic response of frame structures were the fundamental frequency is governing the seismic response.
REFERENCES

FIRE RESISTANCE DESIGN OF LARGE SPACE GRID STRUCTURES BY PERFORMANCE-BASED APPROACH — A CASE STUDY OF THE FIRE RESISTANCE DESIGN OF THE ROOF STRUCTURE OF KUNMING INTERNATIONAL AIRPORT

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KEYWORDS
Fire resistance design; large-space grid structures; performance-based approach; thermal loading condition; Kunming International Airport

ABSTRACT
The objective of fire resistance design for structures is that in the event of fire, the structures’ stability can be maintained for a reasonable period of time. This paper describes the fire resistance design of large space grid structures by performance-based approach (PBA). Heat transfer analysis of large space fire is introduced. The importance of using proper thermal loading condition in the global structural analysis of large space grid structures is illustrated by investigating the fundamental structural behaviour under fire. Three thermal loading conditions are proposed. A case study of the fire resistance design of the roof structure of the Kunming International Airport is given. The proposed thermal loading conditions are used in the project.

INTRODUCTION
Traditionally, prescriptive method is used to ensure fire resistance of structures. Prescriptive method is based on standard fire tests, using isolated elements to be tested in specially designed furnaces. Standard fire test is recognised to have many shortcomings, the main aspects include:
The standard fire curve bears little resemblance to a real fire. A real fire’s type and severity is governed by the geometry of the compartment, amount of combustible material, ventilation conditions and thermal characteristics of the compartment boundary. In large space where flashover is unlikely to happen, the applicable of standard fire is limited.

The behaviour of isolated elements in standard fire tests cannot represent real structural behaviour in fire. Broadgate fire and Cardington full-scale fire tests have shown that structures are more robust in a real fire. The structural elements of a building work interactively with each other. When structures exposed to fire, the “cool” elements will assist the “hot” elements by alternative load-path and the restraint conditions will change with time [1].

As a result, fire resistance design by prescriptive method often yields conservative results and the method’s restrictive nature of prescriptive rules make it inapplicable to the fire design of innovative and complex buildings like sport stadium, airport terminal or railway station [2].

To ensure fire safety of large and complex buildings whilst do fire resistance design scientifically and reasonably, performance-based approach (PBA) as an alternative is also allowed in current codes in many countries (Till now PBA had been used in fire protection design of many challenging projects [2, 3]). The PBA involves the assessment of three basic components comprising the likely fire behaviour, heat transfer to the structure and the structural response. It extends the newest achievements in structural fire engineering to design work, and has the following advantages:

- Be able to model real fire. By launching fire risk analysis, PBA can assess the severity of a real fire. Different fire models from simple nominal fire models to CFD/Field models can be considered.
- A better understanding of the behaviour of structures in fire. By advanced FEA, structure’s integral behaviour can be modelled.
- Generally more economical designs, compared to the simple approaches, whilst still ensure fire safety. Furthermore, PBA can increase the levels of safety.
- Be able to assess the fire safety of large and complex buildings.

This paper describes the fire resistance design of large space grid structures by PBA. By investigating the fundamental structural behaviour under fire, the importance of using proper mechanical and thermal model for fire resistance design of large space grid structures is illustrated. Heat transfer analysis of large space fire is given. Three thermal loading conditions (TLC) for global structural analysis are discussed. A case study of the fire resistance design of the roof structure of the Kunming International Airport is given.

**FUNDAMENTAL STRUCTURAL BEHAVIOUR UNDER FIRE EFFECTS**

*Theoretical Derivation*

Usmani et al. [4] give the fundamental mechanism of thermal effects on structures. When subjected to fire, the total strain of a structure is governed by

\[ \varepsilon_{total} = \varepsilon_{th} + \varepsilon_m \]

(1)

where \( \varepsilon_{th} = \varepsilon_T + \varepsilon_{\phi} \) is thermal strain, in which \( \varepsilon_T \) is the strain caused by thermal expansion and \( \varepsilon_{\phi} \) is the strain caused by thermal bowing; and \( \varepsilon_m \) is mechanical strain caused by applied loading or restrained thermal strains.
Any resistance to free movement of thermal expansion or thermal bowing will induce internal stresses within the structure. Considering an axially restrained steel column, as shown in Figure 1, the increase in the compressive load, $\Delta P$, due to uniform heating is given by Wang [5]

$$\Delta P = \frac{k_s}{k_s + k_c} (\Delta \varepsilon_{th} - \Delta \varepsilon_{mec}) k_c L_c$$  \hspace{1cm} (2)

$$\Delta \varepsilon_{mec} = \frac{P_0}{k_c L_c} - \frac{P_0}{k_{c0} L_c} = \frac{P_0}{E_T A} - \frac{P_0}{E_0 A}$$  \hspace{1cm} (3)

where, $k_s$, $k_c$ and $k_{c0}$ are the stiffness of the restraint, of the column at elevated and room temperature respectively; $\Delta \varepsilon_{th} = \alpha \Delta T$ is the free thermal strain of the column due to the temperature increase; $\Delta \varepsilon_{mec}$ is the increase of mechanical strain in the column at elevated temperature; $E_0$ and $E_T$ are the material’s elastic modulus at room and elevated temperature; and $P_0$ is the initial axial load.

Taking $R = k_c/k_s$ and $E_T = \eta_T E_0$, Eqn. 2 can also be expressed as

$$\Delta P = \frac{1}{1 + R} (\alpha \eta_T E_0 A \Delta T - P_0 + \eta_T P_0)$$  \hspace{1cm} (4)

where, $\eta_T$ is the reduction factor of elastic modulus of steel with temperature elevation given by EC3 [6].

As a result, the axial stress within the axially restrained column at high temperature is given by

$$\sigma = \frac{P_0 + \Delta P}{A} = \frac{1}{1 + R} \alpha \eta_T E_0 \Delta T + \frac{R + \eta_T P_0}{1 + R}$$  \hspace{1cm} (5)

For stocky sections with sufficient stiffness, the material will yield before buckling occurs, thus, considering $\sigma = \sigma_y = \chi_T f_y$, by Eqn. 5 we get

$$\Delta T_y = \frac{(1 + R) \chi_T f_y - (R + \eta_T) \sigma_0}{\alpha \eta_T E_0}$$  \hspace{1cm} (6)

where $\chi_T$ is the reduction factor of yield stress of steel with temperature elevation given by EC3 [6]; $\sigma_0 = P_0/A$ is the axial stress at room temperature; and $\Delta T_y$ is the yield temperature at which material yields.

For slender sections, the column will buckle before the material reaches its yield stress. Taking the Euler buckling load $P_{cr} = \pi^2 \eta_T E_0 I / l^2$ and $P_0 = \beta P_{cr}$, from $\sigma = \sigma_{cr}$ we get

$$\Delta T_{cr} = \frac{\pi^2}{\alpha \chi_T^2} [(1 + R) - (\eta_T + R) \beta] \approx \frac{\pi^2}{\alpha \chi_T^2} (1 + R)(1 - \beta)$$  \hspace{1cm} (7)

where $\Delta T_{cr}$ is the buckling temperature at which column buckles.
Parameter Studies

Considering $\alpha = 1.4 \times 10^{-5} \text{ m/(m}^0\text{C)}$, $E_0 = 2.06 \times 10^5 \text{ N/mm}^2$, $f_y = 295 \text{ N/mm}^2$, $k_s = n E_0 A / L$ ($n = 0.2, 0.5, 1, 2, 5, \infty$) and $P_0 = 0$, from Eqn. 7, we get the relationship between $\Delta T_{cr}$ and $\lambda$ as shown in Figure 2. Also, by Eqn. 6 we get $\Delta T_y$ for different $k_s$ as given by Table 1. From Figure 2 we know the stiffer the axial restraint, the less $\Delta T_{cr}$, whilst $\Delta T_{cr}$ decreases with $\lambda$ grows.

### Table 1

<table>
<thead>
<tr>
<th>$k_s$</th>
<th>0.2$E_0A/L$</th>
<th>0.5$E_0A/L$</th>
<th>1$E_0A/L$</th>
<th>2$E_0A/L$</th>
<th>5$E_0A/L$</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T_y$ (°C)</td>
<td>628</td>
<td>321</td>
<td>218</td>
<td>158</td>
<td>124</td>
<td>102</td>
</tr>
</tbody>
</table>

Figure 2: Buckling temperature increment for thermal expansion against finite axial restraint

Figure 3: Influence of applying load on the buckling temperature

Figure 3 shows the influence of applying load on the buckling temperature of axially restrained column given by Eqn. (7). It shows the buckling temperature decrease rapidly depending on the magnitude of the applying load.

**FUNDAMENTALS OF PBA**

**Design procedure**

Fire resistance design of steel structures by PBA can be carried out by following the
procedure shown in Figure 4. In PBA the worst credible fire scenarios are used for fire protection design.

After determining fire scenarios, key elements (usually the structural elements with least section factor $A/V$) are selected for heat transfer analysis. Then, calculated temperatures are compared with the critical temperature recommended in design practice (usually 550°C are adopted). In most cases when the calculated temperatures are much less than the critical temperature, the design will end with no passive protection needed, as shown by the dot line in Figure 4. In other cases initial fire protection strategy will be determined by ensuring the temperatures of the protected element not exceed the critical temperature.

Upon determining initial fire protection, structural analysis by proper mechanical model is often required to evaluate the initial fire proof’s validity. The mechanical model for structural analysis should be element model, sub-structure model and structure model as given in CIB W14 [7]. If the protected structure has sufficient load-bearing capacity in fire condition, design will end and the initial fire protection is recommended. Otherwise, initial fire protection should be enhanced until the structure can resist fire for a reasonable time.

![Figure 4: Flow chart of fire resistance design of steel structures by PBA](image)

**Thermal action of large space fire**

In large space fire, ‘flashover’ is unlikely to happen. Localized fire model instead of standard compartment fire model is preferable for heat transfer analysis in large space fire. The thermal action of localized fire can be assessed following the expressions recommended in EC1 [8]. Figure 5 shows the behavior of an axi-symmetric localized fire.
By heat balance, the average temperature increase of steel exposed to a localized fire is determined by,

\[ \Delta T_s = \frac{A}{V \rho_s c_s} \left( q_c + q_r \right) \Delta t \]

where, \( A/V \) is the section factor of the steel member; \( \rho_s, c_s \) are density and specific heat of the steel respectively; and \( q_c, q_r \) are convective and radiative heat transferred from localized fire environment to the steel respectively. In which \( q_c \) can be simply determined by

\[ q_c = h_c (T_p - T) \]

where, \( h_c \) is the coefficient of convection, taken as 0.025kW/m\(^2\)K; and \( T_p \) is the temperature of the gas surrounding the steel.

The calculation of radiative heat transfer in localized fire is complex. By idealizing the flame zone as an axi-symmetric cylinder and ignoring the absorption of radiation by plume, the radiation from flame zone to the steel is calculated by

\[ q_r = \phi \varepsilon_s \varepsilon_f \sigma (T_f^4 - T_s^4) \]

where, \( \phi \) is the configuration factor; \( \varepsilon_s \) is the surface emissivity of the member, taken as 0.625; \( \varepsilon_f \) is the effective emissivity of the fire, taken as 0.8; \( \sigma \) is the Stephan Boltzmann constant, taken as 5.67 x 10\(^{-11}\)kW/m\(^2\)K\(^4\); and \( T_f \) is the mean flame temperature.

**Thermal loading condition in Global FE Analysis of Grid Structures in Fire**

When processing global FE analysis to evaluate fire resistance of grid structures, the thermal loading condition (TLC) is the critical factor which influences the reliability and accuracy of the result. Basically, TLC should respect the temperature condition on structures in nature fire. However, nature fire is unlikely to be determined exactly in a real situation. In practice theoretical or empirical fire models are used.

Compartment fire model is the widely used empirical model which is applicable to most building fires (in small or medium compartment). In compartment fire model, the temperature distribution in the compartment is assumed to be uniform because of flashover, as a result LTC in compartment fire is very simple by the way that all exposed structures are assumed to be engulfed by the fire. Or in other words, all structures exposed to the compartment fire should be applied with the same temperature in global FE analysis.

However in large space fire where flashover is unlikely to happen, localized fire model instead of compartment fire model is preferred. In localized fire model, only structures above or near the fire source will be heated. As a result, LTC in localized fire is critical to the
location of the fire source (which is variable in large space because of the scattering of the fire loads). Furthermore for grid structures, elements are always located at different elevation and the cross sections of the elements are non-uniform after optimization design. Considering these aspects, it is difficult to determine TLC in global FE analysis of grid structures exposed to large space fire.

Literally there are three kinds of TLCs in global FE analysis of grid structures which are total heating (marked as M1), zone heating (M2) and isolated elements heating (M3). M1 means all elements should be loaded with temperature in fire analysis, which is adaptive where structures are exposed to compartment fire but too conservative where structures exposed to large space fire. M2 means only elements close to fire should be loaded with temperature in fire analysis. M2 can well model the real situation where structures are exposed to large space fire but the application of it in practice is restricted by the difficulty of determining the heating zone. M3 is the simplest one where only the critical elements which are often the weakest elements from the structural analysis in ambient temperature should be loaded with temperature in fire analysis. Though simple, M3 is the most suitable one from practice point of view to evaluate or design the fire resistance of large space grid structures by the following reasons,

- Using M3 in global FE analysis, the effect of structural continuity on the heating elements can be safely considered.
- From structural analysis in ambient temperature the critical elements can be determined easily. Besides, the temperature applying on the critical elements in M3 can be easily and safely taken as the highest temperature of the key elements in temperature analysis.

Based on the above analysis, M3, for its safe and easy to use, is proposed to be used in practical fire resistance design of large space grid structures.

A CASE STUDY

Figure 6 shows the steelworks of the roof with supporting structures of Kunming International Airport. The roof is assembled by space grid structures and has been divided into 7 parts by temperature joints. The division of the roof structure is illustrated in the top corner in Figure 6. The structural design of the roof in ambient temperature had been achieved by model and analyze different parts individually. As a result, fire resistance design of the roof has been implemented by investigation the fire safety of different parts respectively. In this paper, global FE analysis of roof structures in typical areas (Parts A and B in Figure 6) in fire situation has been introduced to illustrate the fire design of large space grid structures by the aforementioned method. More and detail information about the Kunming international airport project can be found in the project report [9].
Table 2 gives the fire severities of the worst credible fire scenarios and the location of the key elements in temperature analysis. Steady-state fires are conservatively considered. Elements in different structural planes (which are bottom chord plane, web plane and top chord plane) of the grid structures, with lowest section factor (Hp/A=250m⁻¹), are selected as the key elements. Table 1 also gives the calculated highest temperature of the key elements. Temperature analysis recommends all structures can be left unprotected.

### TABLE 2

<table>
<thead>
<tr>
<th>Design fire</th>
<th>Key elements (Height to floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fire severity</td>
</tr>
<tr>
<td>Part A</td>
<td>4.5MW</td>
</tr>
<tr>
<td>Part B</td>
<td>4.5MW</td>
</tr>
<tr>
<td>T\text{max,A}</td>
<td>-</td>
</tr>
<tr>
<td>T\text{max,B}</td>
<td>-</td>
</tr>
</tbody>
</table>

After temperature analysis, structural analysis by global FE models with M3 TLC is processed to check and verify the initial fire protection strategy. Weakest elements which are elements with maximum stress (tension and compression) in structural analysis at ambient temperature in different structural planes are selected as the critical elements in M3. The temperatures, applying on those critical elements, are conservatively taken as the highest temperatures of the key elements in temperature analysis at the same structural plane. For example, the temperatures of the critical elements at bottom chord plane of Part A are taken as 501°C in table 2 which are the temperatures of the key elements at bottom chord plane in temperature analysis.

In global FE analysis under fire, neither global nor local instability has happened, or in other words, the structures can maintain their stability during the whole heating processes. Therefore, the validity of the initial fire protection has been verified by structural analysis.
CONCLUSIONS AND DISCUSSIONS

This paper investigates the fire resistance design of large space grid structures by PBA, plus a simple introduction to the real fire resistance design of the roof structure of the Kunming international Airport by the approach given in this paper. The following conclusions can be drawn:

- The fire resistance design of large space grid structures should comprise two main steps. First determine initial fire protection by temperature analysis then check and verify the initial protection by structural analysis.
- Global FE model should be adopted in evaluating fire resistance of large space grid structures.
- The reliability and accuracy of the results from global FE analysis is critical to the temperature load condition (TLC) in the FE model. Generally, M3 TLC is recommended in daily fire engineering design of large space grid structures.

It should be pointed out that the fundamental structural behavior under fire condition focus on the behavior of the axially restraint elements. In real fire condition, heating elements will “push” the adjacent elements by thermal expansion while providing restraints to the thermal expansion of the adjacent elements. The ‘pushing’ behavior is much complex and hasn’t been considered in current codes. For frameworks, the ‘pushing’ behavior has little influence on the fire resistance of beam/column members but for grid structures ‘pushing’ behavior might have considerable effect on the fire resistance of the structures. However, till now, little work has done on this subject. Further studies are planned by the authors to study those ‘push’ behavior in grid structures.

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REFERENCE


GEOPOLYMER CONCRETE FILLED STEEL TUBES
AT ELEVATED TEMPERATURES

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KEYWORDS
Geopolymer concrete, composite tubular sections, elevated temperatures, fire.

ABSTRACT
Concrete filled tubes have been used in many structural engineering applications, e.g. high rise buildings and bridge piers. During construction the steel tube provides permanent formwork to the concrete. The steel tube can also support a considerable amount of construction loads prior to the placement of wet concrete, which results in quick and efficient construction. The steel tube provides confinement to the concrete core while the infill of concrete delays local buckling of steel tubes. Compared with unfilled tubes, concrete-filled tubes demonstrate increased ductility and energy absorption during earthquake as well as increased fire resistance. There is a rapid development in concrete technology such as high performance concrete. Geopolymer concretes are emerging new materials promising superior fire resistance and durability and potentially cheaper than the widely used high strength concretes. Production of 1 ton of Portland cement releases 1 ton of greenhouse gases. The geopolymer concrete investigated in this paper utilizes fly ash instead of Portland cement. This paper reports the testing of geopolymer concrete-filled steel tubes at both ambient and elevated temperatures. Results are also compared with those of SCC (self-consolidating concrete) filled tubes.

INTRODUCTION
Geopolymer Concrete is a newer kind of concrete which replaces the Portland cement mix with a Fly-Ash based geopolymer mix [1-4]. This involves substituting Portland-Cement and water for Fly-Ash and a liquid activator to make non-cement based concrete. Fly-Ash, because of its ability to produce high compressive strengths and its relative abundance and cheap price compared to other geopolymers, has come to the fore as a modern geopolymer use. As a by-product of coal burning, fly-ash is rich in Silicon and Alum inium and is activated by being mixed with Sodium Silicate (Na₂SiO₃) and Sodium Hydroxide (NaOH). Unlike Portland cement concretes however, where the compressive strength is highly influenced by the water to cement ratio, the liquid to binder ratio of Geopolymers has been found to have little to no effect on the overall compressive strength. Previous investigation
has in fact shown that the compressive strength is more influenced by curing temperature, the curing time, and the types of activators used [5]. Perhaps the most promising features of geopolymer concretes has been their potential for a reduction in environmental impact through a lowered release of CO$_2$ compared to Portland based cement production and its greatly superior fire resistance performance with the concrete showing no spalling effects during elevated temperatures and high residual strength.

Concrete filled steel tubes (CFST) have proven to provide significant benefits both during and post construction [6, 7]. During construction, the steel skin provides a permanent formwork for pouring of wet concrete. Further, the steel is also capable of carrying extremely large construction loads, allowing the lower storeys to confine wet concrete, whilst the upper storeys are being formed, which leads to greatly reduced construction time and costs. Also, the concrete infill acts as a restraining medium, increasing the local buckling coefficient, thus lowering or preventing local buckling from occurring on the steel skin [8].

This paper explores the potential for a further increase in the composite fire resistance of CFST by the use of geopolymer concrete. Specially examined is the behaviour and performance of CFST with geopolymer at temperatures according to the standard fire curve of AS1530.4 [9]. Twelve specimens were constructed. Eight of these were tested in the furnace (four tubes with thermocouples) and the remaining four tubes tested in ambient conditions. Comparisons are also made with those filled with self-consolidating concrete (SCC) [10].

**FIRE EXPOSURE INVESTIGATION**

**Specimens**

The fire exposure study was conducted on 8 tubes. Each of the geopolymer mixes where tested using 4 tubes (2 x SHS 150x5 mm & 2 x SHS 200x6 mm) utilising load densities similar to recent fire testing of SCC [10]. The load density is the ratio of the load applied at fire testing to the section capacity of compression at ambient temperature. Specimen labels are given in Table 1 where AGG stands for aggregate mixes and PST stands for paste mixes.

**Material Properties**

No testing for the steel material properties was conducted as the steel used in this investigation was the same as that for the SCC filled steel tubes [10]. The geopolymer concrete mixes specified for the investigation included an aggregate mix with a similar density (2500 kg/m$^3$) and strength of 85-87 MPa and a lighter weight (1800 kg/m$^3$) and similar strength of 80-85 MPa. Details of the mix proportions are summarised in Table 1.

**TABLE 1**

<table>
<thead>
<tr>
<th>Components</th>
<th>Aggregate Mix (kg/m$^3$) Paste</th>
<th>Mix (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fly-Ash</td>
<td>381</td>
<td>381</td>
</tr>
<tr>
<td>NaOH 49</td>
<td></td>
<td>49</td>
</tr>
<tr>
<td>Na$_2$SiO$_3$ 122</td>
<td></td>
<td>122</td>
</tr>
<tr>
<td>Coarse Aggregate</td>
<td>1294</td>
<td>0</td>
</tr>
<tr>
<td>Sand 554</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
Experimental Procedures

Construction of the specimens involved sealing of one end of the tube using a plate and then pouring the concrete into the tube from the open end. The tubes were cured in an oven at 60°C for at least 48 hours and cylinders were tested for compressive strength on the day of furnace testing. All of the initial 8 Specimens were then tested in the Monash University Civil Engineering Gas Furnace and 500 ton reaction frame with temperatures following the standard ISO fire curve as specified by AS 1530.4 [9]. All the specimens were tested till failure was achieved or if 90 minutes was achieved. For this experiment, failure was defined as the point at which the specimen could no longer support the load without excessive deflection.

Experimental Results

Results of the furnace tests are summarised below in Table 2 and Figure 1. None of the tested specimens made the 90 minutes test time and the test was terminated once failure was achieved.

<table>
<thead>
<tr>
<th>Specimen Label</th>
<th>SHS (B x t)</th>
<th>Load Density</th>
<th>Fire Resistance (minutes) (this paper)</th>
<th>Fire Resistance (minutes) [10]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGG150-1 150x5</td>
<td>0.44</td>
<td>24</td>
<td></td>
<td>26</td>
</tr>
<tr>
<td>PST150-1 150x5</td>
<td>0.44</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGG200-1 200x6</td>
<td>0.38</td>
<td>45</td>
<td></td>
<td>43</td>
</tr>
<tr>
<td>PST200-1 200x6</td>
<td>0.38</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGG150-2 150x5</td>
<td>0.17</td>
<td>85</td>
<td></td>
<td>90</td>
</tr>
<tr>
<td>PST150-2 150x5</td>
<td>0.17</td>
<td>27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGG200-2 200x6</td>
<td>0.26</td>
<td>45</td>
<td></td>
<td>55</td>
</tr>
<tr>
<td>PST200-2 200x6</td>
<td>0.26</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It can be seen from Table 2 that the Paste mixes did not perform as well as the Aggregate mixes, with the paste mixes consistently failing several minutes before the equivalent aggregate mix. Similar to the study of SCC, both the geopolymer concrete mixes underwent stages of deformation during the elevated temperature test. Initially, all the specimens underwent axial expansion where the axial deformation was exceeded by the expansion of the concrete. Then, as the steel and concrete began to weaken, the deformation exceeded the expansion and thus specimen began to deform, which continued until the specimen reached failure.

Failure of the specimens occurred in the central region (see Figure 2) of the columns with the steel sections buckling outwards along the length of the column with the major buckling occurring at the mid section of the column. Cutting open of the sections post testing revealed cracking and crushing of the concrete along the column length. It is interesting to note the significant crushing failure that has occurred in the paste mix specimens compared to the relatively minor crushing in the aggregate mixes (see Figure 3).
Discussion

The results of the furnace experimental investigation yielded results that were comparable to the fire exposure results of SCC in steel tubes. All the tubes failed in a similar manner, with the steel losing strength as the temperature increased, causing the load to transfer to the concrete core. As the steel lost its strength, it also lost its confining effect to concrete, and as the load on the concrete increased, its capacity became further compromised until eventually failing by crushing.

Figure 1: Deflection versus time curves

Figure 2: Specimen axial deformation
Similarly, the fire resistance of the specimens ranged from 19 to 85 minutes. These results did not compare favourably to the SCC SHS with the SCC performing significantly better than the geopolymer paste mix and the marginally better than the geopolymer aggregate mix.

**TEMPERATURE DISTRIBUTION**

*Experimental Setup & Procedures*

Thermocouples were installed in the second set of tubes (denoted by -2) in order to map an initial set of data on the temperature distribution field within the SHS and to determine if there was significant difference between the mixes. The placement of the thermocouples is shown in Figures 4 and 5.

**Observations**

Temperature versus time curves are plotted in Figure 6 for reference points 2, 5 and 8. A comparison between the observed temperature fields indicated that the difference between the aggregate and paste mixes was highly dependent on the size of the SHS used. For the 150mm SHS, the noted difference between the paste and aggregate mix was small. This compares to the 200mm SHS in which the difference between the mixes was larger, with the aggregate mix being higher.
AMBIENT CAPACITY INVESTIGATION

Experimental Setup & Procedures

Four pristine specimens and four specimens that were tested at longer fire exposure (smaller load utilisation) were used for the ambient capacity testing. Construction of the pristine specimens followed the same procedures as the furnace tested specimens. Specimens were tested by placing them in the Monash Civil Engineering Department 500 tonne Amser Testing rig and crushing the specimens until failure. For this testing, failure was determined to be at significant deformation or if the steel ruptured.

Figure 6: Temperatures versus time curves
Experimental Results & Discussion

Test results of the ambient testing is summarised in Table 3 and by Figure 7. Results of the tests indicated that the furnace specimens continued to deform and buckle in the centre of the column length where they had buckled during elevated temperature exposure. In most cases, the specimens eventually failed either by the welds in the steel tubing rupturing, or the corner of the steel tube rupturing. It was interesting to note the increase in ultimate deformation the specimens underwent post fire exposure. The results have indicated that the heating has made the SHS “soften”, which has delayed the eventual rupture of the sections during subsequent ambient loading testing.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Load Density</th>
<th>Pristine Capacity (kN)</th>
<th>Furnace Capacity (kN)</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGG150-2</td>
<td>0.17</td>
<td>2364</td>
<td>2092</td>
<td>88%</td>
</tr>
<tr>
<td>PST150-2</td>
<td>0.17</td>
<td>1995</td>
<td>1761</td>
<td>88%</td>
</tr>
<tr>
<td>AGG200-2</td>
<td>0.26</td>
<td>3770</td>
<td>3980</td>
<td>106%</td>
</tr>
<tr>
<td>PST200-2</td>
<td>0.26</td>
<td>3319</td>
<td>3303</td>
<td>99%</td>
</tr>
</tbody>
</table>

(a) SHS 150x5mm aggregate specimens  (b) SHS 150x5mm paste specimens
(c) SHS 200x6mm aggregate specimens  (d) SHS 200x6mm paste specimens

Figure 7: Load deflection curves
The results have also highlighted some significant similarities and differences between the concrete mixes and SHS sizes. Both results for the 150x5mm SHS specimens, shown in Figure 7 (a) and (b) displayed similar load-deflection behaviour which is supported by the similar temperature field data observed. Further, the 150x5mm SHS displayed lower ductility than the 200x6mm SHS. Interestingly however, the 200x6mm SHS specimens displayed differing results with the post-furnace Paste specimen not displaying any steel rupturing during testing. Rather, the specimen continued to deform until it was decided that the specimen had deformed sufficiently to deem that it had failed. These results compare to the results of the pristine specimens in which the point of buckling, rupture and failure occurred towards the bottom of the specimen. Similarly, the specimens failed either by corner rupture or weld rupture.

All the results with the exception of the 200mm aggregate specimen indicated that the residual strength of the furnace tested specimens remained above 88%. It was later determined, that the specimen was placed into the testing rig slightly off centre which introduced bending moment in the specimen.

CONCLUSION

Eight Geopolymer filled SHS tubes were tested in elevated temperature conditions according to the ISO standard fire curve in order to compare their performance to SCC filled SHS tubes. The results found showed that the aggregate based mix performed well compared to SCC whereas paste mix ones did not perform that well. The investigation also found that the geopolymer concrete filled tubes showed excellent performance in a post fire exposure condition.

ACKNOWLEDGEMENTS

The authors wish to thank Mr. Kevin Nievaart and the late laboratory manager, Mr. Graeme Rundle, for their assistance in conducting the tests.

REFERENCES

BEHAVIOUR OF CONCRETE-FILLED DOUBLE SKIN STEEL TUBULAR BEAM-COLUMNS AFTER EXPOSURE TO FIRE

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KEYWORDS
Concrete-filled double skin steel tubes (CFDST), beam-columns, fire exposure, residual strength, post-fire.

ABSTRACT
Test results of 16 concrete-filled double skin steel tubular (CFDST) beam-columns are presented in this paper. Two types of column with different cross-sections were investigated, i.e. circular hollow section (CHS) inner and CHS outer, CHS inner and square hollow section (SHS) outer, respectively. Other parameters explored were load eccentricity, slenderness ratio and fire exposure time. It was found that the fire exposure had significant influence on the performance of CFDST beam-columns. A finite element (FE) modelling technique was then developed to predict the structural response of the composite members after exposure to fire. Reasonable agreement is obtained between the predicted load-deformation curves and the test results. It is therefore possible to conduct further investigation on the mechanisms of the composite beam-columns by using the FE modelling.

INTRODUCTION
Composite steel-concrete construction has been widely used in structures for a long time. More recently, a creative innovation of composite construction known as concrete-filled double skin steel tubes (CFDST) was put forward [1-3]. They consist of two concentric steel tubes with concrete sandwiched between them. So they have almost all the same advantages as conventional concrete-filled steel tubes (CFST), but with lighter weight and better cyclic performance.

In the past few years, many studies have been performed on this kind of innovative composite column to investigate their performance under static and dynamic loading conditions [4].
More recently, Yang and Han [5] presented a theoretical model to predict the performance of CFDST columns subjected to ISO-834 standard fire. However, no information is available about the post-fire performance of CFDST columns after exposure to fire.

To fill the above gap, a research program has been developed and carried out recently by the authors, where six test results of CFDST stub columns have been reported by Yu et al. [6], and the test results of 16 CFDST beam-columns will be given in this paper. Two kinds of beam-column with different cross-sections were investigated as shown in Figure 1, i.e., circular hollow section (CHS) inner and CHS outer, CHS inner and square hollow section (SHS) outer. Other parameters explored were load eccentricity \( e \) (0, 30 mm), slenderness ratio \( \lambda \) (21.8-42.2) and whether or not exposure to fire. The finite element (FE) program ABAQUS was then adopted to model the CFDST specimens.

![Figure 1: CFDST cross-section configurations](image)

**EXPERIMENTAL INVESTIGATION**

**Specimen Preparation**

All of the 16 CFDST beam-columns in this paper were designed with a same hollow section ratio (\( \chi \)) of 0.51. \( \chi \) was defined as \( D_i/(D_o-2t_{so}) \), where \( D_i \) and \( D_o \) are the major dimensions of the inner and outer tubes, respectively; \( t_{so} \) is the wall thickness of the outer steel tube. More specimen details are presented in Table 1, where \( L_e \) is the effective length of a specimen; \( \lambda \) is the column slenderness ratio which is defined as \( L_e/i \), where \( i = \frac{J}{I_{sc}} \) is the section radius of gyration, \( J \) and \( A_{sc} \) are the second moment of the area and area of a composite cross section, respectively; \( T \) is the fire exposure time; \( N_{ue} \) is the experimental ultimate strength; and \( N_{uc} \) is the predicted ultimate strength using the FE software ABAQUS.

For all the specimens presented in Table 1, their inner and outer steel tubes were all manufactured from mild steel sheets with a measured thickness of 2.49 mm. The diameter or width (\( D_o \)) of the outer tubes is 200 mm, whilst the diameter (\( D_i \)) of the inner tubes is 100 mm. Tension tests were conducted on three coupons to measure the steel material properties. The average yield strength and ultimate strength of the steel were 308.9 N/mm\(^2\) and 450.5 N/mm\(^2\) respectively, whilst the elastic modulus, Poisson’s ratio and elongation were found to be 1.95\(\times10^5\) N/mm\(^2\), 0.288 and 26.2\%, respectively.

All specimens were cast from one batch of concrete. The concrete mix was found to give a compressive cube strength (\( f_{cu} \)) of 37.6 N/mm\(^2\) at the time of fire exposure. The average value of \( f_{cu} \) was 45.6 N/mm\(^2\) at the time of strength tests.

Eight specimens shown in Table 1 were heated by exposing them to heat in a furnace.
specially built for testing building structures in Tianjin, China. The furnace heating was controlled as closely as possible to the ISO-834 standard fire curve [7]. The fire duration time \( T \) was set to be 90 min.

### TABLE 1

**SPECIMEN LABELS AND MEMBER CAPACITIES**

<table>
<thead>
<tr>
<th>Section type</th>
<th>Specimen label</th>
<th>( L_e ) (mm)</th>
<th>( e ) (mm)</th>
<th>( \lambda )</th>
<th>( T ) (min)</th>
<th>( k_e ) (kN/mm)</th>
<th>( D_\lambda )</th>
<th>( N_{ue} ) (kN)</th>
<th>( N_{ue} ) (kN)</th>
<th>( N_{ue}/N_{ue} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHS inner, CHS outer</td>
<td>CCBC-1</td>
<td>1218</td>
<td>0</td>
<td>22.0</td>
<td>90</td>
<td>0</td>
<td>22.0</td>
<td>0</td>
<td>4.23</td>
<td>1598</td>
</tr>
<tr>
<td></td>
<td>CCBC-1F</td>
<td>1218</td>
<td>0</td>
<td>22.0</td>
<td>90</td>
<td>0.62</td>
<td>2918 8.39</td>
<td>871</td>
<td>1024</td>
<td>0.851</td>
</tr>
<tr>
<td></td>
<td>CCBC-2</td>
<td>1218</td>
<td>30</td>
<td>22.0</td>
<td>90</td>
<td>0</td>
<td>—</td>
<td>1804</td>
<td>—</td>
<td>1066</td>
</tr>
<tr>
<td></td>
<td>CCBC-2F</td>
<td>1218</td>
<td>30</td>
<td>22.0</td>
<td>90</td>
<td>0.63</td>
<td>358 15.54</td>
<td>636</td>
<td>747</td>
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<td></td>
<td>CCBC-3</td>
<td>2336</td>
<td>0</td>
<td>42.2</td>
<td>0</td>
<td>—</td>
<td>603 3.56</td>
<td>1428</td>
<td>1458 0.979</td>
<td></td>
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<tr>
<td></td>
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<td>2336</td>
<td>0</td>
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<td>90</td>
<td>0.46</td>
<td>1275 7.06</td>
<td>673</td>
<td>673</td>
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<tr>
<td></td>
<td>CCBC-4</td>
<td>2336</td>
<td>30</td>
<td>42.2</td>
<td>90</td>
<td>0</td>
<td>—</td>
<td>169 5.50</td>
<td>904</td>
<td>983 0.920</td>
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<td></td>
<td>CCBC-4F</td>
<td>2336</td>
<td>30</td>
<td>42.2</td>
<td>90</td>
<td>0.49</td>
<td>90 12.65</td>
<td>472</td>
<td>491</td>
<td>0.961</td>
</tr>
<tr>
<td>CHS inner, SHS outer</td>
<td>SCBC-1</td>
<td>1364</td>
<td>0</td>
<td>21.8</td>
<td>90</td>
<td>0</td>
<td>—</td>
<td>3914 2.50</td>
<td>1816</td>
<td>1658 1.095</td>
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<tr>
<td></td>
<td>SCBC-1F</td>
<td>1364</td>
<td>0</td>
<td>21.8</td>
<td>90</td>
<td>0.68</td>
<td>1232 4.80</td>
<td>902</td>
<td>1129</td>
<td>0.800</td>
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<td>1364</td>
<td>30</td>
<td>21.8</td>
<td>90</td>
<td>0</td>
<td>—</td>
<td>1418 3.29</td>
<td>1267</td>
<td>1193 1.062</td>
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<td></td>
<td>SCBC-2F</td>
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<td>21.8</td>
<td>90</td>
<td>0.86</td>
<td>627 8.96</td>
<td>712</td>
<td>1028</td>
<td>0.693</td>
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<td></td>
<td>SCBC-3</td>
<td>2628</td>
<td>0</td>
<td>41.9</td>
<td>0</td>
<td>—</td>
<td>1315 2.93</td>
<td>1630</td>
<td>1600 1.019</td>
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<td></td>
<td>SCBC-3F</td>
<td>2628</td>
<td>0</td>
<td>41.9</td>
<td>90</td>
<td>0.66</td>
<td>893 3.63</td>
<td>881</td>
<td>1055</td>
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<td>2628</td>
<td>30</td>
<td>41.9</td>
<td>90</td>
<td>0.33</td>
<td>216 3.46</td>
<td>1146</td>
<td>1322 0.867</td>
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<tr>
<td></td>
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<td>30</td>
<td>41.9</td>
<td>90</td>
<td>0</td>
<td>—</td>
<td>104 3.36</td>
<td>511</td>
<td>439</td>
</tr>
</tbody>
</table>

**Test Results**

All beam-columns were tested as pin-ended supported and subjected to single curvature bending. The loading and data collection method is the same as that described by Tao and Han [2].

It was found that the fire exposure had no apparent influence on the failure modes. For most axially loaded specimens, local buckling failure mode was observed since thin-walled steel tubes were used. The local buckling generally occurred near the top and/or bottom ends due to the effect of end conditions. Comparatively, all eccentrically loaded columns showed an overall buckling failure mode, and local buckling occurred near the specimen mid-height.

The dot points in Figures 2 and 3 show the approximate locations of observed local buckling of the steel tubes. It is observed that the steel tubes of fire-damaged specimens bucked earlier than those of undamaged specimens.

The maximum loads \( (N_{ue}) \) obtained in the tests are summarised in Table 1. Figure 2 shows the axial load \( (N) \) versus axial shortening relations for those beam-columns under axial compression, where the specimen axial shortening \( (\Delta) \) is normalised by the undeformed height of the specimens \( (L) \). Axial load \( (N) \) versus mid-height lateral deflection \( (u_m) \) for those eccentrically loaded specimens are presented in Figures 3. It can be seen that a specimen exposed to fire can result in significant loss in terms of ultimate strength and stiffness.
However, the post-peak curves of the undamaged specimens are much steeper compared with those of the damaged specimens. It is worth noting that very significant strength loss was observed for the specimen SCBC-4F. This is due to the fact that severe initial local buckling was formed for the outer steel tube of this specimen because of vapour pressure during the fire exposure.

![Graphs showing axial load versus normalized axial shortening for various specimens.](image)

**DISCUSSION**

For convenience of analysis, a residual strength index ($k_r$) is defined to quantify the residual strength of composite columns subjected to standard fire. It is expressed as:

$$k_r = \frac{N_{u,r}}{N_u}$$

where $N_{u,r}$ is the residual strength of specimens after exposure to fire, and $N_u$ is the ultimate strength of specimens at ambient temperature. The values of $k_r$ for all fire-damaged specimens range from 0.33 to 0.86, as shown in Table 1.

Figure 4 compares $k_r$ for specimens with different parameters. Generally, specimens with CHS inner and SHS outer (SCBC series) had higher residual strength than those with CHS inner and CHS outer (CCBC series) except the specimen SCBC-4F. This trend was also found for those CFDST stub columns presented by Yu et al. [6], which can be explained by the fact that the former specimens had a larger volume of core concrete. It can also be seen from Figure 4 that, the larger the slenderness ratio, the smaller the value of $k_r$ is. This is not surprising since a slender column was affected more significantly by the degradation in material modulus after exposure to fire. The load eccentricity has no significant influence on
the residual strength for specimen ens in CCBC series as shown in Figure 4. However, the influence of load eccentricity on residual strength is not so clear for specimen ens in SCBC series. It seems that more research is needed to clarify this.

![Graphs showing axial load (N) versus mid-span lateral deflection (um) for CCBC and SCBC series.](image)

(a) $\lambda=22.0, e=30\text{ mm}$
(b) $\lambda=42.2, e=30\text{ mm}$
(c) $\lambda=21.8, e=30\text{ mm}$
(d) $\lambda=41.9, e=30\text{ mm}$

Figure 3: Axial load (N) versus mid-span lateral deflection (um) curves

![Bar chart showing the residual strength indexes for CFDST beam-columns.](image)

Figure 4: The residual strength indexes for CFDST beam-columns

Stiffness ($K_e$) in the present context is determined from load versus mid-height deflection curves, which is defined as secant modulus corresponding to axial load of 0.6 $N_{ue}$ in the pre-peak stage. The values of $K_e$ are given in Table 1 except that for the specimen CCBC-1, since this axially loaded specimen had no apparent deflection detected before its peak load was attained. As can be seen from Table 1, generally, a specimen exposed to fire resulted in considerable loss of stiffness. It is also worth to note that $K_e$ decreases with an increasing of slenderness ratio ($\lambda$) or load eccentricity ($e$).

In order to quantify the effect of fire exposure on the ductility of specimens, a ductility index ($DI$) is defined herein for specimens subjected to eccentric loading:
where \( u_{90\%} \) is the mid-height deflection when the load falls to 90% of the ultimate load, \( u_y \) is equal to \( u_{50\%/0.5} \), and \( u_{50\%} \) is the mid-height deflection when the load attains 50% of the ultimate load in the pre-peak stage. Since no apparent deflection was detected for those specimens under axial compression, nominal axial shortening (\( \Delta/L \)) is used to replace the mid-height deflection in Eq. (2) in calculating \( DI \).

All the ductility indexes (\( DI \)) so determined are given in Table 1, and Figure 5 compares the ductility indexes for all tested beam-columns with different parameters. It should be noted that very small mid-height deflection was achieved at the load of 0.5 \( N_{ue} \) for the specimen CCBC-2. Therefore, no meaningful value of \( DI \) can be obtained for this specimen. It is found that the \( DI \) indexes for the CCBC specimens are generally larger than the SCBC specimens, indicating the former displays a better ductile behaviour. It can also be observed from Figure 5 that, the \( DI \) index increases for specimens after fire exposure.

FINITE ELEMENT MODELLING

To further investigate the behaviour of CFDST beam-columns after exposure to fire, the finite element software ABAQUS was used in this paper. In the past, Han [8] proposed a simple concrete model and an elastic-plastic model with isotropic strain hardening for steel material, which can be used to simulate both CFDST and conventional concrete-filled steel tubular members at ambient temperatures. This FE modelling technique will be used in this paper in simulating CFDST beam-columns at ambient conditions.
For those specimens exposed to fire, the temperature distributions were calculated by using ABAQUS at first [6]. Then the results were inputted in the following static analysis by using *LOAD option available in the ABAQUS library in simulating the post-fire behaviour of the specimens. A model for steel after exposure to high temperatures was given by Han et al [9], and used in this paper. As far as the concrete model is concerned, Lin [10] proposed a modified model for core concrete based on the model at ambient temperatures given by Han [8]:

\[
y = \begin{cases} 
2x - x^2 & (x \leq 1) \\
-\frac{x}{\beta_0(x-1)^\eta + x} & (x > 1)
\end{cases}
\]

in which, \( x = \frac{\varepsilon}{\varepsilon_0}; \ y = \frac{\sigma}{\sigma_0}; \)

\[
\sigma_0 = \frac{f'_c}{1 + 2.4(T_{\text{max}} - 20)^6 \times 10^{-17}} \text{(N/mm}^2\text{);} \\
\varepsilon_0 = \varepsilon_c + \frac{800}{\varepsilon_c} \times 10^{-6}; \\
\varepsilon_c = (1300 + 12.5f'_c) \times 10^{-6} \times \left[1 + (1500T_{\text{max}} + 5T_{\text{max}}^2) \times 10^{-6}\right];
\]

\[
\eta = \begin{cases} 
2 & \text{for CFDST column with CHS inner and CHS outer} \\
\frac{1.6 + 1.5}{x} & \text{for CFDST column with CHS inner and SHS outer}
\end{cases}
\]

\[
\beta_0 = \begin{cases} 
\frac{f_c^{0.1}}{1.2\sqrt{1 + \xi}} & \text{for CFDST column with CHS inner and CHS outer} \\
(2.36 \times 10^{-5})^{0.25}((\xi - 0.5)^{0.5})f_c^{0.5} \times 0.5 \geq 0.12 & \text{for CFDST column with CHS inner and SHS outer}
\end{cases}
\]

Where \( \varepsilon \) is the strain; \( \sigma \) is the stress; \( f'_c \) is the cylindrical strength of concrete; \( T_{\text{max}} \) is the historical highest temperature; and \( \xi \) is the confinement factor [2, 3].

The results obtained from the FE modelling were verified against those obtained from the experimental results in Figures 2 and 3 in dashed lines, and the ultimate strengths calculated by ABAQUS were given in Table 1. Reasonable agreement was achieved between the predicted load-deformation curves and the test results, although the simulation for the specimens in CCBC series is a little bit better than that for the specimens in SCBC series.

**CONCLUSIONS**

The following conclusions could be drawn within the limitations of the current research:

1. The strength of the CFDST beam-columns decreased significantly after exposure to fire. Generally, the specimens in SCBC series (section type with CHS inner and SHS outer) shown a higher residual strength than the specimens in CCBC series (section type with CHS inner and CHS outer).
2. Fire exposure resulted in considerable loss of stiffness for CFDST specimens. The stiffness decreased with an increasing of slenderness ratio (\( \lambda \)) or load eccentricity (\( e \)).
3. Fire exposure led to an increase in the column ductility. The specimens in CCBC series displayed a better ductile behaviour than those in SCBC series.
4. FE modelling was developed in this paper, which was able to reasonably predict the axial load versus mid-height lateral deflection on or axial shortening relations for CFDST.
beam-columns. It is therefore possible to conduct further investigation on the mechanisms of the composite beam-columns by using the FE modelling.

ACKNOWLEDGEMENTS

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REFERENCE

TESTS ON FIBRE REINFORCED SCC FILLED DOUBLE SKIN TUBULAR STUB COLUMNS EXPOSED TO STANDARD FIRE

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KEYWORDS
Fire, concrete filled double skin tube (CFDST), self-consolidating concrete (SCC), fibre reinforced concrete

ABSTRACT
This paper reports an experiment investigation into steel tubular stub columns filled with fibre reinforced self-consolidating concrete (SCC) subject to standard fire. Eight specimens were tested. The temperature distribution and axial deformation were measured. The failure modes were observed. The fire resistance was determined accordingly. The experiment mainly aims to provide information of the fire behaviour of such columns to calibrate numerical model in the future research.

INTRODUCTION
Concrete filled double skin tube (CFDST) is an innovative technique in steel and concrete composite construction. Many studies were carried out on the behaviour of CFDST columns at ambient temperature [1]. In addition, studies also showed that the use of self-consolidating concrete (SCC) in composite columns can offer improved cost efficiency and safety of steel-composite structures.

Apart from the behaviour at ambient temperature, the fire behaviour of CFDST columns is also an important aspect due to the direct fire exposure of the outer steel tube. However, there is very limited information available about the fire behaviour of CFDST columns. A serial of fire tests on SCC filled CFDST columns have been completed by the authors [2]. The fire resistance of the columns varied from 30 minutes to two hours [2]. Some measures to increase the fire resistance of the concrete filled steel columns (CFST) include external fire resistance coating or fibre reinforced concrete [3, 4]. The
use of fibre reinforced SCC is adopted in the current investigation with an aim to further increase the fire resistance of CFDST columns.

This paper reports an experiment investigation into the fire performance of the fibre reinforced SCC filled CFDST stub columns. Eight circular columns were prepared for standard fire tests. The temperature distribution, axial deformation and fire endurance of the columns were measured and the failure modes were observed. The tests were carried out on small CFDST tub columns due to the limitation of the test set-up. The primary aim of the tests is to provide information, such as temperature distribution and failure mode, for calibrating the numerical model in the future research. At the same time, the effect of some parameters, such as fibre, geometric size of the concrete and load level, on the fire behaviour of the columns was also investigated.

EXPERIMENT PROCEDURE

Material property

Concrete

The concrete used in the current tests is SCC, steel and polypropylene fibre reinforced SCC. A combination of the ACI recommended mixture design method [5] and a method proposed by Su et al. [6] was used to design the mixture of the SCC. The SCC mixture is shown in Table 1.

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>Cement (kg)</th>
<th>Fly ash (kg)</th>
<th>Slag (kg)</th>
<th>Water (kg)</th>
<th>Fine aggregate (kg)</th>
<th>Coarse aggregate (kg)</th>
<th>Superplasticizer (l)</th>
<th>Steel fibre (kg)</th>
<th>Polypropylene fibre (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCC1</td>
<td>125</td>
<td>157</td>
<td>157</td>
<td>160</td>
<td>865</td>
<td>817</td>
<td>2.77</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SCC1SF</td>
<td>125</td>
<td>157</td>
<td>157</td>
<td>160</td>
<td>865</td>
<td>817</td>
<td>2.83</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>SCC2</td>
<td>380</td>
<td>170</td>
<td>0</td>
<td>178</td>
<td>776</td>
<td>831</td>
<td>2.85</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SCC2SF</td>
<td>380</td>
<td>170</td>
<td>0</td>
<td>178</td>
<td>776</td>
<td>831</td>
<td>3.08</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>SCC2SPF</td>
<td>380</td>
<td>170</td>
<td>0</td>
<td>178</td>
<td>776</td>
<td>831</td>
<td>3.62</td>
<td>42</td>
<td>0.9</td>
</tr>
</tbody>
</table>

In this study, the slump flow and L-box tests were used to evaluate the workability of the SCC. In the slump flow test, the slump flow and the time when the concrete sample flowing to a 500 mm diameter circle was recorded as an index to evaluate the workability of the SCC ($t_{50}$). The L-box is an L-shaped apparatus with a vertical and horizontal section to evaluate the ability of a SCC sample flow through reinforcement obstacle. The ratio of the height of the concrete at the end of the horizontal section ($h_2$) and the height of the concrete left in the vertical section ($h_1$) is called blocking ratio which is an index to evaluate the ability of the SCC flow through obstacle [5]. The flow ability of the SCC sample in this test is summarized in Table 2. Both tests confirmed the satisfaction of workability of SCC developed in this testing program.
Concrete cylinders were prepared to obtain the compressive strength of the concrete. Some of the cylinders were cured under standard condition to test the strength of the concrete at 28 days. Others were cured under conditions similar to the concrete in the CFDST specimens so as to acquire more realistic concrete strength in the CFDST. The concrete strength at 28 day and the average strength of the concrete during the CFDST specimens testing are shown in Table 3. The average concrete strength for SCC1 type is about 48 MPa whereas that for SCC2 type is about 62 MPa.

TABLE 3
CYLINDER STRENGTH OF SCC

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>28 day Standard curing</th>
<th>28 day CFDST curing</th>
<th>Average strength during test</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCC1</td>
<td>50.6</td>
<td>42.7</td>
<td>46.6</td>
</tr>
<tr>
<td>SCC1SF</td>
<td>53.5</td>
<td>41.8</td>
<td>48.6</td>
</tr>
<tr>
<td>SCC2</td>
<td>65.9</td>
<td>60.8</td>
<td>63.4</td>
</tr>
<tr>
<td>SCC2SF</td>
<td>58.9</td>
<td>56.1</td>
<td>61.2</td>
</tr>
<tr>
<td>SCC2SPF</td>
<td>64.4</td>
<td>61.5</td>
<td>62.5</td>
</tr>
</tbody>
</table>

Steel

The circular tubes for the CFDST columns in this test program are cold-formed tubes manufactured according to AS 1163 [7]. Steel coupons were cut longitudinally from each type of the tubes to obtain the material properties, as shown in Table 4. Due to the cold-form processing of the tubes, the steel had not shown obviously yield plateau. The yield stress ($f_y$) in the table is the 0.2% proof stress.

TABLE 4
STEEL PROPERTY

<table>
<thead>
<tr>
<th>Tube label</th>
<th>Profile of the section</th>
<th>$D \times t$ (mm)</th>
<th>$f_y$ (MPa)</th>
<th>$f_u$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Circular</td>
<td>406×8</td>
<td>401</td>
<td>458</td>
</tr>
<tr>
<td>C2</td>
<td>Circular</td>
<td>219.1×5</td>
<td>426</td>
<td>469</td>
</tr>
<tr>
<td>C3</td>
<td>Circular</td>
<td>165.1×3</td>
<td>399</td>
<td>470</td>
</tr>
<tr>
<td>C4</td>
<td>Circular</td>
<td>101.6×3.2</td>
<td>426</td>
<td>476</td>
</tr>
</tbody>
</table>
Specimens

In this study, all specimens consist of circular hollow section as the inner and outer tubes to form a CFDST as shown in Figure 1. The total length of the specimens was 800 mm. The parameters of the specimens are shown in Table 5. The specimen label in Table 5 is arranged as ‘outer tube-inner tube-type of concrete’ in which the outer and inner tubes are defined in Table 4 and the type of concrete is given in Table 3. Totally eight specimens were prepared.

![Figure 1: Profile of the CFDST specimen](image)

<table>
<thead>
<tr>
<th>Specimen label</th>
<th>$\chi$</th>
<th>Load applied (kN)</th>
<th>Load level</th>
<th>Fire endurance (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1-C3-SCC2</td>
<td>0.42</td>
<td>4100</td>
<td>0.37</td>
<td>62</td>
</tr>
<tr>
<td>C1-C3-SCC2SF</td>
<td>0.42</td>
<td>4000</td>
<td>0.37</td>
<td>138</td>
</tr>
<tr>
<td>C1-C3-SCC2SFP</td>
<td>0.42</td>
<td>3400</td>
<td>0.31</td>
<td>&gt;122</td>
</tr>
<tr>
<td>C2-C4-SCC2</td>
<td>0.49</td>
<td>1821</td>
<td>0.5</td>
<td>30</td>
</tr>
<tr>
<td>C2-C4-SCC2SF</td>
<td>0.49</td>
<td>1785</td>
<td>0.5</td>
<td>39</td>
</tr>
<tr>
<td>C2-C4-SCC2SFP</td>
<td>0.49</td>
<td>1821</td>
<td>0.5</td>
<td>27</td>
</tr>
<tr>
<td>C2-C4-SCC1</td>
<td>0.49</td>
<td>1923</td>
<td>0.6</td>
<td>24</td>
</tr>
<tr>
<td>C2-C4-SCC1SF</td>
<td>0.49</td>
<td>1964</td>
<td>0.6</td>
<td>26</td>
</tr>
</tbody>
</table>

Several parameters were chosen as main variables to observe their effect on the fire behaviour of the columns, i.e. type of concrete (SCC and fibre reinforced SCC), concrete strength, load level (i.e. the ratio of the load applied in fire testing to the section capacity of the stub column at ambient temperature) and cavity ratio ($\chi$) which is defined in Eqn. 1:

$$\chi = \frac{D_i}{D_o - 2t_o}$$

where $D_o$ and $D_i$ are the outside diameter of the outer and inner tube, $t_o$ is the thickness of the outer tube. Cavity ratio is a parameter which to some extent quantifies the void in the centre of the CFDST column or the thickness of the concrete in the columns. For a steel tube fully filled with concrete the cavity ratio becomes zero.
Test procedure and conditions

The tubes were firstly cut into 800 mm length. Then, the inner and outer tubes were concentrically welded to a 5 mm thick mild steel end plate. A 20 mm diameter hole was drilled on the outer tube for releasing vapour in the concrete at elevated temperatures. In order to monitor the temperatures in the CFDST columns, three thermocouples were installed in each specimen. Locations of the thermocouples are illustrated in Figure 1. Thermocouple No.1 is located on the inner surface of the outer tube. Thermocouple No.3 is located on the outside surface of the inner tube. Thermocouple No.3 is located at the middle of the concrete. The concrete was added into the gap between the tubes without vibration. The concrete was cured for two weeks and then air dried before testing.

The specimens were tested in a furnace in the Civil Engineering Laboratory at Monash University. The setup consists of a gas furnace, a load reaction frame, a loading system with maximum capacity of 5000 kN, a fire temperature control unit and a data acquisition system. Details of the set up can be found in Lu et al. [8].

The furnace temperature followed the standard ISO fire temperature curve specified in AS 1530.4 [9]. All the specimens were tested to failure except C1-C3-SCC2SFP in which the test had to be stopped after 120 minutes due to some insulation problems. The fire resistance was determined by axial deformation or deformation rate according to AS 1530.4 [9].

TEST RESULTS

Temperature distribution

The temperatures measured from the thermocouples in the CFDST specimens are shown in Figure 2. The temperature curves noted as 1, 2 and 3 in the figure are corresponding to temperatures measured by thermocouples 1, 2 and 3 shown in Figure 1. As can be seen in Figure 2, the temperature in the outer tube (point 1) is significantly higher than that in the inner tube (point 3), whereas the difference in the temperatures at the middle of the concrete (point 2) and at the inner tube (point 3) is less significant.

There is a relative stable stage of the temperature values in the concrete when the temperature is about 100 °C. This phenomenon is caused by the water in the concrete change state from liquid to vapour. At this moment, most of the heat is used to change the state of the water rather than to elevate the temperature in the concrete. Concrete has lower heat conductivity and higher heat capacity than steel. Thus, temperature elevation in concrete is slower than in steel. The presence of the concrete in the CFDST columns slows down temperature elevation in the composite columns and results in non-uniform temperature distribution in the CFDST columns as can be seen in the figure. Therefore, geometric size of the concrete in the columns is an important parameter affecting the temperature distribution in the columns. The concrete thickness and perimeter of outer tube are two parameters among others which can represent the geometric size of the concrete in the CFDST columns. As can be seen in the figure, thicker concrete and larger perimeter of the outer tube result in lower temperature in the columns (by comparing C1C3 series with C2C4 series).

It seems no significant difference between temperatures in the CFDST specimens filled with SCC, steel fibre reinforced SCC and both steel and polypropylene fibre SCC. The polypropylene fibre absorbs heat to melt at about 160 °C. However, the amount of the polypropylene fibre used was 9 kg per cube meter concrete. This quantity is very small compared to the massive concrete. Therefore, such small amount of the polypropylene fibre can not alter too much the thermal property of the concrete nor the temperature distribution in the columns. It is encouraging to see the temperature of the inner
tube is very low (less than 200 degrees) even after 120 minutes fire exposure, which is significantly lower than the critical temperature of steel tubes (about 526 to 711 degrees given in Eurocode 3 Part 1-2 [10]) beyond which the steel tubes start to lose capacity rapidly. Hence the inner tube makes significant contribution to the fire performance of CFDST.

![Temperature Graphs](image)

**Figure 2: Temperature in the CFDST columns**

### Axial deformation and fire endurance

The axial deformation for the stub columns is shown in Figure 3. The axial deformation of the CFDST stub columns has three distinct stages. The first stage is the early stage of fire exposure. There is a little axial deformation and an obvious tensile deformation for most of the specimens in this stage. Then in the second stage, compressive axial deformation gradually increases. Finally in the third stage, the axial deformation dramatically increases in a very short time and the stub columns can no longer sustain the applied load. The fire endurance of the stub columns was determined by the axial deformation versus the fire exposure time relationship. The failure criteria of the axial load columns under fire exposure is either total axial deformation or deformation rate specified in AS-1530.4 [9]. The fire endurance of the columns which varies from 24 to 138 minutes as shown in Table 5.

As can be seen in Table 5, the larger size columns have longer fire resistance than the smaller size columns. This is mainly due to the slower temperature elevation in the larger columns. The load level
has significant influence on the fire resistance of the columns, higher load level results in shorter fire endurance. In addition, the use of the fibres in the SCC can significantly increase the fire resistance of the CFDST stub columns. The CFDST columns filled with steel fibre reinforced SCC generally achieve higher fire resistance than those filled plain SCC. However, the use of both steel and polypropylene fibres seems to give much less benefit to the fire resistance of the columns. More research is needed to explore the combination of different fibres.

![Graphs showing axial deformation over time for different specimens.](image)

**Failure mode**

Figure 4 shows the typical failure modes of the stub columns after fire tests. The failure of the columns is typical compression failure. The failure mode of the outer tube is outwards bulge. As can be seen in Figure 4, failure mode of the inner tube is consecutive local inwards and outwards bulges.

![Images showing typical failure modes.](image)
CONCLUSIONS

Standard fire tests of eight SCC and fibre reinforced SCC filled CFDST stub columns have been conducted. The temperature distribution, axial deformation and failure mode of the columns were obtained. The fire resistance of the columns was determined accordingly. All the acquired information can be used as a frame of reference for future numerical analysis.

In addition, the effect of several parameters on the fire behaviour of the columns was also studied. The geometric size of the concrete, namely concrete thickness and perimeter of outer tube has significant influence on the temperature distribution in the CFDST columns. The combination of steel and polypropylene fibres does not significantly influence the temperature distribution in the columns. The load level significantly affects the fire resistance of the columns. Steel fibre can improve the mechanical properties of the concrete. Therefore, columns filled with steel fibre SCC achieve considerable higher fire resistance compared with those filled with plain SCC. More research is needed to explore the combination of different fibres.

ACKNOWLEDGEMENTS

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REFERENCES

NUMERICAL INVESTIGATION OF COLD-FORMED STEEL SHEETING IN FIRE

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KEYWORDS
Cold-formed steel sheeting, structural fire engineering, catenary action, screwed connection and shot nailed connection

ABSTRACT
Cold-formed steel sheeting is used in industrial buildings as structural deck in roofing system with insulations on its top. A great deal of researches on steel beam in steel framed structures have indicated that a steel beam can have a substantial fire resistance by alternative load carrying mechanisms, such as catenary action in large deformation, if the large deformation of structure is allowed. However, few researches are available for investigating the behavior of cold-formed steel sheeting in fire. A 3-D finite element model incorporating both geometric and material non-linearity is firstly created to investigate the behavior of cold-formed steel sheeting in fire. It has been shown that due to the thinness of the material and degradation of material properties at elevated temperature, the profile buckles early under thermal expansion. The steel sheeting can survive in fire (fire rating is R30 in this analysis) by developing tensile force in large deformation with totally opening of profile. The catenary force in the structure has significant role to resist the transversally applied load. Corrugated steel sheets are normally attached to the underlying purlins or more commonly straight to the steel trusses by self-drilling screws, self-tapping screws or shot nails. The behavior of sheeting connections in fire conditions is important so as to make the catenary action possible. The performance of screwed connections between a thinner plate and a thicker plate are investigated to simulate the structural application of connecting the roof sheeting to underneath truss members. The investigations are carried out via single lap shear tests both at room temperature and at elevated temperature. The aim of this study is to get the load-displacement curves so that they can be used in the further research.
INTRODUCTION

The insulated metal deck roofing system is used for industrial buildings with low-pitched roof. The roofing system is mainly composed of three components: the steel sheeting itself, the insulation and fasteners or adhesive, and the weather resistant roof. The structural deck is the basis for each roofing system. The steel decks are manufactured with cold forming technique from thin steel strip and are attached to the underlying purlins or more commonly straight to the steel trusses by self-drilling or self-tapping screws, or shot nails.

It has been shown from researches [1][2] that in fire axial forces can often be generated in steel deck when the roof sheeting is loaded transversely. Due to the restraints to thermal expansion these forces are initially compressive. At later stages the forces become tensile when the catenary action starts to develop, which helps the sheeting to survive in fire via behaving as a cable hanging from the adjacent structural member. If the large deformation of structure is allowed and the sheeting can survive in the large deflection stage in fire, the expensive fire protection can be removed or reduced due to wide covering area of the roof in this type of buildings. Currently, few researches are available on this topic. One of the major factors affecting the steel sheeting behavior in fire is how the screw fasteners behave when they are used to connect steel deck to its supports.

In this paper, the numerical simulations for two-span sheeting with overlap in the mid-support and for one-span sheeting are carried out, respectively. These simulations provide an initial data for guiding further testing of the same type of roof system and initial understanding of the behaviour of cold-formed roof sheeting in fire. The performance of screwed connections between a thinner plate and a thicker plate are investigated separately via single lap shear tests both at room temperature and at elevated temperature. The purposes of investigation of connections are to get the load-displacement curves so that they can be used in the further researches. Due to the screw connector failure at and above 400°C, the investigation of behavior of shot nailed connections has been also carried out.

NUMERICAL INVESTIGATION OF COLD-FORMED STEEL SHEETING IN FIRE

FE Modeling

The profile of steel sheeting and its dimensions are shown in Figure 1 (a). The thickness of the profile is 0.8 mm. When assembled as load-bearing decking, it is necessary to overlap edges of two sheets to make a side lap joint or to overlap ends of two sheets to make an end lap joint. When simulating two span sheeting, an end lap joint at internal support is modeled with overlapping length of 600 mm. The span length is 6 m. Commercial FE software, ABAQUS Explicit [3], is used as an analysis tool. Only one rib is modeled for the sake of computing efficiency as shown in Figure 1 (b). The profile is connected to rigid plates, which simulate the purlins or trusses underneath the roof system. The fasteners are assumed to be rigid. It has been assumed that the material properties of rigid plates and fasteners are not affected by temperature.

The loads applied to the sheeting are mechanical loading and thermal loading. According to EN 1991 [4], fire is defined as accidental load and the combination factor for permanent load (with characteristic value of 0.4 kN/m²) is 1.0 and for snow load (with characteristic value of 1.8 kN/m²) is 0.5. The nominal fire curve is used to simulate the fire. Since the thickness of the sheeting is thin, the temperature of steel sheeting is assumed to be the same as gas
temperature. The distributions of mechanical loading for two-span and one-span sheeting respectively are shown in Figure 2.

Figure 1: (a) Steel sheeting and its dimensions; (b) FE modelling of sheeting over two spans

Thin shell elements with reduced integration S4R are used to model the steel sheeting. The quasi-static analysis procedure was adopted with a small enough dissipated energy fraction so that the energy fraction has no effects on the deflection behavior of sheeting. The two-step analysis is carried out, in which the mechanical loading was applied first (step 1) and then the temperatures were increased (step 2). The material of steel sheeting is S350GD+Z275 with the nominal yield strength 350 N/mm². The temperature dependent stress-strain curves and thermal elongation are taken according to EN 1993-1-2 [5]. In current FE modelling, the decreasing phase of the material properties is not considered. According to the input requirements, the nominal stress-strain curves are transformed to the true stress-true strain curves. Similarly, total elongation coefficients are used via dividing the thermal elongation by its corresponding temperature.

Results of FE Modeling

Figure 3 shows the displacement histories of one-span and two-span sheeting, respectively. The sheeting deforms upwards initially due to the thermal restrained bottom flanges and freely expanded top flanges. Because of the local buckling of thinner sheeting in compression near supports, along with the degradation of strength and stiffness of materials at elevated temperatures, plastic hinges have been formed near the supports. The sheeting deformed downward rapidly until the tension forces developed at supports, the sheeting is in catenary action. Later, the profile opens; top flange, web and bottom flange deform separately. Thereafter, the deflections increased further with a steady gradient until the sheeting profile along each span opens totally.

When comparing the mid-span displacement of one-span sheeting to that of two-span sheeting, three differences have been observed: the maximum upward displacement is larger in one-span sheeting; the starting time for downward displacement is delayed for two-span sheeting; the displacement, when the profile is opening, is less for one-span sheeting. The main reason for these differences is that continuity in the internal support for two-span sheeting provides the restraints for the displacement.
Figure 3: Displacement histories of one and two span sheeting at elevated temperatures

Figure 4 shows the forces histories developed at end supports for one-span and two-span sheeting, respectively. The value in each figure is the sum of two fasteners at each support. Due to the thermal expansion and restraints from fasteners connected to the supports, the axial compression force developed first. Because of the fasteners are assumed to be rigid, the values of the compression forces increase rapidly with temperature rising until the local buckling occurred at end supports. The compressive axial force is released.

Due to large displacement promoted by post-buckling and the plastic straining (with decreasing material strength), the compression force reduces dramatically. Thereafter, for one-span sheeting the tensile axial force developed because the lateral deflection become sufficiently large, the shortening of sheeting length overtook the thermal expansion. The sheeting is now in catenary action, i.e. a small component of the tension carries the transverse load directly. For two-span sheeting after two hinges developed near two end supports, the sheeting deformed further downwards due to the overtaking of mechanical loading, the hinge developed near the internal support releases the hogging moment in the internal support. Two-span sheeting acted as two independently supported one-span sheeting. Thereafter, the sheeting is in catenary action. Therefore, the restraints from the internal support delay the transforming of compression force to tension force and increase the value of tension force in
two-span sheeting. The maximum compression forces developed at the supports for both cases are the same because of the same dimensions of the profiles and span lengths.

**BEHAVIOUR OF SHEETING CONNECTIONS IN FIRE**

In the previous analyses, it has been shown that the behaviour of connections plays an important role in the behavior of sheeting at elevated temperature. Thus, the connection has been taken out and investigated numerically and experimentally. The connectors, which have been investigated, are self-drilling screws and shot nails, which are two typical connectors when connecting cold-formed steel sheeting. The shot nails are more common when connecting the sheeting to its support.

**Single Lap Shear Tests**

Due to the limitation of dimensions of the furnace available, the dimensions of the specimens are reduced (Figure 5 (a)) when compared to the standard testing set-up according to ECCS recommendations [6][7]. Two kinds of tests with screw connectors have been carried out: one is to validate the testing results of specimens with Reduced Dimensions (RD) with Standard Dimensions (SD); and the other is to do the tests for a group of temperatures, i.e. 20 °C, 200 °C, 400 °C and 600 °C. The displacement of the specimen inside furnace is recorded as the plate end displacement.

![Figure 5: (a) Testing set-up, (b) Results for validation tests](image)

In the room temperature tests, the rate of loading in the initial stage of testing shall not exceed 1 kN/min. Until the ultimate load is reached the rate of straining shall not exceed 1 mm/min [6][7]. The displacement control is used. At elevated temperatures, the tests are carried out in two steps. Firstly, the temperature is raised to a given temperature. Then the loading is applied to the specimen with the rate of 0.1 to 0.3% per minute. In fire cases the large deformation is an alternative mechanism to avoid the failure of the structure. Thus, the tests have been stopped when the displacement of 20 mm is reached at room temperature and 15 mm at elevated temperature. The failure load is recorded as the maximum load in the range of deformation.

In order to show the feasibility of testing results with RD specimens, a load-deflection curve is taken out respectively for SD and RD specimens as shown in Figure 5 (b). According to 7 points (0-6) marked on the curves, it can be seen that the deformation histories of two types of specimens are the same due to the same shapes of two curves. The maximum loads of the
connections (point 5) are at about the same level. However, due to the displacement of SD specimen has been measured at the end of the plate instead of around the connection area, the RD specimens are more stiff than SD ones.

Two failure modes as shown in Figure 6 have been observed: (a) bearing and tearing failure of thinner plate below 200 °C, and (b) shearing failure of screw connector at 400 °C and 600 °C (similar mode to 400 °C). The main factor that determines the final failure mode in the connection might be the relative material strength involved in the connections. At room temperature, the material strength of thinner plate is lower than that of screw connector. Thus the failure mode is Mode 1 failure. When the temperature is above 400 °C, the material properties for screw connector might drop dramatically because of the manufacture process of increasing material strength of screws (for instance heat treatment).

Figure 6: Failure modes of screwed connectors (a), (b) and of shot nailed connectors (c), (d)

The load-displacement curves from tests for screw connectors are shown in Figure 7. When comparing load-displacement curves at 20 °C to that at 200 °C, it can be seen that up to point 2 these two curves are very close because both strength and modulus of elasticity are not much reduced at 200 °C. Then load-deformation curves at 200 °C are higher than those of at 20 °C but with reduced ductility. The stiffness and maximum load at 400 °C and 600 °C are lower than those at room temperature because of the reduction of strength and modulus of elasticity of materials at elevated temperatures. The ductility has been reduced as well due to the brittle failure mode of screw connectors when comparing to the bearing tearing failure of thinner plate at 20 °C and 200 °C respectively. The detailed explanation of failure mechanisms of screwed connectors can be found from previous researches [8].

Due to the screw connector failures at and above 400 °C, the behaviors of connection types with shot nailed connectors are further investigated. The same testing set-up for screw connectors has been used. The failures of shot nails have been observed at 600 °C (Figure 6 (d)), below which the failure modes are tearing failure of the thinner plates (Figure 6 (c)). Figure 7 shows the comparisons of load displacement curves of shot nailed connectors with those of screwed connectors. It can be seen that maximum loads for two types of connectors are very close but connections made by shot nails are much stiffer due to the special installation process of shot nails.
Comparison of Connection Loads with Support Loads from FE Modeling of Sheeting

The horizontal axial forces at two end supports for both single span and two span sheeting have been output as shown in Figure 4. The values of axial forces are the sum of two connectors at each end support. It can be seen the maximum compressive force for each connector in both one-span and two-span sheeting is about 14 kN at about 300 °C. And the maximum tensile force developed are about 2.5 kN for one span sheeting and about 3 kN for two span sheeting at 600 °C. The maximum load bearing capacity of shot nailed connections from testing is 7 – 8 kN at 300 °C and about 1.64 kN at 600 °C. So it seems that it is necessary to increase the number of connectors so as to bear these forces when the sheeting is under nominal fire and with uniform temperature distribution. Or the fire protections are provided to the connection area. However, if the connections are located at area with temperature lower than 400 °C to 600 °C, the connections can provide enough load resistance to help the sheeting to develop catenary action. Further investigations are needed to integrate the connection model to the sheeting model.

CONCLUSIONS

3-D finite element model incorporating geometric non-linearity and non-linear material properties have been used to analyse the behaviour of two-span sheeting with overlaps in the mid-support in fire. In the beginning, the sheeting structure bent and buckled upwards first due to thermal expansion and axial restraints when connecting the sheeting to the supports, which causes high compression force in the structure. After that the structure deforms rapidly downwards due to stiffness degradation with increasing temperature. The catenary force in the structure has significant role to sustain the applied load. Small tensile force can sustain the transversally applied load. No run-away deformation happened before 30 minutes in this analysis. It can be concluded that the sheet itself can stand for 30 minutes fire, assumed that the steel temperature is the same as in air. When comparing the displacement and axial forces at supports with one-span sheeting, the restraints coming from the continuity in the internal support in two-span sheeting delay the transformation to tensile force, lessen the maximum displacement developed in the mid-span and increase the maximum value of tensile force.

The connection plays an important role in the behaviour of sheeting in fire. Thus, the tests have been carried out for single lap shear specimens with 0.8 mm thinner sheet being connected to 5 mm thicker sheet via one connector at such temperatures as 20 °C, 200 °C, 400 °C and 600 °C.
Two types of connectors have been investigated, self-drilling screw connector and shot nailed connector. The maximum load of the connection is defined as the maximum load can be reached in load-deformation curves. For self-drilling screw connection the connector shear failure happened when the temperature was over 400°C and for shot nailed connection it happened at temperature of 600°C. The possible reason is that the metallurgical treatment of screw or nail materials makes the strength and stiffness of the material drop more dramatically over certain temperatures when comparing the steel grade used for thinner sheet. When comparing the loading capacity of shot nailed connections with the compressive and tensile forces developed at the end support output from FE analysis of sheeting in fire, it seems that the number of connector might need to be increased or the fire protections are needed around connection areas. Thus, more research on the connections with multiple connectors might be needed.

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REFERENCES

EFFECT OF ROTATIONAL STIFFNESS AT COLUMN BASE OF PORTAL FRAME AT ELEVATED TEMPERATURE

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KEYWORDS
Dynamic analysis, instability, snap-through-buckling, semi-rigidity, restabilization, non-linearity, large-displacement.

ABSTRACT

In this paper, numerical models are developed to investigate the behavior of portal frames at elevated temperature. These simulation is able to capture the snap-through-buckling and the post-buckling behaviour. It is found that after buckling, the rafter takes the catenary shape whose compressive stress is transformed into tensile stress and pulls the top of the column inward. A parametric study on the rotational stiffness of the column base is carried out. It is found that at a given lower rotational stiffness, the structure experiences snap-through at lower temperature; and increase of rotational stiffness provides stabilization of rafter and which yields a higher buckling temperature. Increasing the partial strength of column base increases the snap-through buckling temperature.

INTRODUCTION

A number of incidences of fires in warehouses have drawn attention to the current lack of understanding of the behavior of steel portal frame structures in fire (Wong 2001). General understanding of the behavior of steel portal frame structure is insufficient as the behavior of steel portal frame is complicated while the structure experiences fire and steel loses ultimate strength and stiffness rapidly (Wang 2002). Wong has conducted an experiment on a scaled portal frame where it is established that a portal frame can have large deformation and large strain while it is in fire (Wong 2001). He has studied the response of industrial pitched portal frame structures in fire both experimentally and numerically while he is extending the capabilities of the finite element code Vulcan. However, his study gives no information about post-buckling behaviour. Song et al. (2007) has continued Wong’s work and investigated the failure mechanism of a single-storey haunched portal frame in fire subject to different boundary conditions at their column bases and proposed a new design method for portal frame. Moss et al. (2009) points out that the structure may collapse in either of the inward or sway mode as shown in Figure 1. The inward mode is considered as acceptable...
whereas the sway is unacceptable mode of collapse. Their method uses static analysis which cannot
describe dynamic behaviour. The Steel Construction Institute (SCI) has developed a simple design
method which avoids the fire protection and allows the rafter to collapse at elevated temperature
(Simms and Newman, 2002). This method is widely used in UK and is the only guideline available to
design portal frame at elevated temperature. The idea of SCI method is to specify a very strong base
connection to resist the overturning moment, $M_{OTM}$, at the column bases. Note, this method is based
on arbitrary assumptions and its accuracy and soundness is discussed in this paper.

This research aims to understand how a portal frame structure behaves while it is at elevated
temperature through a parametric study. A non-linear elasto-plastic dynamic finite element analysis is
presented that can be used to simulate the global buckling and post-buckling behaviour of portal
frames in fire.

**PORTAL FRAME DESCRIPTION**

**Geometry**

Figure 2 shows the details of the frame used in the worked-example from SCI (2002). Table 1
summarises the section properties of the steel sections. According to the SCI method, the value of
$M_{OTM}$ (overturning moment) that needs to be resisted is 54.2 kNm with haunch and fillets, and 61.0
kNm without haunch and fillets. Note, for simplicity, the portal frame in this study does not include
any haunch or fillet.

**Material properties**

Figure 3a shows the engineering stress-strain curves of steel at different temperatures from 20 °C to
1100 °C in accordance to Eurocode 3 (BS EN 1993-1-2, 2005). The strength at 1200 °C is almost
zero. Figure 3b shows the reduction of the normalized ultimate strength and elastic modulus at
different temperatures. It should be noted that the SCI method (2002) assumes that 6.5 % of steel
strength at ambient temperature is retained when the steel is subjected to a temperature of 890 °C
which agrees to the corresponding values in Eurocode 3 (EC3). Figure 3c shows the variation of
coefficient of thermal expansion of steel given by Eurocode 3. A value of 0.3 is used for Poisson's
ratio for all temperatures. The steel is considered to be isotropic material with a density of 7850
kg/m$^3$. Figure 3d shows the ISO834 fire curve used in the numerical model.
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Figure 2: Details of frame used in SCI worked example.

TABLE 1
SECTION PROPERTIES OF PORTAL FRAME

<table>
<thead>
<tr>
<th>Section</th>
<th>$A$ cm$^2$</th>
<th>$I_{maj}$ cm$^4$</th>
<th>$Z_{pl}$ cm$^3$</th>
<th>$M_{pl}$ kNm</th>
</tr>
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<tbody>
<tr>
<td>457x152x52 UB S275</td>
<td>66.6</td>
<td>21370</td>
<td>1096</td>
<td>301.4</td>
</tr>
<tr>
<td>457x152x52 UB S275 (without fillets)</td>
<td>65.8</td>
<td>20969</td>
<td>1077</td>
<td>296.2</td>
</tr>
</tbody>
</table>

where, $A$ is area of the section; $I_{maj}$ is the second moment of area about the major axis; $Z_{pl}$ is the plastic section modulus; $M_{pl}$ is the plastic moment capacity of section.

Finite element modelling

Numerical model and analysis type

A non-linear elasto-plastic finite element model is developed to simulate the true behaviour of a portal frame in fire using the commercial code Abaqus. 96 beam elements of type B21 have been selected in order to keep the model computationally less expensive and accurate based on sensitive study. Implicit dynamic analysis is performed in order to overcome the structural instability and numerical convergence problem. A combination of HAFTOL and ALPHA parameters with *DYNAMIC option are used. These parameters are related to computational time and convergence rate of the model. They help achieve faster convergence by changing the size of increment of time. Both are found to have no impact on the accuracy of analysis results. It is found that lower value of ALPHA parameter reduces the computational time dramatically. Roughly, a HAFTOL value within the range of 10 to 100 helps achieve faster solution.

Loadings and boundary conditions

According to the SCI worked example, the vertical dead load applied to the frame in fire is 1.0 kN/m and the load is applied first and held constant throughout the analysis. An initial ambient temperature of 20°C of the whole structure is applied and increased gradually in accordance to ISO834 fire curve. Rayleigh mass proportional damping is assumed as 5% in order to dissipate kinetic energy. Rayleigh stiffness proportional damping is not considered as the stiffness of the structure is lost at higher temperature that makes the structure mass dependent.

A semi-rigid column base is assumed in order to simulate the real behaviour of column base and is modelled by using a spring element with rotational stiffness which can be either linear or bi-linear a
shown in Figure 4. Non-dimensional rotational stiffness at column base, \( K_b \), can be calculated by using Equation 1 and can have the value between 0 for pinned to 25 for fixed (Salter et al. 2004):

\[ K_b = 25 \]

Figure 3: (a) Engineering stress-strain curve of S275 steel at various temperatures in °C; (b) normalised value of ultimate strength and elastic modulus of S275 steel at different temperatures; (c) coefficient of thermal expansion of S275 steel at different temperatures; (d) ISO834 fire curve.

Figure 4: Idealised (a) linear and; (b) bi-linear moment rotation curve for column base.
\[ K_b = \frac{k_b}{EI_{maj}/h} \]  

where, \( k_b \) is the rotational stiffness at column base; \( E \) is Young’s modulus; \( I_{maj} \) is moment of inertia along the major axis; \( h \) is the height of column from base to the eaves.

**Validation of models**

The numerical model has already been validated with two benchmark models of Song et al. (2007) and Vassart et al. (2007). Song used single bay portal frame and Vassart used double bay portal frame. In both cases, the results are in good agreement. Details of the validation can be found in other papers by Rahman et al. (2009a and 2009b).

**RESULTS AND DISCUSSIONS**

**Overturning moment (OTM), catenary force and horizontal reaction**

Figure 5a shows the variation of overturning moment at the left column base against the temperature where column is protected from fire and column base is fixed. As can be seen, the moment at ambient temperature is -21.5 kNm and the maximum \( M_{OTM} \) is reached as far as 306 kNm within temperature range of 718 °C to 730 °C, and is quite close to the plastic moment \( M_{pl} \) (301 kNm) of the section. This \( M_{OTM} \) is five times higher than the \( M_{OTM} \) predicted by SCI hand calculation method (61 kNm). It clearly shows that the SCI method is highly over-conservative and inaccurate. As the temperature increases the horizontal moment suddenly changes from -241 kNm to +241 kNm while the structure experiences snap-through-buckling at a temperature around 936 °C. The concept of a fixed critical temperature of 890 °C in SCI design guidelines is totally unrealistic as this critical temperature always depends on other factors, e.g., loads on rafter, level of protection of column, column base fixity, etc. After snap-through buckling the rafter will always takes the shape of a catenary (Figure 6d) and an infinite number of fire hinges is formed as contrasted to the SCI assumption of four fire hinges. The figure also shows that the rafter experiences another snap-through-buckling at temperature 1069 °C. The rafter shows stabilization in-between these two temperatures. Figure 5b shows the variation of catenary force at the left column top against temperature where it can be seen that the catenary force at ambient temperature is 14.4 kN and it reaches to a maximum value of 57.7 kN within the same temperature range of 718°C to 730°C as observed in the case of overturning moment. Just before the snap-through buckling, the catenary force reaches 45.5 kN which drops to a value of 5.7 kN and immediately rises for restabilization. Then it further drops to an ultimate value of 1.69 kN.

The catenary force plays a significant role in designing of structure under fire as the compressive force is transformed into tensile force and pulls the top of the column. Figure 5c shows the variation of horizontal reaction at left column base. Similar to the overturning moment pattern, the column base shows a maximum horizontal reaction of 57 kN within the same temperature range of 718 °C to 730 °C. The horizontal reaction, like the overturning moment, suddenly changes from +42.5 kN to -42.5 kN when rafter of the structure experiences snap-through-buckling at the same temperature i.e. 936 °C. Figure 5d shows the horizontal variation of the left eaves deflection against temperature. When temperature increases the eaves shows a maximum horizontal deflection of 0.096 m to the left within the similar temperature range. Prior to buckling, this deflection is due to thermal expansion of steel. At the snap-through temperature, the eave suddenly moves from outward to inward and changes the horizontal eaves deflection from -0.08 m to +0.043 m. At this critical temperature, thermal expansion has no effect on the behavior of the structure rather the stiffness degradation of material plays the dominant role.
Parametric study

Effect of semi-rigidity of column base

In order to investigate the effect of semi-rigidity of the column bases, four different moment-rotation curves with different \( K_b \) values are considered according to Figure 6a, and the results are shown in Figure 6b. Moment values remained fixed at 61.0 kNm in all cases. It is evident that the non-dimensional rotational stiffness has no major influence on the buckling temperature; however, they have influence on stabilization. Figure 6c shows the variation of apex deflection against temperature for different initial values of \( K_b \). When \( K_b = 25 \), the frame experiences snap-through buckling at a temperature of 818 °C. After this temperature, the frame exhibits some post-buckling strength until 1010 °C at which temperature the frame collapses. Figure 6d shows the deformed shapes of the frame at different temperatures. It can be seen that initially rafters expand due to thermal expansion up to 602 °C. The whole frame experiences an acceptable inward collapse rather than an unacceptable outward collapse.

Effect of partial strength of column base

In order to investigate the effect of the partial strength of the column base, four different moment-rotation curves for the column base are considered where \( K_b =25 \) in all cases. The results are compared against the case of a pinned column base. Figure 7a shows details of the moment-rotation curves used for these column bases. Figure 7b shows the variation of apex deflection against temperature for each of the four rotation curves. As can be seen, increasing the moment-capacity of the column base from 0.25 \( M_{pl} \) to 1.0 \( M_{pl} \) increases the snap-through buckling temperature of the rafter from 810°C to 937°C. Figure 7c shows the deformation of a pinned supported column frame. Note, the deformation differs from the frames with rotational stiffness (shown in Figure 6d) and the frame is starting to sway outwards (the unacceptable mode). Immediately after 812.5°C, the frame has undergone snap-through buckling with an acceptable inwards collapse mode. Figure 7d shows a comparison of different column bases. It can be seen that the behavior of the frame with rotational stiffness is within the envelope of pinned and fixed column bases.

Figure 5: a) Overturning moment of column base; (b) catenary force at column top; (c) horizontal reaction force at column base; (d) horizontal deflection of eaves.
Figure 6: (a) Moment-rotation curve for various $K_b$ for a fixed $M_{OTM}$ of 61.0 kNm; (b) variation of apex deflection against temperature for different values of $K_b$; (c) variation of apex deflection against temperature when $K_b = 25$; (d) various deformed shape of portal frame at different temperature when $K_b = 25$.

Figure 7: (a) Different moment-rotation curves with $K_b = 25$; (b) apex deflections for column base; (c) deformed shape for portal frame with pinned column; (d) comparison of apex deflections for different column bases at different temperatures.
The frame with semi-rigidity and partial-strength column bases experiences snap-through-buckling at a temperature higher than that of the pinned column base. However, unlike the frame with pinned column base, the semi-rigid partial-strength column experiences re-stabilization due to the remaining strength of the column base before collapsing at a higher temperature.

CONCLUSIONS

A numerical model is developed for portal frame at elevated temperature. The model is capable of capturing true behavior of portal frame under fire. The study suggests that the SCI method is obviously over-conservative. The compressive stress in the rafter becomes tensile when approaching the buckling temperature, which pull the top of the column inward. At higher temperature, thermal expansion has no effect on the behavior of the structure rather the stiffness degradation of material plays the dominant role. Increasing moment-capacity of the column base increases the snap-though buckling temperature however, the non-dimensional rotational stiffness has no major influence on the snap-through buckling temperature.

REFERENCES

PARAMETER STUDY ON INFILLED STEEL FRAMES WITH DISCRETELY CONNECTED PRECAST CONCRETE PANELS

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KEYWORDS
Infilled frame, steel, precast concrete, structural stability, finite element analysis, parameter study

ABSTRACT

This paper presents a parameter study on infilled steel frames with discretely connected precast concrete infill panels having window openings. In this study, finite element simulations were carried out to study the infilled frame performance by varying several parameters. A recently developed finite element model was used, which had been validated with results from full-scale tests.

The parameters investigated in the parameter study include those that are considered to be the most influential for the structure. These parameters are the frame member dimensions, the rotational stiffness of the steel frame joints, the infilled frame aspect ratio and the panel opening geometry including reinforcement layout.

The results of the parameter study show that the structure exhibits a quite complex composite behaviour. The stiffness and strength of the structure were found to vary with the relative stiffness of the infill to the frame, while the location of connection failure was found to depend on the aspect ratio. The results of the parameter study serve as a base for the development of design guidelines for infilled steel frames with discretely connected precast concrete panels having window openings.

INTRODUCTION

In today’s building practice, growing numbers of hybrid structures are being built [1]. Hybrid construction combines the structural and architectural advantages of components made from different materials. These materials may work integrated or jointly. Hybrid construction is cost-effective as it combines the benefits, derived from using components of different materials. Besides the structural advantages, benefits may be realized in the following areas: aesthetics, function, construction speed, safety and constructability. These result into substantial savings and higher quality buildings.
Taking the advantages of hybrid construction, a hybrid lateral load resisting structure has been designed at Eindhoven University of Technology. It consists of discretely connected precast concrete infill panels within simple steel frames, and is a new application in infilled frames. The infilled frame is a type of structure that has shown to be effective and efficient in bracing low-rise and medium-rise buildings to resist in-plane lateral loads. It acts by composite action between an infill and its confining frame. Structural interaction between the two components produces a composite structure with a complicated behaviour as the infill and frame mutually affect each other.

Since the early fifties extensive investigations have been carried out into the composite behaviour of framed structures with masonry and cast-in-place concrete infills [2, 3]. When connectors or strong bonding at the interfaces between the frame and the infill panel are absent, as for example with masonry infill, the structures are generally known as non-integral infilled frames (Figure 1a). When these structures are subjected to lateral loading, a major portion of the load is taken up by the infill panel at its loaded corner. The provision of strong bonding or connectors at the interface enables the infill and frame to act compositely. These infilled frames are known as fully-integral infilled frames (Figure 1b). Part of the shearing load is transmitted from the frame to the infill panel through the connectors. A variety of methods have been developed for the design and analysis of non-integral and fully-integral infilled frames. An extensive survey of these methods can be found in a state-of-the-art report on analytical modelling of infilled frames by Crisafulli et al. [4].

Discretely connected precast concrete infill panels may provide similar improvements to frame structures as masonry and cast-in-place concrete infills. However, the application of discrete interface connections between the infill and frame results in different behaviour from non-integral or fully integral infilled frames. Infilled frames with discrete connections between frame and panel are termed semi-integral infilled frames (Figure 1c). To provide insight into the composite behaviour of this structure, it has recently been subjected to experimental and numerical analyses.

**Figure 1: Classification of infilled frames**

**FINITE ELEMENT MODEL**

With the finite element program DIANA, the response of 5 full-scale tests on one-storey, one-bay, 3 x 3 m hybrid frame structures, having different window opening geometries, was simulated. For this purpose, a two dimensional finite element model was developed (Figure 2). This model consists of plane stress, beam and two types of spring elements. The two types of spring elements were calibrated on test results. For the concrete, a non-linear material model was used that combines the Drucker-Prager plasticity model for the compressive regime with a smeared cracking model for the tensile regime. Material nonlinearities such as concrete cracking, tension softening, shear retention, concrete plasticity and reinforcing bar yielding were all simulated. The finite element simulations were performed taking into account geometrical non-linearity.
A comparison was made between the experiments and the simulations in order to check the validity of the finite element model. This comparison showed that the finite element model enables simulating the elastic and plastic behaviour of the hybrid lateral load resisting structure. A detailed description of the full-scale experiments and the finite element analyses is available in Teeuwen et al. [5] and in Teeuwen et al. [6] respectively. In this paper, the validated finite element model is used to carry out a parameter study to investigate other configurations of the hybrid infilled frame structure.

**PARAMETER STUDY**

*Parameters considered*

The parameters investigated in the parameter study include those that are considered to be the most influential for the hybrid structure. These parameters are the frame member dimensions, the rotational stiffness of the frame joints, the infilled frame aspect ratio and the panel geometry including reinforcement layout. Each investigated parameter is described below.

*Frame member dimensions*

The specimens tested experimentally had frame members HE180M all around. This frame is applied in the parameter study as well and is denoted hereafter as 'weak frame'. In addition, a more practical frame is studied, consisting of HE400B columns and HE320A beams (denoted as ‘strong frame’). In practice, the increased height of the frame members reduces the space between the columns and beams, requiring smaller panel dimensions. However, in order to evaluate the effect of the frame members only, the height and width of the panel are kept constant in the parameter study. This is possible because the frame is modelled with beam elements which only have sectional properties and no height in the model. The geometrical properties for the frame members used are provided in Table 1. Note that by changing the frame members, the stiffness characteristics of the panel-to-frame connections ($k_c$) change as well, as they are related to the flange thickness.
TABLE 1
GEOMETRICAL PROPERTIES OF FRAME MEMBERS

<table>
<thead>
<tr>
<th>Property</th>
<th>Weak frame</th>
<th>Strong frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia of the column</td>
<td>$I_{\text{column}}$</td>
<td>7.38E+07 mm$^4$</td>
</tr>
<tr>
<td>Moment of inertia of the beam</td>
<td>$I_{\text{beam}}$</td>
<td>7.38E+07 mm$^4$</td>
</tr>
<tr>
<td>Area of the column</td>
<td>$A_{\text{column}}$</td>
<td>1.11E+04 mm$^2$</td>
</tr>
<tr>
<td>Area of the beam</td>
<td>$A_{\text{beam}}$</td>
<td>1.11E+04 mm$^2$</td>
</tr>
<tr>
<td>Transl. stiffness of panel-column con.</td>
<td>$k_{c;\text{column}}$</td>
<td>6.90+05 N/mm</td>
</tr>
<tr>
<td>Transl. stiffness of panel-beam con.</td>
<td>$k_{c;\text{beam}}$</td>
<td>6.90+05 N/mm</td>
</tr>
<tr>
<td>Rotation stiffness of frame joints</td>
<td>$S_{\text{ini}}$</td>
<td>8.29+09 or 0 Nmm/rad</td>
</tr>
</tbody>
</table>

Rotational stiffness frame joints

For the simulation of the full-scale tests, the frame joints were calibrated with experimental results obtained from testing the bare frame. In the parameter study, flexible joints are assumed for the frame, estimating the initial joint stiffness ($S_{\text{ini}}$) using design equations. Values of the joint stiffnesses applied are given in Table 1. In addition to the flexible joints, hinged joints are studied ($S_{\text{ini}} = 0$)

Infilled frame aspect ratio

The infilled frame aspect ratio is defined as $\alpha = H / L$, where $H$ and $L$ are the height and length of the infilled frame respectively. The specimens tested experimentally had dimensions 3.0 m × 3.0 m, equal to an aspect ratio of $\alpha = 1$. In this parameter study, three more aspect ratios are studied: $\alpha = \frac{1}{2}$, $\frac{2}{3}$ and $1\frac{1}{2}$. The height of the infilled frame is kept constant, equal to 3.0 m. Consequently, the infilled frames considered are sized $H \times L = 6.0$ m × 3.0 m, 4.5 m × 3.0 m, 3.0 m × 3.0 m and 2.0 m × 3.0 m respectively (Figure 3).

Figure 3: Investigated aspect ratios with transformed panel opening geometry
Panel opening geometry

The five types of panel opening geometries investigated experimentally are considered in the parameter study as well. This means that each aspect ratio includes five panel opening geometries. The dimensions of the window openings result from keeping the heights of the panel members (indicated with \( x, y \) and \( z \) in Figure 3) constant. The thickness of the panels is kept constant (\( t = 200 \) mm). The reinforcement types are determined in accordance with the panel geometry using strut-and-tie models [7].

Table 2 gives an overview of the analyses performed in the parameter study with the investigated parameters.

**TABLE 2**
OVERVIEW OF PERFORMED ANALYSES WITH INVESTIGATED PARAMETERS

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>Frame members</th>
<th>Frame joints</th>
<th>Panel opening</th>
<th>Aspect ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 4</td>
<td>Weak</td>
<td>Flexible</td>
<td>No panel</td>
<td>½, ⅔, 1, 1½</td>
</tr>
<tr>
<td>5 – 24</td>
<td>Weak</td>
<td>Flexible</td>
<td>Type 1 to 5</td>
<td>½, ⅔, 1, 1½</td>
</tr>
<tr>
<td>25 – 32</td>
<td>Weak</td>
<td>Pinned</td>
<td>Type 1 and 5</td>
<td>½, ⅔, 1, 1½</td>
</tr>
<tr>
<td>33 – 36</td>
<td>Strong</td>
<td>Flexible</td>
<td>No panel</td>
<td>½, ⅔, 1, 1½</td>
</tr>
<tr>
<td>37 – 56</td>
<td>Strong</td>
<td>Flexible</td>
<td>Type 1 to 5</td>
<td>½, ⅔, 1, 1½</td>
</tr>
<tr>
<td>57 – 64</td>
<td>Strong</td>
<td>Pinned</td>
<td>Type 1 and 5</td>
<td>½, ⅔, 1, 1½</td>
</tr>
</tbody>
</table>

Material properties

The applied concrete and reinforcement properties are shown in Table 3. Concrete of grade C50/C60 and reinforcement steel with nominal material properties according to Eurocode 2 are assumed for the infill panels. For the steel frame, elastic-perfectly plastic behaviour is assumed, having the elastic material properties Young’s modulus \( E_s = 2.1E+05 \) N/mm² and Poisson’s ratio \( \nu = 0.3 \) in combination with Von Mises plasticity with yield strength \( f_y = 355 \) N/mm².

**TABLE 3**
MATERIAL PROPERTIES USED FOR INFILL PANEL

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete Young’s modulus</td>
<td>3.7E+04 N/mm²</td>
<td>Fracture energy ( G_f )</td>
<td>0.105 N/mm²</td>
</tr>
<tr>
<td>Poisson’s ratio ( \nu )</td>
<td>0.2</td>
<td>Shear retention factor ( \beta )</td>
<td>0.2</td>
</tr>
<tr>
<td>Tensile strength ( f_{ct} )</td>
<td>4.1 N/mm²</td>
<td>Young’s modulus reinforcement steel ( E_s )</td>
<td>2.0E+05 N/mm²</td>
</tr>
<tr>
<td>Compressive strength ( f_c )</td>
<td>50 N/mm²</td>
<td>Yield strength reinfo. ( f_y )</td>
<td>560 N/mm²</td>
</tr>
</tbody>
</table>

Results parameter study

The major findings for each investigated parameter are reported below.

Influence of frame member dimensions

The load-deflection response of two infilled frames having a weak frame (grey line) and strong frame (black line) respectively are shown in Figure 4 (\( \alpha = 1 \), panel opening type 3). The dashed line in the matching colour shows the bare frame behaviour of the corresponding infilled frame.
Figure 4: Load-deflection response of (infilled) frames having a weak or strong frame

It is shown that, despite the considerable higher bare frame stiffness of the strong frame, the variation in the initial infilled frame stiffness is low. Furthermore, comparable ductility is achieved. However, applying the strong frame, the ultimate strength of the infilled frame structure increases.

Influence of rotational stiffness frame joints

Figure 5 shows the load-deflection response of two infilled frames with small window openings (panel opening type 1, black lines) and large window openings (panel opening type 5, grey lines), having flexible (solid lines) and pinned frame joints (dashed lines) respectively ($\alpha = 1$). It is shown that the application of pinned joints results in a lower ultimate strength compared to the infilled frame with flexible frame joints. Moreover, for a frame with pinned frame joints, the relative stiffness of the frame to the panel has no influence on the ultimate strength. This is demonstrated by the ultimate loads of the pinned infilled frames, which are approximately the same (nearly 600 kN), independently of the applied panel (panel opening type 1 or 5) or frame (weak or strong).

Influence of the aspect ratio

The infilled frames were designed to fail by failure of the discrete interface connections. From the simulations, the locations of connection failure were derived. Evaluation of the results showed that the aspect ratio determines the location of failure. For $\alpha = \frac{1}{2}$ and $\frac{2}{3}$, failure occurs at the column-to-panel connection at the upper loaded corner of the infilled frame. For $\alpha = 1$ and $1\frac{1}{2}$, the beam-to-panel connection at the lower loaded corner of the infilled frame fails first.

Influence of the panel opening geometry
Figure 6 shows the load-deflection response for infilled frames with panel opening types 1 (smallest window opening) to 5 (largest window opening), having a weak and strong frame respectively. The results show that a decrease of the relative stiffness of the infill to the frame results in a decrease of the lateral stiffness and in an increase of the ultimate strength of the infilled frame. This effect is more significant considering the strong frame.

![Figure 6: Load-deflection response of infilled frames with varied window openings size](image)

**Influence stiffness discrete panel-to-frame connection**

The influence of the stiffness of the discrete panel-to-frame connections is demonstrated by considering infilled frames with pinned frame joints. The lateral load-deflection response of two infilled frames with panel geometry type 1 and type 5, having a weak (grey line) and strong (black line) frame respectively are shown in Figure 7, for \( \alpha = \frac{2}{3} \) and for \( \alpha = 1\frac{1}{2} \). Despite the fact that both bare frame stiffnesses are zero, the strong frame gives a higher infilled frame stiffness for \( \alpha = \frac{2}{3} \). However, for \( \alpha = 1\frac{1}{2} \), the infilled frame having the weaker frame acts stiffer. This effect is to be attributed to the stiffness of the panel-to-frame connection and demonstrates that the infilled frame exhibits quite complex behaviour.

![Figure 7: Load deflection response of infilled frames with pinned frame joints](image)

From the 64 simulated load-deformation curves, strength and stiffness characteristics have been derived. These serve as a verification for the development of analytical models for prediction of the ultimate lateral load carrying capacity and lateral stiffness of the semi-integral infilled frame.
CONCLUSIONS

A parameter study was performed on infilled steel frames with discretely connected precast concrete panels. In this study, finite element simulations were carried out to study the infilled frame performance by varying different parameters. A finite element model was used which had been validated using results from full-scale experiments. Different variables have been studied with respect on their influence on the structural response. These parameters are the frame member dimensions, the rotational stiffness of the frame joints, the infilled frame aspect ratio and the panel geometry. The major findings are summarized below:

- For infilled frames with flexible frame joints, an increase of the relative stiffness of the frame to the infill results in an increase of the ultimate strength and stiffness.
- For infilled frames with flexible frame joints, an increase of the window opening and thus a decrease of the relative stiffness of the infill to the frame results in a decrease of the stiffness and in an increase of the ultimate strength and deformation capacity.
- For infilled frames with pinned frame joints, the ultimate strength is independent on the relative stiffness of the infill to the frame.
- The aspect ratio of the infilled frame defines the location of connection failure.
- Partly because a change in frame members goes together with a change in the panel-to-frame connection stiffness, the structure exhibits quite complex behaviour.

The results of the parameter study serve as a base for the development of analytical models for the design of infilled steel frames with discretely connected precast concrete panels having window openings.

REFERENCES

CONTRIBUTION TO SUSTAINABILITY IN STEEL STRUCTURES

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KEYWORDS
Steel, structures, material properties, sustainability, natural fire, after fire, cooling phase.

ABSTRACT
The load bearing behaviour of steel structures after fire is examined. Considerations and numerical studies, in this work phase, have been made for members under tension. During fire (pinned) members under restraint experience significant compression forces due to thermal expansion. This leads to residual internal forces, which can reach yield limit, and residual displacements. Statically undetermined structures show inelastic load carrying behaviour after fire. Residual displacements often are in a range that they can be left without repair or compensated by affordable means of repair. In the planning phase of a building sensitivity to fire damages can be checked and the structural detailing can be done in a way, that no or little means are necessary to compensate for fire damages, e. g. residual displacements.

INTRODUCTION
Many efforts have been spent on determining the load carrying behaviour of steel structures under fire during the heating phase. Much lesser results are given for structures under fire comprising the cooling phase (Roben et al. 2007, Wang et al. 2008). Few results are given for the load bearing behaviour and bearing capacity of structures after fire (Hamme 1989). Recently the authors have investigated the load bearing capacity of steel structures after fire. Due to fire considerable residual stresses and strains occur after fire which affect the load bearing behaviour and thus the serviceability of steel structures after fire. Numerical studies done permit to better evaluate whether fire damaged structure may be reinstated and reused rather than deereeted. Thus, this paper contributes to evaluate and improve sustainability of steel structures.
**ASSUMPTIONS AND BASIC REMARKS**

**Abbreviations**

\( A \) cross sectional area

\( E, E_\theta \) modulus of elasticity at room temperature (20° C), at steel temperature \( \theta \)

\( N_{Ed} \) design normal force

\( N_{p,Rd,\theta} \) design resistance of a tension member at steel temperature \( \theta \)

\( N_{b,Rd,\theta} \) design resistance of a compression member at steel temperature \( \theta \)

\( P_{Ed} \) design load

\( P_{u,Rd} \) design ultimate limit load for permanent situation

\( P_{u,Rd,fi} \) design ultimate limit load for fire situation

\( P_{u,Rd,\theta} \) design ultimate limit load under fire conditions

\( a \) distance

\( f_{y,y,\theta} \) yield strength at room temperature (20° C), effective yield strength at steel temperature \( \theta \)

\( k_{E,\theta} \) reduction factor for the modulus of elasticity at steel temperature \( \theta \) (see EC3-1.2)

\( k_{\sigma,\theta} \) reduction factor for the yield strength at steel temperature \( \theta \) (see EC3-1.2)

\( l \) length of a member

\( w \) deformation of a member, displacement of a node

\( \gamma_G \) partial safety factor for permanent loads for permanent situation, \( \gamma_G = 1.35 \)

\( \gamma_{G,fi} \) partial safety factor for permanent loads for fire situation, \( \gamma_{G,fi} = 1.0 \)

\( \gamma_P \) partial safety factor for variable loads for permanent situation, \( \gamma_P = 1.50 \)

\( \gamma_{P,fi} \) partial safety factor for variable loads for fire situation, \( \gamma_{P,fi} = 1.0 \)

\( \gamma_M \) partial safety factor for material properties for permanent situation, \( \gamma_M = 1.1 \)

\( \gamma_{M,fi} \) partial safety factor for material properties for fire situation, \( \gamma_{M,fi} = 1.0 \)

\( \epsilon, \epsilon_{pl} \) strain, plastic strain

\( \theta, \epsilon_{\text{pl}} \) steel temperature, maximum steel temperature reached during a fire [°C]

\( \theta_{cr} \) critical steel temperature [°C]

\( \lambda \) slenderness ratio

\( \rho \) degree of utilisation (before fire, after fire)

\( \rho_{cr} \) critical degree of utilisation

\( \sigma_{Ed}, \sigma_{i,\text{res}} \) normal stress, residual normal stress in member i

\( \chi \) reduction factor for buckling

**Material behaviour**

For the numerical studies the temperature dependant functions for yield strength, modulus of elasticity and thermal elongation (appr. 0.000012) as given in EC3-1-2 2005 have been used. It is noted that mild steel grades show the same yield strength before and after fire (Smith et al. 1981), whereas fine grain high strength steel grades lose up to 40% of the yield strength depending on the maximum temperature reached during fire, see figure 1 (Wohlfeil 2006). Material behaviour in the cooling phase of natural fire is unknown and assumed similar to the heating phase. No results are known to the authors concerning the modulus of elasticity, therefore it is considered to be unaltered by fire.

The ultimate strain after fire can decrease down to 16% and is not needed to be considered. Effects of embrittling are not important, because structures exposed to natural fire always are cooling down slowly even by using water for fire fighting.

For the numerical studies material behaviour is taken into account by assuming a linearly elastic – ideally plastic stress-strain-relationship.
Member behaviour

Members are considered to show elastic behaviour until the characteristic tension or compressive resistance is reached.

Characteristic resistance of tension members at steel temperature $\theta$ is given by

$$N_{p, R, \theta} = \frac{A \cdot f_y \cdot k_{y, \theta}}{\gamma_{M, f_i}}$$

(1)

It is independent from geometrical and physical imperfections. Reduction in bearing capacity after fire is proportional to the reduction of yield strength accordingly.

Characteristic resistance of compression members at steel temperature $\theta$ is taken into account by

$$N_{c, R, \theta} = \chi(\lambda(\theta)) \cdot \frac{A \cdot f_y \cdot k_{y, \theta}}{\gamma_{M, f_i}}$$

(2)

This is a simplified approach but a sufficient one for these studies. Members of a frame under tension can considerably be compressed during the heating and cooling phase due to thermal expansion and even afterwards due to plastic deformations occurred. Due to buckling partial plastification may occur and lead to higher physical and geometrical imperfections after fire situation than before (Hamme 1989). This effect is allowed for by using a less favourable buckling curve. Reduction in bearing capacity is thus considerably higher than just only reduction in yield strength. Further studies will deal more detailed with compression members in fire.

Frame behaviour

Fire causes high internal forces and moments in statically indeterminate structures, which often reach yield limit. Plastic deformations are irreversible and thus after fire high residual forces are encountered or, already in the cooling phase of a fire, members or connections are overstressed (Roben et al. 2007). Due to plastic deformations during fire frame behaviour is different before and after fire.
Numerical studies have been done for a structure as shown in figure 1. It is the same structure as used by Wang et al 2008, so results can be compared. It is considered to be representative for structures under tension. Degree of utilisation $\rho$ and maximum temperature reached during fire $\max \theta$ have been varied. Members are assumed either stocky or, to allow for stability effects, very slender. First order analysis has been used.

**Time-temperature curves**

For the numerical studies uniform steel temperature in the whole cross sections and along the length of the members is assumed (steady state conditions). Further studies will deal with local heating. These studies do not require time-temperature curves.

**LOAD BEARING CAPACITY AFTER FIRE**

For room temperature the design ultimate limit load $P_{u,Rd}$ is given by:

$$P_{u,Rd} = \frac{A \cdot f_y}{\gamma_M} \left(1 + \sqrt{2}\right)$$

and

$$P_{u,Rd,fi} = \frac{A \cdot f_y}{\gamma_{M,fi}} \left(1 + \sqrt{2}\right)$$

(3a), (3b)

The degree of utilisation $\rho$ is given by:

$$\rho = \frac{P_{ld}}{P_{u,Rd}} \leq 1.0$$

(4)

As the yield strength can be different before and after fire this leads to different ultimate limit loads and degrees of utilisation, indicated by $\rho^{bf}$ and $\rho^{af}$ (e.g.: $\rho^{bf}, \rho^{af}$) if necessary.

Taking a ratio variable/permanent loads of 2.0 at room temperature for a common building the degree of utilisation under serviceability loads is $\rho^{bf} = 0.69$. For the numerical studies a range from 0 to 0.7 has been chosen. It is noted, that the degree of utilisation refers to the ultimate limit load of the structural system. The elastic limit load (at room temperature) is reached when $P_{ld} = 0.414 \cdot P_{u,Rd}$. For a. m. common building, designed using elastic theory, this would be for $\rho^{bf} = 0.29$.

In a first step internal forces (stresses) and deformations (strains), occurring during fire situation, have been analysed. Frame analysis is made by using plastic hinge theory. Whenever a cross section is fully plastified, a plastic hinge and a couple of forces $N_{pl,Rd}$ is introduced in the statical system. Whenever, in the cooling phase, an internal force in a plastified member is decreasing, the plastic hinge is extracted but the plastic strain is imposed. The plastic strain is given by the strain $\epsilon$ (dis-
placement \( w \) taken from frame analysis) reached at maximum steel temperature minus mechanical strain and minus thermal strain:

\[
\varepsilon_{\text{pl}}(\max \theta) = \frac{w(\max \theta)}{l} - \frac{N_{\text{pl,Rd},\max \theta}}{A \cdot E \cdot k_{E,\max \theta}} - 1.2 \cdot 10^{-5} \cdot \max \theta
\]  

(5)

There is one load-displacement curve for the heating phase but, as the plastic strain depends on the maximum steel temperature reached, there are innumerable load-displacement curves for the cooling phase. Regarding residual normal stresses for upper limit of \( \rho = 0.7 \) all members stay under tension after fire while for small \( \rho \) the normal stresses could change from tension to compression, as shown in figure 2 for \( \rho = 0.166 \) (same value as used by Wang et al. 2008) assuming no stability effects occur. As the members designs for tension before considering of stability effects could be necessary after fire.

Reduced yield strengths, after fire lower than before fire, lead to reduced design resistances and thus to lower residual forces, see figures 2 and 3. For the residual forces and stresses given in this paper this effect has been neglected, i.e. upper limits are given here.

![Figure 2: Relative normal stresses for \( \rho = 0.166 \)](image)

![Figure 3: Relative normal stresses for \( \rho = 0.7 \)](image)

Using equilibrium conditions residual forces and displacement as well as residual stresses and strains can be derived from the internal forces of the members. They differ with the maximum temperature reached during fire. The residual normal stresses in function of maximum temperature reached during fire \( \max \theta \) and degree of utilisation before fire \( \rho_{bf} \) are given in figure 4. For low maximum temperatures and low degrees of utilisation the structure remains fully in the elastic range, no residual stresses
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and strains occur. Residual normal stresses and strains are nonlinear in function of maximum temperature and degree of utilisation. Buckling effects reduce significantly the residual normal stresses and strains.

Figure 4: Relative residual stresses $\sigma_{2,\text{res}} / f_y$ using slender members (left) and stocky members (right).

Figure 5: Load-displacement curves for $f_y = f_y^{bf}$

Figure 6: Load-displacement curves for $f_y^{af} = 0.80 f_y^{bf}$
The load-displacement curves are shown in figure 5 and 6 for both the situation before and after fire. They start with zero, so residual deformations are not included. Although there are considerably high residual normal stresses, the ultimate load is not affected by these. The sequence of forming plastic hinges can be different. To illustrate the effect of reduced yield strength after fire, in figure 6 a loss of 20% is assumed. The loss of ultimate limit load capacity after fire is proportional to the loss of yield strength.

With regard to reuse of the structure in figure 8 the relative differences in deformation $\Delta w/a$ are given in function of the maximum temperature reached in fire. They are defined by

$$\Delta w = \frac{w^{bf} (\max \theta, \rho) - w^{bf} (\max \theta, \rho)}{a}$$

(6)

Figure 7: Temperature-displacement curves

Figure 8: Residual displacement after fire depending on the degree of utilization and maximum temperature reached during fire

The residual deformation comprises the plastic deformation and the elastic deformation due to residual forces but not the elastic deformation due to external load. As for low maximum temperatures reached in fire there are no residual stresses and thus no plastic strains after fire there are no residual deformations. For high maximum temperatures reached in fire the differences are high as the structure tends to collapse.
These curves show the increase in deformation due to fire. With regard to limit values for serviceability they easily permit to decide whether a structure can still be used without repair, whether and how it should be repaired to be used further. In severe cases costs of repair will be too high in economic terms. These curves can also be used, for a given degree of utilisation and by measuring the residual deformation, to determine the maximum temperature reached in fire by.

The curves show that for low degrees of utilisation the residual deformations are in the range of less than 1/100. Overall deformations will thus not or only little exceed limit values for serviceability. For high degrees of utilisation the residual deformations are in the range of up to 1/25. It has to be taken into account that, as mentioned before, for a common building designed using elastic theory, the degree of utilisation is less than 0.29. High degrees of utilisation can only be acceptable when deformations do not govern the design, thus, high residual deformations may be acceptable and buildings may be used after fire without or with little repair.

CONCLUSIONS AND FINAL REMARKS

After fire in general residual normal forces and displacements appear. An elastic frame analysis will not reflect sufficiently the load bearing behaviour.

For low maximum temperatures reached in fire no residual displacements appear. For low degrees of utilisation residual displacements may reach limit values of serviceability. For high degrees of utilisation, structures designed using plastic theory, serviceability limit states are less important and thus residual displacements in general can be left without repair.

Results are based on uniform temperature distribution. For non-uniform temperature distribution residual normal forces and displacements tend to be lesser. The load bearing capacity tends to be unaffected.

With regard to sustainable construction, buildings should be checked in the planning phase on sensitivity to residual displacements due to natural fire. Examinations show, that in many cases structures can be left without repair after fire. If structures show to be more sensitive to residual deformations due to fire, the constructional detailing should be done in a way, that residual displacements can compensated by simple means, e. g. long holes, fitting plates etc. Steel structures are inherently sustainable or can easily be designed to fulfil requirements of sustainability.

REFERENCES


APPLICATION OF TEMPERATURE CRACK WITH SINGLE COLUMN IN MULTI-SPAN AND SINGLE-STOREY STEEL FRAMES

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KEYWORDS
Steel-frame building, transverse temperature crack, rolling support, substructure, main structure, horizontal displacement

ABSTRACT
In order to reduce temperature actions in multi-span single storey steel frame with super-wide, the single column temperature crack is employed to break the frame in construction, which divides the multi-span steel frame with super-wide into main structure and substructure. The span in general of the main structure meets the limited geometrical dimension given in the Code of China mainland, and temperature actions which affect the structural bearing capacity need not be considered. The substructure connects the main structure with rolling supports which are set in the top of columns. The displacement of rolling supports must be limited according to the horizontal displacement of the main structure at the top of columns under wind load combination. The substructure and the main structure would work together if the displacement of the rolling support reaches the limit, and check the bearing capacity of the substructure. According to the structural analysis the single column transverse temperature crack can effectively reduce the temperature actions in multi-span single storey steel frames with super-wide without increasing the cost of the structure, and the simple structure model can be used by engineers conveniently. The transverse temperature crack with single column for the multi-span factory building and its detail can be employed by the same projects.

INTRODUCTION
An union dockyard steel-framed building has been erected along the Yangzi downriver covers about 120,000 m². The “L” plan is divided into area “A” and area “B” with a crack between axial F1 and E1 shown in Figure 1a. The area “A” is 180m width (four spans of 45m),
460.1 m long and 32 m height, and area “B” is 181 m wide (four spans of 36 m and one span of 37 m), 221 m long and 23 m height. The steel frames are spaced at 9 m, and location at 18 m, 20 m and 23 m for special manufacture process. In area “A” there are two 150 t capacity crane bridges, two 50 t capacity crane bridges and five 10 t capacity gantry in each spans at 23.5 m, 16.75 m and 11.25 m level respectively shown in Figure 1b.
In area “B” there are one 32t capacity magnetic crane and 10t capacity gantry in each span at 16.75m, 11.25m level respectively shown in Figure 1c. Steel columns are supported concentrically on reinforced concrete pile caps. The precast piles in circle cross-section of 500mm diameter are driven until meeting the supporting soil below natural ground about 24m–36m. It is important to design a preliminary structure scheme for this huge plant, and develop a simple calculation method for engineers.

**STRUCTURAL SCHEMES**

The rule defined by the Code GB50017-2003[1] in China mainland demands temperature cracks for the building which dimension beyond the limit, otherwise temperature actions must be concerned. The “resistance” structural systems without temperature cracks take account of temperature actions within load combination, and “release” structural systems with temperature cracks ignore temperature actions. This building geometry is beyond the limit dimension in the longitudinal and the transverse. Usually the temperature crack should be set in the transverse which divides area “A” to two parts and regardless temperature action in the longitudinal direction shown as Figure 1a. However the temperature crack setting in the longitudinal will increase the structure materials largely. Then how to consider the temperature actions along spans of frame should be put forward as follows.

**Frames Without Temperature Cracks in Transverse**

The frames without the temperature cracks have four spans of 45m up to 180m width in the area “A” and four spans of 45m adding one span of 37m up to 181m width in the area “B” respectively shown in Figure 2, Figure 3. Their spans in general are beyond the limit according to the Code GB50017-2003, and the temperature actions must be considered within the load combination. In Figure 2, Figure 3 the dotted lines describe the column deformations which cause moments at the bottom of columns.
Frames With Temperature Cracks in Transverse

Temperature cracks can divide structures to two parts and reduce temperature actions in frames. There are two details of temperature cracks. Two rows of columns replacing one row in the longitudinal direction is widely used by engineers for its simple construction, but two rows of columns will induce more materials and decreased the virtual space. The other detail of the temperature crack can be design as beam-column connections with rolling supports at the top of columns.

Temperature crack with two arrays of columns in transverse

The structure systems with two arrays of columns in the transverse formed the temperature crack are shown as Figure 4, Figure 5. The temperature crack decreases the span of the frames largely, and the temperature action can be ignored.

Where \( L_n \) is the distance from the middle point of the span in general to the external column of the frame, \( \Delta_n \) is the deformation at the top of the column induced temperature expansion.
Temperature cracks with rolling supports

The whole structure is separated into the main structure and the substructure at the beam-column connection shown as Figure 6 and Figure 7. The beam-column connection is constructed as the rolling support and the detail shown as Figure 8. When the displacement of the rolling support reaches the limit, the main structure and the substructure act together. This process of the rolling support moving belongs to the nonlinear touch field, and structure models with rolling supports can’t be calculated by PKPM software \(^1\) which is widely used by engineers in China mainland. So the beam-column connections with rolling supports are seldom adopted by engineers in China mainland. However PKPM software can analyze the structural response under forced displacement, based on this function a simple calculation method is developed. First the displacement of the main structure at the top of the column induced by horizontal wind load is obtained from PKPM software. Then assumed that the value of the limit displacement of rolling supports is equal to the displacement of the main structure at the top of the column, and check the substructure load capacity working with the main structure in the case of the forced displacement and horizontal wind load.

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Figure 6: The temperature crack with limited rolling support for area “B”

Figure 7: The temperature crack with limited rolling support for area “B”

Figure 8: The rolling support detail
STRUCTURAL ANALYSIS

Calculation of Internal Forces and Moments

A number of computer programs are available for calculating the load capacity of structural members or frameworks (e.g. ANSYS, SAP2000, ETABS finite-element program). However the commercial design software PKPM developed by China academy of building research is popular in China mainland for its super capacity of computer aiding drawing, and is available for this project also. Based on the load fundamental combinations for design according to the Code GB50009-2001[3] in China mainland, forces and moments are determined from the frame analysis with PKPM software. The design situation is defined as persistent situations corresponding to normal conditions during design reference period. The permanent actions for buildings include the weight of building materials and the variable actions for buildings include the wind loads, roof loads and crane loads. The actions concerned above and load combination should be assessed by the Code GB50009-2001 in China mainland for ultimate limit state and serviceability limit states.

Calculation of Temperature Internal Forces

Because PKPM software can’t carry out the temperature action, hand calculation method for calculating temperature action based on classical mechanics theory is employed. The beam expansion at the elevated temperature causes deformations at the top of columns and induces moments at the bottom of columns. Assuming that the beam stiffness is infinite and rigid-jointed to column, the moment at the bottom of the column can be gotten by Equation 1[4] listed in Table 1.

\[ M_t = \frac{KEI}{H^2} \cdot \Delta_n = \frac{KEI}{H^2} \cdot C \cdot \alpha \cdot \Delta t \cdot L_n \]  

(1)

Where \( H \) is the height of the column, \( E \) is the modulus of elasticity (\( E = 206 \times 10^3 \) N/mm\(^2\)), \( \alpha \) is the coefficient of linear thermal expansion (\( \alpha = 12 \times 10^{-6} \) m/(m·℃)), \( \Delta t \) is the temperature interval, \( K \) is the coefficient (\( K = 3.5 \)), \( C \) is the reduction factor for the temperature internal force (\( C = 0.85 \)), \( I_c \) is the second moment of column area.

MEMBERS DESIGN

The load-carrying capacity of columns should be designed for the limit states the code requires. Along the height of columns the cross-sections of columns are built-up with two H-sections in the bottom segment and the single H-section in the upper segment respectively shown as Figure 9. According to GB50017-2003 [1], Equation 2 is employed to ensure the buckling resistant design of columns with combined axial force and moment determined from the frame analysis, which is isolated from the frame. The limited horizontal deflection at the top of columns caused by wind load is height/400 taken from GB50009-2001 [2].

\[ \frac{N}{\varphi_x A} + \frac{\beta_{\text{ax}} M_x}{W_{1x} \left( 1 - \varphi_x \frac{N}{N'_{\text{Ex}}} \right)} \leq f \]  

(2)

and

\[ N'_{\text{Ex}} = \frac{\pi^2 EA}{1.1 \lambda_x^2} \]
Where $N$ is the axial force, $\lambda_s$ is the equivalent slenderness ratio for built-up member, $M_x$ is the bending moment about the relevant axis, $A$ is the area of the gross cross-section of members, $\varphi_s$ is the reduced coefficient for buckling resistance, $W_{sx}$ is the section modulus of the cross-section about the relevant axis, $\beta_{mx}$ is the reduced coefficient for the bending moment, $f$ is the design value of strength ($f=295\text{N/mm}^2$).

Design of the flange and weld dimension, the buckling resistance of columns can be gotten by Equation 2 listed in Table 1. In the meantime deflections at the top of columns must be checked in the case of the serviceability load combination. All design values concerned above listed in Table 1 for different structures.

**TABLE 1**

INTERNAL FORCES AND SECTIONS OF COLUMNS

<table>
<thead>
<tr>
<th>Column number</th>
<th>Internal forces $M+M_t$</th>
<th>Stress $N$</th>
<th>Sections of columns $f=295\text{N/mm}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z_{1a}</td>
<td>5506.4kN·m + 1523.5kN·m</td>
<td>4487.9kN</td>
<td>$h=2.0\text{m,2H900*300*16*28}$</td>
</tr>
<tr>
<td>Z_{1b}</td>
<td>3119.3kN·m + 920.97kN·m</td>
<td>7059.2kN</td>
<td>$h=2.2\text{m,2H900*300*16*28}$</td>
</tr>
<tr>
<td>Z_{2a}</td>
<td>2593.0kN·m + 941.3kN·m</td>
<td>963.0kN</td>
<td>$h=1.6\text{m,2H600*200*11*17}$</td>
</tr>
<tr>
<td>Z_{2b}</td>
<td>1686.2kN·m + 714.4kN·m</td>
<td>1140.9kN</td>
<td>$h=1.8\text{m,2H600*200*11*17}$</td>
</tr>
<tr>
<td>Z_{3a}</td>
<td>6725.8kN·m + 761.8kN·m</td>
<td>2474.2kN</td>
<td>$h=2.0\text{m,2H900*300*16*28}$</td>
</tr>
<tr>
<td>Z_{3b}</td>
<td>4075.1kN·m + 0.0</td>
<td>7073.7kN</td>
<td>$h=2.2\text{m,2H900*300*16*28}$</td>
</tr>
<tr>
<td>Z_{3c}</td>
<td>4212.6kN·m + 0.0</td>
<td>4463.5kN</td>
<td>$h=2.0\text{m,2H900*300*16*28}$</td>
</tr>
<tr>
<td>Z_{4a}</td>
<td>3292.8kN·m + 0.0</td>
<td>207.3kN</td>
<td>$h=1.6\text{m,2H600*200*11*17}$</td>
</tr>
<tr>
<td>Z_{4c}</td>
<td>2184.9kN·m + 0.0</td>
<td>937.5kN</td>
<td>$h=1.6\text{m,2H600*200*11*17}$</td>
</tr>
<tr>
<td>Z_{4d}</td>
<td>1584.0kN·m + 0.0</td>
<td>1230.6kN</td>
<td>$h=1.8\text{m,2H600*200*11*17}$</td>
</tr>
<tr>
<td>Z_{5a}</td>
<td>7533.7kN·m + 0.0</td>
<td>2505.0kN</td>
<td>$h=2.0\text{m,2H900*300*16*28}$</td>
</tr>
<tr>
<td>Z_{5b}</td>
<td>5412.4kN·m + 0.0</td>
<td>4471.0kN</td>
<td>$h=2.0\text{m,2H900*300*16*28}$</td>
</tr>
<tr>
<td>Z_{5c}</td>
<td>2131.5kN·m + 0.0</td>
<td>7017.6kN</td>
<td>$h=2.2\text{m,2H900*300*16*28}$</td>
</tr>
<tr>
<td>Z_{6a}</td>
<td>3032.5kN·m + 0.0</td>
<td>943.1kN</td>
<td>$h=1.6\text{m,2H600*200*11*17}$</td>
</tr>
<tr>
<td>Z_{6b}</td>
<td>1427.6kN·m + 0.0</td>
<td>1065.2kN</td>
<td>$h=1.8\text{m,2H600*200*11*17}$</td>
</tr>
<tr>
<td>Z_{6c}</td>
<td>1032.6kN·m + 0.0</td>
<td>2819.2kN</td>
<td>$h=1.8\text{m,2H600*200*11*17}$</td>
</tr>
</tbody>
</table>

By comparing the buckling resistance of columns of different structures listed in Table 1, it could be obtained that the columns of frames without temperature crack subject higher stress than those with rolling supports crack based on the same cross-sections. On the other hand the cost of frames without temperature crack will increase based on the same loading capacity of frames with rolling supports. The columns of frames with two arrays columns crack have the approximate stress as those with rolling supports crack, but the temperature crack with two arrays columns increase the cost of the structure for about seventy-seven additional columns. The size of the reinforced concrete pile cap and the number of the precast pile will increase due to additional columns.
CONCLUSIONS

Overall the structure with limited rolling supports is determined to construction for its economic and loading capacity satisfactory, and the simple calculation model which is suitable for PKPM software is developed. The temperature crack with single array of columns and the structural analysis method can be employed by the same projects.

REFERENCES

BEHAVIOR OF STEEL FRAME WITH VARIOUS TYPES OF DIAGONAL BRACING UNDER LATERAL LOADING

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KEYWORDS
Steel frame, semi-rigid joint, steel pipe, light gauge, gusset plate, stiffener

ABSTRACT
Steel frame has merits in recycling. In this paper, three specimens constructed with light gauge steel tubes with different bracing schemes are experimentally studied to examine the behavior of the steel frame systems in terms of strains, displacements and strength. Based on the experimental results, appropriate arrangement of members and connection method is suggested.

INTRODUCTION
Steel structures have been widely constructed in the one-story building like convenience stores. However, steel structures have constructional and ecological problems. For example, it is difficult to construct and to demolish. As far the environment is concerned, it is estimated that recycling of 1 ton of steel reproduces about 0.222-0.250 tons of carbon dioxide. In this regard, steel frame can be used due to lightweight and recycling in reconstruction. In this paper, the stiffness and the strength of the light gauge steel frames are analyzed via laboratory experiment on three specimens. Appropriate thickness, position of joints, and the connection method of steel profiles are presented.

In the experiment, three specimens of steel frame system are examined to obtain the behavior of strains, displacements and strength. Some specimens have diagonal bracings connected by the stiff gusset plates. The others have diagonal bracings without gusset plates. The effects of vertical stiffeners placed on the web of the beam are also examined. The experimental results show the importance of appropriate connection condition of the light gauge steel frame structure.
EXPERIMENTAL OVERVIEW

Description of the specimens

The wall panel is composed of two H-beams (LH200x100x3.2x4.5) and two square-shaped steel pipes (√100x100x3.2). Two H-beams are used as a beam and a foundation beam, respectively, and two square-shaped steel pipes are used as columns. The frame is composed of three rectangular-shaped steel pipes (√75x45x2.3). Three rectangular-shaped steel pipes are used as two studs and a diagonal brace, respectively. The studs and diagonal brace are connected with the flat bar. They are fixed to the beam and the foundation of the H-beam. Three types of frame, Models A, B, and C are provided to examine the performance of elements.

Model A is shown in Figure 1. The frame of Model A has the gusset plate in order to increase strength at the position where the diagonal brace, the studs and the flat bar are welded. The gusset plate is removed in Model B and Model C. A stiffener is installed on the H-beam in Model C. Figure 2 and Figure 3 show the photos of the stiffener and gusset plate. Configuration of the specimens is summarized in TABLE 1.

![Figure 1: Model A (unit : mm)](image1)

![Figure 2: Stiffener](image2)
The experiment method

The H-beam of the wall panel at the bottom is fixed with a bolt and alternative load is applied at the top of the H-beam. Moreover, the strain gauges and the displacement transducers are mounted in each member of the specimens. As shown in Figure 4, the strain gauges are installed 30 cm away from the end of each member, and the displacement transducer is installed 30 cm away from the foundation.

The loading process is as follows:
a) Alternative load is applied. Relative story displacement of the specimens is induced considering the loading path regarding the ratio of displacement to height, which are 1/200 rad, 1/100 rad, 1/75 rad, and 1/50 rad, respectively. The corresponding displacement is 15 mm, 30 mm, 40 mm, and 60 mm, respectively.
b) The alternative load is applied thrice to achieve identical deformation.
c) The deformation of 1/15 rad is defined as the ultimate deformation. The corresponding load is defined as the maximum load.
EXPERIMENTAL RESULTS

Load under allowable relative story displacement of the structure

TABLE 2 shows the applied load to the frame in every loading cycle under the alternative load when the servo actuator jack moves 15 mm in both compressive and tensile directions. The displacement of 15 mm denotes 1/200 of the height of specimen, which is the allowable relative story displacement of the steel frame structures. TABLE 2 shows that the load decreases with increase of the loading amplitude when the frame deforms in the compressive direction. The load does not change with increase of the loading amplitude when the frame deforms in the tensile direction. When the frame deforms in the compressive direction, the deformation is small because residual deformation arises in a flat bar after the frame deforms in the tensile direction.

The frame in the paper is assumed to have the shear strength of 8 kN per a wall panel. Although two frames are used for the wall panel on the actual construction site, only a frame is used in the present specimens. Therefore, in this research, the shear strength of a wall panel is assumed to be 4 kN. Considering this, Model A possesses the sufficient strength at the deformation of 1/200, as shown in TABLE 2. On the other hand, Model B and Model C represent the insufficient strength. Especially, the strength of Model B without both the gusset plate and stiffener does not show the strength of 4 kN which. From the point of view of strength of the wall panel, the strength decreases due to the local buckling when the frame does not have the gusset plate. Also the stiffener improves the strength of the wall panel.

<table>
<thead>
<tr>
<th>Model A</th>
<th>Compressive load at displacement of 15 mm</th>
<th>Tensile load at displacement of 15 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/200 rad(15 mm)</td>
<td>-7.0 kN</td>
<td>5.0 kN</td>
</tr>
<tr>
<td>1/100 rad(30 mm)</td>
<td>-5.2 kN</td>
<td>4.2 kN</td>
</tr>
<tr>
<td>1/75 rad(40 mm)</td>
<td>-4.7 kN</td>
<td>4.1 kN</td>
</tr>
<tr>
<td>1/50 rad(60 mm)</td>
<td>-2.7 kN</td>
<td>4.3 kN</td>
</tr>
<tr>
<td>greater than 1/15 rad</td>
<td>-2.3 kN</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model B</th>
<th>Compressive load at displacement of 15 mm</th>
<th>Tensile load at displacement of 15 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/200 rad(15 mm)</td>
<td>-2.4 kN</td>
<td>5.2 kN</td>
</tr>
<tr>
<td>1/100 rad(30 mm)</td>
<td>-2.9 kN</td>
<td>5.1 kN</td>
</tr>
<tr>
<td>1/75 rad(40 mm)</td>
<td>-3.8 kN</td>
<td>5.1 kN</td>
</tr>
<tr>
<td>1/50 rad(60 mm)</td>
<td>-3.2 kN</td>
<td>6.6 kN</td>
</tr>
<tr>
<td>greater than 1/15 rad</td>
<td>-1.7 kN</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model C</th>
<th>Compressive load at displacement of 15 mm</th>
<th>Tensile load at displacement of 15 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/200 rad(15 mm)</td>
<td>-4.1 kN</td>
<td>4.0 kN</td>
</tr>
<tr>
<td>1/100 rad(30 mm)</td>
<td>-4.4 kN</td>
<td>4.2 kN</td>
</tr>
<tr>
<td>1/75 rad(40 mm)</td>
<td>-2.6 kN</td>
<td>4.1 kN</td>
</tr>
<tr>
<td>1/50 rad(60 mm)</td>
<td>-1.9 kN</td>
<td>5.2 kN</td>
</tr>
<tr>
<td>greater than 1/15 rad</td>
<td>0.1 kN</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5: Buckling at H-beam web

Figure 6: Deformation of the flat bar

Figure 7: Load-displacement relation of the models

The upper beam under the ultimate load

Model B and Model C have no change in the web of the H-beam because H-beam is fixed at the upper beam when the ultimate deformation is obtained. However, as shown in Figure 5, local compressive buckling appears in Model A. This is because restraint effect of the gusset plate increases the stress of Model A. That is, the load applied to a diagonal brace and a stud is concentrated at web of the H-beam.
The load displacement relation

The relation between load and displacement of each frame is shown in Figure 7. Model A with the gusset plate has a steeper slope than Model B and Model C without the gusset plate. Model A has small displacement under alternative load because Model A has larger constraint by the gusset plate.

CONCLUSION

In this paper, the light gauge steel tubes are employed to form the wall panel with opening. Connection method of the steel tube structure is examined to show sufficient ultimate strength. From the experimental results, following conclusions can be drawn:

(1) When the wall panel with opening using a light gauge steel tube is fabricated, the displacement of the frame becomes smaller due to installation of the gusset plate. Consequently the ultimate strength of the steel frame is improved.

(2) The applied load to the studs and the bracing is concentrated in the H-beam connected with these members, and produces local buckling at the H-beam web. In the case, sufficient reinforcement of the H-beam web is required.

(3) The residual displacement at the flat bar of the frame occurs under tensile load rather than under compressive load. Therefore, appropriate stiffener of the flat bar is required.

REFERENCES

APPLICATIONS OF BUILT-UP SECTIONS IN LIGHTWEIGHT STEEL TRUSSES

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KEYWORDS

Cold-formed, lightweight, thin-walled, roof trusses, long span, built-up section.

ABSTRACT

Lightweight steel structures have been widely used in the construction industry. It is flexible and thus can be designed to cater for different usages. To enhance the application of cold-formed steel structures, innovative configurations are developed. These include increasing the load bearing capacity of the structure and stretching it over a larger span. Built-up section of lipped channels, either back-to-back or boxed-up for the critical elements of a truss is often adopted when it may have practical limitations in increasing the truss depth. This paper presents some of such applications in the construction industry. The built-up section possesses apparent improvement in lateral stiffness. Although these sections act together, they are currently designed individually. Current design codes do not have comprehensive provision for the design of built-up sections reflecting the improvement in design strength. The modified slenderness ratio for built-up sections in the American Iron and Steel Institute (AISI) North American Specification for the Design of Cold-Formed Steel Structural Members is adopted from researches and recommendations for hot-rolled sections. From literature search, field observation and preliminary test conducted, it reveals that the modification rule can be further developed to better reflect the improvement in the slenderness ratio of the built-up sections.
INTRODUCTION

With the improvement in the research and development works done, application of cold-formed steel structures in the construction industry has increased significantly in the recent years. This is because cold-formed steel structures have many advantages over other construction methodology. It is lightweight, flexible, higher strength to weight ratio and is easily formed to the required profile shapes. The most common section profiles are Z- (zee) and C- (channel) sections. These sections are commonly used on roof and wall systems, floor decking, framing of residential, industrial, commercial and agricultural buildings.

Cold-formed steel sections behave differently from hot-rolled steel sections. These thin-walled sections are characterized by local instabilities; hot-rolled sections rarely exhibit local buckling. Thus, the slenderness of a section plays an important role in the design of cold-formed steel structures. Local buckling is expected in most cold-formed sections and often ensures greater economy than a heavier section that does not buckle locally. However, the presence of local buckling of an element does not necessarily mean that its load capacity has been reached as it is strengthened by post-buckling strength (edge and intermediate stiffeners).

Current practice to increase the load bearing capacity of a cold-formed steel structure stretching over a vast span is to use built-up section of lipped channels. This section could be either back-to-back or boxed-up. This section is widely used in local projects namely the Trinity Methodist Church, Kuching (Figure 1), the Curtin University of Technology, Miri (Figure 2), the Sarawak International Medical Centre, SIMC, Kota Samarahan (Figure 3) and a food court in Miri (Figure 4) (Mei et al, [1, 2]).

![Figure 1: Trinity Methodist Church, Kuching](image1)

![Figure 2: Curtin University of Technology, Miri](image2)
In these projects, the primary trusses are required to span up to a maximum clear span of 32m. Under this condition, the major considerations that need to be accounted for besides load carrying capacity include deflection, lateral stability of the structure and constructability.

Instead of increasing the depth of the truss or using bigger sections, built-up section was adopted. This may be due to space limitation in the design and practicality in construction. This approach utilizes two lipped channels placed back-to-back at the top and bottom chords of a truss as shown in Figure 5. These channels are spaced apart to receive the web members which act as lacing on the structure forming a gap. This configuration is adopted in the design as it will improve lateral stiffness making handling especially lifting for installation becomes easier as seen in Figure 6. Besides that, it also simplifies the connection detailing of the web members to the chord members. The use of gusset plates is eliminated. Figure 7 shows the connections of the web members to the top chord.

Figure 3: SIMC, Kota Samarahan
Figure 4: Food Court, Miri

Figure 5: Top and bottom chords built-up sections
Figure 6: Lifting of truss
While it is apparent that the built-up sections with gap have improved lateral stiffness, the design codes do not present comprehensive guidelines for estimating the modification of slenderness ratio. In the absence of provision in design codes for built-up sections using this configuration, practitioners will adopt a conservative design by simply assuming the built-up sections to act as two individual members without modifying the chord slenderness, basing on AS/NZS 4600. Table 1 summarizes requirements of various Codes.

**TABLE 1**

<table>
<thead>
<tr>
<th>Code</th>
<th>Material</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS/NZS 4600:1996 [3]</td>
<td>Cold-formed</td>
<td>• Provision for built-up section without gap only</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• specify max spacing of welds or screws connections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• slenderness of the connected element shall not be less than 0.5 of the</td>
</tr>
<tr>
<td></td>
<td></td>
<td>slenderness of the built-up I section</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• do not propose guidelines for modifying slenderness</td>
</tr>
<tr>
<td>2007 AISI Specification</td>
<td>Cold-formed</td>
<td>• Provision for built-up section without gap</td>
</tr>
<tr>
<td>Section D1.2 [4, 5]</td>
<td></td>
<td>• specify max spacing of welds or screws connections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• slenderness of the connected element shall not be less than 0.5 of</td>
</tr>
<tr>
<td></td>
<td></td>
<td>the slenderness of the built-up I section</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• propose guidelines for modifying slenderness adopted from</td>
</tr>
<tr>
<td></td>
<td></td>
<td>researches on hot rolled section</td>
</tr>
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<td></td>
<td></td>
<td>• provisions for built-up member with and without gap</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• detail provisions for design of batten members</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• guidelines on minimum slenderness of batten members</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• minimum dimensional requirements especially that of minimum thickness</td>
</tr>
<tr>
<td></td>
<td></td>
<td>exceed material thickness of cold-formed steel</td>
</tr>
</tbody>
</table>

Figure 7: Connections
RECENT RESEARCHES OF BUILT-UP SECTIONS

Literature search further reveal that there are researches conducted to further understand the modification of slenderness ratio of built-up sections in hot-rolled and cold-formed steel. Liu et al [7] investigated the slenderness ratio of built-up compression members and tested various configurations of built-up hot rolled channel sections. They concluded that the load bearing capacities calculated using modified slenderness is conservative provided the required connector spacing is met.

Whittle and Ramseyer [8] identify that the estimated load-bearing capacities of axially loaded, cold-formed, built-up sections from AISI Specification is based on research adopted from hot rolled steel. Stone and LaBoube [9], Brueggen and Ramseyer [10], Sukumar et al [11] and Whittle and Ramseyer [2] all concluded that the axial load capacities estimated using modified slenderness ratio in the AISI Specification is conservative.

Young and Chen [12] show that the design strength of built-up closed sections with intermediate stiffeners is consistently higher than that of a single member. The experimental results also show an average improvement of 19% load bearing capacities compared to that of a single member. The research findings are summarized in Table 2.

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Topic</th>
<th>Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sukumar et al.</td>
<td>Buckling behavior of I-shaped built-up members formed with angles</td>
<td>AISI design strengths found slightly conservative or equal to the actual strength. Developed design curve for built-up columns undergoing distortional or local-distortional buckling</td>
</tr>
<tr>
<td>Stone and LaBoube</td>
<td>Behavior of cold-formed steel built-up I-sections (studs) formed with C-channels and screw attachments</td>
<td>AISI Section D1.2 (slenderness modification) is conservative on average for thin members and exceedingly for thick members</td>
</tr>
<tr>
<td>Brueggen and Ramseyer</td>
<td>Buckling of closed and open-built-up sections with channels and intermediately welded attachments</td>
<td>AISI Section D1.2 on average conservative for compact sections but potentially unconservative for members with slender elements</td>
</tr>
<tr>
<td>Whittle and Ramseyer</td>
<td>Behavior of cold-formed steel built-up closed sections formed with intermediately welded C-channels</td>
<td>Use of the modified slenderness ratio was exceedingly conservative. Capacities based on the unmodified slenderness ratio and C4.5 fastener and spacing provisions were consistently conservative</td>
</tr>
<tr>
<td>Young and Chen</td>
<td>Behavior of cold-formed steel built-up closed sections with intermediate stiffeners using self-tapping screws</td>
<td>Use of direct strength method to obtain the buckling stresses is conservative using single section. Experimental results of the built-up section show an average improvement of 19% compared to the design of the member individually</td>
</tr>
</tbody>
</table>
PRELIMINARY EXPERIMENT ON BUILT-UP SECTIONS

As it has been concluded by many researches that the estimation of load bearing capacities of built-up section is conservative, a simple preliminary test was carried out at the Curtin University of Technology, Sarawak Campus to assess the effect on load bearing capacities by varying the spacing of connectors. Two C10016 lipped channel connected back-to-back by self-drilling screws were used to build the built-up section. The nominal dimensions for C10016 lipped channel are web width of 100mm, lip of 20mm and flange width of 50mm with 1.6mm thickness. In the preliminary test, the spacing of the connectors used was greater than that specified in the AS4600 standard. Observation of the experiment shows that buckling occurs in between the connectors. This was because the spacing of the connector was too big and there was no rigidity in the connectors (two screws per stitching). The connected elements behaved as individual column and as expected, there was no strength improvement in the built-up section. Figure 8 shows the mode of buckling of the specimen.

DISCUSSION

The theory adopted in the AISI specification on the modified slenderness ratio was based on research done on hot-rolled steel. Researches show that the performance of a structure is improved using built-up sections (Salem et al [13], Young and Chen [12] and Whittle and Ramseyer [8]). The modified slenderness ratio recommended in AISI was proven to be conservative for cold-formed built-up members. Current research identified the
shortcoming and hence further researches is necessary to propose a design rule change to better estimate the load bearing capacities of built-up cold-formed steel sections.

Furthermore, the Codes do not have comprehensive provisions to cater for various configurations of built-up sections such as back-to-back with gap, battened and laced members as shown in Figure 9. It hence does not give practitioners flexibility to vary the spacing of the lace and battens to suit design requirements and limitations. These provisions are needed as the application of cold-formed steel is becoming innovative and demanding.

Salem et al [13] demonstrate that load bearing capacities of the column depends on the following factors:

- gap in between the chords – the greater the gap, the lower the slenderness of the column.
- connectors spacing – load bearing capacities is inversely proportional to the spacing of connecting member
- batten plate thickness and width - the stiffer the battens, the higher it is the strength of the column.

Salem's research finding confirmed the field observation mentioned earlier. The configuration of the built-up sections will ultimately affect the load bearing capacities estimate of the section. It is evident from Salem et al that the buckling capacity of a built-up section shall be a function of the gap in between the chord, spacing and stiffness of connectors.

CONCLUSION

The demand of the construction industry call for innovation in the design of cold-formed steel structures and built-up sections is one such innovation. There are insufficient guidelines in the current codes and design standards to better estimate the load bearing capacities of built-

Figure 9: Different types of lacing (Shu and Fan [14])
up section s. Research s h o w that the modified slenderness ratio used is conservative and inappropriate. More design rules is needed to provide guidelines for the design of built-up sections such as back-to-back without gap, back-to-back with gap, batten, and laced columns. The load bearing capacities of built-up sections is governed by the gap in between the chords, spacing and stiffness of connectors. Rules for modification of the design of cold-formed steel needs to be developed to be tier reflect the behavior of cold-formed built-up sections.

REFERENCES


FAILURE MODE CONTROL OF DISSIPATIVE TRUSS MOMENT FRAMES

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KEYWORDS

Truss moment frames, special devices, plastic design, kinematic theorem of plastic collapse.

ABSTRACT

In this paper, an innovative design approach for Dissipative Truss Moment Frames (DTMFs) able to guarantee, under seismic forces, the development of a collapse mechanism of global type is presented and applied. In particular, DTMFs consist in a special Truss Moment Frame where the energy dissipation is provided by means of special devices located at the ends of truss girders at the bottom chord level. The proposed design methodology is based on the kinematic theorem of plastic collapse. The method is based on the assumption that both the sections of truss elements and the yield threshold of dissipative devices are known, so that the column sections are the unknowns of the design problem which are obtained by imposing that the equilibrium curve corresponding to the global mechanism has to lie below those corresponding to all the undesired mechanisms within a displacement range compatible with local ductility supply of dissipative elements. The design methodology has been implemented in a computer program and applied for the design of some DTMFs with different number of storeys. Aiming to validate the effectiveness of the proposed approach, pushover analyses have been carried out to assess the collapse mechanism actually developed. The analyses have been repeated for different values of the yield threshold of the dissipative devices aiming to strike a balance between the economical point of view, expressed in terms of structural weight, and the energy capacity dissipation which are related to the yield threshold of the devices.

INTRODUCTION

Truss moment frames are often used in the design of steel structures to withstand gravity loads and lateral forces due to wind or earthquakes. This structural typology has been developed in last years, because of its economy and for the simple details required by the
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truss girders. In addition, the structural system provides architectural benefits which allow its use in a large variety of mid-rise structures. As a consequence, researchers have been encouraged to investigate on the seismic performance of this structural typology. Aiming to improve the dissipation capacity of traditional TMFs, Special Truss Moment Frames have been proposed by Basha and Goel (1995). This typology is able to dissipate the seismic input energy by means of special segments located in the midspan of the truss girders.

A design procedure for failure mode control with reference to a new typology of TMF, referred to as Dissipative Truss Moment Frames, has been developed and applied in this work. DTMFs can be considered as deriving from Special Truss Moment Frames by properly modifying the location of dissipative zones and the topology of the dissipative element. In DTMFs, energy dissipation is provided by means of special devices located at the ends of truss girders at the bottom chord level.

The proposed design methodology, already developed for failure mode control of Moment Resisting Frames (Mazzolani and Piluso, 1997), Eccentrically Braced Frames (Mastrandrea and Piluso, 2009) and Knee Braced Frames (Conti et al., 2006) is herein extended to this new structural typology aiming to the optimization of the seismic performance of the structure. Therefore, the aim of the proposed design methodology is the development of a global collapse mechanism assuring the participation of all the dissipative devices to the dissipation of the earthquake input energy. In fact, even though dissipative devices are located at the ends of each truss girder, common hierarchy criteria do not assure that all of them are involved in the energy dissipation process, due to the development of partial mechanisms which engage the devices of only a limited number of storeys.

The best seismic performance of DTMFs is reached when, at collapse, all the dissipative devices are in the yielded condition whereas all the other members are in the elastic range, i.e. when a global mechanism is developed (Fig. 1). Therefore, the development of a collapse mechanism of global type is a primary design goal in plastic design of seismic–resistant structures.

In order to obtain a collapse mechanism of global type, a sophisticated design procedure for DTMFs has been developed and is briefly presented in this paper. The proposed design procedure is based on the kinematic theorem of plastic collapse and on second order plastic analysis. It starts from the observation that the collapse mechanisms for the considered structural typology subjected to horizontal forces can be considered as belonging to three main typologies (Fig. 1), where the collapse mechanism of global type is a particular case of type-2 mechanism. The control of the failure mode can be performed by means of the analysis of $3n_s$ mechanisms (being $n_s$ the number of storeys). The method starts from the knowledge of truss girder sections and of the resistance of the dissipative devices. The truss elements are designed to resist vertical loads, while the threshold of the dissipative devices is chosen to be less than the axial resistance of the chords. The unknowns of the design problem are the column sections whose plastic modulus has to be defined so that the kinematically admissible multiplier of horizontal forces corresponding to the global mechanism has to be less than those corresponding to the other $3n_s - 1$ kinematically admissible mechanisms. According to the upper bound theorem, the above stated multiplier is the true collapse multiplier, so that the global failure mode is the mechanism actually developed. In particular, it is imposed that the mechanism equilibrium curve ($\alpha-\delta$) corresponding to the global mechanism has to lie below the equilibrium curves corresponding to all the other undesired mechanisms within a displacement range compatible with the local ductility supply of
dissipative elements (Fig. 2). This approach allows to take into account also second order effects (Mazzolani and Piluso, 1997).

EQUILIBRIUM CURVES OF ANALYSED MECHANISMS

Notation:
- \( n_s, n_c, n_b \) number of storeys, number of columns, number of bays respectively;
- \( k, i, j, i_m \) storey index, column index, bay index and mechanism index, respectively;
- \( L_j \) span of the \( j^{th} \) bay;
- \( h_{tr,jk} \) depth of truss girder of the \( j^{th} \) bay of \( k^{th} \) storey, measured from center line of chord;
- \( N_{d,jk} \) yield force of the dissipative devices of \( j^{th} \) bay of \( k^{th} \) storey;
- \( N_{Rd,jk} \) axial resistance of the truss girder chord of \( j^{th} \) bay of \( k^{th} \) storey;
- \( \bar{n}_{jk} = \frac{N_{d,jk}}{N_{Rd,jk}} \) non dimensional axial resistance of the dissipative device of \( j^{th} \) bay of \( k^{th} \) storey;
- \( M_{ik} \) plastic moment, reduced due to interaction with axial internal force, of the \( i^{th} \) column of the \( k^{th} \) storey;
- \( q_{jk} \) uniform vertical load acting on the \( j^{th} \) truss girder of \( k^{th} \) storey or equivalent uniform load equal to \( F_{jk}/(N_{jk}+1)/L_j \) where \( F_{jk} \) is the concentrated force acting on the \( j^{th} \) truss girder of \( k^{th} \) storey and \( N_{jk} \) represents the number of application points of \( F_{jk} \) for the same truss girder (the limitation \( q_{jk} < 4N_{d,jk} : h_{tr,jk} / L_j \) has to be satisfied in order to assure that yielding of chords of truss girder is prevented);
- \( R_{d,jk} \) coefficient related to the participation of the \( j^{th} \) dissipative device of \( k^{th} \) storey to the collapse mechanism equal to \( R_{d,jk} = h_{tr,jk} \) when the device participates to the collapse mechanism otherwise \( R_{d,jk}=0 \);
- \( R_{c,ik} \) coefficient accounting for the participation of \( i^{th} \) column of \( k^{th} \) storey to the collapse mechanism. In particular \( R_{c,ik}=2 \) when the column is yielded at both ends, \( R_{c,ik}=1 \) when only one column end is yielded, and \( R_{c,ik}=0 \) when the column does not participate to the collapse mechanism;
- \( F^T \) is the vector of the design horizontal forces equal to \( \{F_1, ..., F_k, ..., F_{n_s}\} \), where \( F_k \) is the horizontal force applied to the \( k^{th} \) storey;
- \( h^T \) is the vector of storey heights, equal to \( \{h_1, ..., h_k, ..., h_{n_s}\} \), where \( h_k \) is the height of the \( k^{th} \) storey;
- \( s \) is the shape vector of the storey horizontal virtual displacements (\( d\mathbf{u}=s \cdot d\theta \), where \( d\theta \) is the virtual rotation of the plastic hinges of the columns involved in the mechanism);
- \( V^T \) is the vector of storey vertical loads, equal to \( \{V_1, ..., V_k, ..., V_{n_s}\} \), where \( V_k \) is the total load acting at \( k^{th} \) storey given by \( V_k = \sum_{j=1}^{n_b} q_{jk} \cdot L_j \);
- \( M^T_{ek} \) is the vector of plastic moment of columns of \( k^{th} \) storey reduced due to influence of axial force, equal to \( \{M_{c,1k}, ..., M_{c,ik}, ..., M_{c,n,bk}\} \), where \( M_{c,ik} \) is the plastic moment of \( i^{th} \) column of \( k^{th} \) storey;
- \( D \) matrix of order \( n_b \times n_s \) whose elements \( D_{jk} \) are equal to yield resistance of devices \( (D_{jk}=N_{d,jk}) \);
- \( C \) matrix of order \( n_c \times n_s \) whose elements \( C_{ik} \) are equal to the plastic moments of columns \( (C_{ik}=M_{c,ik}) \).
• \( R_d \) matrix of order \( n_b \times n_s \) of \( R_{d,jk} \) coefficients;
• \( R_c \) matrix of order \( n_c \times n_s \) of \( R_{c,jk} \) coefficients;
• \( D_v \) matrix of order \( n_b \times n_s \) of \( D_{v,jk} \) coefficients;
• \( q \) matrix of order \( n_b \times n_s \) whose elements \( q_{jk} \) are equal to the equivalent uniform loads acting on the beams.

Aiming to evaluate the kinematically admissible multiplier of horizontal forces corresponding to the generic mechanism, the internal and the external work (due to the horizontal seismic forces and the equivalent uniform loads acting on the truss girders) can be expressed as:

\[
W_i = \left[ tr(C^T \cdot R_c) + 2tr(D^T \cdot R_d) \right] d\theta 
\]  

\[
W_e = \alpha \cdot F^T \cdot s \cdot d\theta
\]  

where \( d\theta \) is the virtual rotation of the plastic hinges of the columns involved in the mechanism and \( tr \) denotes the trace of matrix.

Using the virtual work principle, the kinematically admissible multiplier is expressed by:

\[
\alpha = \frac{tr(C^T \cdot R_c) + 2tr(D^T \cdot R_d)}{F^T \cdot s}
\]  

The second order work due to vertical loads and the slope of the mechanism equilibrium curve are provided by the following relationships (Mazzolani and Piluso, 1997):

\[
W_v = V^T s \cdot \delta / H_o \cdot d\theta
\]  

\[
\gamma = \left(1 / H_o \right) \cdot \left(V^T s / F^T s \right)
\]  

As a consequence, the mechanism equilibrium curve is defined as:

\[
\alpha_c = \alpha - \gamma \delta
\]  

Figure 1: Type of mechanisms for DTMFs.
In the following, the notations $\alpha^{(g)}$, $\gamma^{(g)}$ and $\alpha^{(1)}_{im}$, $\gamma^{(1)}_{im}$ denote the kinematically admissible multiplier of horizontal forces and the slope of the softening branch of $\alpha-\delta$ curve, corresponding to the global type mechanism and to the $i_{m}$th mechanism of $t$th typology (with $t=1$ to $3$), respectively. In the case of global mechanism (Fig. 1), as all the stories participate to the collapse mechanism, the shape vector of horizontal displacements is given by $s^{(g)} = h$. In addition, all the dissipative devices are involved in the mechanism. Therefore, the kinematically admissible multiplier and the slope $\gamma^{(g)}$ are given by:

$$\alpha^{(g)} = \frac{[M_{1}^{T} \cdot I + 2tr(D^{T} \cdot R)^{(g)}]}{F^{T} \cdot s^{(g)}}$$  \hspace{1cm} (7)

$$\gamma^{(g)} = \frac{1}{h_{ns}} \cdot \frac{V^{T} s^{(g)}}{F^{T} s^{(g)}}$$  \hspace{1cm} (8)

where $I$ denotes the unit vector of order $n_c$.

In the case of $i_{m}$th mechanism of type-1, the shape vector of horizontal displacements is equal to $s^{(1)}_{im} = \{h_{1}, h_{2}, \ldots, h_{im}, h_{im}, h_{im}\}^{T}$ where the first element equal to $h_{im}$ corresponds to the $i_{m}$th component. The kinematically admissible multiplier corresponding to the $i_{m}$th mechanism of type 1 is given by:

$$\alpha^{(1)}_{im} = \frac{[M_{1}^{T} \cdot I + 2tr(D^{T} \cdot R)^{(1)}]}{F^{T} \cdot s^{(1)}_{im}}$$  \hspace{1cm} (9)

In addition, only the first $i_{m}$ stories participate to the collapse mechanism, so that $H_{0} = h_{im}$. As a consequence, the slope of the mechanism equilibrium curve is still computed through Equation (5), but assuming $s = s^{(1)}_{im}$ and $H_{0} = h_{im}$. The expressions of kinematically admissible multipliers $\alpha^{(2)}_{im}$ and $\alpha^{(3)}_{im}$ and of the slopes $\gamma^{(2)}_{im}$ and $\gamma^{(3)}_{im}$ of the mechanism equilibrium curves corresponding to type 2 and type 3 mechanisms, respectively, can be obtained by means of the same procedure.

**DESIGN CONDITIONS FOR FAILURE MODE CONTROL**

In order to assure the development of the desired global mechanism, the column sections have to be designed according to the upper bound theorem. In particular, accounting for
second order effects, it is required that the mechanism equilibrium curve corresponding to the global mechanism has to lie below those corresponding to all the other undesired partial mechanisms. However, the fulfillment of this requirement is needed only up to a selected ultimate displacement $\delta_u$ compatible with the plastic deformation capacity of the structural elements. Therefore, the proposed design procedure provides also the control of local ductility supply of structural elements. The resulting design conditions are expressed by the following relationships:

$$\alpha^{(g)} - \gamma^{(g)} \cdot \delta_u \leq \alpha^{(i)} - \gamma^{(i)} \cdot \delta_u$$

with $i_m = 1, 2, n_s$ and $t = 1, 2, 3$ (10)

Substituting the values of $\alpha^{(g)}$, $\gamma^{(g)}$ and $\alpha^{(i)}$, $\gamma^{(i)}$ in equations (10), the design conditions to be satisfied to avoid the undesired mechanisms are obtained, where the unknowns are the plastic moments of columns, reduced due to the axial loads, which can be obtained by means of a numerical algorithm (Mazzolani and Piluso, 1997).

Therefore, the application of the presented design procedure allows the definition of column plastic moments, reduced due to the presence of the axial force. The following step is the evaluation of the column axial forces at collapse state. In particular, axial forces in the columns can be easily determined as the sum of the shears forces transmitted by the truss girders when a collapse mechanism of global type is completely developed.

**APPLICATIONS**

In order to evaluate the accuracy of the proposed design methodology, an adequate number of DTMFs having different numbers of storeys (4÷12) have been dimensioned. In particular, for sake of shortness, only the results of a six-storey DTMF will be discussed in this paper.

The building plan configuration is symmetrical with reference to the two orthogonal directions; as a consequence, neglecting the accidental torsion due to the variability of location of live loads, the distribution of the seismic horizontal forces among the vertical seismic resistant schemes is immediately obtained (Fig. 3). S275 steel has been adopted. For each floor the dead load ($G_k$) is equal to 3 kN/m$^2$ and the live load ($Q_k$) is equal to 2 kN/m$^2$. The seismic horizontal forces have been determined according to Eurocode 8, assuming a peak ground acceleration equal to 0.35g, a seismic response amplification factor equal to 2.5, soil type A and behaviour factor $q$ equal to 5.85, i.e. the $q$ factor of MRFs (CEN, 2005).

The design displacement $\delta_u$ has been determined assuming that the maximum plastic interstorey drift is equal to 0.04 rad; as a consequence, $\delta_u = 0.04 \times 2700 = 108$ cm.

In Table 1 the member sections of truss girders and the column sections resulting from the application of the proposed design procedure are given. Regarding the elements of the truss girder, the spacing between UPN profiles (i.e. the thickness of the gusset plate) is equal to 15 mm. In the same table, the plastic resistance of the dissipative devices are also pointed out with reference to the limit values explained in the following. The design methodology has been applied for different values of $n$ which represents the ratio between the plastic resistance of the dissipative devices and the axial resistance of the chords. For the examined case, the range of possible values assumed by $n$ varies from 0.4 to 0.7.
The lower limit has been established with the aim to prevent the attainment of the threshold resistance of the dissipative devices when the structure is subjected to the vertical loads combination. In fact, it can be easily verified that for a value of $\bar{n}$ less than 0.4 the vertical load combination (i.e. $1.3 G_k + 1.5 Q_k$) gives rise to an axial load action in the dissipative devices greater than the threshold resistance.

The upper limit of $\bar{n}$ value is due to the buckling phenomenon occurring in the upper chord of the truss girders. In fact, considering the vertical load acting in the seismic load combination (i.e. $G_k + \Psi Q_k$) and increasing the seismic horizontal forces, it is easy to verify that, for a value of $\bar{n}$ greater than 0.7 the axial force in the upper chord at the ends of the truss girders exceeds its buckling resistance.

**TABLE 1. RESULTS FOR THE DESIGNED 6TH STOREY DTMF BOTH IN THE CASE OF $\bar{n} = 0.4$ AND IN THE CASE OF $\bar{n} = 0.7$.**

<table>
<thead>
<tr>
<th>Storey</th>
<th>Chords</th>
<th>Diagonals</th>
<th>External columns</th>
<th>Internal columns</th>
<th>External columns</th>
<th>Internal columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2UNP 280</td>
<td>2UNP 180</td>
<td>SHS 65x1.7</td>
<td>SHS 76x2.0</td>
<td>SHS 75x2.0</td>
<td>SHS 81x2.2</td>
</tr>
<tr>
<td>2</td>
<td>2UNP 280</td>
<td>2UNP 180</td>
<td>SHS 61x1.7</td>
<td>SHS 72x1.9</td>
<td>SHS 69x1.9</td>
<td>SHS 75x2.0</td>
</tr>
<tr>
<td>3</td>
<td>2UNP 280</td>
<td>2UNP 180</td>
<td>SHS 61x1.6</td>
<td>SHS 70x1.9</td>
<td>SHS 69x1.9</td>
<td>SHS 75x2.0</td>
</tr>
<tr>
<td>4</td>
<td>2UNP 280</td>
<td>2UNP 180</td>
<td>SHS 57x1.6</td>
<td>SHS 67x1.8</td>
<td>SHS 68x1.8</td>
<td>SHS 73x2.0</td>
</tr>
<tr>
<td>5</td>
<td>2UNP 280</td>
<td>2UNP 180</td>
<td>SHS 53x1.4</td>
<td>SHS 61x1.6</td>
<td>SHS 63x1.7</td>
<td>SHS 69x1.8</td>
</tr>
<tr>
<td>6</td>
<td>2UNP 280</td>
<td>2UNP 180</td>
<td>SHS 42x1.2</td>
<td>SHS 49x1.3</td>
<td>SHS 52x1.4</td>
<td>SHS 57x1.5</td>
</tr>
</tbody>
</table>

|               | Total weight = 81.38 tons |               | Total weight = 90.10 tons |
| Device $\bar{n} = 0.4$ | $N_d=700.4$ kN | Device $\bar{n} = 0.7$ | $N_d=1226$ kN |

This phenomenon precedes the attainment of the threshold resistance of all the dissipative devices, so that the desired mechanism is not achieved. The reason of such behaviour is due to the fact that the axial force in the upper chord under compression continues to increase after the attainment of the threshold resistance of the dissipative device. For this reason the maximum allowable value of $\bar{n}$ is always less than one, but the exact maximum value can be set only by making a push over analysis. Clearly, the range of $\bar{n}$ depends both on the geometrical configuration and on the magnitude of gravitational loads.

In order to validate the proposed design procedure, the seismic response of the designed structure has been investigated by means of push-over analyses carried out using SAP 2000 program (CSI, 2007) which allows to evaluate the actual pattern of yielding. The analyses have been led under displacement control, taking into account both geometrical and mechanical non-linearities. In addition, out-of-plane stability checks of compressed members have been performed at each step of the non-linear analysis.

The structure has been modelled by means of both non-linear and elastic elements. In particular, columns are modelled using beam-column elements with the possibility of developing plastic hinges at their ends. In addition, the truss girders have been modelled using truss elements having the possibility of yielding under axial forces. Finally, the dissipative devices are also modelled by means of non-linear truss elements whose yield axial load represents the threshold resistance of the device.
In Figures 4 and 5 the push over curves and the distribution of plastic hinges and yielded elements corresponding to a top sway lateral displacement equal to the design one are depicted. In particular, Fig. 5 shows a comparison between the horizontal force multiplier versus top sway displacement curve obtained by means of push over static analysis and the global mechanism equilibrium curve providing a first confirmation about the accuracy of the proposed design methodology.

Figure 3: The analyzed structure (dimensions in cm).

Figure 4: Distribution of plastic hinges for 6 storey DTMF for displacement greater than \( d = 108 \text{cm} \) for \( \bar{n} = 0.4 \) and \( \bar{n} = 0.7 \) respectively.

Figure 5: Push over curves for 6 storey DTMF both for DTMF with \( \bar{n} = 0.4 \) and \( \bar{n} = 0.7 \).
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Figure 6: Constructional steel weights and interstorey drift values of the designed DTMFs versus $n$

The second and most important validation of the fulfillment of the design goal is represented by yielded elements and plastic hinges developed when the design ultimate displacement is achieved. In fact, Fig. 4 shows the pattern of yielding corresponding to $\delta_u = 108$ cm, testifying the development of a global type mechanism. These results have been confirmed for all the structures analysed, therefore the fulfilment of the design goal is achieved.

Regarding the fulfilment of the serviceability requirements, i.e. the need to limit the interstorey drift demands under seismic events having a return period comparable with the service life of the structure, the designed DTMFs show, for the examined case, an interstorey drift ratio (equal to 0.0024 in case of DTMF with $n = 0.4$ and 0.002177 in case of DTMF with $n = 0.7$) lower than the limit value provided by Eurocode provisions equal to 0.005 (CEN, 2005).

Regarding the influence of the threshold resistance of the dissipative devices, it is important to know that the application of the global mechanism design criterion has allowed for all the analysed cases the design of structures able to develop a collapse mechanism of global type. In addition, Fig. 6 shows the results in terms of structural weight and in terms of maximum interstorey drift angle versus the ratio $\bar{n}$. It is evident that the structural weight increases as far as the $\bar{n}$ ratio increases, on the contrary, the interstorey drift angle decreases as far as the $\bar{n}$ ratio increases. However, all the designed DTMFs fulfill the interstorey drift limitation provided by Eurocode 8 (equal to 0.005). In Fig. 6, also cases with $\bar{n}$ greater than 0.7 have been reported, but, such values of $\bar{n}$ do not allow the development of a mechanism of global type due to the buckling of the upper chord; there the corresponding line is hatched. However, it is important to underline that the maximum $\bar{n}$ value which can be adopted without the occurrence of upper chord buckling can be predicted by means of capacity design criteria.

Finally, the application of the proposed design approach leads to the fulfillment of the design goal with the involvement of all the dissipative zones, i.e. all the special dissipative devices, preventing the yielding of columns and of truss members. It is important to underline that the proposed design procedure is able to guarantee the development of a collapse mechanism of global type and, the fulfillment of the requirements concerning the serviceability limit state.
CONCLUSIONS

A design methodology, already suggested for MRFs (Mazzolani and Piluso, 1997), EBFs (Mastrandrea and Piluso, 2009) and for KBFs (Conti et.al, 2006), has been implemented and applied in this work with reference DTMFs. It constitutes a rigorous approach, based on the kinematic theorem of plastic collapse, which allows the control of failure mode for any type of seismic resistant structure. In the case of DTMFs, static push over analyses have pointed out the accuracy of the design methodology, because all the dissipative zones, i.e. all the special dissipative devices, are involved in the collapse mechanism, leading to a global failure mode. Finally, in order to definitively validate the design procedure, dynamic inelastic analyses for increasing values a ground motion intensity measure (Incremental dynamic analysis) will be carried out, with reference to different structural schemes, in the forthcoming developments of this research activity.

REFERENCES

SECOND-ORDER ANALYSIS AND BUCKLING BASED DESIGN OF TRANSMISSION TOWER WITHOUT EFFECTIVE LENGTH ASSUMPTIONS

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KEYWORDS
Second-order analysis, buckling, angles

ABSTRACT
Second-order analysis has been widely used for structures with members composed of symmetrical sections but its application to members of asymmetric sections is rather limited. In a recent project by the authors on design of a transmission tower in Myanmar which uses starred angles as main members and single angles as bracing members, the method is used and compared with the linear analysis. The initial imperfections recommended by design code are used and it was found that the method reduces significantly the design time required. Also, the result was found to be more rational than the linear analysis such as the complexity in assuming the buckling length of the legs which are continuous but intermittently tied by bracings and also under axial forces varying along the lengths. The proposed second-order analysis and design method eliminates the uncertainty of assumption for effective length which may lead to significant error in many practical structures.

INTRODUCTION
Transmission line towers are widely used in different parts of the world. The structures are commonly made of angle and double angle sections in the form of a star shape as shown in Figure 1 and their design involves significant factors of buckling. The monosymmetry or asymmetry of an angle section complicates its design procedure when compared with design of members of doubly symmetric sections. Inaccurate estimation of the capacity of angle members would result in inconsistent factors of safety among its members and finally to unsafe and uneconomical design with some members over-designed and others under-designed. Several steel codes such as LRFD [1], BS5950 [2], CoPHK [3] and Eurocode 3 [4]
can be applied to the design of the single angle members with different end conditions. Many codes provide simplified formulae to consider end moments due to the eccentric connections. These formulae modify the slenderness ratio of angle members in order to account for the effect of end eccentricity and end fixity. The codes like LFRD [1] and Eurocode 3 [4] do not permit the use of the simplified method for angle members with single bolted connections, hence the interaction equations must be used to consider the combined effect due to axial force and bending moments arise from end fixities and eccentricity of the angle members. These code methods use the first-order linear analysis which ignores the non-linear effects such as the initial imperfection, $P-\Delta$ and $P-\delta$ effects in analysis process and use some factors to amplify the linear moments and to reduce the member capacity, and thus their use become inappropriate in some scenarios including the case when loads are along the member length affecting the buckling strength but not the effective length factor. It must be emphasized that imperfections are essential for the validity of the proposed second-order analysis otherwise the code results cannot be reproduced by the second-order analysis for some benchmark examples (Machowski and Tylek [5]). As an alternative, the second-order analysis and design method (i.e. the direct analysis) for design of transmission tower is proposed in this paper and the important non-linear effects are included. As the method considers directly the true behavior, the complicated and sometimes irrational design process is eliminated.

![Figure 1: Cross-sections of single angle (left) and starred angles (right)](image)

**CONVENTIONAL DESIGN METHOD**

The conventional design method is based on the first-order linear theory with use of effective length to consider buckling effects. The member forces and moments are first calculated from the linear analysis and the design formulae in codes are then employed to check the buckling strength of each member individually by applying the effective length factor (i.e. the $K$-factor). The structure would be considered safe if the resistance of every member is larger than its internal forces. The reliability of this design method depends highly on the accuracy of the effective length factor assumed. However, the ideal and simple end conditions like pin and rigid ends are unrealistic in practical structures and therefore the determination of the effective length depends heavily on the experience of the engineer. Discrepancies and controversy among designers will appear when different assumed effective length factors are used. The first-order linear analysis ignores the non-linear effects such as $P-\Delta$ and $P-\delta$ effects...
in analysis process, and uses factors to amplify the linear moments and forces and to reduce the resistance of the member. For example, LRFD [1] uses the factor $B_1$ which is employed to include the moment due to the $P-\delta$ effect, and $B_2$ which is used to consider the moment due to the $P-\Delta$ effect. BS5950 [2] and CoPHK [3] use the factor of $\lambda_{cr} / (93.9\varepsilon)$, in which $\lambda_{cr}$ is the elastic critical load factor, to amplify the moment so as to consider the sway effects. These non-linear effects change critically the ultimate load when a member is slender or when the deflection or end rotation is large that the non-linear effects are significant. As a result, using the first-order analysis could be non-conservative if these effects are not carefully considered.

Monosymmetric or asymmetric cross sectional property of single angle members complicates the design when compared with the design of members of doubly symmetric cross-sections. In many design codes, the effects due to end fixity and end eccentricity are implicitly included by modifying the slenderness ratio and thus the compressive strength and the effective length factor under different conditions are included in the slenderness modification equations. Some design codes also provide interaction equations to consider the interactive effect of axial force and bending moments. These interaction equations are mostly derived on the basis of doubly symmetric sections, and the moment ratios in these formulae are evaluated for the case of maximum stresses about each principal axis. For doubly symmetric sections, the most critical locations for maximum moments about the two principal axes occur at one of the four corners. However, for monosymmetric and asymmetric angle sections, the points having maximum bending stress about the two principal u-u and v-v axes may not be coincided and it depends on the load location. Hence, the interaction equations may underestimate considerably the loading resistance of angle sections. In CoPHK [3], a simplified method is provided for calculating the effective length of angle sections in triangulated structures. Buckling about principal minor axis and axes parallel to the legs should be considered. For angle sections connected by two or more bolts, the effective slenderness ratio should be calculated from larger of the actual slenderness ratio and the following:

\[
\lambda = 0.35 + 0.7 \lambda_{cr} / (93.9\varepsilon) \\
\lambda = 0.5 + 0.7 \lambda_{cr} / (93.9\varepsilon) \\
\lambda = 0.5 + 0.7 \lambda_{cr} / (93.9\varepsilon)
\]

The compressive resistance is then calculated from the effective slenderness. Under most circumstances, the compressive strength is controlled by buckling about principal minor axis except when the strut is very stocky.

**SECOND-ORDER ANALYSIS AND DESIGN METHOD**

In addition to the above deficiency of not considering member buckling effectively, following analysis design aspects are observed in the conventional or traditional approach.
1. Redundant or secondary members are normally not included in the analysis model with the assumption that they do not attract any force but to be used to get effective length.
2. Guideline to predict the redundant member force such as 2.5% of connected member force or $L/r$ based distribution are available but use of them requires lot of judgment and experience.
3. Linear analysis cannot capture the influence of steel angle yield stress (S405 or S245).
Due to the drawbacks of the first-order linear analysis method mentioned above, a theory based on the second-order analysis and design method using NIDA [6] is proposed in this paper and brought to predict the tower behavior and member forces accurately without any of the above indicated problems. Extensive researches on development and application of the second-order analysis and design method have been conducted for doubly symmetric sections (see Chan and Zhou [7], Chen [8], Chan and Chui [9] and Liu and Li [10]). In the method, the change of geometry and member stiffness due to the presence of force and load are considered in the analysis part that the effective length is not required to be assumed in the member design part. Thus, the individual member check is replaced by the section capacity check as,

\[
\frac{P}{A_y p_y} + \frac{M_y + P(\delta_y + \Delta_y)}{M_{cy}} + \frac{M_z + P(\delta_z + \Delta_z)}{M_{cz}} = \phi \leq 1
\]  

in which \(p_y\) is the design strength of the member, \(P\) is the axial force in the member, \(A_y\) is the cross-sectional area, \(M_y\) and \(M_z\) are the first-order moments about the y- and z- axes, \(M_{cy}\) and \(M_{cz}\) are the moment capacities about the y- and z- axes. \(P(\delta_y + \Delta_y)\) and \(P(\delta_z + \Delta_z)\) are the second-order moments about the y- and z- axes of which the consideration allows us to include automatically the bending effect due to axial force and second-order deflections. Therefore, the use of effective length is not required as the P-\(\Delta\) and P-\(\delta\) effects have been included in the Eqn. 4 for section capacity check. Moreover, the initial imperfection is also included in analysis so that the Perry-Robertson formula for imperfect columns can be directly applied in this integrated analysis and design procedure. In the analysis procedure, a small load increment of, say 5 to 10% of the expected design load, is applied to the structure by an incremental-interactive procedure and the design load is attained when the section capacity factor \(\phi\) in the Eqn. 4 is equal to unity. The initially curved stability function element is used with initial imperfection at mid-span equal to \(L/300\) in which \(L\) is the member length. This value follows Table 6.1 of the CoPHK (2005).

**DESIGN OF TRANSMISSION TOWER**

The tower of type RCS-90 SUSPENSION TOWER as shown in Figure 2 is selected here for demonstration [11]. The height of the tower is 110 m and grade S405 and S245 steel is used in the design. The second-order effect due to axial force is considered by the use of curved and initially imperfect stability function element with initial curvature set to the code requirement as one-300\(^{th}\) of the length between nodes. The lateral-torsional buckling can be considered by the use of a reduced elastic modulus but as the moment is small here and the approach is not quite applicable here because of the dominance of axial force, this effect is ignored.

Different load cases are assumed in the analysis and they include wind at different angles, self-weight and load from hanging conductors. The case for broken cable is also considered as it introduces a large twist to the structure. The main legs are assumed continuous whereas the bracing members are assumed pinned at their ends. All support conditions are pinned to the foundation. A 250 year return wind speed is considered in the design. For starred angle sections, section properties about geometric axes are used despite the principal axes are diagonal to the geometric axes. Figure 3 compares the compressive strength of pin-ended starred angle struts comprise of two 200×200×20 (S405) sections bending about geometric axis and principal minor axis calculated from the Perry-Robertson formula. It can be seen that
the compressive strength for bending about principal minor axis is considerably smaller than that for bending about geometric x-x axis. Despite this discrepancy, the bending about geometric x-x axis is assumed because the gusset plates at both ends and the battens along the member to some extent restrain the starred angles from bending about the principal minor axis. If the section properties about the principal axes are used, the compression resistance calculated will be too conservative and also the slightly larger resistance makes the buckling curve closely to the curve in the CoPHK [3] plotted in Figure 3 below.

Figure 2: Model of RCS-90 SUSPENSION TOWER before and after deformation

Figure 3: Compressive strength vs slenderness ratio of 200×200×20 starred angles (S405)
For single angle sections, bending about principal minor axis is assumed and thus section properties about principal axes are used. Figure 4 compares the compressive strength of a 200×200×20 (S275) single angle calculated from the design formulae given in CoPHK [3] assuming the angles are connected with more than two bolts at each end and from Perry-Robertson formula using assuming the end support is pinned. It can be seen that except when the equivalent slenderness about its principal minor axis is smaller than approximately 0.60, the use of Perry-Robertson formula always give a close and conservative result. However, when single angle member with equivalent slenderness smaller than 0.60 is used, a term for eccentric moment should be added. This paper proposes a simple technique of limiting the value of initial imperfection to the one at equivalent slenderness equal to 0.6 based on which the buckling curve is plotted in Figure 4. A total of 19 combined load cases have been considered including ultimate limit states and serviceability limit states and the member size is selected to be adequately safe to resist loads from these load cases. The result produced varies with the linear analysis result which very much depends on the assumed effective for the members.

![Compressive strength vs slenderness ratio of a 200×200×20 single angle (S275)](image)

The following example compares the conventional design method in accordance with CoPHK [3] with second-order analysis method on the most critical member of section 200×200×20 (S405) starred angles in the tower.

**Section Properties:**

\[ A = 15200 \text{mm}^2, \quad I_y = I_z = 1.226 \times 10^8 \text{mm}^4, \quad Z_y = Z_z = 5.892 \times 10^5 \text{mm}^3, \]

\[ S_y = S_z = 9.936 \times 10^5 \text{mm}^3 \]

**Linear Analysis Output:**
\[ F_c = 3832 \text{kN}, \ M_{y1} = 23.5 \text{kNm}, \ M_{y2} = 2.8 \text{kNm}, \ M_{z1} = 61.9 \text{kNm}, \ M_{z2} = -6.3 \text{kNm} \]

Cross-section Capacity Check:
\[
\frac{F_c}{A_g p_y} + \frac{M_y}{M_{cy}} + \frac{M_z}{M_{cz}} = \frac{3832 \times 10^3}{15200 \times 395} + \frac{23.5 \times 10^6}{9.936 \times 10^5 \times 395} + \frac{61.9 \times 10^6}{9.936 \times 10^5 \times 395} = 0.856 \leq 1 \ (\text{OK})
\]

Member Buckling Resistance:
\[
\frac{F_c}{P_c} + \frac{m_y M_y}{M_{cy}} + \frac{m_z M_z}{M_{cz}} \leq 1
\]
Assuming the effective length is equal to the member length, i.e. \( L_E = 2.02 \text{m} \)
\[ P_c = 376.3 \text{N/mm}^2 \]
\[ m_y = 0.64 \]
\[ m_z = 0.58 \]

Moment Amplification Factor = \[
\frac{1}{1 - \frac{F_c^2 L_E^2}{\pi^2 EI}} = 1.07
\]
\[
\frac{3832 \times 10^3}{15200 \times 376.3} + \frac{0.64 \times 1.07 \times 23.5 \times 10^6}{589200 \times 395} + \frac{0.58 \times 1.07 \times 61.9 \times 10^6}{589200 \times 395} = 0.904 \leq 1 \ (\text{OK})
\]

However, from second-order analysis by NIDA [6], the section capacity factor calculated from Eqn. 4 is 0.952 which is greater than that found from linear analysis with the moment amplified to account for the second-order effects. In other words, the assumption of taking the member length as the effective length will lead to an overestimate of the member buckling resistance.

**CONCLUSIONS**

A second-order analysis and buckling based design method without assumption of effective length is proposed in this paper and is used for designing the size of members of a 110 m high transmission tower to be built in Myanmar. For starred angles, section properties about geometric axes are used because of the presence of gusset plates and battens that provide certain degree of rotational restraints. In contrast, for single angle, section properties about principal axes are used because under most circumstances the failure of single angle under compression is bending about minor axis. 19 load cases were analyzed in which both ultimate limit states and serviceability limit states are taken into account and the member size is then selected according to the critical load cases. The most critical member is selected to demonstrate the difference between the conventional design method and the proposed second-order analysis method. It is found that the result produced varies with the linear analysis result which very much depends on the assumed effective for the members.
ACKNOWLEDGEMENT

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REFERENCES

STRUCTURAL PERFORMANCE OF STEEL BUILDINGS WITH SEMI-RIGID CONNECTIONS

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ABSTRACT

In the present analytical work, a parametric study is performed narrowing the range of building configurations for which semi-rigid connections (SRCs) are appropriate in high seismic regions. A comparison is made between two types of buildings, namely a traditional moment resisting frame and a SRC building employing end plate connections. Both buildings are expected to maintain a life-safety limit state (severe damage without collapse). A detailed comparison is made between the static and dynamic linear and nonlinear response of the sample buildings. The results of the static nonlinear (pushover) analyses are presented hereafter. It is shown that buildings with SRCs yield both an economical and safe design. The forced formation of plastic hinges at the connections reduces the required column size in the traditional strong column-weak beam design theory.

INTRODUCTION

The present research builds on and advances previous research that has sought to qualify semi-rigid connections as having a stable hysteretic behaviour and sufficient ductility to be used in areas of high seismicity. Comprehensive experimental and numerical work [1, 2, 3] proposed that semi-rigid connections can safely and effectively be used in high seismic regions. There is a reduction in frame stiffness due to the implementation of semi-rigid joints and sufficient evidence that over-strength factors for columns should be placed on connection capacity as opposed to beam capacity. In addition to physical work on connections practitioners such as [4] have offered suggestions that a range of buildings exist in which semi-rigid connections could prove beneficial due to both their economics and their ability to influence the performance of a building, by controlling the distribution of moments throughout a frame. It is believed, however, that a serious economic study needs to ensue with a unifying resource of design methodology and practice to truly take into account what he believes to be the synergistic effects by accounting for the connection behaviour in frame design. Nevertheless, it is thought that the synergistic effects can only improve in high seismic areas and that there also must be work done in code modification to truly reap the benefits of semi-rigid connections. Still large gaps remain to be filled for designers to successfully begin using semi-rigid connections on a production level. Additionally, poor design approaches such as flexible-moment connections must be replaced by modern methodologies of design that account for changes in knowledge and computation power.
To render the conclusions of this research the most general and applicable, the sample structures were designed in compliance with the most common building codes and design provisions, with the help of common design guides. In this case IBC2000 [5] was used for all loading combinations, with ASCE 7-02 [6] and the AISC seismic design provision [7]. Though [5] is not the applicable building code of California, the location of the building was chosen to be in Los Angeles where a peak ground acceleration of 0.4g was imposed by code. It should be noted that there are very strong correlations between the California Building Code and Unified Building Code which have been merged into the creation of the International Building Code. Therefore, it has been deemed appropriate to model and analyze these buildings with IBC at the proposed location. To draw conclusions of the code adequacy and conservatism, three sets of frames where designed that enforce varying levels of code rigidity. The details of the frames are discussed further in subsequent sections.

SAMPLE FRAME DESIGN

To make the sample frames the most economical, it was found that the beam sizing to resist the gravity loading needed to closely match the beam sizing required for the applied lateral loads in the frame design. To effectively do this two design approaches were applied. The first is that frame beams should be used as collector girders for all fill beams between column lines. The second framing decision is to use longer spans between column ns. To keep with the realistic modeling of actual buildings, inter-storey heights were chosen as 14 feet for the first level and 12 feet for each level above that. The largest first storey will slightly affect the distribution of the seismic forces vertically in the frame. Wind loading on the building is heavily influenced by the inter-storey heights due to a greater surface area. But, wind loading does not control the design of building s less than 10 stories in this region. Though structurally the change in cost due to inter-storey heights may not be great, the designer cannot overlook the change in cost of interior and exterior finishes that would result.

For all frames and buildings serviceability requirements including drift and deflection limits were satisfied in accordance with [5]. All frames and buildings are also capable of resisting all applied loads both vertical and horizontal, though not all frames member sizes conform to all the seismic design provisions set forth in order to test the true behaviour of semi-rigid frames.

SAMPLE FRAME COMPARISONS

In total nine frames were designed. In addition to varying the level of code compliance these nine frames also varied the rotational strength of the connections between a fully fixed connection, with 100% of the beam capacity, to a semi-rigid connection with only 50% of the beam plastic moment capacity. To begin, a typical moment resistant frame was designed. This frame like all nine was designed to loadings imposed by [5] for a fictitious building location in Los Angeles, California. In accordance with the traditional moment frame approach, all connections were designed to transfer the beams full rotational moment capacity to the column. To do this, full penetration welds are used on the top and bottom flanges, with fillet welds on the each side of the web of the beam. In addition web doublers and continuing plates are added to the column to assist in the distribution of forces and to approach a fully fixed connection, one with no differential rotation between column and beam. Beams were initially sized to resist the gravity loading under the assumption of a fixed/fixed support restraint. Columns were then sized using the over-strength requirements of the seismic design provisions based on the beam capacity. This fully restrained frame with type I connection according to [7] serves as the base line building which all semi-rigid building responses are compared. The building will here on be referred to as “100-Fixed” (see Table 1).

The next set of frames designed uses partially restrained connections at all columns and beam joints. For these frames a range of column fixities were employed between 80% and 50% of the beams plastic capacity. For the upper bound, 80% was chosen based on the definition of a fully restrained connection of 90% of the beam capacity. A total of three frames were used in this next set, this set enforced the code and specification requirement strictly. These included the frame moment connection (FMC) methodology of using pin/pin analysis in the design of the beams, and enforcing
the seismic design provisions requirement with a maximum 20% reduction in beam capacity for column design. The resultant frames were titled “80-Code”, “65-Code” and “50-Code” with 80%, 65% and 50% connection strengths respectively (see Table 2).

Two additional frames were designed, similar to those above, except with a relaxation of the seismic design provisions maximum 20% reduction in beam capacity. These frames however, still maintained the FMC beam design methodology. The relaxation of the provisions results in two new frames being created with a 65% and a 50% connection rotational strength. Each of these frames has smaller column sizes than the frames that enforce the provisions maximum reduction, because column size is specified to be stronger than the beams capacity to force ideal plastic hinge location. An additional 80% frame is not created because this frame would be no different than the 80-Code frame designed above since the seismic design provisions already allow for the use of a beam with 80% of the original section capacity. As a result, the two new frames will be titled “65-Code2” and “50-Code2” (see Table 3) with respect to the rotation strength of the connections employed.

The final set of frames designed, are completely non-conforming frames. They neither apply the seismic design provisions requirements for column over-strength in the strong-column weak-beam theory, nor do they use the FMC pin/pin beam design to resist the gravity loading. Instead the beams are sized to resist the gravity loading by modelling the beams with rotational springs on each end, with rotational stiffness determined from the component based approach of each connection as discussed above. The resulting frames created are, “80-NC”, “65-NC” and “50-NC”, where NC stands for non-conforming (see Table 4).

A total of nine frames are designed and modelled and will be used for the analysis. The sample comprises one rigid frame and eight semi-rigid or partially restrained frames. The modelling of the connections is the most critical aspect of this research, for improper modelling would invalidate the results of the behaviour and performance of the buildings. Two types of connections were used in the frame design. First is a base line frame using the traditional fixed end or fully restrained connections.
The second is the partially restrained connection, which may be implemented using many different connections methods.

**TABLE 4**

<table>
<thead>
<tr>
<th>Frame</th>
<th>Connection Restraint (%)</th>
<th>Inside Column (W14x--)</th>
<th>Outside Column (W14x--)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80-NC</td>
<td>80</td>
<td>109, 99, 74</td>
<td>74, 68, 53</td>
</tr>
<tr>
<td>65-NC</td>
<td>65</td>
<td>99, 90, 68</td>
<td>68, 61, 48</td>
</tr>
<tr>
<td>50-NC</td>
<td>50</td>
<td>90, 82, 61</td>
<td>61, 53, 43</td>
</tr>
</tbody>
</table>

**PERFORMANCE ASSESSMENT**

**Global Deformations and Forces**

To begin comparison of the frames at the global level two values of interest are plotted. These are the roof displacement of the frame at yield and the roof displacement of the frame at ultimate. First to understand these results it is necessary to define the criteria used to find the yield and ultimate points along the push-over curve (Figure 1).

The first curve, on the top, has a well defined yield point at 5.7 inches of displacement, but indicates no clear ultimate value. The curve on the bottom indicates a clear ultimate displacement of 42.2 inches but multiple yielding points. It was decided that the yield point would be defined as the first yield in the frame, this does not necessarily correspond to the first yield of a member of the frame and in fact generally occurs several displacement steps later. The ultimate point on the graph was chosen based on location of complete plastic hinging of the columns in the frame. The global deformation values can be compared using the definition of the yield and ultimate of the frames as specified above. First a comparison of the yield verse roof displacement is made between the nine frames. Figure 2 shows the displacement at first yield of the nine frames. Note that the figure has ten data points instead of nine. The extra data point is a result of frame “80-Code” being used in both the code based and relaxed code set of frames. Upon inspection of the graph below an interesting result is noted between the 100-Fixed frame and the first set of code based semi-rigid frames shaded in purple. The result is that the code based semi-rigid frames share a striking resemblance in yield point to the rigid frame. The average of the three values of the code based semi rigid frames is within less than 1% of the rigid frame, in addition there is not a significant trend in the displacement at yield that is visible in the other two sets of frames.

Taking the second set of frames, the relaxed code based set, a clear trend can be seen. The trend shows that with decreasing rigidity of the connection there is also a decrease in roof drift at first yield. Interestingly the 80% connection frame has a higher level of displacement at yield than the rigid frame. The third set of frames, the non-conforming frames, follow exactly the pattern noted with the second set of frames. This is the decrease in roof displacement at first yield of the frames with the decrease in connection rigidity. Both the 80% and 65% non-conforming frames showed roof drifts higher than the rigid frame at yield. The second set of data points of interest are those of roof displacements at ultimate levels in the frames. Figure 2 depicts also the results of each of the nine frames drift levels at ultimate. Similar results as to those seen in the frame yielding graph above are found. First, the code based set of frames again has striking similarities in performance to the rigid frame with no clear trend in the data with respect to connection stiffness. The average of the roof...
displacements of these frames is within 1% of the roof displacement of the rigid frame. The uncertainty in the performance of semi-rigid frames has lead to conservatism in the code, in essence trying to force the behaviour of semi-rigid frames to mimic those of a rigid frame. Frames that contained reduced levels of code enforcement had a steadily decreasing roof displacement at ultimate with respect to the rotational stiffness of the connection. These results resemble those from the displacement at yield above. The reduction in column sizes with respect to reduced connection capacities should result in frames with less capacity in a static analysis. Caution should be used to not hastily assume that reduced capacity is undesirable; it will later be verified in dynamic analysis that less capacity is needed from semi-rigid frames than rigid frames under identical loading scenarios.

**TABLE 3**

<table>
<thead>
<tr>
<th>Frame</th>
<th>Connection Restraint (%)</th>
<th>Inside Column (W14x--)</th>
<th>Outside Column (W14x--)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80-Code</td>
<td>80</td>
<td>159, 145, 120</td>
<td>109, 99, 90</td>
</tr>
<tr>
<td>65-Code-2</td>
<td>65</td>
<td>132, 120, 99</td>
<td>90, 82, 68</td>
</tr>
<tr>
<td>50-Code-2</td>
<td>50</td>
<td>109, 99, 82</td>
<td>74, 68, 53</td>
</tr>
</tbody>
</table>

Figure 1. Yield and ultimate point definition of push-over curves for rigid and semi-rigid frames

Figure 2. Roof displacement at yield (left) and at ultimate (right) limit state of frames in static pushovers
The final set of frames, the non-code conforming frames had the smallest roof displacements at ultimate. The values of the three frames where within 5% of each other. A slight trend could be seen that followed the other frame trends of reduced roof drift with connection capacity, though the trend was not nearly as defined as the previous set of frames. These results are also logical. The three frames where designed to resist the forces imposed on them with no over-strength, resulting in three frames of similar periods and weights, with different member sizes and connection capacities. Clearly the difference between these three frames lies in displacement at yield, where significant changes in the displacement capacity where noted. In total the yield and ultimate capacities of the frames begin to define a clear picture of the behaviour of the frames. In addition the results begin to verify some of the initial assumption of the work with respect to code conservatism. To conclude the global response of the frames a final look at the base shear will be made, comparing the base shear at yield in the frames, the supply, to the demand required using a codified approach available in IBC2000. Figure 3 depicts both of these data sets for each of the frames. The code values should be taken with caution, because they are representative of applied forces to the buildings based on the periods of the frames, and in no way represent other over-strength requirements and seismic provisions that are imposed in design.

![Figure 3. Comparison of frames base shear from code verse analysis](image)

It can be seen that there is little difference in the code required base shear between the frames. The difference that is visible is based solely on the period elongation of the frames. Therefore the reduced force in the longest period, 50% non-conforming frames is logical compared to the stiffest fixed frame which has a higher required base shear at first yield. Though the demand on the buildings from code is fairly uniform, the supply of the buildings is not. The first set of semi-rigid frames, the code based frames, once again share similar results to the fixed frame. This is not surprising because of the coded specifications forcing similar behaviour of the frames. It can also be noted that the supply is almost twice that of the demand. It would not be appropriate to write this off completely. Often US code provides larger force reductions compared to international codes, which one might assume leads to lighter frames, but once all seismic provisions are applied to the design of the frame it becomes much heavier and similar to frames that would be designed using a foreign code. Therefore the two times over-strength as implied in the graph should be taken lightly, the true over-strength should be determined using required forces from the dynamic analysis.

A clear reduction of base shear at yield is observed in the second set of semi-rigid frames. Finally the last set of semi-rigid frames exhibit base-shear levels very similar to those required by code.

**DUCTILITY AND OVER-STRENGTH**

Two critical values to the performance of buildings in a seismic event are the displacement ductility and over-strength of the building. In using a code based approach, the force reduction factor, equivalent to the displacement ductility, is used to reduce the required forces for design under the assumption that the inelastic frame will dissipate energy equal to the dissipated energy of an elastic frame, see Figure 4. For special moment resistant frames the code allows for a force reduction factor R (or behaviour factor, q) equal to 8.0. Such factor is the inclusive factor of the ductility factor along
with the damping factor and over-strength factor. To verify the use of 8.0 as the force reduction factor used in the design of all nine of the frames the ductility and over-strength are plotted for each of the frames in Figure 4, respectively. The displacement ductility was calculated from the results of the static push-over analysis. The value is equal to the ultimate roof displacement divided by the roof displacement at yield. The resulting values ranged from 7.28 to 8.59. Within each set of semi-rigid frames it was found that the softer the connection the higher the displacement ductility. Only two semi-rigid frames exceeded the ductility level of the rigid frame that was the code based 65 and the code base 50 frames. Initial indications show that non-code based frames might find the necessity to use a slightly lower force reduction factor, ones proportional to the frames connection stiffness.

Table 5 – Frame Force Reduction Factors
From Push-Over Analysis

<table>
<thead>
<tr>
<th>Frame</th>
<th>Force Reduction Factors</th>
</tr>
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<tbody>
<tr>
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<tr>
<td>80-Code</td>
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</tr>
<tr>
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<tr>
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<tr>
<td>80-NC</td>
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<tr>
<td>65-NC</td>
<td>9.03</td>
</tr>
<tr>
<td>50-NC</td>
<td>9.83</td>
</tr>
</tbody>
</table>

To calculate the over-strength factor or the relationship between the ultimate and yield capacity of the frame, the base shear at ultimate and yield displacements as defined above in the displacement ductility calculation had to be recorded from the static push-over analysis. The over-strength is calculated simply by dividing the base shear at ultimate by the base shear at yield of the frame. This is the inherent over-strength defined in [8], as opposed to the conventional over-strength factor defined as the ratio between actual and design strength. This over-strength value is also used as part of the force reduction factor $R$ calculation from the code. Interestingly the semi-rigid frames always had an equal or higher over-strength factor than the rigid frame. Within each set of semi-rigid frames it was noted that over-strength values where higher for connections of greater strength, an opposite characteristic than that noted for the displacement ductility. Using solely the over-strength factor it could be determined that the codified force reduction factor is sufficient. This would not be entirely correct because the majority of the force reduction factor in moment resistant frames is based on the displacement ductility. To resolve the influence of the displacement ductility and over-strength factor and make a comparison to the code force reduction factor the product of the two values is examined. Results of this calculation can be seen in Table 5. Since it is assumed that all buildings of this work
have the same damping value, this portion of the reduction factor will be ignored. As a result, it is found that both the first and second set of the semi-rigid frames have values exceeding that of the rigid frame. The third set of semi-rigid frames, the non-conforming frames, has values up to 15% less than the rigid frame. Within the third set the stiffest connection has the lowest calculated reduction factor while the softer connection approaches the fixed frame value. It may therefore be suggested that the code based and relaxed code frames continue to use an 8.0 force reduction factor while the non-conforming frames would need to use factors ranging from 6.5 for the 80% connection to 7.5 for the 50% connection.

CONCLUSION

The present work seeks to extend all previous research to the physical application of semi-rigid and partially restrained connections as the only source of lateral resistance in a frame. At the same time it tries to provide a designer with a complete step by step approach to designing and using the connections that is not available using United States design codes. The paper also tries to show the over-conservative nature of code, especially as it applied to using semi-rigid connections, not just connections that seek to mimic rigid frames, but those that look to force plastic hinging in connections at capacities well below the plastic limit of the beam. There is still, however, much work ahead in the successful implementation of semi-rigid connections in high seismic areas. Work must be done to improve seismic provisions so that they can both insure safety to the occupants of buildings while at the same time allow designers the freedom to explore unconventional yet proven design techniques.

REFERENCES

BUCKLING BEHAVIOUR OF LOCALLY AND GLOBALLY BRACED THIN-WALLED STEEL FRAMES

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KEYWORDS

Thin-walled steel frames, Space frames, Local and global buckling behaviour, Local and global bracing, Generalised Beam Theory (GBT), GBT-based beam finite element.

ABSTRACT

This paper reports results concerning the effect of different bracing arrangements on the local and global buckling behaviour of thin-walled steel frames. These results are obtained through (i) beam finite element models, based on a Generalised Beam Theory (GBT) formulation recently developed and validated by the authors, and (ii) shell finite element analyses carried out in the code ANSYS. After briefly reviewing of the concepts and procedures involved in determining the finite element and frame linear and geometric stiffness matrices (incorporating the influence of the frame joints, loading, support conditions and bracings), one presents and discusses numerical results dealing with the buckling behaviour of locally and globally braced thin-walled steel space frames – in particular, one investigates how the bracing arrangement affects the frame critical buckling load and mode shape.

INTRODUCTION

Thin-walled steel frames are often formed by slender open-section members, a feature rendering them highly prone to geometrically non-linear effects, namely those related to global (flexural or flexural-torsional) and local (e.g., see Figs. 1(a) and (b)) instability phenomena. One possible way of improving the frame resistance against the failure prompted by global buckling consists of using diagonal tie-rods (global bracing – see Fig. 1(a)), which transfer part of the horizontal and vertical forces from the members to the frame foundation (e.g., the widespread approach of employing “X” or “V” bracing arrangements in some plane frames). On the other hand, the local critical buckling stress can be increased through the introduction of batten plates (local bracing – see Fig. 1(b)) joining the wall ends of some open cross-sections (i.e., “closing” them) – such battens
may be evenly or unevenly spaced along the member length. However, the structural efficiency of these local and global bracing procedures can only be adequately assessed after acquiring in-depth information about the braced frame buckling behaviour, a task involving (i) the identification of the relevant local and global buckling modes and (ii) the evaluation of the associated bifurcation stresses. This fact explains why assessing the structural response of these frames constitutes such a complex task (e.g., [1]), involving the performance of either (i) carefully planned and very costly experimental tests or (ii) sophisticated, computer-intensive and time-consuming (including data input and result interpretation) shell finite element analyses – while the latter approach is still prohibitive for routine applications, the former one is obviously restricted to research purposes.

In order to render the buckling analysis of thin-walled steel frames computationally simpler, but without sacrificing too much the accuracy of the results obtained, one must develop easy-to-use numerical tools, which involve necessarily beam finite element analysis. A very promising approach is the use of Generalised Beam Theory (GBT), originally formulated by Schardt [2] and significantly improved in the last few years (e.g., [3, 4]). In particular, the authors recently developed and implemented GBT-based beam finite element models that make it possible to analyse the (elastic) local, distortional and global buckling behaviour of plane and space thin-walled steel frames [5-7].

The main objective of this work is to present and discuss the results of an ongoing investigation concerning the use of a GBT-based beam finite element approach to assess how different bracing arrangements affect the local and global buckling behaviour of thin-walled steel space frames (i) built from identical I-section members, (ii) acted by arbitrary loadings and (iii) exhibiting commonly used joint configurations. Some GBT-based critical buckling loads and mode shapes are compared with values yielded by ANSYS shell finite element analyses [8].

**GBT-BASED FINITE ELEMENT FORMULATION – BRIEF OVERVIEW**

Since the cross-section displacement field is expressed as a linear combination of mechanically meaningful deformation modes, GBT analyses involve the solution of equilibrium equations written in a very convenient modal form, thus leading to in-depth insight on the frame structural response. Performing a frame buckling analysis involves two main tasks: (i) a cross-section
analysis, aimed at identifying the deformation modes and evaluating the associated modal mechanical properties, and (ii) a frame analysis, to solve the buckling eigenvalue problem and obtain the corresponding modal results.

Figure 2(a) concerns the I-section exhibited by the frame members dealt with in this work. It shows the dimensions and the GBT discretisation adopted (involving natural, end and intermediate nodes). On the basis of this discretisation, the GBT cross-section analysis leads to 15 deformation modes – Figure 2(b) displays the in-plane shapes of the 7 most relevant ones: axial extension (1), major/minor axis bending (2-3), torsion (4) and local (5-7) modes.

![Figure 2](image)

In thin-walled steel frames acted by arbitrary loadings, the buckling problem may be solved by means of the GBT-based beam finite element approach recently developed and implemented by the authors [6, 7]). Next, one briefly describes the concepts and procedures involved in determining the frame overall linear and geometric stiffness matrices (on the basis of the GBT-based finite element matrices):

(i) After discretising the frame, one handles separately the degrees of freedom associated with the member (i₁) internal or end support nodes and (i₂) end nodes corresponding to frame joints. In the former, GBT (modal) degrees of freedom are always considered, as the compatibility between them is trivially guaranteed [9]. The same does not hold for the joint nodes, where ensuring GBT degrees of freedom compatibility is no longer a straightforward matter – this stems from their modal nature and the fact that they are referred to distinct (member) coordinate systems. To overcome this difficulty, one (i₁) “transforms” these modal degrees of freedom into nodal ones (generalised displacements of the point where the joint is deemed located – usually the intersection of the connected member centroidal axes), by resorting to a joint element concept, and (i₂) imposes joint compatibility conditions to enforce compatibility between the end section displacements and rotations stemming from warping (due to torsion and/or distortion) and wall transverse bending.

(ii) Concerning the modelling of the local and global bracing systems, GBT analyses incorporate the batten plates (local bracing) and rigid tie-rod (global bracing) effects by imposing constraint conditions (varying from case to case) in the frame stiffness matrices [7]. For the batten plates, these constraints amount to imposing a fixed distance between the flange free ends of the restrained cross-sections (e.g., points $A_l^j$ and $A_L^j$ in Fig. 3(a)) – note that this arrangement restrains only the symmetric local deformation modes.1 Concerning

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1 Increasing the batten plate width amounts to “closing” larger beam segments, thus leading to a much higher bending and (mostly) torsional stiffness values. However, this effect is not accounted for by the GBT analyses, since no restraint is imposed to the corresponding deformation modes (2-4).

2 If the member wall thickness is smaller than the batten plate one, one assumes that the latter exhibits in-plane rigidity.
the inclusion of tie-rods into the analysis, one must impose constraint conditions that ensure the full restraint of the transverse flexural displacement of a member mid-surface point located within a wall (e.g., point $P$ in Fig. 3(b)).

(iii) Once the constraint conditions (associated with the joints and bracing systems) are properly modelled, the frame linear and geometric stiffness matrices are readily obtained – they are expressed in terms of constrained and mixed degrees of freedom (there are GBT modal degrees of freedom in all member internal nodes and “conventional” generalised displacements in all frame joints).

After knowing the frame total stiffness matrix, performing its buckling analysis consists merely of solving a standard eigenvalue problem – however, since (i) the “mixed” eigenvectors combines generalised displacements with GBT degrees of freedom and (ii) one aims at a modal representation of the frame buckling modes, it is necessary to “transform back” the joint (nodal) generalised displacements into (modal) GBT degrees of freedom of each connected finite element end section. Once the GBT degree of freedom are known in all member nodes, it becomes possible to obtain the modal representation of the frame buckling modes, i.e., to identify and quantify the individual contributions of the various member deformation modes – this feature provides fresh insight and in-depth understanding on the frame buckling mechanics.

Figure 3: Buckling modes of members with (a) local (batten plates) and (b) global (tie-rod) restraints

NUMERICAL RESULTS: BRACED SPACE FRAMES

In this section, one presents and discusses numerical results concerning the assessment of how different bracing arrangements influence the local and global buckling behaviour of the symmetric space frame shown in Figure 4(a) – it comprises two portal frames ($F1$ and $F2$) joined by a transverse beam ($TB$) and is acted by five equal vertical loads $P$ applied at the four column tops and at the centroid of the transverse beam mid-span cross-section. All members have identical cross-sections, with the dimensions given in Figure 2(a), and are made of steel with $E=205\text{ GPa}$ (Young’s modulus) and $v=0.3$ (Poisson’s ratio). While all column-to-beam joints are box-stiffened (web continuity), the beam-to-beam joints exhibit flange continuity. As for the support conditions, (i) the column bases are fixed and (ii) the transverse displacements along $X$ are prevented at all column-to-beam joints (see Fig. 4(b)).
One analyses the unbraced frame shown in Figure 4(a) and also frames exhibiting the three bracing arrangements depicted in Figures 5(a)-(c), which are characterised as follows:

(i) **BR1** – displacements along $\bar{Z}$ prevented at all column-to-beam joints by means of “rigid” tie-rods positioned diagonally (X-configuration) and transversely$^3$.

(ii) **BR2** – in addition to **BR1**, one also prevents the displacement along $\bar{X}$ at the transverse beam mid-span cross-section, again trough “rigid” tie-rods positioned diagonally (now with a V-configuration).

(iii) **BR3** – in addition to **BR2**, one also uses two batten plates to fix the distance separating the flange free ends at the transverse beam mid-span cross-section.

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$^3$ All tie-rods are connected to the corresponding member cross-section mid-web points.
Figure 5: Frame bracing arrangements dealt with in this work: (a) BR1, (b) BR2 and (c) BR3

The buckling results presented (critical loads and mode shapes) are obtained by means of (i) the GBT beam finite element formulation briefly described earlier and also (ii) shell finite element analyses carried out in ANSYS [8], adopting frame and bracing discretisations into fine meshes of SHELL181 and BEAM189 elements (ANSYS nomenclature).4

While Table 1 shows the frame GBT and ANSYS critical loads concerning the braced and unbraced frames, Figures 6 to 9 provide two representations of the corresponding buckling mode shapes, namely (i) 3D-views yielded by the ANSYS analyses and (ii) the amplitude functions of the GBT deformation modes. At this stage, it is worth noting that the GBT analyses were based on frame discretisations involving 80 finite elements (8 per column and 16 per beam), which corresponds to 1030 degrees of freedom – their ANSYS counterparts require the consideration of more than 25000 degrees of freedom.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>GBT AND ANSYS FRAME CRITICAL LOADS (KN)</th>
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<tr>
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<td>GBT</td>
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<tr>
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<tr>
<td>Braced</td>
<td></td>
</tr>
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<td>BR1</td>
<td>73.25</td>
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<tr>
<td>BR2</td>
<td>161.83</td>
</tr>
<tr>
<td>BR3</td>
<td>167.33</td>
</tr>
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</table>

The observation of the results shown in Table 1 and Figures 6 to 9 prompts the following remarks:

(i) First of all, there is a virtual coincidence between the critical loads yielded by the GBT and ANSYS finite element analyses (all differences below 2.2%). Moreover, there is also very close agreement between the buckling mode shapes provided by ANSYS analyses and the GBT modal amplitude functions – however, the latter representations provide a deeper insight on the mechanics of frame buckling, as well as on the influence of the bracing arrangement.

(ii) Table 1 shows that the presence of the bracing arrangements leads to critical buckling load increases amounting to 69% (BR1), 274% (BR2) and 287% (BR3).

4 The tie-rod and batten plate modelled exhibited (i) a 6cm square cross-section and (ii) 30×7.5×0.7cm dimensions, respectively.
Figure 6: Unbraced frame – GBT and ANSYS critical buckling mode representations

Figure 7: BR1 braced frame – GBT and ANSYS critical buckling mode representations
(iii) The unbraced frame critical buckling mode involves mainly minor axis bending displacements (mode 3) of the portal frames and axial translation of the transverse beam, together with lateral motions of all frame joints. Therefore, it is clear that restraining the lateral motions of the joints will enhance the frame buckling resistance.
(iv) The BR1 braced frame critical buckling mode combines global (flexural-torsional) and local deformation modes. The former, which are clearly dominant, consist of (iv1) a major contribution from mode 3 (minor axis bending) and (iv2) a relevant participation of mode 4 (torsion). In addition, there are also non-negligible contributions from the local modes 5, 6 and 7 in the vicinity of the transverse beam mid-span cross-section. Since buckling is clearly triggered by the transverse beam lateral-torsional instability, the frame resistance can be increased by preventing its the mid-span lateral displacement (global bracing).

(v) The BR2 braced frame critical buckling mode is triggered by the transverse beam and combines participations of both global (3 and 4) and local (5, 6 and 7) deformation modes—while the latter occur mainly in the beam mid-span region (higher bending moments), the influence of the former is felt along the whole beam length. A further improvement of the frame buckling resistance can be achieved by reducing the beam local deformations.

(vi) The BR3 braced frame buckles in a mode that involves very small local deformations in the transverse beam. Indeed, the frame critical buckling is predominantly associated with the lateral-torsional behaviour of all members (modes 3 and 4) due to the mid-span local bracing, the transverse beam exhibits now two half-waves. The major contribution to the frame buckling mode comes from mode 3, with similar maximum values occurring at the portal frame columns and beams.

CONCLUSION

This paper reported the results of an investigation concerning the effect of the bracing arrangement on the local and global buckling behaviour of thin-walled steel frames—these results were obtained by means of a GBT-based beam finite element model recently developed and validated by the authors. After a brief review of the main concepts and procedures involved in determining the finite element and frame linear and geometric stiffness matrices, one presented and discussed numerical results dealing with the local and global buckling behaviour of thin-walled steel space frames built from I-section members, subjected to arbitrary loading and exhibiting three bracing arrangements with an increasing degree of complexity. Taking advantage of the GBT modal features, which provide fresh insight on the frame local and global buckling mechanics, it was possible to make an in-depth assessment of the influence and effectiveness of the bracing arrangements considered—in this particular case, they caused (i) critical buckling load increases ranging from 69% to 287% and (ii) significant changes in the buckling mode shape nature. For comparison and illustration purposes, the buckling results yielded by ANSYS shell finite element analyses were also included in the paper. Despite the disparity in the numbers of degrees of freedom involved (orders of magnitude apart), the critical buckling loads and mode shapes provided by the GBT-based and ANSYS analyses virtually coincide—this confirms the versatility and numerical efficiency of the GBT approach to analyse the buckling behaviour of thin-walled frames.

ACKNOWLEDGEMENTS

The first author gratefully acknowledges the financial support provided by the CAPES Foundation (Brazilian Ministry of Education), through the doctoral scholarship nº BEX 3932/06-0.

5 Note that the GBT normalisation procedure adopted renders the mode 4 amplitude function “artificially small”.

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REFERENCES


PLASTIC DESIGN OF MRF-CBF SYSTEMS

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KEYWORDS
Moment frames, concentrically braced frames, plastic design, kinematic theorem of plastic collapse.

ABSTRACT
In this paper, a new design approach for moment resisting frame (MRF) – concentrically braced frame (CBF) dual systems is presented. It is aimed to design structures failing in global mode by applying the kinematic theorem of plastic collapse. Beam and diagonal sections are assumed to be known quantities, as they are designed to resist vertical loads and horizontal forces, respectively. Therefore, the column sections are the only unknowns of the design problem, obtained by imposing that the equilibrium curve corresponding to the global mechanism has to lie below all the equilibrium curves corresponding to the undesired mechanisms within a displacement range compatible with local ductility supply of dissipative elements. Such procedure has been applied to design several MRF-CBF dual systems. Non-linear static analysis have been carried out in order to check the actual pattern of yielding. In addition, the influence of the seismic action percentage withstood by diagonals has also been investigated.

INTRODUCTION
Moment resisting frame (MRF) - Concentrically braced frame (CBF) dual systems constitute a reliable alternative for designers, as they combine the advantages of both structural typologies. The dissipative capacity of beam end sections and the stiffness provided by diagonals allow to obtain high global ductility and limited interstorey drifts, so that both the ultimate and the serviceability limit state requirements can be easily satisfied. Nevertheless, in order to obtain high global ductility, the need to control the location of dissipative zones, i.e. the control of the failure mode, is of primary importance. In particular, partial mechanisms have to be prevented as they undermine the global ductility supply and the energy dissipation capacity of the structure. To this scope, modern seismic codes, among which Eurocode 8 (2003), provide simple requirements for traditional structural schemes...
based on the hierarchy criterion. Even though such requirements are sufficient to prevent soft
storey mechanisms, they don’t lead to structures failing in global mode. To this scope, more
sophisticated design procedures need to be applied. In this paper, with reference to MRF-
CBF dual systems, a design procedure aimed at the failure mode control, already successfully
applied with reference to MRFs (Mazzolani and Piluso, 1997), EBFs (Mastrandrea et al.
2003) and KBFs (Conti et al. 2006), is briefly presented.

DESIGN PROCEDURE

The proposed design procedure, aimed at the failure mode control, is based on the application
of the kinematic theorem of plastic collapse. It starts from the observation that the collapse
mechanism of dual systems under seismic horizontal forces can be considered as belonging to
three main typologies (Fig. 1), where the collapse mechanism of global type is a particular
case of type-2 mechanism. The global mechanism is achieved when plastic hinges are
developed at all the beam ends and at the base of first storey columns, while all the tensile
diagonals are yielded and the compressed ones are in buckled conditions. The control of the
failure mode can be performed by means of the analysis of $3n_s$ mechanisms (being $n_s$ the
number of storeys). The beam and diagonal sections are assumed to be known quantities as
they are designed to withstand vertical loads and horizontal actions, respectively. Therefore,
the only unknowns of the design procedure are the column sections. They can be obtained by
applying the kinematic theorem of plastic collapse, i.e. by imposing that the kinematically
admissible multiplier of horizontal forces corresponding to the global mechanism is the
smallest among kinematically admissible multipliers corresponding to all the other undesired
partial mechanisms. Therefore, according to the upper bound theorem, the above stated
multiplier is the true collapse multiplier and, as a consequence, the global mechanism is the
true failure mechanism.

Nevertheless, this condition is sufficient to assure the development of the desired mechanism
provided that the structural material behaves as rigid-plastic, so that the horizontal
displacements are equal to zero up to the complete development of the collapse mechanism.
Conversely, the actual behaviour is elastoplastic, so that structures exhibit significant
displacements before failure, giving rise to second order effects which cannot be neglected in
the design process. To this scope, a linearized mechanism equilibrium curve can be used:

$$\alpha_c = \alpha - \gamma \cdot \delta$$

where $\alpha$ is the kinematically admissible multiplier of seismic horizontal forces and $\gamma$ is the
slope of the mechanism equilibrium curve. The kinematically admissible multiplier of
horizontal forces corresponding to the generic mechanism is obtained by means of the virtual
work principle. For a virtual rotation $d\theta$ of the columns involved in the mechanism, the internal work is given by:

$$ W_i = \left[ \text{tr}(C^T R_e) + 2\text{tr}(B^T R_b) + \text{tr}(N^T T_e) + \text{tr}(N^T E_e) \right] d\theta $$

(2)

where:

- $\text{tr}$ denotes the trace of the matrix;
- $C$ is a matrix of order $n_c \times n_s$ (number of columns × number of storeys) whose elements $C_{ik}$ are equal to the plastic moments of columns ($C_{ik} = M_{c,ik}$);
- $R_e$ is a matrix of order $n_c \times n_s$ whose elements, $R_{e,ik}$, are coefficients accounting for the participation of $i$th column of $k$th storey to the collapse mechanism. In particular $R_{e,ik} = 2$ when the column is yielded at both ends, $R_{e,ik} = 1$ when only one column end is yielded, and $R_{e,ik} = 0$ when the column does not participate to the collapse mechanism;
- $B$ is a matrix of order $n_b \times n_s$ (number of bays × number of storeys) whose elements $B_{jk}$ are equal to plastic moments of beams ($B_{jk} = M_{b,jk}$);
- $R_b$ is a matrix of order $n_b \times n_s$ whose elements, $R_{b,jk}$, are coefficients accounting for the participation of $j$th beam of $k$th storey to the collapse mechanism. In particular $R_{b,jk} = 0$ when the beam does not participate to the collapse mechanism (and for beams of braced bays pin jointed to the columns), otherwise $R_{b,jk} = L_j / (L_j - x_{jk})$, where $L_j$ is the span of $j$th bay and $x_{jk}$ is the abscissa of second plastic hinge of $j$th beam of $k$th storey. This abscissa is given by $x_{jk} = L_j / (L_j - M_{b,jk} / q_{jk})^{1/2}$ when the uniform vertical load $q_{jk} > 4 \cdot M_{b,jk} / L_j^2$ and $x_{jk} = 0$ in the opposite case (Mazzolani and Piluso, 1997);
- $N_t$ and $N_c$ are matrices of order $n_b \times n_s$ (number of braced bays × number of storeys) whose elements $N_{t,bk}$ and $N_{c,bk}$ are equal to the yield axial forces in tensile diagonals and the axial force accounting for post-buckling behaviour in compressed diagonals of $bh$ braced bay and $k$th storey, respectively, with reference to the collapse condition (Longo et al., 2008);
- $E_t$ and $E_c$ are matrices of order $n_b \times n_s$ whose elements, $e_{t,bk}$ and $e_{c,bk}$, are coefficients representing, respectively, the elongation of the tensile yielded diagonal and the shortening of the buckled compressed one belonging to the $bh$ braced bay and $k$th storey due to a unit rotation of columns. They are given by $l_{bk} \cos\alpha_{bk}$ ($l_{bk}$ is the brace length) when the diagonal participate to the collapse mechanism, conversely $e_{t,bk} = e_{c,bk} = 0$.

The external work due to the horizontal forces and uniform loads acting on the beams, for a virtual rotation $d\theta$ of the columns involved in the mechanism, can be written as:

$$ W_e = [\alpha \cdot F^T s + \text{tr}(q^T D_v)] d\theta $$

(3)

where:

- $F^T$ is the vector of the design seismic horizontal forces equal to $\{F_1, F_2, \ldots, F_k, \ldots, F_{ns}\}$, where $F_k$ is the horizontal force applied to the $k$th storey;
- $s$ is the shape vector of the storey horizontal virtual displacements ($du = s \cdot d\theta$, where $d\theta$ is the virtual rotation of the plastic hinges of the columns involved in the mechanism);
- $q$ is the matrix of order $n_b \times n_s$ (number of bays × number of storeys) of uniform vertical loads acting on the beams;
− $D_v$ is a matrix of order $n_b \times n_s$ whose elements, $D_{vjk}$, are coefficients related to the external work of the uniform load acting on $j$th beam of $k$th storey. In particular $D_{vjk} = L_j \cdot \chi_{jk} / 2$ when the beam participates to the collapse mechanism and $D_{vjk} = 0$ in the opposite case.

Therefore the application of the virtual work principle provides the kinematically admissible multiplier:

$$\alpha = \left[tr(C^T R_e) + 2tr(B^T R_b) + tr(N^T E_e) - tr(q^T D_v)\right] F^T s$$  \hspace{1cm} (4)

In order to evaluate the slope of the mechanism equilibrium curve, the second order work related to uniform vertical loads can be expressed as:

$$W_v = V^T s \cdot \frac{\delta}{H_0} d\theta$$  \hspace{1cm} (5)

where:

− $V$ is the vector of the storey vertical loads $\{V_1, V_2, ..., V_b, ..., V_{ns}\}$, where $V_k$ is the total load acting at $k$th storey;
− $s \cdot \delta / H_0$ is the shape vector of the storey vertical virtual displacements, with $s$ shape vector of the storey horizontal virtual displacements, $\delta$ top sway displacement of the structure and $H_0$ sum of the interstorey heights of all the storeys involved in the generic mechanism.

Finally, the slope $\gamma$ is related to the ratio between the second order work due to vertical loads and the work due to horizontal forces:

$$\gamma = \frac{V^T s \cdot 1}{F^T s \cdot \frac{\delta}{H_0}}$$  \hspace{1cm} (6)

The kinematically admissible multiplier and the slope of the mechanism equilibrium curve can be evaluated for each one of the considered mechanisms by means of Eqs. (4) and (6). In the following, the notations $\alpha^{(g)}$, $\gamma^{(g)}$ and $\alpha^{(im)}$, $\gamma^{(im)}$ denote the kinematically admissible multiplier of horizontal forces and the slope of the softening branch of $\alpha-\delta$ curves corresponding to the global type mechanism and to the $i_{th}$ mechanism of $t$th type (with $t=1$ to 3), respectively. With reference to the global mechanism the following relationships are obtained:

$$\alpha^{(g)} = \left[M_{c1}^T I + 2tr(B^T R_b^{(g)}) + tr(N^T E_e^{(g)}) + tr(N^T E_c^{(g)}) - tr(q^T D_v^{(g)})\right] F^T s^{(g)}$$  \hspace{1cm} (7)

$$\gamma^{(g)} = \frac{V^T s^{(g)} \cdot 1}{F^T s^{(g)} \cdot h_{ns}}$$  \hspace{1cm} (8)

where the $M_{c1}^T$ is the vector of plastic moments of first storey columns, reduced due to the influence of axial forces, $I$ is the unit vector of order $n_c$ and $s^{(g)} = h = \{h_1, h_2, ..., h_{ns}\}^T$ as all the storeys participate to the mechanism.

With reference to the $i_{th}^{th}$ mechanism of type-1 only the beams and the diagonals of the first $i_{th}$ storeys are involved in the mechanism, whereas plastic hinges in columns develop at the base of first storey and at the top of $i_{th}$ storey, so that the following relationships are obtained:

$$\alpha^{(1)}_{i_{th}} = \frac{M_{c1}^T I + 2tr(B^T R_b^{(1)}_{h,im}) + M_{c1}^T I + tr(N^T E_e^{(1)}_{i_{th}}) + tr(N^T E_c^{(1)}_{i_{th}}) - tr(q^T D_v^{(1)}_{i_{th}})}{F^T s^{(1)}_{i_{th}}}$$  \hspace{1cm} (9)
\[
\gamma_{\text{im}}^{(1)} = \frac{V^{T} s_{\text{im}}^{(1)}}{F^{T} s_{\text{im}}^{(1)}} \cdot \frac{1}{h_{\text{im}}}
\]

with \( H_0 = h_{\text{im}} \) and \( s_{\text{im}}^{(1)} = \{h_1, h_2, \ldots, h_{\text{im}}, h_{\text{im}}\}^T \), where the first element equal to \( h_{\text{im}} \) corresponds to the \( i_{\text{im}} \)th component. The relationships for evaluating the kinematically admissible multipliers \( \alpha_{\text{im}}^{(2)} \) and \( \alpha_{\text{im}}^{(3)} \) and the slopes \( \gamma_{\text{im}}^{(2)} \) and \( \gamma_{\text{im}}^{(3)} \) of the mechanism equilibrium curves corresponding to type 2 and type 3 mechanisms, respectively, can be obtained by means of the same procedure.

In order to assure the development of the desired global mechanism, the column sections have to be designed according to the upper bound theorem. In particular, accounting for second order effects, it is required that the mechanism equilibrium curve corresponding to the global mechanism has to lie below those corresponding to all the other undesired partial mechanisms. However, the fulfillment of this requirement is needed up to a selected ultimate displacement \( \delta_u \) compatible with the plastic deformation capacity of members. Therefore, the proposed design procedure provides also the control of local ductility supply of dissipative members. The resulting design conditions are expressed by the following relationships:

\[
\alpha^{(g)} - \gamma^{(g)} \cdot \delta_u \leq \alpha_{\text{im}}^{(1)} - \gamma_{\text{im}}^{(1)} \cdot \delta_u
\]

with \( i_{\text{im}} = 1, 2, 3, \ldots, n_i \) and \( t = 1, 2, 3 \). It means that there are \( 3n_i \) design conditions to be satisfied for a structure having \( n_i \) storeys. These design conditions combined with technological limitations (Mazzolani and Piluso, 1997), provide the column plastic moments, reduced due to the influence of axial forces, required at each storey in order to prevent the developments of partial mechanisms. The final step consists in the evaluation of axial forces occurring in columns in the collapse condition. This step can be easily carried out accounting for the shear forces transmitted by beams and the vertical components of axial forces in yielded tensile diagonals and buckled compressed diagonals.

It has to be underlined that structures designed according to the failure mode control procedure could not exhibit sufficient lateral stiffness to fulfill code requirements dealing with serviceability limit state (Eurocode 8, 2003). In such case, some iterations are necessary. The stiffening of structures can be obtained by applying the design procedure, described above, either increasing beam sections or increasing the design displacement level, for a given percentage of seismic action to be entrusted to diagonal braces.

**APPLICATIONS**

In order to check the accuracy of the proposed design methodology for MRF-CBF dual systems aimed at the failure mode control, an adequate number of structures has been designed and their inelastic behaviour has been investigated by means of non-linear static push-over analyses carried out using SAP 2000 computer program (1998). Structures having different numbers of storeys (4÷12) have been designed with the same geometrical scheme, having three bays being the span of un-braced bays equal to 4.0m and the span of the central braced bay equal to 6.0m. Interstorey height is equal to 3.0m. In addition, beams of braced bays are pin-jointed to the columns. Also two different design criteria for dimensioning bracing elements have been investigated. The first one is based on the fulfillment of brace slenderness limitation required by Eurocode 8 for CBFs (2003). In such case, bracing members are able to provide a contribution to the energy dissipation capacity of the structure.
The second criterion does not satisfy the aforesaid limitation, so that bracing members are dimensioned with the aim to provide only an increase of lateral stiffness, without significant improvement of dissipation capacity.

In this paper, for sake of synthesis, only the results concerning the 12-storeys MRF-CBF dual system are presented with reference to the second criterion for designing bracing elements. The design seismic actions have been determined according to Eurocode 8 (2003), assuming a peak ground acceleration equal to 0.35g, a seismic response factor equal to 2.5, soil type A and behaviour factor q equal to 4, i.e. the q factor of CBFs. Bracing members have been designed to withstand the whole design seismic actions. The design ultimate displacement $\delta_u$ has been determined assuming that the maximum plastic rotation $\theta_u$ of beams is equal 0.04 rad, so that $\delta_u = \theta_u \times h_{ns}$. Diagonal, beam and column sections obtained from the design procedure are summarized in Table 1. The constructional steel weight is equal to 15.73 tons. The structure has been modeled with SAP 2000 computer program by means of non linear elements. In particular, beams and columns have been modeled using beam-column elements with the possibility of developing plastic hinges at their ends. Tension diagonals have been modeled accounting for the possibility of yielding, whereas the compressed ones have been neglected, because their post-buckling resistance is negligible due to their slenderness (the design criterion for diagonals does not account for Eurocode 8 provisions about normalized slenderness limitation). In addition, for each step of the analysis, the member stability checks have been performed according to Eurocode 3 provisions (1993). Fig. 2a shows the pushover curve (top sway displacement versus horizontal forces multiplier) obtained from the analysis. The global mechanism equilibrium curve, expressed by Eq. (1) with $\alpha = \alpha^{(b)}$ and $\gamma = \gamma^{(b)}$, is also depicted. By comparing these results, a first confirmation of the accuracy of the design methodology can be deduced. In addition, in Fig. 3a the pattern of yielding developed at the design ultimate displacement is depicted, where hinges in the midspan of bracing elements denotes their yielding. It can be observed that all the diagonals are yielded and plastic hinges develop only at beam ends, confirming the fulfilment of the design goal, i.e. the failure mode control. For displacements exceeding the selected design ultimate displacement, the mechanism evolves towards a global type mechanism.

### Table 1

<table>
<thead>
<tr>
<th>Storey</th>
<th>CBF beams*</th>
<th>MRF beams**</th>
<th>Diagonals</th>
<th>External columns</th>
<th>Internal columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>IPE 220</td>
<td>IPE 140</td>
<td>bar Φ42</td>
<td>HEB 500</td>
<td>HEB 220</td>
</tr>
<tr>
<td>2</td>
<td>IPE 220</td>
<td>IPE 140</td>
<td>bar Φ41</td>
<td>HEB 500</td>
<td>HEB 220</td>
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<td>3</td>
<td>IPE 220</td>
<td>IPE 140</td>
<td>bar Φ41</td>
<td>HEB 450</td>
<td>HEB 200</td>
</tr>
<tr>
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<td>bar Φ40</td>
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<td>HEB 200</td>
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<td>5</td>
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<td>IPE 140</td>
<td>bar Φ39</td>
<td>HEB 340</td>
<td>HEB 200</td>
</tr>
<tr>
<td>6</td>
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<td>IPE 140</td>
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</tr>
<tr>
<td>7</td>
<td>IPE 220</td>
<td>IPE 140</td>
<td>bar Φ36</td>
<td>HEB 280</td>
<td>HEB 160</td>
</tr>
<tr>
<td>8</td>
<td>IPE 220</td>
<td>IPE 140</td>
<td>bar Φ33</td>
<td>HEB 240</td>
<td>HEB 160</td>
</tr>
<tr>
<td>9</td>
<td>IPE 220</td>
<td>IPE 140</td>
<td>bar Φ31</td>
<td>HEB 220</td>
<td>HEB 140</td>
</tr>
<tr>
<td>10</td>
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<td>IPE 140</td>
<td>bar Φ27</td>
<td>HEB 180</td>
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<tr>
<td>11</td>
<td>IPE 220</td>
<td>IPE 140</td>
<td>bar Φ23</td>
<td>HEB 160</td>
<td>HEB 120</td>
</tr>
<tr>
<td>12</td>
<td>IPE 220</td>
<td>IPE 140</td>
<td>bar Φ17</td>
<td>HEB 120</td>
<td>HEB 100</td>
</tr>
</tbody>
</table>

* Beams of braced bays  ** Beams of un-braced bays
Nevertheless, the structure is not able to satisfy the requirements regarding the damage limitation limit state provided by Eurocode 8 (2003). In fact, for this limit state a maximum interstorey drift equal to 0.0083 is obtained, which exceeds the prescribed limit value equal to 0.005. Therefore, in order to satisfy such requirement, an iterative procedure has been applied. The proposed design methodology has been repeated by assuming increased sections of beams for unbraced. In order to guarantee the development of a global mechanism, this configuration requires more robust column sections, so that lateral stiffness increases. The iterative procedure will stop when the designed structure satisfies also damage limitation requirements. Diagonal, beam and column sections corresponding to the convergence situation are summarized in Table 2. The constructional steel weight is equal to 27.69 tons. It is important to underline that in this structural solution the same percentage of seismic horizontal forces of the previous design solution is entrusted to diagonal braces. In Fig. 2b the
top sway displacement versus horizontal forces multiplier curve and the global mechanism equilibrium curve are depicted. Finally, in Fig. 3b the pattern of yielding developed at the design ultimate displacement is depicted, showing the development of the global mechanism. Nevertheless an increase of more than 75% of the constructional steel weight with respect to the first solution has been obtained. Therefore, structures satisfying both the failure mode control goal and the damage limitation requirements of Eurocode 8 can be obtained by means of the described iterative procedure, but considerable increase in constructional steel weight is due to drift limitation.

**TABLE 2**
BEAM, DIAGONAL AND COLUMN SECTIONS OF THE 12-STOREYS MRF-CBF DUAL SYSTEM SATISFYING DAMAGE LIMITATION REQUIREMENTS.

<table>
<thead>
<tr>
<th>Storey</th>
<th>CBF beams*</th>
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<th>Diagonals</th>
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<td>bar Φ42</td>
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</tr>
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<td>IPE 330</td>
<td>bar Φ41</td>
<td>HEB 450</td>
<td>HEB 340</td>
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<tr>
<td>3</td>
<td>IPE 220</td>
<td>IPE 330</td>
<td>bar Φ40</td>
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<td>HEB 240</td>
</tr>
</tbody>
</table>

* Beams of braced bays  ** Beams of un-braced bays

**TABLE 3.**
CONSTRUCTIONAL STEEL WEIGHTS AND INTERSTOREY DRIFTS FOR THE DESIGNED 12-STOREYS FRAMES SATISFYING DAMAGE LIMITATION REQUIREMENTS

<table>
<thead>
<tr>
<th>Structure</th>
<th>Constructional steel weight (tons)</th>
<th>Interstorey drift (DLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBF</td>
<td>26.05</td>
<td>0.0050</td>
</tr>
<tr>
<td>MRF-CBF 100%</td>
<td>27.69</td>
<td>0.0046</td>
</tr>
<tr>
<td>MRF-CBF 75%</td>
<td>30.02</td>
<td>0.0047</td>
</tr>
<tr>
<td>MRF-CBF 50%</td>
<td>38.05</td>
<td>0.0038</td>
</tr>
<tr>
<td>MRF-CBF 25%</td>
<td>37.83</td>
<td>0.0046</td>
</tr>
<tr>
<td>MRF</td>
<td>45.11</td>
<td>0.0039</td>
</tr>
</tbody>
</table>

Finally, the influence of the seismic action percentage withstood by diagonals has been investigated. To this scope three values have been assumed: 75%, 50% and 25%. For each seismic action percentage, column sections have been design according to the aforesaid iterative procedure until interstorey drifts satisfy code requirements for serviceability limit state. In addition, nonlinear static pushover analyses have been carried out in order to check the pattern of yielding corresponding to the design ultimate displacement. All the structures have exhibited global mechanisms, satisfying the design goal. In Table 3 the constructional steel weights and the maximum interstorey drifts for the damage limitation state, exhibited by the designed MRF-CBF dual systems, are summarized with reference to the percentage of the
design seismic action withstood by bracing members. The results are also compared with those obtained for a 12-storeys moment resisting frame (MRF) and a 12-storeys concentrically braced frame (CBF). For these structures, the same geometry defined for MRF-CBF dual systems has been assumed. They have been dimensioned according to the design methodologies, aimed at the failure mode control, proposed for MRFs (Mazzolani and Piluso, 1997) and CBFs (Longo et al., 2008), respectively. In both cases, iterative procedures, similar to that one described for MRF-CBF dual systems, have been implemented in order to satisfy serviceability requirements. It can be observed that by reducing the percentage of the design seismic actions entrusted to diagonals, the weight of MRF-CBF dual systems increases. This result is the consequence of the reduction of the contribution of the bracing members to the lateral stiffness, so that more robust beams and columns are needed to satisfy serviceability requirements, leading to the increase of constructional steel weight. Therefore, the greatest saving in constructional steel weight can be gained by designing bracing elements to withstand the whole seismic action.

![Figure 4. Constructional steel weights of the designed 12-storeys frames satisfying damage limitation requirements.](image)

### CONCLUSIONS

A design methodology aimed at the failure mode control, already successfully applied with reference to MRFs (Mazzolani and Piluso, 1997), EBFs (Mastrandrea et al. 2003) and KBFs (Conti et al. 2006), has been implemented and applied to MRF-CBF dual systems. Non-linear static push-over analyses have been carried out for the designed structures in order to check the accuracy of the proposed methodology. The results have pointed out the fulfillment of the design goal, showing the development of global mechanisms characterized by yielding of diagonals and plastic hinge formation at beam ends.

An iterative procedure has also been described in order to obtain structures satisfying both the goal of failure mode control and the damage limitation requirements. In this case a considerable increase in constructional steel weight is obtained with respect to the simple application of the proposed design methodology without any drift limitation. Finally, the influence of the percentage of the design seismic action entrusted to diagonals has been investigated. The results have pointed out that, fulfilling code requirements dealing with serviceability limit state, the greatest saving in constructional steel weight is gained by designing bracing members to withstand the whole seismic action. As future development of the presented research activity, MRF-CBF dual systems where diagonal braces are designed to satisfy normalised slenderness limitation requirements will be investigated.
REFERENCES

RECENT DEVELOPMENT OF NON-LINEAR COMPUTATIONAL DESIGN BY SOFTWARE “NIDA”

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KEYWORDS
Second-order analysis, Buckling, Structures, Imperfections, NIDA

ABSTRACT
With the rapid development of computers in the past decade, design making the full use of computer speed and memory has become the trend of structural design in the world. To date, the simulation-based design of various types of structures is common and has been found to be successful in providing a practical engineering solution to safety and stability of most structures like buildings, towers, scaffolds, domes, pre-tensioned steel structures, glass structures and others. This paper describes the experience and new findings in the course of using the new design concept for quick and reliable check of various forms of structures made of different materials. The high accuracy as well as the underlying philosophy of the method of 'Nonlinear Integrated Analysis and Design (NIDA)', which allows for the second-order effects and initial imperfections for design of steel structures, is presented.

INTRODUCTION
Rapid development of computers and internet have made the design and checking of structural safety and stability more demanding with higher skill requirements. The conventional design using the linear analysis and effective length method has been handicapped in dealing with this type of structural design involving high degree of nonlinearity such as pre-tensioned steel trusses, long span domes and scaffolds where the elastic critical load factor is generally less than 3 such that the effective length method cannot be used (Eurocode-3 [1]). In Hong Kong, advanced engineering consultants have already made frequent and regular use of the nonlinear analysis in assessing structural safety to-date. Whenever the elastic critical load factor is greater than 5 or when a structure is irregular, the linear analysis and effective length design method is not allowed.
SECOND-ORDER ANALYSIS

In second-order analysis allowing for both $P-\Delta$ and $P-\delta$ effects, a structure is simulated under an expected maximum load in its design life with safety inspected. As a result, the design code does not need to be referred to. This is the dream of the structural engineer that his design can be visualized through simulation with reliable and accurate stability and strength checks. This idea of advanced analysis has been proposed and extensive research has been conducted on the topic. Liew et al. [2], White [3] and Chen and Chan [4] proposed methods for analysis and design of portal frames allowing for various second-order and yielding effects. Advanced analysis is considered as an analysis technique utilizing conventional nonlinear analysis with straight elements cannot fulfill the aim of integrating design into an analysis and meeting the requirements of advanced element. An advanced element should be able to re-produce the buckling curve of a member in design codes. This concept follows the idea of advanced analysis, which specifies any analysis capturing the behaviour of a realistic structure and does not need a separated member capacity check. Whilst straight elements derived by various formulations do not allow for member curvature, which is mandatory in most design codes, they cannot be classified as a design element and design check at element, not section, level is needed. The use of several elements to model a member not only leads to complexity in modelling, but also to difficulty in assessing the direction of member initial imperfection and inconsistency in modelling.

The most direct method for efficient and robust implementation of advanced analysis is to formulate an advanced element capturing the actual behaviour of a member under axial force. This automatically considers the $P-\delta$ effect so that efficiency and convenience in data preparation can be gained. Further, it provides a clearer and more direct concept of structural modelling.

NONLINEAR INTEGRATED ANALYSIS AND DESIGN (NIDA)

In the direction of formulating a design element capturing the second-order behaviour of a member, Chan and Zhou [5-8] developed several elements with different features to simplify the analysis procedure and make the advanced analysis practical and usable in a design office to date. This paper describes the method of applying the curved stability function [9] as an advanced element capturing the actual behaviour of a member to the design of practical structures. This simulation-based concept is envisaged as a trend for future design philosophy. This paper is devoted to a design approach for practical structures where failure load is sensitive to initial imperfection. Although it is not feasible to reconstruct the actual imperfection mode of some roof which collapsed in the past for further investigation, it appears that many collapses were partly due to an overlooking of the most dangerous imperfection mode in the structure when under an unsymmetrical buckling mode. Its design is not generally covered in national design codes.

There are basically two approaches making use of computers to solve design problems. Firstly, the formulae in the design code are programmed and thus the tedious procedure can be carried out by the machine. This method still requires the assumption of effective length and cannot allow for the reduction in the stiffness of the supporting members under loads as the design and analysis procedures have been separated. In this procedure, the analysis part is linear and simple, but the design part is complicated and inaccurate. The second approach makes use of a rigorous second-order analysis simulating the true behavior of the structure itself. The computed stress, allowing for second-order terms such as the $P-\delta$ and the $P-\Delta$
effects, can be directly compared with the design strength of the factored yield stress or the section capacity check, as shown in Figure 1 and Figure 2. In this approach, the analysis is complicated and required the consideration of all features of a practical structure but the analysis result can be directly used for design.

Figure 1: Conventional procedure

Figure 2: Proposed procedure

Unlike the conventional design method based on the first-order linear theory with extensive use of code tables and charts, the simulation-based design relies on the use of structural mechanics to predict the stability and strength of structures. To this, imperfection must be assumed on a statistical basis and reasonable assessment on the loads and various effects in the life of a structure should be made. Consideration must be given to the handling of hundreds or even more than a thousand load cases and scenarios as well as temperature and support settlement effects. On the contrary, the use of the old cubic elements is inadequate when dealing with buckling and large displacement problems.

AN ADVANCED ELEMENT FOR SECOND-ORDER ANALYSIS

The theory and procedure of using regularly the second-order analysis and design method is summarized in this paper. The imperfect stability element (see Figure 3) derived by Chan and Gu [9] is selected for illustration of the new method. It should be pointed out that an advanced element for second-order analysis must allow for initial member bowing so that the P-δ effect due to initial conditions and applied loads can be captured.

Figure 3: The basic force vs displacement relations in an element

The equilibrium equation along the element length can be expressed as,

$$EI \frac{d^2 v_1}{dx^2} = -P(v_0 + v_1) + \frac{M_1 + M_2}{L} x - M_1$$

in which $EI$ is the flexural rigidity, $M_1$ and $M_2$ are the nodal moments and $v_1$ is the lateral displacement induced by loads. Making use of the boundary conditions and via a series of
mathematical procedure, we have

$$M_1 = \frac{EI}{L} \left[ c_1 \theta_1 + c_2 \theta_2 + c_0 \left( \frac{v_{mo}}{L} \right) \right]$$

(2)

$$M_2 = \frac{EI}{L} \left[ c_2 \theta_1 + c_1 \theta_2 - c_0 \left( \frac{v_{mo}}{L} \right) \right]$$

(3)
in which $\theta_1$ and $\theta_2$ are the nodal rotations, $c_1$, $c_2$ and $c_0$ are stability functions and more details can be found in the original reference [9].

The shortening due to bowing of initial imperfection and deflection is coupled into the axial force $P$ as below,

$$P = EA \varepsilon = EA \left[ \frac{u}{L} - b_1 (\theta_1 + \theta_2)^2 - b_2 (\theta_1 - \theta_2)^2 - b_{vs} \frac{v_{mo}}{L} (\theta_1 - \theta_2) - b_{vv} \left( \frac{v_{mo}}{L} \right)^2 \right]$$

(4)
in which $u$ is the nodal shortening, $b_1$, $b_2$, $b_{vs}$ and $b_{vv}$ are curvature functions (more details see reference [9]) and $A$ is the cross sectional area. These functions need to be repeatedly derived for the cases of positive, zero and negative values of axial force.

To consider the member initial curvature at the element level, the imperfection is first assumed to be in a half sine curve with an assigned amplitude at mid-span as follows (see also Figure 3).

$$v_0 = v_{mo} \sin \frac{\pi x}{L}$$

(5)
in which $v_0$ is the lateral deflection, $v_{mo}$ is the magnitude of imperfection at the mid-span, $x$ is the distance along the element longitudinal axis and $L$ is the element length.

It is obvious that the magnitude of imperfection in the proposed method affects the buckling strength of a column or a beam-column. The values of imperfections have been rationally calibrated against the existing buckling curves in design codes and the following values were obtained and used in the new Hong Kong Steel Code [10]. The four different $a$, $b$, $c$ and $d$ curves are checked and below shows the buckling curve generated by NIDA [11] against the code result for a pin-pin column using one element per member. Figure 4 shows the two sets of results match closely with the second-order analysis slightly more conservative.

In the design context, one needs to identify a condition for failure of a member which is the yield function of the cross section. When dealing with the plastic design, the geometric
nonlinear equilibrium path is traced by incrementing the load in Equation 6 until the sectional capacity factor $\phi$ given below is larger than 1 as,

$$\frac{P}{p_y A} + \frac{(M_y + P\Delta_y + P\delta_y)}{M_{y_y}} + \frac{(M_z + P\Delta_z + P\delta_z)}{M_{z_y}} = \phi \leq 1$$

where, $\Delta = $ nodal displacement due to out-of-plumpness frame imperfections plus sway induced by loads in the frame; $\delta = $ displacement due to member curvature / bowing due to initial imperfection and load at ends and along member length of a member; $A = $ cross sectional area; $p_y = $ design strength; $M_{y_y}, M_{z_y} = $ plastic or yield moments about principal y- and z-axes (i.e. $M_i = p_y S_i$, $S = $ plastic or elastic modulus); $M_y, M_z = $ external moments about principal y- and z-axes; $\phi = $ section strength factor. If $\phi > 1.0$, the member fails in section strength check which means that this member is inadequate.

ANALYSIS AND DESIGN OF A 220M X 90M ROOF STRUCTURES

The structure shown below has irregular form and the assumption of effective length becomes difficult. The proposed method is used for checking the buckling strength of the structure under various load scenarios. The contribution of the cladding element can also be considered in the analysis, which, however, is normally taken to a reduced dimension such as using 60% of the thickness of the panel for conservative design. Even with this reduced dimension, the contribution can improve significantly the stability of the structure and reflects more closely and accurately the actual response of the structure.

Figure 5: Three-dimensional view of the roof structure

Figure 6: Deformed shape of the structure at plastic stage with mark “•” representing plastic hinges (deflection enlarged)
CONCLUSIONS

A review on previous work on second-order analysis is reported in this paper. Some recent applications are described and the advantages of the new method are illustrated. One advantage of the method is on its automatic computation of primary linear and secondary nonlinear stress that manual estimation of a K-factor or effective length is not needed. The reduction of member stiffness in the presence of axial load is also allowed for in the stress computation and analysis. Global and local instability can be accounted for automatically in the computer approach. It has been demonstrated that this method can be used for design of practical steel structures efficiently, accurately and economically. The successful application of the proposed “NIDA” indicates the era of integrated design and analysis by second-order analysis has come and a revolution in the design office is expected.

ACKNOWLEDGEMENT

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REFERENCES

SIMULATION OF THE IMPACT EFFECT IN PROGRESSIVE COLLAPSE OF MULTI-STOREY STRUCTURES

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KEYWORDS
Progressive collapse, Impact, Virtual spring, Virtual damper

ABSTRACT

A simple method that builds on virtual mass-spring-damper system is proposed to analyze the pancake-like progressive collapse of multi-storey structures. In the model, columns and beams are considered separately to analyze various collapse modes. A virtual spring and a virtual damper are placed in between two storeys to simulate the impact effect. Local failure, such as beam-to-beam connection failure and beam-to-column connection failure are considered. Although the process of progressive collapse involves a large number of complex mechanisms, the proposed model provides a simple numerical tool to assess the overall behavior of collapse arising from a few initiating causes.

INTRODUCTION

Progressive collapse occurs when some members are loaded beyond their intended capacities due to the change of loading pattern or boundary conditions. The behavior of structures in progressive collapse has attracted much attention since last century and several approaches have been proposed to include structural resistance to progressive collapse in building design (Pretlove et al. [1], Virdi et al. [2]). Briefly, the current methods used to study progressive collapse can be classified into two main categories, viz. quasi-static approach (Jones [3]) and structural dynamics (Izzuddin et al. [4], Bao et al. [5], Colombo [6], Fu [7]). Compared with a full dynamics analysis, the formulation in a quasi-static approach is the same as regular static analysis, thus the computer time required to obtain numerical solutions is reduced significantly due to a lack of dynamic terms in the governing equations. However, quasi-static approaches may not be conservative because they ignore the impulsive effects of energy released in the process of overloaded members failing and impacting upon the storeys
Below. Therefore, numerical simulations based on structural dynamics have been developed and become an alternative and possibly more accurate approach in the past two decades (Itoh et al. [8], Kaewkulchais et al. [9], Kim et al. [10], Tosaka et al. [11], Isobe et al. [12], Isobe et al. [13]). On the other hand, such kind of extensive numerical treatment of progressive collapse may not be required by engineers at the preliminary design stage since the details of structural collapse modes are very complex and their clarification would require considerable computational effort. To achieve a good balance between accuracy and efficiency, a simplified model based on one dimensional spring-mass-damper system was developed to explore the effects of a few relevant parameters in the progressive collapse analyses (Yuan et al. [14]). To make the problem more tractable and amendable by mechanics, this one-dimensional model adopts some simple assumptions. For instance, in the collapse of a multi-storey structure, each storey is treated as an equivalent component including a mass, a virtual spring and a virtual damper. However, it was reported that columns and beams may not fail simultaneously in reality. An example of progressive collapse shows that a portion of the floor on the 17th storey concrete slab caved in and plummeted downwards, sparking off a domino effect that brought down the 16 slabs below (Lim et al. [15]). Hence, a new one dimensional mass-spring-damper model in which columns and beams can fail individually is proposed in this study. By this model, the nature of progressive collapse once initiated, and whether the fall is self-arresting or propagating leading to domino effect can be simulated.

**MASS-SPRING-DAMPER SYSTEM**

As shown in Fig. 1, it is assumed in this study that a complex multi-storey structure can be idealized as a typical frame which consists of three components, viz. columns, beams and beam-to-column connections. If this frame is described by a mass-spring-damper system given in Fig. 2, the following two dynamical equations can be established:

\[
\begin{align*}
    m_{i(i)} \ddot{x}_{i(i)} &+ k_{i(i)} (x_{i(i)} - x_{i(i-1)}) + c_{i(i)} (\dot{x}_{i(i)} - \dot{x}_{i(i-1)}) + k_{2(i)} x_{2(i)} + c_{2(i)} \dot{x}_{2(i)} \\
    -k_{i(i-1)} (x_{i(i-1)} - x_{i(i)}) - c_{i(i+1)} (\dot{x}_{i(i+1)} - \dot{x}_{i(i)}) &= -m_{i(i)} g
\end{align*}
\]

\((1)\)

Figure 1: A typical multi-storey frame  
Figure 2: A mass-spring-damper system
where the letters \(m\), \(c\) and \(k\) represent mass, damping and stiffness, respectively. \(x\) denotes the vertical location of structural members in the coordinate system. The terms \(\dot{x}\) and \(\ddot{x}\) are the first and the second derivatives of \(x\) with respect to time \(t\). In the subscripts of these terms, the letters inside the brackets denote a particular storey, while the numerals outside the brackets are used to indicate the properties of these components. In this manuscript, 1, 2 and 3, indicate column-to-column, beam-to-column and beam-to-beam connections, respectively. As an example, the term \(k_{i(l-1)}\) represents the stiffness of the column-to-column spring located at the \((i-1)^{th}\) storey.

Assuming a structure has \(N\) storeys, Eq. (1) and (2) can be expressed by a compact form that behaves like

\[
[M\ddot{X} + C\dot{X} + KX = F, \quad \text{where} \quad M, \quad C \quad \text{and} \quad K \quad \text{are mass, damping and stiffness matrices. The term} \quad F \quad \text{denotes external forces. If only gravity force is considered,} \quad F_i = -m_{i(i)}g \quad \text{and} \quad F_{i,N} = -m_{2(i)}g \quad \text{for} \quad (i = 1, N). \quad \text{Up to today, quite a number of schemes have been developed to solve Eq. (1) and (2). In this study, the standard central difference scheme is applied in time domain to convert Eq. (1) and (2) into algebraic equations.}
\]

**ASSUMPTIONS FOR STIFFNESS AND DAMPING**

To overcome the difficulty in the calculation of contact force between different storeys, the terms \(k_{j(i)}\) and \(c_{j(i)}\) \((i, N \text{ and } j = 1, 2)\) are defined by Eq. (3) and (4):

\[
c_{j(i)} = \varepsilon c_{j(i)}[1 - H(u_{j(i)} - u_{j-min})] + \varepsilon c_{j(i)} \alpha H(u_{j(i)} - u_{j-max}), \quad (i = 1, N \text{ and } j = 1, 2) \tag{3}
\]

\[
k_{j(i)} = \varepsilon k_{j(i)}[1 - H(u_{j(i)} - u_{j-min})] + \varepsilon k_{j(i)} \beta H(u_{j(i)} - u_{j-max}), \quad (i = 1, N \text{ and } j = 1, 2) \tag{4}
\]

where \(u_{j(i)}\) indicates the change of the length of the \(j^{th}\) spring located at the \(i^{th}\) storey. The terms \(\varepsilon c_{j(i)}\) and \(\varepsilon k_{j(i)}\) represent the equivalent damping and stiffness for a particular damper and spring, respectively. \(H(\cdot)\) is the Heaviside function, in which \(u_{j-min}\) and \(u_{j-max}\) are two constants. \(\alpha\) and \(\beta\) are also constant factors and will be explained later.

According to Eq. (3), the damping \(c_{j(i)}\) is equal to \(\varepsilon c_{j(i)}\) if \(u_{j(i)} \leq u_{j-min}\). However, it becomes zero once \(u_{j-min} < u_{j(i)} \leq u_{j-max}\). If \(u_{j(i)} > u_{j-max}\), \(c_{j(i)}\) takes the value of \(\alpha \varepsilon c_{j(i)}\). From Eq. (4), one obtains a similar interpretation for \(k_{j(i)}\). Based on Eq. (3) and (4), the failure of the \(i^{th}\) storey of a multi-storey structure can be classified into three categories:

(a) only \(k_{j(i)}\) becomes zero under \(u_{t-min} < u_{j(i)} \leq u_{t-max}\). This is to simulate that the \(i^{th}\) storey fails to resist any applied loading. At this situation, \(c_{j(i)}\) is also set to zero to simulate a free-falling procedure. However, when the distance between the \(i^{th}\) and \((i-1)^{th}\) storeys
approaches to a small value ($u_{i(i)} > u_{i-max}$), which means the two storeys come into contact with each other, $k_{i(i)}$ takes on the value of $\beta \cdot c_{i(i)}$, and $c_{i(i)}$ becomes $\alpha \cdot c_{i(i)}$, where $\beta$ and $\alpha$ may be very large.

(b) only $k_{2(i)}$, becomes zero under $u_{2-min} < u_{2(i)} \leq u_{2-max}$. This is to simulate the failure of the beam-to-column connection at the $i$th storey. At this situation, $c_{2(i)}$ is also set to zero to guarantee that the $i$th slab becomes a free-falling object before it impacts onto the $(i-1)$th slab. If the $i$th and $(i-1)$th storeys contact each other ($u_{3(i)} > u_{3-max}$), $k_{3(i)}$ and $c_{3(i)}$ become $\beta \cdot c_{3(i)}$ and $\alpha \cdot c_{3(i)}$, respectively.

c) both $k_{1(i)}$ and $k_{2(i)}$ fail. This is a combination of (a) and (b).

**NUMERICAL EXAMPLES**

A 9-storey frame with a local damage at the 5th story is taken as a numerical example. The consequent dynamic response of the frame will be investigated. To make the simulations more reasonable, a realistic steel frame is designed to withstand conventional dead and live loads. For simplicity, the geometric dimensions and material properties of all stories are the same. The relevant information of the structure is listed in Table 1. In this example, an equivalent stiffness $E_{i(i)}$ ($i = 1, 9$) is defined by $E_{i(i)} = 2E_i \cdot c_{i(i)}^2 / c_{i(i)} = 29.25 \times 10^7$ N/m for reference. Meanwhile, the uniformly distributed load applied to each story is set to $q = 8\text{kN/m}$. Hence, the total mass of the $i$th story is assumed to be $m_{1(i)} + m_{2(i)} = 4.9 \times 10^3$ kg ($i = 1, 9$). In this manuscript, it is assumed without loss of generality that $m_{1(i)}/m_{2(i)} = 0.25$.

As a reference, a critical load for the $i$th story is defined to be $P_{i-cr} = 8\pi^2 E_i \cdot c_{i(i)}^2 L_{i(i)} / (c_{i(i)}^2)$, which is equal to Euler buckling load of two columns with fixed ends. In total, six typical scenarios listed in Table 2 are studied. Among the case studies, the cause of the collapse in each of the first 3 scenarios is the failure of the 5th storey beam-to-beam connection. Both the columns and the beam-to-column connections are stiff, but the bearing capacity of the beam-to-column connections varies. In each of the last 3 scenarios, the collapse of the frame is induced by the failure of the 5th storey column. The bearing capacity of the beam-to-column connections is constant, but their stiffness changes.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>MEMBER AND MATERIAL PROPERTIES OF STEEL FRAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column (UC 152×152×23)</td>
<td>Beam (UB 254×146×31)</td>
</tr>
<tr>
<td>$c_A = 29.25\text{cm}^2$, $c_I = 1249.8\text{cm}^4$</td>
<td>$b_A = 39.69\text{cm}^2$, $b_I = 4413.4\text{cm}^4$</td>
</tr>
<tr>
<td>$c_L = 4.0\text{m}$, $E = 2.0 \times 10^{11}\text{Pa}$</td>
<td>$b_L = 6.0\text{m}$, $E = 2.0 \times 10^{11}\text{Pa}$</td>
</tr>
</tbody>
</table>
TABLE 2
PARAMETERS USED IN CASE STUDIES

<table>
<thead>
<tr>
<th>Case No.</th>
<th>$k_{i(j)}$ and $p_{i(j)-cr}$ ($i = 1,9$ and $j = 1,3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$k_{i(j)} = \text{ref}k_{i(j)}$, $k_{2(i)} = \text{ref}k_{i(j)}$, $k_{3(i)} = 0$; $p_{i(j)-cr} = \text{ref}p_{i(j)-cr}$, $p_{2(i)-cr} = 0.1\text{ref}p_{i(j)-cr}$, $p_{3(i)-cr} = \infty$</td>
</tr>
<tr>
<td>2</td>
<td>$k_{i(j)} = \text{ref}k_{i(j)}$, $k_{2(i)} = \text{ref}k_{i(j)}$, $k_{3(i)} = 0$; $p_{i(j)-cr} = \text{ref}p_{i(j)-cr}$, $p_{2(i)-cr} = \text{ref}p_{i(j)-cr}$, $p_{3(i)-cr} = \infty$</td>
</tr>
<tr>
<td>3</td>
<td>$k_{i(j)} = \text{ref}k_{i(j)}$, $k_{2(i)} = 0.1\text{ref}k_{i(j)}$, $k_{3(i)} = 0$; $p_{i(j)-cr} = \text{ref}p_{i(j)-cr}$, $p_{2(i)-cr} = 0.2\text{ref}p_{i(j)-cr}$, $p_{3(i)-cr} = \infty$</td>
</tr>
<tr>
<td>4</td>
<td>$k_{i(j)} = \text{ref}k_{i(j)}$, $k_{2(i)} = 0.5\text{ref}k_{i(j)}$, $k_{3(i)} = 0$; $p_{i(j)-cr} = \text{ref}p_{i(j)-cr}$, $p_{2(i)-cr} = 0.8\text{ref}p_{i(j)-cr}$, $p_{3(i)-cr} = \infty$</td>
</tr>
<tr>
<td>5</td>
<td>$k_{i(j)} = \text{ref}k_{i(j)}$, $k_{2(i)} = 0.3\text{ref}k_{i(j)}$, $k_{3(i)} = 0$; $p_{i(j)-cr} = \text{ref}p_{i(j)-cr}$, $p_{2(i)-cr} = 0.8\text{ref}p_{i(j)-cr}$, $p_{3(i)-cr} = \infty$</td>
</tr>
</tbody>
</table>

Scenario 1: Stiff beam-to-column connection with low bearing capacity

The result is depicted in Fig. 3. It can be seen that all columns remain intact after the failure of the beam-to-column connection at the 5th storey. However, all the beams beneath the 5th storey collapse due to domino effect.

Scenario 1 indicates that in the case of a beam-to-column connection failure, column failure may not occur if the bearing capacity of beam-to-column connections is very low. In fact, this observation has happened in reality. It was reported that the columns in a building were kept intact when all 16 floors crashed through the building in a domino effect to slam onto the ground due to the falling of a portion of the floor on the 17th storey [15].

Scenario 2: Stiff beam-to-column connection with high bearing capacity

Only the beam-to-column connection at the 5th storey fails at the beginning. However, as shown in Fig. 4, this does not lead to subsequent progressive collapse. Compared with Scenario 1, the stiffness of beam-to-column connection does not change, but its bearing capacity is ten times higher than that in Scenario 1. Since the beam-to-column connection is much stronger, they are not easily damaged. Hence, the falling of the 5th storey is stopped by the 4th storey. At the same time, the impact between the 5th and 4th beams is transferred to columns, but the frame is still able to remain intact.

Scenario 3: Stiff beam-to-column connection with moderate bearing capacity

Only the beam-to-column connection at the 5th storey fails at the beginning. However, as shown in Fig. 5, the columns also collapse due to the impact between beams. Compared with Scenarios 1 and 2, the stiffness of beam-to-column connection does not change, but its bearing capacity is two times higher than that in Scenario 1. After the 5th storey beam hits on the 4th storey beam, the 4th storey beam-to-column connection fails due to impact since it is not as strong as that in Scenario 2. Following this failure, all beams below the 4th storey
experience a domino collapse. Due to the failures of the beam-to-column connections, columns at the 3rd, 4th and 5th storeys merge to form a long column but still remain intact. After that, this long column is further weakened when the 2nd storey beam-to-column connection fails. Subsequently, the whole structure collapses when the 2nd storey impacts onto the 1st storey.

**Scenario 4: Very flexible beam-to-column connection with high bearing capacity**

In this scenario, the local damage is the failure of the 5th storey column. Although the beam-to-column connections are much more flexible than the columns, the curves in Fig. 6 show that the progressive collapse does not happen.

**Scenario 5: Stiff beam-to-column connection with high bearing capacity**

The local damage is the same as in Scenario 4. Compared with Scenario 4, the bearing capacity of beam-to-column connection does not change, but its stiffness is five times greater than that in Scenario 4. From Fig. 7, one finds that the whole structure collapses. It can be seen that some beam-to-column connections also fail during the collapse.

**Scenario 6: Flexible stiff beam-to-column connection with high bearing capacity**

The local damage is the same as in Scenarios 4 and 5. However, the beam-to-column connections are stiffer than that in Scenario 4, but more flexible than that in Scenario 5. Similar to Scenario 5, the whole structure collapses due to impact as shown in Fig. 8. However, it should be noted that no beam-to-column connection fails as the collapse progresses.

Based on the analyses about Scenario 1, 2 and 3, one concludes that in the case of a beam-to-column connection failure, the bearing capacities of beam-to-column connections dominate the ensuing structural response of the multi-storey frame. It can be seen that progressive collapse of beams occurs but it does not cause column failure if beam-to-column connections are very weak. Generally, as shown in Scenario 2, beam-to-column connections with high ultimate strength are helpful to prevent progressive collapse. However, one should apply this concept restrainedly in structural design since Scenario 3 shows that beam-to-column connections with higher ultimate strength lead to a completed collapse as compared with Scenario 1. Scenarios 4, 5 and 6 are used to investigate the cases in which a column fails at the beginning. Numerical results reveal that flexible beam-to-column connections are helpful to mitigate the risk of progressive collapse. It is found that stiffer beam-to-column connections, under particular bearing capacities, make the structure more critical.

![Figure 3: Collapse in Scenario 1](image1)

![Figure 4: Collapse in Scenario 2](image2)
CONCLUSIONS

This manuscript proposes a numerical mode to simulate progressive collapse of multi-storey buildings. The model is established based on simplifying assumptions. Although such a model can hardly make detailed analyses at the member level, it can be used to study qualitatively the overall behavior of buildings. For instance, several observations have been obtained through the numerical examples: (1) a local member failure in a multi-storey structure may lead to a global progressive collapse. (2) if the initial local damage is the falling of a beam, beam-to-column connections with high bearing capacities are not always good. (3) if the initial local damage is the failure of a column, flexible beam-to-column connections are more helpful to structural safety than stiff beam-to-column connections when they have same bearing capacities. (4) in dynamic cases, flexible structures with high strength have advantages to keep away from progressive collapse.

Obviously, the proposed model is far from a mature approach. To make it more substantial, it is suggested that the future work focuses on the following areas, viz. (1) develop a method to determine the ultimate loads of columns and beams under various conditions, and then incorporate the criteria into the proposed model. (2) extend the present model to 3 dimensional problems.
REFERENCES


ITERATIVE METHOD FOR ESTIMATING COLLAPSE LOADS OF STEEL CABLE-STAYED BRIDGES

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KEYWORDS
Cable-stayed bridge, Structural stability, Collapse loads, Buckling, Inelastic buckling analysis, Beam-column interaction, Tangent stiffness

ABSTRACT
This paper proposes a new and simple method for estimating the collapse load of a steel cable-stayed bridge. A new convergence criterion for iterative eigenvalue computations is suggested to consider the beam-column effect of a cable-stayed bridge system. The collapse loads of example bridges, which have center spans of 600-m, 900-m and 1200-m with different girder depths, are evaluated by the proposed method and a nonlinear inelastic analysis. The results demonstrate that the proposed method is a good substitute for a complex nonlinear inelastic analysis to approximately evaluate the collapse loads as well as failure modes of steel cable-stayed bridges.

INTRODUCTION
A cable-stayed bridge is distinguished from conventional highway bridges by special features, including the use of the longer center span without intermediate piers, as well as light-weight girders and high strength cables. In an economical sense, bridge engineers and researchers had considered this kind of a bridge to be superior to medium and short-span bridges with several intermediate piers, for constructing long-span bridge crossings. In recent years, engineers have also planned diverse variations of a typical cable-stayed bridge system to fulfill aesthetic desires for scenic views encompassing landmark structures within their hometown surroundings, as well as the economical aspects of the bridge system. In the first decade of this century, pioneers of bridge engineering had continuously developed innovative design and construction technologies to extend the practical span limitation of a cable-stayed...
bridge system. As a consequence of these efforts, two projects constructing super long-span cable-stayed bridges, which have a center span of more than 1000 m without intermediate piers, were planned and are now close to completion: the Sutong Bridge (center span of 1088 m, China) and the Stone Cutters Bridge (center span of 1018 m, Hong Kong).

As the center span of cable-stayed bridges increases and the weight of steel girders is reduced, two major issues are naturally addressed: buckling instability and wind instability. The buckling instability of steel girder and tower members is caused by the large axial forces that are transmitted by cables under dead and live loads, whereas the wind instability of steel girder members comes from lateral wind loads. In particular, the buckling instability of girder and tower members may be a fundamental problem that should be checked in the preliminary design of a cable-stayed bridge because it directly controls the geometric dimensions of structural members and the practical limitation of the center span length.

This paper proposes a new method for estimating the collapse loads of steel cable-stayed bridges. Based on the fundamental concept of the inelastic buckling analysis previously established by the authors, we widen the range of application for the method by suggesting a new criterion of each structural member in the bridge system. The proposed method determines the tangent stiffness of each structural member in the bridge system by iterative eigenvalue computations with the classical tangent modulus theory. In addition, an improved convergence criterion for girder and tower members is proposed to take into account the beam-column interactions. After summarizing theoretical substances, we analyze the example bridges that have center spans of 600-m, 900-m, and 1200-m with different girder depths. To show the validity and applicability of the method, the results of the proposed method are compared with those of the established inelastic buckling analysis and a nonlinear inelastic analysis. Some discussions are also made about the effect of the girder depth on the collapse load and the failure modes of the example bridges.

PROPOSED INELASTIC BUCKLING ANALYSIS

In this section, we briefly illustrate the fundamental theory of the established inelastic buckling analysis as an alternative for a nonlinear inelastic analysis. We propose a new criterion for a beam-column member for the inelastic buckling analysis by incorporating the axial-flexural interaction equations. Analysis procedures for the inelastic buckling analysis are also given, utilizing the proposed criterion for a beam-column member.

Proposed criterion for a beam-column member

For a typical beam-column member, the stability of a member is usually checked by the axial-flexural interaction equation as

$$\frac{P}{P_a} + \frac{M_y}{M_{yy}} + \frac{M_z}{M_{zz}} = 1.0$$

(1)

where the terms $P_a$ represent the axial resistance of a beam-column member. After some modifications, we may use Eqn. 1 as a new criterion for the inelastic buckling analysis. At a specific load state, the terms of axial force and moments of a member in Eqn. 1 may be written with the eigenvalue from a conventional eigenvalue analysis as
where the terms $P^0$, $M_y^0$, and $M_z^0$ are the member force and moments that are calculated from a linear stress analysis. The axial resistance $P_x$ of a member is also described as $P_x = P_x^c$, where the term $P_x^c$ is the inelastic critical load of a member calculated from the column strength curve at the $i$-th iteration. We assumed that the plastic moments of a member are not affected by the iterations in iterative eigenvalue computations. By substituting Eqn. 2 and the term of the axial resistance into Eqn. 1, we obtain the criterion for a beam-column member as

$$E_i^{+1} = \frac{\kappa' P^0 + \kappa' M_y^0 + \kappa' M_z^0}{P_x^c + M_y^0 + M_z^0} \approx 1.0$$

(3)

By reversing the numerator terms and denominator terms of Eqn. 3, the tangent modulus of a beam-column member can be obtained at $i$-th iteration steps as

$$E_i' = \frac{P_x^c M_y M_z}{\kappa' P^0 M_y M_z + P_x^c \kappa' M_y^0 M_z + P_x^c M_y^0 M_z} E_i^{+1}$$

(4)

It can be seen that the criterion of Eqn. 4 for a beam-column is equivalent to the criterion of a column member when the moments are not exerted on a member. Therefore, we can say that Eqn. 4 is the general convergence criterion for a column and beam-column member in the inelastic buckling analysis, and may be used for structural members in a steel cable-stayed bridge system without any theoretical discrepancy.

**Procedures of the inelastic buckling analysis**

As previously mentioned, the inelastic buckling analysis adopted in this paper is based on the bifurcation-point stability concept. The basic equation of the inelastic buckling analysis is similar to that of conventional elastic buckling analysis [1, 3] except for the elastic stiffness matrix term of the structure, which is

$$\det \left[ [K_s(E_i)] + \kappa [K_x] \right] = 0$$

(5)

where $[K_s(E_i)]$ and $[K_x]$ are the modified stiffness matrix and geometric stiffness matrix of the global bridge system corresponding to eigenvalues of $\kappa$, respectively. The element stiffness matrices, which are combined into the global stiffness matrices in Eqn. 5, can be derived by using the principle of virtual work [2]. Figure 1 presents the flow-chart for the computer program developed in this study.
STABILITY ANALYSIS OF THE EXAMPLE CABLE-STAYED BRIDGES

Example bridge models

In order to validate the proposed inelastic buckling analysis, we analyzed 2-dimensional example bridge models using both the nonlinear inelastic analysis and the proposed method. The center spans of the example bridges are 600-m, 900-m and 1200-m. The side span length is about half that of the center span in each bridge model. The cables are arranged as a fan type and fixed in the girders at 20-m intervals. The ratio of tower height measured from the decks to the center span is selected as one fifth. The yield strength of the steel for girder and tower members is 450 MPa and it is assumed to be 1600 MPa for cables. The intersection between the left tower and the girder is supported by a fixed hinge, whereas other supports are modeled by movable hinges. Some intermediate supports are also installed in the side spans of each example bridge. Figure 2 presents the elevation view of the numerical models of example bridges used in this study. Table 1 shows the geometric and material properties of structural members. In addition, Table 2 gives the values of the dead and live loads applied to example bridges.
Figure 2: Numerical models of the example cable-stayed bridges

TABLE 1
GEOMETRIC PROPERTIES OF EXAMPLE CABLE-STAYED BRIDGES

<table>
<thead>
<tr>
<th>Model</th>
<th>Girders</th>
<th>Tower</th>
<th>Cables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H (m)</td>
<td>A (m²)</td>
<td>I (m⁴)</td>
</tr>
<tr>
<td>600-m</td>
<td>1.0</td>
<td>1.296</td>
<td>0.296</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.396</td>
<td>1.239</td>
</tr>
<tr>
<td>900-m</td>
<td>3.0</td>
<td>1.496</td>
<td>2.880</td>
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<tr>
<td></td>
<td>4.0</td>
<td>1.596</td>
<td>5.270</td>
</tr>
<tr>
<td>1200-m</td>
<td>5.0</td>
<td>1.696</td>
<td>8.457</td>
</tr>
<tr>
<td></td>
<td>6.0</td>
<td>1.796</td>
<td>12.492</td>
</tr>
</tbody>
</table>

TABLE 2
DEAD LOADS AND LIVE LOADS OF THE EXAMPLE CABLE-STAYED BRIDGES

<table>
<thead>
<tr>
<th>Models</th>
<th>Girders load (DC)</th>
<th>Girders live load (LL)</th>
<th>Towers live load (LL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>(kN/m)</td>
<td>(kN/m)</td>
<td>(kN/m)</td>
</tr>
<tr>
<td>600-m</td>
<td>1.0 239.60</td>
<td>144.07</td>
<td>129.54</td>
</tr>
<tr>
<td>900-m</td>
<td>2.0 252.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1200-m</td>
<td>3.0 265.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.0 278.40</td>
<td>185.46</td>
<td>129.54</td>
</tr>
<tr>
<td></td>
<td>5.0 291.34</td>
<td>226.84</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.0 304.27</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Collapse loads of example bridges**

We evaluated collapse loads of all example bridges by analyzing bridge models with inelastic buckling analyses and the nonlinear inelastic analysis. Table 3 shows the collapse loads and buckling mode shapes of example bridges with a girder depth of 2-m.

### TABLE 3
BUCKLING MODE SHAPES OF EXAMPLE BRIDGES BY INELASTIC BUCKLING ANALYSES (GIRDER DEPTH = 2 M)

<table>
<thead>
<tr>
<th>Models</th>
<th>Criterion for a column (Eqn. 14)</th>
<th>Proposed criterion for a beam-column (Eqn. 18)</th>
</tr>
</thead>
<tbody>
<tr>
<td>600 m</td>
<td>$\kappa_{\text{conv}}=3.90$</td>
<td>$\kappa_{\text{conv}}=3.24$</td>
</tr>
<tr>
<td>900 m</td>
<td>$\kappa_{\text{conv}}=2.35$</td>
<td>$\kappa_{\text{conv}}=2.16$</td>
</tr>
<tr>
<td>1200 m</td>
<td>$\kappa_{\text{conv}}=1.82$</td>
<td>$\kappa_{\text{conv}}=1.78$</td>
</tr>
</tbody>
</table>

The buckling mode shapes in Table 3 do not indicate the real deformed state at the collapse of the bridge system in that the buckling analysis with eigenvalue computations merely gives the information of shapes in the post-buckling state, but does not determine the amplitudes of shapes. Nevertheless, we can at least recognize the fact that the girder members, which are near to the left tower, are the most critical members in the example bridges from these buckling mode shapes.

On the other side, the collapse loads by the nonlinear inelastic analysis were calculated from the load-displacement curve of the global bridge systems in Figure 3. The vertical axis in Figure 3 indicates the load factor that means the ratio of the current load amplitudes to the initial load amplitudes. In addition, Table 4 summarizes collapse load factors of example bridges explicitly with respect to analysis methods, girder depths and center spans for the sake of completeness.

![Figure 3: Load-displacement curves of example bridges by nonlinear inelastic analysis](image)
### TABLE 4
COLLAPSE LOAD FACTORS OF EXAMPLE CABLE-STAYED BRIDGES

<table>
<thead>
<tr>
<th>Model Depth</th>
<th>Inelastic buckling analysis</th>
<th>Nonlinear elasto-plastic analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column criterion</td>
<td>Proposed criterion</td>
</tr>
<tr>
<td>600-m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.11</td>
<td>2.77</td>
</tr>
<tr>
<td>2</td>
<td>3.90</td>
<td>3.24</td>
</tr>
<tr>
<td>3</td>
<td>4.00</td>
<td>3.02</td>
</tr>
<tr>
<td>4</td>
<td>3.94</td>
<td>2.97</td>
</tr>
<tr>
<td>5</td>
<td>3.87</td>
<td>2.93</td>
</tr>
<tr>
<td>6</td>
<td>3.81</td>
<td>2.86</td>
</tr>
<tr>
<td>900-m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.78</td>
<td>1.71</td>
</tr>
<tr>
<td>2</td>
<td>2.35</td>
<td>2.16</td>
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<tr>
<td>3</td>
<td>2.57</td>
<td>2.31</td>
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<td>4</td>
<td>2.71</td>
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<td>5</td>
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</tr>
<tr>
<td>6</td>
<td>3.04</td>
<td>2.47</td>
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<tr>
<td>1200-m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.22</td>
<td>1.18</td>
</tr>
<tr>
<td>2</td>
<td>1.82</td>
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<tr>
<td>3</td>
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<td>1.95</td>
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<tr>
<td>5</td>
<td>2.15</td>
<td>1.98</td>
</tr>
<tr>
<td>6</td>
<td>2.22</td>
<td>1.92</td>
</tr>
</tbody>
</table>

**SUMMARY AND CONCLUSIONS**

This paper proposed an alternative for complex nonlinear inelastic analysis to estimate the collapse load of a steel cable-stayed bridge. A new criterion for a beam-column member was proposed based on the axial-flexural interaction equation in combination with the classical tangent modulus theory and the column-strength curve. The example bridge models, which had different center-spans with different girder depths, were studied for verification.

**ACKNOWLEDGEMENTS**

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REFERENCES


ASSESSMENT OF PROGRESSIVE COLLAPSE IN MULTI-STOREY BUILDINGS – INFLUENCE OF MATERIAL RATE SENSITIVITY

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KEYWORDS
Progressive collapse, composite construction, material rate sensitivity, connection modelling

ABSTRACT
A simplified framework for progressive collapse assessment of multi-storey buildings, considering sudden column loss as a design scenario, was previously developed at Imperial College London. This framework accounted for the nonlinear dynamic (pseudo-static) structural response but ignored the effects of material rate sensitivity. This paper proposes a modification of the previous method to consider the duration of the action and thus quantify the overstress resistance of the beam and joint components as a function of the strain rate, through models such as those of Malvern or Cowper-Symonds. A novel method is proposed for determining the displacement rate for different types of gravity loading that would act under sudden column loss. The resulting velocity can be converted to the level of connections, allowing the evaluation of representative deformation rates for the connection components. In turn, these deformation rates can be used to evaluate the enhanced resistance of the connections due to rate-sensitivity. This enhanced approach has been incorporated within the previously developed framework for progressive collapse assessment, where it is shown that the influence of material rate sensitivity can be included while maintaining the simplicity of the original method. The resulting enhanced framework is verified against detailed nonlinear finite element analysis, and is applied in several studies. In this respect, it is shown that the strain rate effect under progressive collapse conditions driven by gravity loading, while not as severe as under blast loading, can still be significant, enhancing the effective resistance by as much as 30%. Importantly, however, it is also shown that the strain rate effect can lead to undesirable brittle failures in typically designed connections due to the increased resistance of ductile components which can over-stress other less ductile components.
INTRODUCTION

A design-oriented framework for the assessment of building robustness under sudden column loss scenarios has been developed at Imperial College London by Izzuddin et al. [1], which provides a simplified approach based on energy balance that, starting from the nonlinear static response of the lowest level of structural idealisation of the affected bay, determines the maximum dynamic response for specific levels of gravity loading. Within this framework, the progressive collapse susceptibility of a building becomes a function of the ductility of the joints, which can be subject to large axial and rotation deformations, especially in partial strength connections. In view of such deformation demands, which occur over a relatively short duration, material rate sensitivity may play a significant role in enhancing structural resistance, though the counter effect is the well-established reduction in steel material ductility at high strain rates.

This paper aims at enhancing the previous progressive collapse assessment framework to account for rate sensitivity in the connections, thus enabling a more realistic assessment of the robustness limit state.

MATERIAL RATE SENSITIVITY

The rate sensitivity of steel is now well established, where dynamic overstress models (e.g.: Malvern or Cowper-Symonds) are widely used. Viscoplastic models are typically used, where the main features of the strain-rate effect for steel are (Figure 1):

1. the elastic modulus is unaffected,
2. the ultimate tensile strength increases slightly with strain rate,
3. the yield strength has a much higher increase, in comparison, and
4. the ultimate tensile strain can reduce with strain rate.

![Figure 1: Monotonic and reverse flow curves at low and high strain rates - [2]](image_url)

The progressive collapse assessment method developed by Izzuddin et al. [1] did not consider the duration over which the maximum dynamic deformations are developed, and hence was not concerned with deformation rates. This would be an important requirement for dealing with the influence of material rate sensitivity, as elaborated next.
TIME PREDICTION

The method developed in [1] determines the member capacity based on the simplifying principle of work balance for a SDOF system. Thus, through the static and pseudo-static responses [1] it is straightforward to predict the full response time from the initial stage to maximum dynamic deformed configuration. Because the dynamic strength enhancement is proportional to $a^{0.25}$ (acc. to Cowper-Symonds model), it is the order of the strain rate rather than its exact value, that is required and therefore the use of an average strain rate within the time domain is sufficiently accurate for application in the constitutive equation, as recommended in some design manuals (e.g. TM5-1300).

Physically, the kinetic energy should be equal to the difference between the work done by the load and the internal energy at a given displacement:

$$E_{k,n} = \alpha \lambda_n \frac{P_b}{\alpha} u_{d,n} - \int_{u_n}^{u_{d,n}} \frac{1}{2} \alpha \lambda_n \frac{P_b}{\alpha} \; du$$  \hspace{1cm} (1)

Using Eqn. 1, it is possible to obtain the kinetic energy profile for $n$ levels of gravity loading. Note that when the kinetic energy comes down to zero, the maximum dynamic displacement $u_{d,n}$ is achieved. Also, all the profiles obtained herein are over the displacement domain. Knowing the kinetic energy allows the time prediction for point load or UDL through the following four methods, presented in a descending order of accuracy:

5. Based on the equation $E_{k,n} = \frac{1}{2} \alpha \lambda_n \frac{P_b}{\alpha} u_{d,n}^2$, the velocity profile can be obtained, considering that $m = \alpha \lambda_n \frac{P_b}{\alpha}$. Accordingly, the time to the maximum dynamic displacement $u_{d,n}$ is given by:

$$t_{d,n} = \sum_{k=1}^{d} \frac{u_{d,n} - u_{d-1,n}}{v_{d,n} + v_{d-1,n}}$$ \hspace{1cm} (2)

6. A faster way to determine the time is to divide the maximum dynamic displacement by the average velocity from the velocity profile over the displacement domain:

$$t_{d,n} = \frac{u_{d,n}}{v_{d,n}}$$ \hspace{1cm} (3)

7. An incremental method that saves computational time and avoids the step division for every new dynamic displacement in the load-displacement curve is suggested. Observing the same curve, it is possible to realize that, for the next level of gravity loading $i$, the increment on kinetic energy is as represented in Figure 2a, which means that the kinetic energy profile is rotated, as seen in Figure 2b. The time can then be given by:

$$t_{d,n} = \frac{u_{d,n}}{\int_{u_{d,n}}^{u_{d,n-1}} \frac{E_{k,n} \; du}{\alpha \lambda_n \frac{P_b}{\alpha}}}$$ \hspace{1cm} (4)

where, $\int_{u_{d,n}}^{u_{d,n-1}} E_{k,n} \; du = \int_{u_{d,n-1}}^{u_{d,n-1}} E_{k,n-1} \; du + \frac{SP \times u_{d,n-1} \times u_{d,n}}{2}$
Figure 2: The third method consists in the incremental consideration of the integral of the kinetic that can be obtained for every new displacement. Areas 1 and 2 on figure (a) are equal and correspond to the increment of the kinetic energy for the n-1 dynamic displacement and to work done by $\delta P$.

Finally, the simplest way to determine the representative duration is to consider the maximum velocity arising from a linearly varying acceleration profile (1.5 or 1 $g$ reducing to 0):

$$\int_{u_n}^{u_{n+1}} a \cdot du = 1.5 \alpha \cdot r \cdot 1 \cdot g \times \frac{v_n^2}{2} = \frac{1}{2} m \cdot a = \sqrt{\frac{1}{1.5} \alpha \cdot r \cdot 1 \cdot g \times v_n}$$

(5)

Case study 1 (see Section 5) was used to produce the pseudo-static curve and then compare the previous four methods for a UDL load, as seen in Figure 3. The results show that relative to the first method at a load of around 33kN (for which the deviation appears to be the highest), the second method underestimates the total time by 12%, the third by 11% and the fourth clearly by 68%.

The recommendation is then to use the third method, with clear gains in processing time (took in average a quarter of the time taken by the first and second methods) and has a low loss in accuracy. The last method may present better results if the pseudo-static response curve is more linear than the one used in the example (see Figure 5b).
PROGRESSIVE COLLAPSE ASSESSMENT ACCOUNTING FOR RATE SENSITIVITY

The case study was developed in [3] and is used here for considering the influence of material rate sensitivity. The detailed bare steel bay model has the properties shown in Figure 4:

![Figure 4: Two dimensional bay model with L=6 m, universal steel beam S355 UB406x140x39 subjected to distributed load, bolt-rows idealized as springs with bilinear behaviour in tension (k₁ = 1%kₒ). This strain hardening factor was used to produce the same response as the one obtained in [3], but actual t-stubs present lower strain hardening factors (average of 0.2%), as in [4]](image)

Two cases were considered, one with bolt-rows rigid in compression and no axial restraint by the adjacent structure in both tension and compression (case 1), and another limiting the connection resistance in compression and considering the adjacent structure member axial stiffness and connection axial stiffness (case 2). The pseudo-static response (Figure 5 b)) was obtained from the nonlinear static response, which was established for proportionally varied UDL using ADAPTIC [5]. The midspan and support joints models, minor axis joints with a partial depth flexible end-plate connection, are described in [3].

![Figure 5: Geometry of the joint used by Vlassis [3] in beam-to-column connections and nonlinear static and pseudo-static responses obtained for the two cases considered](image)

Component level

According to EN 1993-1-8, for the minor axis beam to column end-plated connections, the elastic tensile stiffness of each bolt row is given by the assembly in series of the flexibility of the end-plate in bending and the bolts in tension (6.3.3.1(5) of EN 1993-1-8), and the yield limit is given by the minimum of the resistances of the end-plate in bending and the beam
web in tension. In this case study, the end-plate in bending is the critical component in the resistance of the connection and so, the detailed modelling of an endplate t-stub was carried on in ADAPTIC.

First, a rate insensitive shell model was created and calibrated according to the experimental tests conducted by Zandonini et al. [4], for different geometries of t-stubs. A viscoplastic shell model was not available and so, to overcome this issue, a beam model with equivalent width and length was created from the shell model, fitting not only the load-displacement curve but also the strains next to the bolt and web. Finally, this beam model was used in the rate sensitive studies, as seen in Figure 6 a).

According to Figure 6a, the resistance of the end-plate in bending achieves a substantial increase. This tensile yield strength obtained for the shell and beam models developed in ADAPTIC for a rate insensitive analysis is substantially higher than the one obtained from EN 1993-1-8, as the code formula is based on a plastic hinge failure beam model and, for the ADAPTIC models, carefully calibrated to the experimental tests in [5], the authors considered the yield force by checking the plateau on the load- displacement curve.

As for the ultimate strains, two different criteria are shown as possible for the failure of the joint:

9. For a S275JR end plate and according to BS EN 10025-2, the tying capacity of the end plate is 224kN, which, if divided equally by the four bolt-rows, leads to an axial elongation limit of 25.93mm (see [3]). This code accounts for group effect of the t-stubs.

10. However, the numerical analysis, again calibrated to the experimental tests, shows that, for a t-stub with such geometry, the top fiber on the section close to the web achieves an average of 32% strains (very high ultimate strain for a normal steel specimen) in the planar direction for an axial elongation of 6.22mm under a rate insensitive analysis and close to 25 mm for the considered range of strain rates. Even if the ultimate strain drops for higher strain rates (loss of ductility - see Figure 1), this decrease is marginal and so it expected to see the t-stub to provide an axial elongation closer to 25mm than 6.22mm.

It would still be interesting to perform some t-stub specimen tests subjected to different strain rates to confirm this behaviour. Therefore, the two situations (6.22 and 25.93mm) supply a reasonable bound.

The highly nonlinear behaviour of the end-plate induces catenary action that translates into shear force in the resisting bolts. It is known that this is a brittle mode of failure and therefore may be critical for the joint behaviour. The bolts used in these connections are M20 8.8 bolts, with a shear force resistance of 94.09kN, according to EN 1993-1-8. For all the experimental tests and for the shell model of this connection in particular, a shear failure of the bolts was never observed, so it can be concluded that for the general geometry of the t-stubs used in common practice, this problem can be ignored.
Figure 6: (a) The solid lines correspond to the load-vertical displacement curves for the different models and the dotted lines to the strains on axial direction-vertical displacement. Note these are nonlinear static responses for different rates. (b) A rigid rotation of the minor axis joint is assumed and so the ratio on the displacement rate achieved in the bolt rows below the outermost tensile bolt-row is 0.72, 0.44 and 0.16, respectively.

**Beam level**

One of the assumptions made for partial strength connections is that the beam itself does not yield (plasticity is restricted to the connection); therefore, a nonlinear static analysis is performed in ADAPTIC for a rate insensitive bilinear elasto-plastic steel model. The rate sensitivity is considered only in the yield limit of the springs modelling the t-stubs.

The following procedure synthesizes the determination of a beam rate sensitive pseudo-static response:

1. Create a new set of curves which correspond to a different elongation rate on the outermost tensile t-stub with a log variation (e.g.: $\delta_{t-stub} = 0 \text{ mm/s}$, $\delta_{t-stub} = \frac{1}{3} \text{ mm/s}$, $\delta_{t-stub} = 2 \frac{1}{3} \text{ mm/s}$, $\delta_{t-stub} = 4 \frac{1}{3} \text{ mm/s}$, $\delta_{t-stub} = 8 \frac{1}{3} \text{ mm/s}$, ...) – see Figures 7a-b, without iterations yet. The other three bolt rows follow the ratio elongation ratio given in Figure 6b.
2. Based on the incremental method presented in the section entitled “Time Prediction”, obtain the time and elongation of the outermost tensile bolt-row corresponding to the first load factor, using the rate insensitive pseudo-static curve, so as to establish a first estimate of the elongation-rate for the t-stub at this load factor. It is assumed that the relationship between the beam vertical displacement and t-stub axial elongation is independent of the elongation rates and, therefore, equal to the one obtained in the rate insensitive case.
3. Perform a logarithmic interpolation to obtain the pseudo static load-displacement curve corresponding to the t-stub elongation rate for the aforementioned load factor.
4. Repeat Step 2, with the pseudo-static curve obtained in Step 3, until the time converges. According to the computational runs, a maximum of two iterations is
typically required. Add the new load factor-dynamic displacement point to the pseudo-static rate sensitive load-displacement curve – see Figure 7c.

5. Repeat Steps 2 to 4 for different load factors in order to accurately define the pseudo-static rate sensitive curve – see Figures 7a-b iterations for the pseudo-static rate sensitive response for cases 1 and 2, respectively.

![Figure 7: (a) Pseudo-static responses of the bare steel beam for the rate insensitive model of the case study, that defines the bolt-rows yield strengths as shown in Figure 6a for different elongation rates at the component (t-stub). The dotted vertical lines correspond to the 6.22mm and 25.93 mm t-stub elongations, respectively (joint failure criteria). First and final iterations for the pseudo-static rate sensitive response of the steel beam, for case 1.](image)

<table>
<thead>
<tr>
<th>Load - 50 kN</th>
<th>Time (secs)</th>
<th>T-stub elongation (mm)</th>
<th>Dynamic Displacement (mm)</th>
</tr>
</thead>
<tbody>
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<td>Rate insensitive</td>
<td>0.4569</td>
<td>35.8025</td>
<td>867.8356</td>
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<tr>
<td>First iteration</td>
<td>0.4885</td>
<td>26.0011</td>
<td>673.4434</td>
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<tr>
<td>Second iteration</td>
<td>0.4878</td>
<td>25.9000</td>
<td><strong>671.4421</strong></td>
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<table>
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<th>Dynamic Displacement (mm)</th>
<th>Capacities (kN)</th>
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</thead>
<tbody>
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<td>180.36</td>
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</tr>
<tr>
<td></td>
<td>615.64</td>
<td>40.56</td>
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</tbody>
</table>

Figure 7: (a) Pseudo-static responses of the bare steel beam for the rate insensitive model of the case study, that defines the bolt-rows yield strengths as shown in Figure 6a for different elongation rates at the component (t-stub). The dotted vertical lines correspond to the 6.22mm and 25.93 mm t-stub elongations, respectively (joint failure criteria). First and final iterations for the pseudo-static rate sensitive response of the steel beam, for case 1.
CONCLUSIONS

This paper outlines the consideration of material rate sensitivity within an enhanced framework for progressive collapse assessment. The proposed method is simple and fast, and is applicable to steel members with known joint properties.

At first, various procedures for predicting the time to maximum dynamic displacement of the affected floor system are presented, where the kinetic energy incremental approach is found to be the most effective. Minor discrepancies in comparison with a more accurate approach do not have practical significance on the dynamic overstress, as it is proportional to the fifth root of the strain rate for steel.

Subsequently, the calibration of shell and beam models to determine the rate sensitive response of a t-stub lead to satisfactory results, although the need to extend rate sensitivity to a shell element is established, so as to realistically describe the behaviour of a t-stub subjected to high strain rates. The numerical investigations on a t-stub from the case study joint has revealed that high elongation rates induced an ultimate elongation four times higher than in the rate insensitive model, allowing for a marginal variation on the ultimate strain for high strain rates.

The case study has been chosen to comprehensively compare the results previously obtained without accounting for material rate sensitivity with the new results accounting for the same phenomenon. The two cases, axially restrained and unrestrained beams, do not differ considerably in their load-displacement curves. An increase of 20-26% (axially unrestrained beam) and 11-26% (axially restrained beam) has been observed when material rate sensitivity is accounted for.

The weight of these values demonstrates that material rate sensitivity clearly should be taken into account for a more realistic assessment of progressive collapse under sudden column loss. Further developments of the proposed method include an extension to reinforced concrete structures.

ACKNOWLEDGEMENTS

This work is part of a research study on progressive collapse undertaken by Miguel Pereira under the supervision of Prof. Bassam Izzuddin at Imperial College London with the financial support of Fundação para a Ciência e a Tecnologia (FCT).
REFERENCES


DEFLECTION SOLUTIONS OF A SPHERICAL MEMBRANE SHELL FOR MICRO-BUBBLES UNDER A POINT LOAD

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ABSTRACT

The action of concentrated loads on spherical shell has been widely studied. The solution for a concentrated normal force was first obtained by E. Reissner in 1946 by using the classical theory of shallow shells. This problem was also considered by S.P. Timoshenko (1946), Stanislaw Lukasiewicz (1979), W.T. Koiter (1963), W.C. Young (1989) etc. In deriving these solutions, it is assumed that the stresses and displacements are very small at some distance from the loading point. In this paper, the displacement of non-shallow spherical membrane shell for micro-bubbles under a concentrated load has been considered. Applying the above all kinds of methods, we have calculated the deflections under the point of application of concentrated loads on spherical membrane shells with R/t=10~50 and R/t=100~500. The results are analyzed and all kinds of methods are compared, the calculation precisions and covering ranges of all kinds of methods are discussed.

KEYWORDS

Spherical membrane shell, Point load, Non-shallow spherical shell, Calculation precisions, Covering ranges

INTRODUCTION

The action of concentrated loads on spherical shell has been widely studied. The solution for a concentrated normal force was first obtained by E. Reissner in 1946 by using the classical theory of shallow shells. This problem was also considered by S.P. Timoshenko (1946), Stanislaw Lukasiewicz (1979), W.T. Koiter (1963), W.C. Young (1989) etc. In deriving these solutions it is assumed that the stresses and displacements are very small at some distance from the loading point.
DEFLECTION SOLUTIONS

Timoshenko(1946) take the fact into account that the effect of transverse shear $Q_t$ on membrane forces can be neglected in the case of a shallow shell and the deflection at the center of such a shell is affected very little by the respective conditions on the outer edge. He considered a shallow shell with a very large radius subjected to a point load $P$ at the apex $r=0$ (Figure 1), while the normal displacements $w$ must be finite at $r=0$, and $w$ must vanish for $r=\infty$. He obtained for the deflection of the shell at the point of the application of the load the value (Figure 1):

Figure 1: The shell at the point of the load

Figure 2: $P$ uniformly distributed
When the central load $P$ is uniformly distributed over a circular area of a small radius $c$ (Figure 2), the following results hold at the center of the loaded area $r=0$:

$$w_0 = \frac{\sqrt{3(1-\nu^2)}}{4} \frac{PR}{Et^2}$$

Timoshenko(1946)

$$w_0 = \frac{\sqrt{12(1-\nu^2)}}{\pi} \frac{PR}{Et^2} \left[ \frac{1}{\mu^2} - \frac{\pi}{2\mu} \psi'_4(\mu) \right]$$

Timoshenko(1946)

where $\mu = \frac{c}{l}$, $l = \frac{\sqrt{Rt}}{4\sqrt{12(1-\nu^2)}}$; the function $\psi'_4(\mu) = -(2/\pi)\text{ker}\mu$, numerical values of the function $\psi'_4$ are given in Table 86 of Timoshenko(1946).

Koiter(1963)

The non-shallow spherical shell loaded at the vertex has been considered by Koiter(1963) in spherical coordinates, and based on the equations of the theory of non-shallow spherical shell. Koiter obtained the following expression for the normal deflection under the point of application of the load $P$ (Figure 1):

$$w_0 = \frac{\sqrt{3(1-\nu^2)}}{4} \frac{PR}{Et^2} \left[ 1 + \frac{2(1+\nu)}{\pi\lambda^2} \left( \ln \lambda + \gamma_0 - 1 + \frac{\ln 2}{2} \right) + \frac{4}{3\pi\lambda^2} + O(\lambda^{-3}) \right]$$

Koiter(1963)

where $\lambda^2 = \frac{\sqrt{3(1-\nu^2)}}{R} \frac{R}{t}$; $\gamma_0$ is Euler’s constant, $\gamma_0 = 0.5772$.

Lukasiewicz(1979)

Lukasiewicz(1979)[3] also obtained an approximate formula for the displacement of non-shallow shell under a concentrated load $P$ (Figure 1):

$$w = \frac{\sqrt{3(1-\nu^2)}}{4} \frac{PR}{\piEt^2} \left[ \pi - 4(1+\nu)k_R \ln \sqrt{2k_R} + 2k_R + \left( r^2 - 4(\eta - \varepsilon) \right) \left( \ln \frac{r}{2} + \gamma_0 \right) - r^2 - \eta \right]$$

where $k_R = \frac{l^2}{R^2}$; $\eta = \frac{l^2}{5(1-\nu)l^2}$; $\varepsilon = \frac{ul^2}{10(1-\nu)l^2}$

However, the equation cannot be used to calculate the deflection at those points of the shell that are very close to the point of application of the load ($r < t$). If we neglect the effect of transverse shear deformation, we can obtain the following expression for the normal deflection under the point of application of the load $P$ (Figure 1):

$$w_0 = \frac{\sqrt{3(1-\nu^2)}}{4} \frac{PR}{\piEt^2} \left[ \pi - 4(1+\nu)k_R \ln \sqrt{2k_R} + 2k_R - \eta \right]$$

Lukasiewicz(1979)
Lukasiewicz(1979) also gave an expression for the normal deflection under the point of application of the load $P$ when the central load $P$ is uniformly distributed over a circular area of a small radius $c$ (Figure 2):

$$w_0 = \frac{12(1 - \nu^2)}{\pi^2} \frac{PR}{Et^2} \left[ \frac{1}{\mu^2} + \frac{1}{\mu} \frac{\ker'(\mu)}{2} \frac{1 + \nu}{k_R} \frac{\ln \sqrt{2k_R}}{k_R} \right]$$

$$+ \frac{3}{5\pi} (1 + \nu)(2 - \nu) \frac{P}{Et} \left[ \frac{\ker \mu + \frac{\mu \ker' \mu}{2(2 - \nu)}}{k_R} \right]$$

Lukasiewicz(1979)b

**Young(1989)**

Young(1989) gave a simple formula for partial spherical shell with any edge support and load $P$ concentrated on small circular area of radius $c$ at pole. The deflection does not include any deflection due to the edge supports or membrane stresses remote from the loading.

The deflection under the center of the load $P$ (Figure 2):

$$w_0 = A \frac{PR\sqrt{1 - \nu^2}}{Et^2} \text{......... Young(1989)}$$

where $A$ is a numerical coefficient that depends upon $\mu$ and its values can be obtained from Table 30 of Young(1989).

**Comparison of solutions**

Applying the above all kinds of methods, we have calculated the deflections under the point of application of concentrated loads on spherical shells with $R/t=10$ to $50$ and $R/t=100$ to $500$. The results are shown as the following Figure 3 and Figure 4.

**CONCLUSIONS**

From the comparison of solutions, we can see that the results of all kinds of solution differ little, specially for $R/t=100$ to $500$. The results of Koiter(1963) and Lukasiewicz(1979)a are very close, the results of Timoshenko(1946)b and Young(1989) are very close too. The solution of Koiter(1963) is most precise, the solution of Young(1989) differ from Koiter(1963) furthest, but the formula is simplest.
Figure 3: Comparison of solutions for the deflections under the point of application of local load \( P(\frac{R}{t}=10^{-50}) \)

Figure 4: Comparison of solutions for the deflections under the point of application of local load \( P(\frac{R}{t}=100^{-500}) \)
REFERENCES

BUCKLING MODES AND OPTIMAL STIFFENER
ARRANGEMENT OF RECTANGULAR STIFFENED PLATES
UNDER UNIFORM LATERAL LOADS

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KEYWORDS
Structural Optimization, Buckling, Mode Shapes, Stability, Strength, Plates, Stiffener

ABSTRACT
Longitudinal stiffeners attached to the plates may significantly increase the overall buckling loads of the resultant stiffened structures. Stiffened plates are selected from the deck structure of real sea going ships and inland waterway vessels. The main portion of ship’s structure is usually composed of stiffened plates. Between girder and floors, stiffeners are furnished to plates usually in the longitudinal direction. Under various loads applied to a ship, such as those due to cargo, buoyancy and waves, these stiffened plates are subjected to combined inplane and lateral loads. Longitudinal stiffeners attached to the plates may significantly increase the overall buckling loads of the resultant stiffened structures. The stiffener more or less remains in its original position at the onset of buckling while the buckling pattern of the stiffened plate is dominated by local buckling modes of the plate itself. The ratio of the length to the width of plate is defined as aspect ratio of plate. In present work a series of detailed numerical analysis of stiffened plates subjected to inplane longitudinal compressive load is performed for different aspect ratios. Eigen-buckling responses for a series of rectangular perfectly flat plates and longitudinally stiffened plates subjected to inplane axial forces for different aspect ratios are derived by finite element approach using general purpose software (ANSYS). The solution technique makes use of simplified displacement computations that involve the elastic buckling load (eigen-value) and finally stress computations using large deflection theory in combination with strength assessment using von Misses’ yield criterion as applied to membrane stresses. Finally an optimized stiffener arrangement is expressed in explicit form based on the results of numerical investigations of the non-linear behaviour of plates and stiffened plates.
INTRODUCTION

In structural design it is necessary to obtain an appropriate geometric shape for the structure so that it can carry the imposed loads safely and economically. The optimal design of structures subject to dynamic loading can be extremely useful, leading to improved dynamic characteristics. The minimum natural frequency of the structure should be higher than the predominant excitation frequencies in order to prevent resonance. This may be achieved by the use of structural shape optimization procedures. The shape or thickness of the components of the structures are varied to achieve a specific objective satisfying certain constraints. Such procedures are iterative and involve several re-analyses before an optimum solution can be achieved.

The optimal buckling design of stiffened plates may be defined as finding the maximum value of the critical buckling load for a given structural weight, or alternatively it may be to minimize the structural weight that satisfies a prescribed buckling load. Maximization of the buckling load is essential to enhance the overall structural stability by decreasing the possibility of reaching an unstable equilibrium position under any contemplated loading. Buckling of plate involves two planes and two boundary conditions on each edge of plate. A plate is normally supported at edges. It continues to resist the additional axial load even after primary buckling load is reached and does not fail even when the load reaches a value 10-15 times the buckling load. Thus for a plate post buckling load (elastic) is much higher. In order to increase the buckling load of a plate, longitudinal and transverse stiffening is adopted as in the case of ship hull and aircraft wing. Plates are of different sizes and shapes with respect to design requirement.

This paper presents applications in determining frequencies, mode shapes and buckling loads of different stiffened panels. A finite element model is developed to describe the statics and dynamics of Mindlin plates which are stiffened with arbitrary oriented stiffeners. The model is used as a basis for optimizing separately or simultaneously the critical buckling loads and natural frequencies of plates per unit volume of the plates/stiffeners assemblies. The orientations of stiffeners are arranged in the form of isogrid configuration over a flat plate. These are selected to optimize the static and dynamic characteristics of plates/stiffeners assemblies. The stiffener more or less remains in its original position at the onset of buckling while the buckling pattern of the stiffened plate is dominated by local buckling modes of the plate itself. The ratio of the length to the width of plate is defined as aspect ratio of plate. A series of detailed numerical analysis of stiffened plates subjected to in-plane longitudinal compressive load is performed for different aspect ratios. In this work, eigen-buckling responses for a series of rectangular perfectly flat plates and longitudinally stiffened plates subjected to inplane axial forces for different aspect ratios are derived by finite element approach using general purpose software (ANSYS). The solution technique makes use of simplified displacement computations that involve the elastic buckling load (eigen-value) and stress computations using large deflection theory. Finally an optimized stiffener arrangement is expressed in explicit form based on the results of numerical investigations of the non-linear behaviour of stiffened plates. The static characteristics are optimized by maximizing the critical buckling loads of the isogrid plate, while dynamic characteristics are optimized by maximizing multiple natural frequencies of the stiffened plate. Eigen-value optimization algorithm has been applied to discretized finite-element structural models for stiffened plate analysis.
BUCKLING OPTIMIZATION PROBLEM

Three principal phases in an optimization problem are defined as; (i) definition and measure of design objectives, (ii) definition of design constraints, (iii) definition of design variables and preassigned parameters. The proper definition of the design variables is of great importance in formulating a design optimization model. In the present study the objective function is represented by maximization of the critical frequency of modal analysis subject to a specified stiffener arrangement. The pre-assigned variables are those that do not change in the optimization process. They are chosen to be the material of construction, type and location of supports, type and shape of the cross section and the width of the plate. To accurately define the true design variables that have a direct bearing on buckling optimization, let us first examine, as fundamental case study, of a simply supported plate without stiffener in following section 3.

EQUILIBRIUM EQUATIONS GOVERNING THE BUCKLING OF THIN PLATES

For a simply supported rectangular plate axially compressed in one direction (Fig.1) the governing equation is:

\[
\left( \frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{N_x}{D} \frac{\partial^2 w}{\partial x^2} \right) = 0
\]

Introducing non-dimensional parameters \( \xi \) and \( \eta \) for \( x = a\xi \) and \( y = b\eta \) the above Equation 1 can be written as

\[
\left( \frac{\partial^4 w}{\partial \xi^4} + \frac{a^4}{b^4} \frac{\partial^4 w}{\partial \eta^4} + \frac{2a^2}{b^2} \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \frac{N_x}{D} a^2 \frac{\partial^2 w}{\partial \xi^2} \right) = 0
\]

Let \( p = \text{aspect ratio} = a/b \) and \( \lambda^2 = N_x \frac{a^2}{D} = \sigma_x \frac{ta^2}{Et^3} \left( 12 \left( 1 - v^2 \right) \right) \)

Equation 2 reduces to

\[
\left( \frac{\partial^4 w}{\partial \xi^4} + p^4 \frac{\partial^4 w}{\partial \eta^4} + 2p^2 \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \lambda^2 \frac{\partial^2 w}{\partial \xi^2} \right) = 0
\]

Since all the edges are simply supported, the boundary conditions are;
The solution of the differential Equation 3 with boundary conditions 4 and 5 is of the form:

\[ w(\xi, \eta) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin m\pi \xi \sin n\pi \eta \quad (m = 1, 2, \ldots; n = 1, 2, \ldots) \]  

where,

- \( m \) = number of half waves that the plate buckles in X-direction
- \( n \) = number of half waves that the plate buckles in Y-direction
- \( A_{mn} \) = amplitudes of mode shapes

Nontrivial solution Equation 3 after substituting Equation 6 in Equation 3, leads to

\[ \lambda^2 = \frac{N_x}{D} a^2 = m^2 \pi^2 + 2p^2 n^2 \pi^2 + p^4 \frac{n^4}{m^2} \pi^2 \]  

\[ N_x = \frac{D\pi^2}{b^2} \left( \frac{m}{p} + \frac{n^2}{m \cdot p} \right)^2 \]

For critical value of \( N_x \) at which equilibrium configuration of the plate changes to bent configuration, values of ‘m’ and ‘n’ should be such that they minimize Equation 8. This implies ‘n’ must be equal to one and the plate buckles with one half sine wave along Y – direction. With \( n \) equal to one taking derivative of Equation 8 with respect to ‘m’, number of sine waves in X-direction: \( m = p \). Thus

\[ (N_x)_{cr} = 4D\pi^2/b^2 \]  

Thus a simply supported plate buckles with one half wave in Y-direction and ‘p’ half waves in X-direction in critical buckling condition. The critical buckling load is minimum (9) for each integer value of ‘p’. It is higher for non-integer value of ‘p’ and varies as follows:

\[ (N_x)_{cr} = kD\pi^2/b^2 \]  

where  \( k = (m/p + n^2 p/m)^2 \)
Fig. 2 shows variation of ‘k’ with aspect ratio ‘p’. For ‘p’ greater than 4 variation of ‘k’ is negligible and it has a fairly constant value 4.0 resulting in critical buckling load (9). The values m = 1, 2, . . . indicates number of half waves that the plate will buckle in the X-direction for a given aspect ratio. For $p = \sqrt{2}, \sqrt{6}, \sqrt{12}, \ldots$ transition occurs from one to two, two to three, three to four half waves . . . respectively.

In the design of deck structure of ships and inland waterway vessels as well as aircraft structures, it is necessary to optimize the oscillatory behaviour to guard against failure due to resonance. In the present study the objective function is represented by maximization of the critical frequency of modal analysis subject to a specified stiffener arrangement and aspect ratio. The constraints in the present study are geometric constraints, imposing limits on the sizes and thicknesses of plates and stiffeners, stiffeners arrangements.

### NUMERICAL ANALYSIS OF THIN PLATES

This section is confined to the determination of the numerical critical buckling response of a thin plate made of any arbitrary aspect ratio having same cross sectional properties and dimensions. Parameters for the current analyses are presented in schematic illustrations in and Tables 1 to 3.

#### TABLE 1
FLOW STRESS VALUES OF STEEL IN UNIAXIAL ISOTHERMAL

<table>
<thead>
<tr>
<th>True Strain (MPa)</th>
<th>False Stress (MPa)</th>
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<tr>
<td>0.003</td>
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<td>0.51</td>
<td>939.27</td>
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#### TABLE 2
PROPERTIES OF STRUCTURAL STEEL

<table>
<thead>
<tr>
<th>Elastic Modulus (GPa)</th>
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<tr>
<td>Rigidity Modulus(GPa)</td>
<td>65.6</td>
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<tr>
<td>Melting point in Celsius</td>
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<tr>
<td>Co-efficient of friction</td>
<td>0.47</td>
</tr>
<tr>
<td>Poisson’n ratio</td>
<td>0.28</td>
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<tr>
<td>Density (gm/cc)</td>
<td>7.85</td>
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</table>
OUTPUT FROM NUMERICAL ANALYSIS

The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions. Modal analysis has been carried out for rectangular plates subjected to uniform axially compressive force per unit length along edges $x=0$ and $x=a$ to determine the natural frequencies (Table. 4) and mode shapes of plates with three different aspect ratios. The boundary conditions for all the analyses has fixed restraints at $x=a$ and simply supported at $x=0$. Edges along $y=0$ and $y=b$ are kept free.

OPTIMIZATION OF STIFFENED PLATES

Responses of natural frequencies against aspect ratios are illustrated in Figs. 3 to 6 respectively. The first, second, third and fourth mode responses exhibit monotonic decrease in the values of natural frequencies with increase aspect ratios. However the values of natural frequencies increased remarkably with increase in the number of stiffeners for low aspect ratios. The increase in the values of frequencies with increase in the number of stiffeners is not remarkable at higher aspect ratios. Solutions indicate that optimum zones must be around low aspect ratios to have design control in respect of resonance.

TABLE 3
PARAMETERS CONSIDERED FOR INVESTIGATIONS

<table>
<thead>
<tr>
<th>Plate Designation</th>
<th>Series</th>
<th>Plate Thickness (mm) $t_p$</th>
<th>Length of Plate (mm) $a$</th>
<th>Breadth of Plate (mm) $b$</th>
<th>Aspect ratio ($p = a/b$)</th>
<th>Depth of stiffener (mm) $t_s$</th>
<th>Thickness of stiffener (mm) $t_w$</th>
</tr>
</thead>
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<td>Plate without Stiffener</td>
<td>PS01</td>
<td>6</td>
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<td>1000</td>
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<td>NA</td>
<td>NA</td>
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<td>1000</td>
<td>4</td>
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<td>NA</td>
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<td>4000</td>
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<td>4</td>
<td>50</td>
<td>4</td>
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<td>1000</td>
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<td>4</td>
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<td>1000</td>
<td>3</td>
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<td>4000</td>
<td>1000</td>
<td>4</td>
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<td>2000</td>
<td>1000</td>
<td>2</td>
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<td>4</td>
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<td></td>
<td>PS32</td>
<td>6</td>
<td>3000</td>
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<td>PS33</td>
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<td>4000</td>
<td>1000</td>
<td>4</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>Plate with Four Stiffeners</td>
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<td>2</td>
<td>50</td>
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<td></td>
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<tr>
<td>Plate Designation</td>
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<td>Frequency of 1st Mode</td>
<td>Frequency of 2nd Mode</td>
<td>Frequency of 3rd Mode</td>
<td>Frequency of 4th Mode</td>
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**Table 4**

Modal Responses of Plate Series from ANSYS Output

**Figure 3**

1st Modal response of plates with aspect ratio

**Figure 4**

2nd Modal response of plates with aspect ratio
Responses of natural frequencies against number of stiffeners are illustrated in Figs. 7 to 10 respectively. The first, second, third and fourth mode responses exhibited monotonic increase in the values of natural frequencies with increase in number of stiffeners. Comparing the results given herein, it can be noticed that optimum plates need not be provided with higher number of stiffeners for large aspect ratios. They can be economically designed with fewer numbers of stiffeners for large aspect ratios. By optimizing the longitudinal and transverse dimension of plates the overall stability can be substantially improved for lower aspect ratios.
CONCLUSION

In view of the importance of improving overall stability level of thin plates, an appropriate optimization model has been formulated by considering thin plates of multiple aspect ratios and stiffener arrangements. The object is to obtain a more stable plate by maximizing frequency of eigen-buckling mode for a given total mass and length. General purpose software ANSYS provides an approximate numerical analysis for uniform Euler’s plate geometry. Based on this fact, the buckling mode and frequency are obtained for any geometry, type of cross section and type of boundary conditions. It has been shown that the actual design variables that have a direct bearing on buckling optimization must include thickness of plate, aspect ratio of plate, stiffener arrangements, depth and thickness of stiffener. The model has been applied successfully to stiffened plates with restrained transverse sides and free longitudinal sides. ANSYS outputs indicate that the buckling modes and hence frequencies are well behaved, monotonic and defined everywhere in the selected design space. The buckling modes and hence frequencies are found to be very sensitive to the variation in the stiffener arrangements and aspect ratios.

Figure 9
3rd Modal response of plates with number of stiffeners

Figure 10
4th Modal response of plates with number of stiffeners
REFERENCES


Software Ref: ANSYS VERSION 8.1

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TRANSVERSE SHEAR STRENGTH OF A BI-DIRECTIONAL CORRUGATED-STRIP-CORE STEEL SANDWICH PLATE

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KEYWORDS
Sandwich, Modelling, Corrugated core, Homogenisation, Stiffness, Shear strength

ABSTRACT

The need for large structures with higher strength and stiffness, but lower weight and material is increasing. The concept of a corrugated-core sandwich plate could offer a solution here. A corrugated-core sandwich plate consists of two thin faceplates and one corrugated sheet with varied corrugation patterns. The conventional corrugated-core sandwich plate is evidently strong in only one direction; it needs to be stiffened in the other direction. Arranging the corrugated plate in both x- and y-direction is a method to stiffen such sandwich plates in both directions. This paper aims to present a new alternative corrugated-core steel sandwich plate named the bi-directional corrugated-strip-core steel sandwich plate. The core consists of a series of corrugated-strip plate which are arranged in both x- and y-direction. This paper also aims to present an analytical model to address the efficiency in transverse shear stiffness and strength of this proposed corrugated-core sandwich plate. According to preliminary analysis results, the transverse shear stiffness and strength of this alternative sandwich plate is shown to be increased.

INTRODUCTION

The need for large structures with higher specific strength and stiffness is increasing. This is especially true in transportation and tall-building structures where there is an interest in increasing service load to structure weight ratios. To deliver such structures, engineers can design a new structural topology such as a corrugated-core sandwich structure. A corrugated-core sandwich structure consists of two main parts: (1) the two thin, stiff plates of a high-strength material, located at top and bottom layer of a sandwich structure, called 'sandwich faces', and (2) a corrugated sheet located between two sandwich faces, called...
'corrugated core'. A conventional corrugated core sandwich plate, as shown in Figure 1(a), is theoretically strong in the x-direction, but less so in the y-direction [1]. Instead of filling expanded materials in the voids [2], this simple arrangement of corrugated core could be stiffened by arranging the corrugated core in both x- and y-directions. The simple arrangement is shown in Figure 1(b). Other arrangements named bi-directional corrugated core [3] and cross corrugated core [4] are shown in Figure 1(c) and Figure 1(d), respectively. These cores consist of a series of corrugated strip plate units which are arranged in a bi-directional format. The stiffness of these alternative corrugated sandwich plates can be controlled in both x- and y-directions. It was found by Ray [5] that the cross-corrugated core was more efficient in transverse shear stiffness than that of conventional one; its shear stiffness was 173% higher.

![Figure 1: Sketch of corrugated-core sandwich construction (a) one-way corrugated core, (b) two-way corrugated core, (c) bi-directional corrugated core [3] and (d) cross corrugated core [4]](image)

To analyse the stiffness of web-like cored sandwich structure, a simplified approach referred to as a force-distortion relationship technique may be used. Libove and Hubka [6] presented the well-known formulas for evaluating the elastic constants of the corrugated-core type of sandwich plate. Nordstrand et al. [7] proposed theoretical solutions for a curved, corrugated core based on curved beam theory. Ray [5] further adapted concept of Libove and Hubka [6] to determine the transverse shear stiffness of his innovative offset-corrugated core sandwich beam. The force-distortion relationship technique is also used to derive the equivalent elastic constants of C-cored sandwich panel [8], Z-cored sandwich panel [9]. The equivalent stiffness parameters of an extruded truss-core sandwich panel were also studied based on this technique by Lok and Cheng [10]. Nevertheless, the force-distortion relationship technique would be too complex for an indeterminate structural topology core because it needs to perform complex equilibrium, constitutive and compatibility equations.

This paper aims to present a new alternative to sandwich construction, named bi-directional corrugated-strip-core sandwich construction and to outline an analytical method to model the transverse shear stiffness and strength of the proposed sandwich type based on a simplified force-distortion relationship technique in which a deformed shape of the sandwich beam is assumed in advance. The equivalent shear stiffness is separated into two parts: face and core stiffness. Then, the equivalent homogeneous solid-core sandwich structure is analysed using classical sandwich beam theory to deduce the global response of the proposed sandwich beam.
**BRACED FRAME ANALOGY APPROACH TO DERIVE TRANSVERSE SHEAR STIFFNESS**

**Configuration of Bi-Directional Corrugated-Strip-Core Sandwich Beam**

Figure 2 shows a bi-directional corrugated-strip-core sandwich beam which is cut from a sandwich plate by two parallel planes. The configuration of a bi-directional corrugated-strip-core of a repetitive module is designed to provide nine face-core contact areas whereas the cross-corrugated core provides less. The dimensions of the bi-directional corrugated-strip-core sandwich beam are defined by 6 parameters. The thickness of top faceplate, bottom faceplate, and sandwich core are defined, in the conventional terms of sandwich structure, as \( t_{e}, t_{b}, \) and \( h_{c} \), respectively. The thickness of the sandwich core is equal to the depth of corrugated strip plate. The corrugated strip plates are linearly arranged in equal spacing, \( s_{e} \), in both \( x \)- and \( y \)-direction. The spacing of the corrugated strip plates and the half length of one corrugation part are identical. The width of sandwich beam, \( b \), is twice as \( s_{e} \), i.e. equal to full length of one corrugation part. The length of beam is \( n \) times as \( s_{e} \).

![Figure 2: Configuration of bi-directional corrugated-strip-core sandwich beam](image)

**Braced Frame Model and a Unit Cell**

The 2-dimensional braced frame model (BFM), as shown in Figure 3(a), is used to represent the 3-dimensional corrugated-strip-core sandwich beam. The part of sandwich beam is represented by a single straight line which passes through the neutral axis of such part. The BFM consists of four basic elements: top chord, bottom chord, vertical chord, and inclined bracing chord. They are compatible in material properties and geometrical dimensions with top faceplate, bottom faceplate, transverse corrugated strip core, and longitudinal corrugated strip core, respectively. The top, bottom, and vertical chords are assumed to be beam elements. Thus, they are able to withstand forces and moments. On the other hand, the inclined bracing chord is assumed as a truss element [1]. Therefore, it is able to be subjected to only axial forces. The unit cell, as shown in Figure 3(b), is used to represent the periodical substructure of the bi-directional corrugated-strip-core sandwich beam. The length of inclined part, \( L_{e} \), and other geometrical parameters can be simply defined from the geometry of the unit cell; \( s_{sc} \) is defined as \( s_{sc} = x_{e} - x_{b} \).
Transverse Shear Load Transfer Mechanism

In these derivations, some assumptions have to be made as follows: (1) materials have linear elastic properties; its tensile property are also the same as its compressive property, (2) the deformation of the beam is so small that a linear superposition method can be applied, (3) the corrugated strip cores are sufficiently stiff in the vertical direction so that the depth of core is always constant, (4) there is no relative horizontal movement between end nodes of the vertical and inclined bracing chord; this means that the distances $\lambda_a$ and $\lambda_b$ will remain constant, and (5) during shear effort, the shear panel is displaced in either vertical or horizontal direction only.

Considering the equilibrium of force in $x$-direction of FBD of the right part of unit cell, as shown in Figure 4(a), we can see that $F_{xz} = F_{xz} = F_v$. As a result, the equilibrium of force in $z$-direction can be expressed as follows

$$V_{xz} = F_{xz} + F_{bz} + 2F_{iz}\sin\theta \tag{1}$$

Figure 4(b) illustrates the deformation of the unit cell subjected to transverse shear force. According to assumptions, the top and bottom chords would perform as fixed-end beam. The relative vertical displacement between both ends of top chord and of bottom chord are identical, i.e. $\Delta_{xz} = \Delta_{bz} = \Delta_z$. The extended length $\Delta_{C1}$ of the inclined bracing chord C1 and the contracted length $\Delta_{C2}$ of the inclined bracing chord C2 can also be expressed as a function of $\Delta_z$. Since the initial geometry, axial stiffness and internal axial force of both inclined bracing chords are identical, we can show that

$$\Delta_{C1} = -\Delta_{C2} = \Delta_z = \sqrt{(d + \frac{s_{xz}}{s_c}\Delta_z)^2 + s_c^2} - L_c \tag{2}$$

Applying Taylor's series to Eqn. 2, $\Delta_z$ can be approximately expressed as another function.
of $\Delta_2$ as follows

$$\Delta_2 = k_2 \Delta_2$$  \hspace{1cm} (3)$$

where

$$k_2 = \begin{cases} \frac{2s_c^2 s_c}{s_c} \sum_{n=1}^{\infty} \frac{(-1)^n(2n)!}{(1-2n)!n^2} \frac{n^2}{4^n} \frac{d}{d^2} \left( \frac{d^2}{dx^2} \right)^{-1} & \text{if } s_c \leq d \\ \frac{2s_c s_c}{s_c} \sum_{n=1}^{\infty} \frac{(-1)^n(2n)!}{(1-2n)!n^2} \frac{n^2}{4^n} \frac{d}{d^2} \left( \frac{d^2}{dx^2} \right)^{-1} & \text{if } s_c \geq d \end{cases}$$  \hspace{1cm} (4)$$

From kinematic conditions, $\Delta_{xz}$, $\Delta_{bz}$ and $\Delta_c$ can be expressed as functions of fixed-end shear forces $F_{xz}$ and $F_{bz}$ and internal axial force $F_c$, respectively, as follows

$$\Delta_{xz} = \frac{1}{12 E_{b1} h_1} F_{xz} \quad \Delta_{bz} = \frac{1}{12 E_{b1} h_1} F_{bz} \quad \Delta_c = \frac{F_c}{E_c A_c}$$  \hspace{1cm} (5)$$

Utilising Eqn. 1 and 5, we can show that

$$\Delta_2 = \frac{V_{xz}}{D_2}$$  \hspace{1cm} (6)$$

$D_2$ is the shear deflection stiffness of a sandwich beam which is separated into two parts, namely face stiffness $D_{2F}$ and core stiffness $D_{2c}$, and expressed as follows

$$D_2 = D_{2F} + D_{2c} = \left( \frac{12 E_{b1} I_1}{s_c^2} + \frac{12 E_{b1} I_1}{s_c^2} \right) + 2\frac{E_c A_c}{k_2} \delta_1 n_0$$  \hspace{1cm} (7)$$

It should be noted that Eqn. 7 can be further developed to deliver the shear strain stiffness.

According to the small displacement assumption, i.e. $\tan \psi \approx \psi \approx \frac{\Delta_2}{s_c}$, we can derive the shear strain stiffness by multiplying $s_c$ to $D_2$. Meanwhile, the flexural stiffness caused by bending moment can be calculated using the same procedure.

VALIDATION OF PROPOSED EQUIVALENT SHEAR STIFFNESS

To validate the proposed shear stiffness equation, the theory of a truss-core sandwich panel developed by Lok and Cheng [10] is selected as a reference. To compare with the truss-core, the proposed bi-directional corrugated-strip-core needs to be transformed so that its configuration is the same as that of the truss core, as illustrated in Figure 5. In addition, the proposed shear stiffness equivalent developed here also needs to be included in the horizontal shear displacement due to effect of a couple of horizontal force $\frac{\delta_c}{d}$ so that it would be compatible with Lok and Cheng’s model. To derive such an effect, the horizontal shear stiffness is calculated using the same procedure as for the vertical shear stiffness.
Figure 5: Transformation from bi-directional corrugated-strip-core to truss-core sandwich beam (a) proposed core topology, (b) remove all transverse CSC, (c) remove two adjacent longitudinal CSC and (d) extend width of remaining longitudinal CSC

Figure 6: Comparison between transverse shear stiffness of bi-directional corrugated-strip-core and of truss-core sandwich beam

Figure 6 shows the comparison between transverse shear stiffness of bi-directional corrugated-strip-core and of truss-core sandwich beam. It can be seen from this figure that both models yield the same developing trend. However, they are still different in shear stiffness value if considering at the same point of $s/d$. These differences arise from different assumptions of the core element in which Lok and Cheng [10] assume the corrugated-core as a beam element whereas the assumption here is that of a core as a truss element. Owing to this difference in assumptions, it can be seen that Lok and Cheng's model yields a lower shear stiffness than the proposed model if $s/d < 1.0$, approximately. On the other hand, Lok and Cheng's model yields higher shear stiffness than the authors' model if $s/d > 1.0$.

APPLICATION

Configuration of the Case Study

The studied beam is simply supported and subjected to a point load at its midspan; the position of load is on the line of transverse corrugated-strip plate. The beam is assumed to consist of $n$ corrugations of CSC core, as shown in Figure 7. The parameter $s/d$ is selected for this study since the effects of other material stiffness and sandwich face/core geometrical parameters are well documented. This parameter can present the angle of inclined part of the CSC core in this sandwich beam model. All of base parameters are kept
constant except for $b_c$ and $s_c$. $b_c$ is presented as $b_c = k_{cb} b$ where the $k_{cb}$ is coefficient of width of the CSC plate. The parameter $s_c$ is presented in terms of the ratio, i.e. the inclined angle of CSC core, as given here as $s_c = \frac{f_c}{d}(h_c - t_c) + 2f_c$.

![Figure 7: Configuration of studied beam subjected to point load at midspan](image)

**Analytical Procedures**

Since the equivalent flexural and shear stiffnesses are known, the stress and deflection of the beam can be calculated from the classical solid-core sandwich beam (such as [2]). The shear stress is calculated at the top face-core interface instead of at the neutral axis of sandwich cross section. This is because the load transfer mechanism of this proposed core topology would be different from that for a solid-core; horizontal shear force at face-core interface would be transformed into another form of internal load, such as an axial force, in diagonal member of the CSC core. This paper, therefore, selects the face-core interface to study the effect of $s_c$ to shear stress. The shear deflection here can be calculated by summing up the vertical shear deflection of each unit block, i.e. $\Delta_{ss, max} = n\Delta_{ss}$.

**Analytical Results and Discussion**

**Shear Stiffness**

The relationship between the shear stiffness $D_{sc}$ and $\frac{s_c}{d}$ is represented in Figure 8(a). Increasing $\frac{s_c}{d}$ from 0 to 1.0 yields a sharply increasing value of $D_{sc}$. As $\frac{s_c}{d}$ continues to increase from 1.0 to 5.0, $D_{sc}$ gradually reduces nonlinearly. This trend is identical for every studied value of $t_f$. They all reach the maximum point of $D_{sc}$ at 0.027 and $\frac{s_c}{d}$ nearly 0.75. It can be inferred that the inclined part of the CSC core which is aligned near 45 degrees yields the maximum value of shear stiffness for this core topology.

Figure 8(b) shows the relationship between $\frac{s_c}{d}$ ratio and $\frac{D_{sc}}{D_{sf}}$ ratio. $\frac{s_c}{d}$ ratio is increased to reach the maximum value of 320, for $\frac{t_f}{t_p} = 0.25$, at an approximate value of $\theta = 21.8$ degrees, and then gradually increases. A similar trend can be found for other values of $\frac{t_f}{t_p}$. It could be seen that the CSC core makes a significant contribution to the overall shear stiffness of this sandwich beam model. In addition, the contribution of this
proposed core topology would still be of more benefit to the overall shear stiffness of this sandwich beam since the transverse CSC core is now not included in the current proposed shear stiffness equation.

![Graphs showing relationships between shear stress and core interface properties.](image)

Figure 8: Relationship between $\frac{s_x}{d}$ and (a) $D_{s0}$, (b) $\frac{D_{sf}}{D_{s0}}$, (c) $\tau$ at top face-core interface and (d) $\Delta_s$ at midspan

**Shear Stress**

Figure 8(c) shows the relationship between $\frac{s_x}{d}$ ratio and maximum shear stress at the top face-core interface at the end of the beam. A nonlinear trend can be found in this figure. The shear stress reduces rapidly by increasing $\frac{s_x}{d}$ from 0.3 to 1.0. The shear stress at $\frac{s_x}{d} = 1.0$ is approximately three times less than that of $\frac{s_x}{d} = 0.3$. Then, it is gradually reduced. The shear stress at $\frac{s_x}{d} = 5.0$ is about a quarter of at $\frac{s_x}{d} = 1.0$. It is evident from Figure 8(c) that the $\frac{s_x}{d}$ ratio has little effect on the shear stress at face-core interface.
Shear Deflection

Figure 8(d) presents the relationship between the $\frac{s_x}{d}$ ratio and maximum shear deflection at midspan. The shear deflection is sharply reduced while $\frac{s_x}{d}$ increased from 0.3 to 1.2 to reach the minimum point value of $2.1 \times 10^{-6}$ at $\frac{s_x}{d} \approx 1.2$. Then, the shear deflection is gradually increased to a value of $4.5 \times 10^{-5}$ at $\frac{s_x}{d} = 5.0$. This may imply that the optimum point of shear deflection occurs at $\frac{s_x}{d} \approx 1.2$; or, in other words, at inclined angle of CSC core about 50 degrees.

CONCLUSION

An alternative corrugated-core sandwich construction named bi-directional corrugated-strip-core sandwich structure is proposed. Transverse shear load transfer mechanisms in the bi-directional corrugated-strip-core sandwich beam are modelled using a braced frame analogy method. Deriving the force-distortion relationship, the periodical unit cell is homogenised to an equivalent solid core. The sandwich beam response could be studied by classical solid-core sandwich beam theory using this equivalent flexural stiffness and shear stiffness.

In the case study of a simple beam subjected to point load, based on $\frac{s_x}{d}$ parameter, it could be seen that the shear stiffness is increased by approximately 300%. Considering the alignment of the inclined part of the CSC core, it could be concluded that the maximum shear stiffness of CSC core is found at an $\frac{s_x}{d}$ ratio of about 1.2, in other words, at an angle of 50 degrees approximately. The shearing stress is nonlinearly reduced while $\frac{s_x}{d}$ ratio is increased. The optimum shear deflection is at $\frac{s_x}{d} \approx 1.2$, i.e. at angle about 50 degrees.

REFERENCES


EXPERIMENTAL BEHAVIOUR OF PLATES WITH AND WITHOUT HOLES SUBJECTED TO LOCALISED LOADS

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KEYWORDS

Localised load, Plate stability, Experimental behaviour, Perforated plate.

ABSTRACT

The results of an experimental investigation on square and rectangular perforated and non-perforated panels are shown. The hole is the centre of the square plates and both in the maximum and nodal point of the deformed critical shape corresponding to the first mode of deformation of the rectangular plates. Symmetrical localised loads have been applied to every panel. Both square and rectangular panels have been realised with slender and thick geometrical configurations to observe the different behaviour in elastic and plastic field. The experimental investigation has concerned, in particular, the case of square and rectangular panels with \( a/b = 2 \), diameter hole with \( d/b = 0.5 \), centred and eccentric hole for rectangular panels, slenderness \( \lambda = 50 \) (thick panel) and 100 (slender panel), and short length of localised load with \( s_x/a = 0.2 \) was studied. The results of the experimental tests in terms of ultimate resistance and failure mode give some elements to understand and validate the numerical results and could give some insights for a validated design basis for plate element with holes subjected to localised loads.

INTRODUCTION

Plates are commonly used in a number of steel structures, traditionally in naval and aeronautical filed as important civil structures like bridges, viaducts and off-shore structures. When the loaded plates are subjected to compressive stresses in their own plane and the thickness reduces, that could cause the loss of stability, which can be solved with the classical methods. The solution has been generally found in a closed form or with numerical methods for plain elements, subjected to uniform axial load, flexure and shear. Particularly
attention has been given to the effect of the tolerances and imperfections on the elastic critical load and the ultimate resistance; see Maiorana et al. [1].

Plates subjected to symmetrical and localised eccentric loads have been studied by Maryland et al. [2], Granath and Lagerqvist [3], whereas Granath et al. [4], Maiorana et al. [5] have treated the interaction of biaxial loads or the interation between localised load and bending moment.

Due to inspection, maintenance and also aesthetic purposes, holes are often unavoidable in webs of steel beams and in plates. The effects of holes on the behaviour of plates has been studied by, El-Sawy and Martin [6], Komur and Sonmez [7], for circular hole shape and by Narayanan et al. [8], Maiorana et al. [9] for rectangular shapes. Some studies have been developed on localised symmetrical load and shear on perforated plates; see El-Sawy et al. [10], Paik [11], Pellegrino et al. [12].

In this framework, the present work shows the main results of an experimental investigation on some particular cases of the wide numerical study described in Maiorana et al. [13], [14]. In particular, the case of square and rectangular plates with \( a/b = 2 \), diameter hole with \( d/b = 0.5 \) and short length of applied load with \( s_s/a = 0.2 \) is studied herein. The considered slenderness are \( \lambda = 50 \) and \( \lambda = 100 \). The hole has been positioned in significant points of the plates: the “nodal point” where the sinusoidal shape of the critical deformed mode does not show any out-of-plane displacement (centre of the plate) and the “maximum point” where the sinusoidal shape shows the maximum out-of-plane displacement (eccentric position). Both for the case of square and rectangular panel, the results have been compared with those of the respective whole plates (control plates).

MATERIAL CHARACTERIZATION

The tests were developed at the Materials Testing Laboratory of the Department of Structural and Transportation Engineering, University of Padova. Carbon steel grade S355J0 with Young’s modulus \( E = 206000 \) N/mm\(^2\) and Poisson’s ratio \( v = 0.3 \) was used for all the panels.

The maximum eccentricities were under the tolerance according to ENV 1090-1 [15]. The tests on the panels were carried out monotonically, under a 10000kN loading capacity machine, with loads increased between 0.5 and 2.5kN/s. Pressure transducer mounted on the loading machine was used to measure the applied loads. The load has been applied through a rigid plate of thickness 40mm (to obtain a load length vs. panel width ratio \( s_s/a = 0.2 \)) at the centre of the upper and lower flanges. The panels were instrumented with twelve linear variable differential transducers (LVDT) placed orthogonally to the web to measure out-of-plane web displacements. Fig. 1 shows the typical plate with eccentric hole before the test (Fig. 1a), the plate without hole during the test (Fig. 1b). During the loading phase, the flanges were allowed to rotate about their longitudinal axis.
EXPERIMENTAL RESULTS

Tab. 1 list the geometric characteristics of the square and rectangular panels respectively (see notation). The panels marked with the letter “A” are the slender ones ($\lambda = 100$), while the stocky ones ($\lambda = 50$) are identified with the letter “B”.

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The slender panels (1A and 2A) showed the maximum displacement in the points located on the vertical axis of symmetry of the panel with a global failure involving the entire surface of the plate; the presence of the hole reduced the ultimate resistance. The stocky plate 1B (without hole) showed a local failure due to excessive out-of-plane displacements in the zone under the applied load, whereas a global failure involving the entire surface of the plate occurred for the stocky plate with the hole (plate 2B). Figs. 2a, 2b, 2c and 3d show the deformed shape of the plates corresponding to various percents (50%, 75% and 100%) of the maximum load along the central measurement vertical line for panels 1A, 2A, 1B and 2B respectively.

Panel 1A, 2A and 2B showed a progressive global half-wave failure involving the entire plate along both central (1A) and lateral (1A, 2A, 2B) cross-sections of the plate whereas stocky panel 1B show a local failure under the applied load. Therefore, on one hand the failure mode
was changed from “local” to “global” due to the presence of the hole for stocky panels (1B and 2B), on the other hand it was not changed by the hole for slender panels involving the entire surface of the plate for both panels 1A and 2A. Looking at the experimental deformed configurations of the panel 1A shown in Fig. 2a, an effect related to the constraint to the free rotation due to the presence of the flanges was observed. This effect was less visible for the perforated plates.

Figs. 3a, 3b, 3c and 4a, 4b, 4c show the deformed shape of the plates corresponding to various percent (50%, 75% and 100%) of the maximum load along the central measurement vertical line for panels 3A, 4A, 5A and 3B, 4B and 5B respectively.

A different behaviour was observed for the plates with centred hole (plates 4A and 4B) with respect to those with eccentric hole (plates 5A and 5B). Slender panels 4A and 5A showed in a few points an entrance in plastic field. The out-of-plane displacements with the maximum amplitude were mainly located in the zone under the applied load for the plates without hole (3A and 3B) and those with centred hole (4A and 4B). The presence of the centred hole did not substantially change the failure mode and the maximum load because the zone with significant out-of-plane displacements was far from the hole. Conversely the presence of the eccentric hole near the zone under the localised symmetrical load involved significant out-of-plane displacements around the hole and caused the reduction of the maximum load.

In Tab. 2 experimental ultimate loads are listed for all the panels.

<table>
<thead>
<tr>
<th>Panel n.</th>
<th>$F_{u,exp}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A 357</td>
<td>3A 912</td>
</tr>
<tr>
<td>2A 308</td>
<td>4A 883</td>
</tr>
<tr>
<td>1B 837</td>
<td>5A 696</td>
</tr>
<tr>
<td>2B 591</td>
<td>5B 3089</td>
</tr>
</tbody>
</table>

TABLE 2
EXPERIMENTAL ULTIMATE RESISTANCES
Figure 2: Deformed shape. a) Plate 1A; b) Plate 2A; c) Plate 1B; d) Plate 2B

C = central vertical line; L = lateral vertical line
Figure 3: Deformed shape. a) Plate 3A; b) Plate 4A; c) Plate 5A
Figure 4: Deformed shape. a) Plate 3B; b) Plate 4B; c) Plate 5B

C = central vertical line; L = lateral vertical line
DISCUSSION

As expected, the stocky plates ($\lambda = 50$) show greater ultimate loads than the corresponding slender ones ($\lambda = 100$). The hole causes a significant reduction of the plate ultimate load only if its centre is located in the maximum point of the critical deformed configuration.

Regarding the deformed configuration at failure of the square panels, those without hole showed different failure modes: the deformed configuration at failure of the slender panel (1A) involved its entire height with a half-wave shape having the maximum out-of-plane displacement in the central zone of the panel, whereas that of the stocky panel (1B) mainly developed in the zone immediately under the localised symmetrical load. The presence of the centred hole, on one hand, did not significantly modify the deformed configuration of the slender panel since the maximum out-of-plane displacements occurred again in the middle height of the slender panel (2A) with the hole, on the other hand, the hole causes the modification of the deformed configuration at failure of the stocky panel (2B) since the maximum out-of-plane displacements moved from the zone immediately under the localised symmetrical load, for the entire panel, to the central zone when the centred hole is present.

Regarding the deformed configuration at failure of the rectangular panels, it mainly developed in the zone under the localised symmetrical loads, in which maximum out-of-plane displacements occurred, both for slender and stocky panels. Hence the hole did not significantly modify the deformed configuration of slender and stocky rectangular panels but generally caused greater out-of-plane displacements for the perforated panels. Considering the rectangular panels as divided in two square sub-panels, the maximum out-of-plane displacements were, as expected, in the central zone of each sub-panel, both for slender and stocky panels without holes. On one hand the presence of the centred hole did not substantially change the maximum load because the zone with significant out-of-plane displacements was far from the hole, on the other hand the presence of the eccentric hole near the zone under the localised symmetrical load, where significant out-of-plane displacements occurred, caused the reduction of the maximum load.

CONCLUSIONS

The experimental investigation described in this paper was developed to further validate some previous numerical studies on linear and non-linear behaviour of perforated plates subjected to localised symmetrical loads developed by the authors; see Maiorana et al. [13] and [14].

The conclusions of this work can be summarised in the following points.

- The presence of the hole causes a significant reduction of the plate resistance only if its centre is located in a maximum point of the critical deformed configuration.

- The deformed configuration at failure of the entire square panels showed the maximum out-of-plane displacement in the central zone of the panel for the slender panel, whereas the stocky panel failed with out-of-plane displacements mainly developed in the zone immediately under the localised symmetrical load.

- The presence of the centred hole did not significantly modify the deformed configuration of the slender square panel, whereas the hole causes the modification of the deformed configuration at failure of the stocky panel.
- The deformed configuration at failure of the rectangular panels showed a “local buckling” mechanism both for slender and stocky panels.
- The hole did not significantly modify the deformed configuration of the slender and stocky rectangular panels and generally caused greater out-of-plane displacements for the perforated rectangular panels. The centred hole did not substantially change the maximum load, whereas the eccentric hole caused the reduction of the maximum load of the perforated rectangular panels.

ACKNOWLEDGMENTS

The writers wish to thank OMBA Impianti & Engineering SpA of Podenzano (Piacenza, Italy) for supplying the panels and Alberto Maiorana for contributing to the experimental tests developed during the preparation of his degree thesis.

REFERENCES


THEORETICAL RESEARCH OF ELASTIC THIN RECTANGULAR PLATE PINNED AT FOUR CORNERS

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ABSTRACT

The equation in this paper is derived from the general theory of displacement of bending rectangular plate proposed by Fu Baolian, which can be used to calculate the deflection and internal forces of elastic thin rectangular plate pinned at four corners bearing transverse uniform loads. The convergence and accuracy of this equation are verified by examples.

KEYWORDS

General theory of displacement, pinned at four corners, bending thin rectangular plate, theoretical research.

INTRODUCTION

A wide range of research on sandwich composite panel treats it as a load-bearing wall, such as in the Composite-Lightweight structural system, and much useful conclusions are obtained. However, few studies are about precast sandwich composite panels, which are prefabricated in advance and hanged over steel frame by using bolts at four points, served as the exterior cladding of steel residential houses. There are some problems for these panels, such as the indefinite calculation method of ultimate strength and the incomplete design theory, both of the two problems bring difficulties in application.

To this end, it is necessary to provide a simplified approach to calculate the internal forces of the panels supported at four corners and design the panels. According to the loads and boundary conditions of panels, equation used to calculate the deflection of elastic thin rectangular plate pinned at four corners bearing transverse uniform load is proposed based on the theory of bending thin plate of elasticity and references of interrelated research results at
home and abroad [1~5], aiming to solve the problem of calculation of deflection and internal forces of rectangular plate pinned at four corners and provide theoretical basis for simplified calculation of energy-saving composite sandwich plates.

**GENERAL DISPLACEMENT SOLUTION OF BENDING THIN RECTANGULAR PLATE**

Based on the small deflection of elasticity, the differential equation of bending thin plate is [6]:

\[
\frac{\partial^4 \omega}{\partial x^4} + 2 \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{q(x,y)}{D}
\]  

(1)

Where \( \omega \) is deflection function, \( q \) is load per unit area, \( D = \frac{Eh^3}{12(1-\nu^2)} \) is flexural rigidity of the plate, \( E \) is elastic modulus and \( \nu \) is poison ratio.

By applying the amended reciprocal theorem of work, the general displacement solution for system in Figure 1 is obtained and can be expressed by trigonometric series mixed with hyperbolic functions [2].

![Figure 1: Rectangular plate with general bearing edge](image)

\[
\omega(x,y) = \frac{4q}{Da} \sum_{n=1}^{\infty} \left[ \frac{1}{2} \left( \frac{1}{\alpha_n} \right) \sin \alpha_n x \right]
\]

\[
+ \sum_{n=1}^{\infty} \left[ \frac{1}{2D_n} \left( \frac{1}{\beta_n} \right) \sin \beta_n y \right]
\]

\[
+ \sum_{n=1}^{\infty} \left[ \frac{1}{2D_n} \left( \frac{1}{\beta_n} \right) \sin \beta_n y \right]
\]

\[
+ \sum_{n=1}^{\infty} \left[ \frac{1}{2D_n} \left( \frac{1}{\beta_n} \right) \sin \beta_n y \right]
\]

\[
+ \sum_{n=1}^{\infty} \left[ \frac{1}{2D_n} \left( \frac{1}{\beta_n} \right) \sin \beta_n y \right]
\]

\[
+ \sum_{n=1}^{\infty} \left[ \frac{1}{2D_n} \left( \frac{1}{\beta_n} \right) \sin \beta_n y \right]
\]

\[
+ \sum_{n=1}^{\infty} \left[ \frac{1}{2D_n} \left( \frac{1}{\beta_n} \right) \sin \beta_n y \right]
\]

(2)
In this equation, the following formula can be replaced by:

$$\frac{4q}{Da} \sum_{m=1,3} \left[ 1 + \frac{1}{2\alpha_m b} \left( 2 + \frac{1}{2} \alpha_m b \right) \right] \left( y-b \right) \frac{1}{\alpha_m^5} \sin \alpha_m x$$

and

$$\frac{4q}{Db} \sum_{m=1,3} \left[ 1 + \frac{1}{2\beta_n a} \left( 2 + \frac{1}{2} \beta_n a \right) \right] \left( x-a \right) \frac{1}{\beta_n^3} \sin \beta_n y$$

where $\alpha_m = \frac{m\pi}{a}$ and $\beta_n = \frac{n\pi}{b}$; $k_1$, $k_2$, $k_3$ and $k_4$ are the given displacement of the four points; $x = 0$, $y = 0$; $x = a$, $y = 0$; $x = a$, $y = b$ and $x = 0$, $y = b$ respectively.

The boundary displacements and bending moments of rectangular plate with general bearing edge as shown in Fig.1 are respectively expressed as:

$$\omega_{x0} = \sum_{n=1,2} a_n \sin \frac{n\pi y}{b} + \frac{b-y}{b} k_1 + \frac{y}{b} k_4$$

$$\omega_{xa} = \sum_{n=1,2} b_n \sin \frac{n\pi y}{b} + \frac{b-y}{b} k_2 + \frac{y}{b} k_3$$

$$\omega_{y0} = \sum_{m=1,2} c_m \sin \frac{m\pi x}{a} + \frac{a-x}{a} k_1 + \frac{x}{a} k_2$$

$$\omega_{ya} = \sum_{m=1,2} d_m \sin \frac{m\pi x}{a} + \frac{a-x}{a} k_3 + \frac{x}{a} k_4$$

$$M_{xx} = \sum_{n=1,2} A_n \sin \frac{n\pi y}{b}$$

$$M_{xx} = \sum_{n=1,2} B_n \sin \frac{n\pi y}{b}$$

$$M_{y0} = \sum_{m=1,2} C_m \sin \frac{m\pi x}{a}$$

$$M_{xx} = \sum_{m=1,2} D_m \sin \frac{m\pi x}{a}$$

where $\omega_{x0}$, $\omega_{xa}$, $\omega_{y0}$, $\omega_{ya}$; $M_{xx}$, $M_{xx}$, $M_{y0}$ and $M_{yy}$ are the given displacements and the given bending moments of four bearing edges respectively; $x = 0$, $x = a$, $y = 0$ and $y = b$, and $a$, $b$, $c$, $d$, $A$, $B$, $C$ and $D$ are unknown parameters to be solved.

**EQUATION OF DEFLECTION OF BENDING THIN RECTANGULAR PLATE PINNED AT FOUR CORNERS**

The elastic thin rectangular plate pinned at four corners is shown in fig.2. The deformation of plate is symmetrical bending, so the respective displacement of four edges is:
Figure 2: Thin rectangular plate pinned at four corners bearing transverse uniform loads

\[
\omega_{x0} = \omega_{yo} = \sum_{n=1,3} a_n \sin \beta_n y + k_4, \quad \omega_{yo} = \omega_{yb} = \sum_{n=1,3} c_m \sin \alpha_m x + k_2
\]

That is, \(a_n = b_n, \quad c_m = d_m\)

The boundary conditions of the plate are:

\[
(0,0,0,0) = \left(\begin{array}{c}
\omega(x,y) = 0, \quad \omega_{y0} = 0, \quad \omega_{yo} = 0, \quad \omega_{yo} = 0
\end{array}\right)
\]

Based on the boundary conditions and the boundary displacement and bending moment of the general bearing edge mentioned above, the results can be obtained:

\[
A_n = B_n = C_m = D_m = 0, \quad k_1 = k_2 = k_3 = k_4 = 0
\]

According to Equation (1) mentioned above for general displacement solution, therefore, the deflection equation for thin rectangular plate pinned at four corners bearing transverse uniform loads is:

\[
\omega(x,y) = \frac{4q}{Da} \sum_{n=1,3} \left[ 1 + \frac{1}{2ch^2} \alpha + \left( \begin{array}{c}
\alpha_n \left( y - \frac{b}{2} \right) \right) \right] \frac{1}{a_n} \sin \alpha_n x
\]

\[
+ \frac{1}{2} \sum_{n=1,3} \left[ 1 + (1 - \nu) \left[ \beta_n \left( x_n - \frac{b}{2} \right) \right] \frac{1}{sh \beta_n x} \sin \beta_n y(a_n)
\]

\[
+ \frac{1}{2} \sum_{n=1,3} \left[ 1 + (1 - \nu) \left[ \beta_n \left( x_n - \frac{b}{2} \right) \right] \frac{1}{sh \beta_n x} \sin \beta_n y(a_n)
\]

\[
+ \frac{1}{2} \sum_{n=1,3} \left[ 1 + (1 - \nu) \left[ \alpha_n \left( x_n - \frac{b}{2} \right) \right] \frac{1}{sh \alpha_n x} \sin \alpha_n y(c_n)
\]

\[
+ \frac{1}{2} \sum_{n=1,3} \left[ 1 + (1 - \nu) \left[ \alpha_n \left( x_n - \frac{b}{2} \right) \right] \frac{1}{sh \alpha_n x} \sin \alpha_n y(c_n)
\]

\[
(6)
\]
In Equation (6), the following formula:

\[
\frac{4q}{Da} \sum_{m=1,3}^{\infty} \left\{ 1 + \frac{1}{2ch \alpha_m b} \left[ \alpha_m \left( y - \frac{b}{2} \right) sh \alpha_m \left( y - \frac{b}{2} \right) \right] \right\} \frac{1}{\alpha_m^5} \sin \alpha_m x
\]

Can be replaced by:

\[
\frac{4q}{Db} \sum_{n=1,3}^{\infty} \left\{ 1 + \frac{1}{2ch \beta_n a} \left[ \beta_n \left( x - \frac{a}{2} \right) sh \beta_n \left( x - \frac{a}{2} \right) \right] \right\} \frac{1}{\beta_n^5} \sin \beta_n y
\]

Due to symmetrical bending, the following equations can be obtained by summarizing four boundary conditions mentioned above.

\[
-G_n q + F_n a_n - \sum_{m=1,3}^{\infty} H_{mn} c_m = 0 \tag{7}
\]

\[
-K_m q + Q_m c_m - \sum_{n=1,3}^{\infty} S_{mn} a_n = 0 \tag{8}
\]

In the two equations,

\[
G_n = \frac{2}{b} \left( 3 - \nu \right) th \frac{1}{2} \beta_n a - \left( 1 - \nu \right) \frac{\beta_n a}{2ch^2 \frac{1}{2} \beta_n a} \frac{1}{\beta_n^2}
\]

\[
F_n = \frac{1}{2} D \left( ch \beta_n a - 1 \right) \left[ 2 \left( 1 - \nu^2 \right) + \left( 1 - \nu \right)^2 \left( 1 - \frac{\beta_n a}{sh \beta_n a} \right) \right] \frac{\beta_n^3}{sh \beta_n a}
\]

\[
K_m = \frac{2}{a} \left( 3 - \nu \right) th \frac{1}{2} \alpha_m b - \left( 1 - \nu \right) \frac{\alpha_m b}{2ch^2 \frac{1}{2} \alpha_m b} \frac{1}{\alpha_m^2}
\]

\[
Q_m = \frac{1}{2} D \left( ch \alpha_m b - 1 \right) \left[ 2 \left( 1 - \nu^2 \right) + \left( 1 - \nu \right)^2 \left( 1 - \frac{\alpha_m b}{sh \alpha_m b} \right) \right] \frac{\alpha_m^3}{sh \alpha_m b}
\]

\[
H_{mn} = \frac{4}{b} D \left( 1 - \nu \right)^2 \frac{\alpha_m^3 \beta_n^3}{K_{mn}^2}, \quad S_{mn} = \frac{4}{a} D \left( 1 - \nu \right)^2 \frac{\alpha_m^3 \beta_n^3}{K_{mn}^2}
\]

\( a_n \) and \( c_m \), which can be obtained by overlay computing of Equation (7) and (8) with appropriate \( m \) and \( n \), can be substituted into the Equation 6, the deflection of each point can be obtained, and then bending moment, shearing force, stress and strain of the elastic thin rectangular plate pinned at four corners bearing transverse uniform loads are obtained.
CONVERGENCE AND ACCURACY OF THE EQUATION

Convergence of the Equation

According to Equation (6),

1. The mid-span deflection of plate is shown as below (when \( x = \frac{a}{2}, y = \frac{b}{2} \)):

\[
\omega \left( x = \frac{a}{2}, y = \frac{b}{2} \right) = \frac{4q}{Da} \sum_{m=1,3}^{\infty} \left\{ 1 + \frac{1}{2ch} \alpha_m b \left[ - \left( 2 + \frac{1}{2} \alpha_m b th \frac{1}{2} \alpha_m b \right) \right] \right\} \frac{1}{\alpha_m^5} \sin \frac{1}{2} \alpha_m a
\]

\[
+ \sum_{n=1,3}^{\infty} \left\{ 2 + (1 - \nu) \left[ \beta_n \beta_n a - \frac{1}{2} \beta_n a \right] \beta_n a \right\} \frac{1}{sh} \beta_n a \sin \frac{1}{2} \beta_n b (a_n)
\]

\[
+ \sum_{n=1,3}^{\infty} \left\{ 2 + (1 - \nu) \left[ \alpha_n b th \alpha_n b - \frac{1}{2} \alpha_n b th \frac{1}{2} \alpha_n b \right] \right\} \frac{1}{sh} \beta_n b \sin \frac{1}{2} \alpha_n a (c_m)
\]

(9)

2. Mid-span deflection of plate is shown as below (when \( x = \frac{a}{2}, y = 0 \)):

\[
\omega \left( \frac{a}{2}, 0 \right) = \sum_{m=1,3}^{\infty} \sin \frac{1}{2} \alpha_m a (c_m)
\]

(10)

3. Deflection of plate is shown as below (when \( x = a, y = 0 \)):

\[
\omega \left( a, \frac{b}{2} \right) = \sum_{n=1,3}^{\infty} \sin \frac{1}{2} \beta_n b (a_n)
\]

(11)

Take a rectangular plate with \( a = 3000 \text{mm}, b = 2400 \text{mm}, \nu = \frac{1}{6}, E = 2.3 \times 10^4 \text{N/mm}^2, h = 70 \text{mm}, \)

\( q = 0.1 \text{kN/m}^2 \) for example, and then \( \frac{b}{a} = 0.8 \), \( D = \frac{Eh^3}{12(1 - \nu^2)} = 6.7620 \times 10^8 \text{N} \cdot \text{mm} \). The deflections of the plate are illustrated in Table 1.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>CONVERGENCE OF EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stacking fold of deflection</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
</tr>
</tbody>
</table>
It can be concluded from the Table 1 that the equation has rapid convergence in terms of evaluation. Convergence meets the demand when both m and n are 13, that is 7 times of overlay computing of deflection.

**ACCURACY OF THE EQUATION**

To verify the accuracy of the equation mentioned above for deflection, calculation results for Equation (6) are compared with the exact solution of those of elastic thin rectangular plate pinned at four corners \(^7\). A rectangular plate bearing transverse uniform load \(q\) with four pinned corners is chosen for comparison.

1. **Example 1**

Take a rectangular plate with \(a=2000\text{mm}, \ b=2000\text{mm}, \ \nu = \frac{1}{6}, \ E=2.3\times10^4\text{N/mm}^2, \ h=70\text{mm}, q=0.3\text{kN/m}^2\) for example, and then \(\frac{b}{a} = 1.0, \ D = \frac{Eh^3}{12(1-\nu^2)} = 6.7620\times10^8\text{N\cdot mm}\). The deflections of the plate are illustrated in Table 2.

<table>
<thead>
<tr>
<th>Calculation point</th>
<th>Analysis of results</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>x</td>
<td>y</td>
<td>x</td>
</tr>
<tr>
<td>Calculated value</td>
<td>(\omega)</td>
<td>0.1854</td>
<td>0.1218</td>
<td>0.1218</td>
</tr>
<tr>
<td>(mm)</td>
<td></td>
<td>a/2</td>
<td>b/2</td>
<td>a/2</td>
</tr>
<tr>
<td>Theoretical value</td>
<td>(\omega')</td>
<td>0.1867</td>
<td>0.1221</td>
<td>0.1221</td>
</tr>
<tr>
<td>(mm)</td>
<td></td>
<td>a/2</td>
<td>0</td>
<td>a</td>
</tr>
<tr>
<td>Error</td>
<td>(\frac{\omega - \omega'}{\omega} \times 100%)</td>
<td>-0.70%</td>
<td>-0.25%</td>
<td>-0.25%</td>
</tr>
</tbody>
</table>

2. **Example 2**

Take a rectangular plate with \(a=3000\text{mm}, \ b=2400\text{mm}, \ \nu = \frac{1}{6}, \ E=2.3\times10^4\text{N/mm}^2, \ h=70\text{mm}, q=0.3\text{kN/m}^2\) for example, and then \(\frac{b}{a} = 0.8, \ D = \frac{Eh^3}{12(1-\nu^2)} = 6.7620\times10^8\text{N\cdot mm}\). The deflections of the plate are illustrated in Table 3.

3. **Example 3**

Take a rectangular plate with \(a=2000\text{mm}, \ b=1000\text{mm}, \ \nu = \frac{1}{6}, \ E=2.3\times10^4\text{N/mm}^2, \ h=70\text{mm}, q=0.5\text{kN/m}^2\) for example, and then \(\frac{b}{a} = 0.5, \ D = \frac{Eh^3}{12(1-\nu^2)} = 6.7620\times10^8\text{N\cdot mm}\). The deflections of the plate are illustrated in table 4.
It can be concluded from the table 2 to 4 that the relative error of deflection of the same plate calculated by two different methods with the same boundary conditions and loads is less than 3%. Results also show that the equation for calculating deflection of elastic thin rectangular plate pinned at four corners is highly accurate.

### TABLE 3

<table>
<thead>
<tr>
<th>Calculation point</th>
<th>Analysis of results</th>
<th>1</th>
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<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>x y</td>
<td>x y</td>
<td>x y</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a/2</td>
<td>b/2</td>
<td>a/2</td>
</tr>
<tr>
<td>Calculated value</td>
<td>ω</td>
<td>0.2211</td>
<td>0.1894</td>
<td>0.0938</td>
</tr>
<tr>
<td>Theoretical value</td>
<td>ω'</td>
<td>0.2156</td>
<td>0.1881</td>
<td>0.0946</td>
</tr>
<tr>
<td>Error</td>
<td>$\frac{ω - ω'}{ω} \times 100%$</td>
<td>2.49%</td>
<td>0.68%</td>
<td>-0.85%</td>
</tr>
</tbody>
</table>

### TABLE 4

<table>
<thead>
<tr>
<th>Calculation point</th>
<th>Analysis of results</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>x y</td>
<td>x y</td>
<td>x y</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a/2</td>
<td>b/2</td>
<td>a/2</td>
</tr>
<tr>
<td>Calculated value</td>
<td>ω</td>
<td>0.1665</td>
<td>0.1688</td>
<td>0.0190</td>
</tr>
<tr>
<td>Theoretical value</td>
<td>ω'</td>
<td>0.1656</td>
<td>0.1668</td>
<td>0.0189</td>
</tr>
<tr>
<td>Error</td>
<td>$\frac{ω - ω'}{ω} \times 100%$</td>
<td>0.54%</td>
<td>1.18%</td>
<td>0.53%</td>
</tr>
</tbody>
</table>

### CONCLUSIONS

The equation for calculating deflection of elastic thin rectangular plate pinned at four corners bearing transverse uniform loads is derived from the general theory of displacement of bending thin rectangular plate proposed by Fu Baolian. The verifications show that the equation is better in terms of consistency and accuracy.

### REFERENCES

THE MIRACLE OF POST-BUCKLED BEHAVIOUR IN THIN-WALLED STEEL CONSTRUCTION AND ITS PARTIAL “EROSION” DUE TO REPEATED LOADING

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KEYWORDS

Thin-Walled Construction, Buckling, Breathing, Fatigue, S-N curves.

ABSTRACT

The promising trend of thin-walled construction can be materialised just thanks to the beneficial effect of post-buckled behaviour. However, as a great part of steel structures are subjected to many times repeated loads (bridges, crane-supporting girders and the like), it is demonstrated, by means of the results of numerous experiments carried out by the authors in Prague, how the post-critical reserve of strength is affected by the cumulative damage process generated by the many times repeated character of loading, and how this phenomenon influences the design of structures.

THIN-WALLED CONSTRUCTION AND POST-BUCKLED BEHAVIOUR IN IT

One of the most promising trends in our striving to save steel is to use thin-walled structures, i.e. structural systems made of slender (usually plate) elements. Of course, here it can be argued that such elements are liable to buckle so that then the limit state of the system is substantially reduced by stability phenomena. The situation is however remedied by the miracle of post-buckled behaviour, in the light of which a thin-walled plated system subjected to quasi-constant loading behaves like a (so called) super-smart structure, i.e. like one which is able not only to diagnose its own situation, but also to generate a means of powerful self-defence, thanks to which the ultimate strength of the system is usually very significantly higher than the linear-buckling-theory critical load.

That is why a great attention has been internationally paid to research on the post-buckled behaviour and ultimate strength of slender webs, flanges and other plate elements, the Czech research always striving to play a useful role in these activities.
For example, the authors of this paper and their co-workers spent about three decades in investigating the post-critical reserve of strength and ultimate load behaviour of steel plate girders, box girders, thin-walled columns etc.

**PARTIAL “EROSION” OF THE POST-BUCKLED BEHAVIOUR IN THIN-WALLED CONSTRUCTION Subjected TO MANY TIMES REPEATED LOADING**

Although a great part of steel plated structures used in building construction can be listed among structures under the action of quasi-constant loading, this cannot be said about steel bridgework, crane supporting girders and similar systems. Such structures are exposed to many times repeated loading. Then, if their webs are slender, they repeatedly buckle out of their plane. This phenomenon, being now usually termed web breathing, induces significant cumulative damage process in the breathing webs and we can ask the obvious question of whether the breathing phenomenon leads to a significant „erosion“ of the post-critical reserve of strength described above.

And to research on this problem the authors turned their attention several years ago.

Given the complex character of the cumulative damage process in breathing webs, it was crystal clear that a very important role should be played by experiments. The tests, their number already exceeding two hundred, were conducted in three laboratories, viz. at (i) the Institute of Theoretical and Applied Mechanics of the Czech Academy of Sciences, (ii) Klokner Institute of the Czech Technical University, and (iii) the Research Institute of Materials.

The large number of tests proved to be indispensable in view of the large scatter which is characteristic of all breathing experiments.

But as this juncture it is useful to say a few words about the character of the test girders used.

Like most girders tested now by the writers at the Institute of Theoretical and Applied Mechanics in Prague, they are fairly large, having a web 1000mm deep, so that their character is not far from that of ordinary girders.

All the girders were fabricated in the steel fabricator of Division 7 of the Company METROSTAV plc., using the same technological procedures as are applied there in the fabrication of ordinary steel bridges. It is important to note that, in the fabrication of the test girders, no attempt was made to diminish (by heat treatment) the initial curvature of the web generated during the process of girder fabrication.

Here only some of the conclusions drawn during the experimental investigation can briefly be mentioned. Other results and conclusions can be found for example in [1].

For the purpose of practical design it is important to know what portion of the post-buckled strength, i.e. of the maximum load that a girder is able to sustain, is “eroded” when the girder is exposed to many times repeated loads.
The authors were able to shed some light on this interesting question because for each series of their fatigue tests, they also carried out a few static experiments on girders having the same dimensions. Given the character of the Prague test girders, their webs were subjected to combined shear and bending, with the effect of shear predominating. Let us therefore measure the loading of each web panel by shear force $V$, its intensity being determined by ratio $V/V_{cr}$, when $V_{cr}$ is the critical shear force (given by the linear buckling theory), calculated for a web panel clamped into the flanges and hinged on the transverse stiffeners. The ratio $V/V_{cr}$ then also determines the post-buckled reserve of strength.

The results of the writers’ experiments are shown in Figures 1 and 2. The corresponding ratios $V_{max}/V_{cr}$ are plotted on the vertical axis, the related numbers $N$ of loading cycles on the horizontal axis.

As all of the Prague breathing tests are conducted to failure, the authors are able to study the whole history of the development of each fatigue crack from its initiation to the collapse of the whole girder. Figure 1 is then related to the initiation of the first crack and Figure 2 to the fatigue failure of the whole girder.

![Figure 1: The cumulative-damage induced “erosion” of the maximum load – with respect to the initiation of the first fatigue crack.](image)

Each of the two figures is divided into two parts: that denoted by $a$ concerns $0 < N \leq 10^5$ (i.e. the interval of $N$ where the gradient of the test results is greatest), the part $b$ then holds for $N > 10^5$.

A very significant impact of the cumulative damage process on the fatigue tests can easily be seen in the figures. As the Prague breathing experiments exhibited, like most fatigue tests of all kinds, a large scatter, an average line of the breathing tests is also given in the figures. The
area above it, \( \frac{V_{\text{max, st}}}{V_{\text{cr}}} - \frac{V_{\text{max, fat}}}{V_{\text{cr}}} \) \text{average}, gives an average value of the cumulative-damage induced “erosion” of the maximum sustainable load.

\[ \text{a)} \]

\[ \text{b)} \]

Figure 2: The cumulative-damage induced “erosion” of the maximum load – with respect to the fatigue failure of the girder.

**IMPACT ON DESIGN**

It follows from the above analysis that the problem of web breathing can play a very important role and therefore cannot be disregarded; on the contrary, it can significantly affect the design of steel bridges, crane-supporting girders and other structural systems under the action of many times repeated loads.

And to establish a reliable method for the analysis of the breathing webs of thin-walled girders was the main objective of the authors’ research.

First it should be mentioned here that during the first stage of our research, we deliberately postponed any attempt to establish a design procedure, desiring first to “map” in detail all aspects of the breathing phenomenon (the initiation and propagation of fatigue cracks and their role in the failure mechanism of the girders, and a suitable definition of the limit state of the whole system) and the part of all factors influencing it.

It is also worth mentioning that, unlike some tests carried out by other authors, in which the experiments were stopped when the first observable crack was detected, all of the Prague tests were conducted to failure. Thus we were able to “map” the whole history of all fatigue cracks – from their initiation to the failure of the girder. Thereby we avoided being “fascinated” by the very phenomenon of crack appearance, but were able to study the further
development of each crack, to see whether it propagated or stopped, and to find out how far away was the initiation of the first fatigue crack from the fatigue failure of the whole girder.

Based on an analysis of the experimental results obtained and thanks to having thus “mapped” for all test girders the whole regime of fatigue crack growth from the initiation of the first fatigue fissure to the failure of the whole girder, the authors were able to establish a design procedure based on S-N curves.

**S-N CURVES ESTABLISHED BY THE WRITERS ON THE BASIS OF THEIR BREATHING TESTS**

The authors follow the general features of the design philosophy proposed by Maquoi and Škaloud [2], according to which two limit states are introduced in the analysis, viz. (i) the fatigue limit state, (ii) that of serviceability.

While the fatigue limit state can be related to the failure of the girder (i.e. to unreparable damage – which is acceptable in view of the fact that the fatigue limit state can never be attained during the planned life of the girder), the limit state of serviceability should be related to a much more limited, easily repairable degree of damage. In the case of steel girders with breathing webs, this means that, in the course of the useful fatigue life of the girder, either no or very small fatigue cracks can develop, such as to be easily kept under control, or easily retrofitted in case of need.

The two limit states can be ascertained directly on the basis of the authors´ breathing tests, following the statistic procedure recommended in Appendix Z of EUROCODE 3. In so doing, the range of the stress state in the breathing web can be measured simply by the range $\Delta \tau$ of the average shear stress $\tau_0$ (= shear force $V$/web area $A_w$) in the web. Only those of the writers´ experiments were considered in which the girder went through all stages of fatigue crack growth until the very failure of the girder.

Before proceeding to the derivation of the S-N curves, let us define the range of their application.

The breathing phenomenon is usually linked with high-cycle fatigue. The frontier, measured by the number $N$ of breathing cycles, between high-cycle and low-cycle fatigue is between $10^4$ and $10^5$ cycles. As the authors also obtained enough results in this domain, the S-N curves can be regarded as applicable for $N > 10^4$. As regards the other boundary, i.e. for high values of $N$, the S-N curves established herebelow hold for all $N < 10^m$ cycles. For $N = 10^m$, the S-N curves are assumed to reach their threshold value, i.e. for $N > 10^m$ the S-N curves are parallel to the N-axis. For the fatigue limit state $m = 6.75$, for the serviceability limit one $m = 6.25$. However, for these very high values of $N$, the S-N curves shall yet be the objective of further investigation.

As far as web slenderness $\lambda$ and aspect ratio $\alpha$ are concerned, let us mention that the authors´ experiments were carried out for $\lambda = 117, 175, 250$ and $\alpha = 1, 1.43, 2$, which are the parameters of the webs of most steel plated girders subjected to breathing. The influence of these parameters is reflected in the S-N curves by the role of the quantity $\tau_{cr}$, the (linear-buckling theory) critical load of the web. Given the fact that the writers´ tests were conducted...
on girders with various flange dimensions, \( \tau_{cr} \) can also take account of the boundary conditions of the web.

A similar statement can be made in regard to web loading. The webs of the writers’ test girders were under the action of combined shear \( \tau \) and bending \( \sigma \), with shear predominating, i.e. their \( \sigma / \tau \) ratios < 1.0. This means that the S-N curves are applicable to webs subjected to shear or to combined shear \( \tau \) and bending \( \sigma \) provided \( \sigma / \tau \leq 1.0 \). For larger \( \sigma / \tau \) ratios the S-N curves shall be the objective of further research.

![Figure 3a: S-N curve for the fatigue limit state.](image)

\[
\log (\Delta \tau / \tau_{cr} + 1) = -0.1027 \log N + 0.7537
\]

Figure 3a: S-N curve for the fatigue limit state.

![Figure 3b: S-N curve for the serviceability limit state](image)

\[
\log (\Delta \tau / \tau_{cr} + 1) = -0.0756 \log N + 0.5265
\]

Figure 3b: S-N curve for the serviceability limit state

Let us now establish, via the statistic procedure recommended in Appendix Z EUROCODE 3, the S-N curves for both limit states.

148 test results, i.e. the data resulting from all of those authors’ experiments in which the authors were able to study the whole cumulative damage process in the breathing webs from the initiation of the first fatigue crack to the complete fatigue failure of the whole girders, were used in the analysis.

All test results related to the fatigue failure of the test girders, i.e. to their fatigue limit state, are plotted in Figure 3a. Also two straight lines are given there: one of them shows average
values of the experimental results obtained (their scatter being large) and the other one is the fatigue limit state S-N curve proposed by the authors.

Mathematically it can be expressed as follows:

$$\log(\Delta \tau/\tau_{cr} + 1) = -0.1027\log N + 0.7537$$  \hfill (2a)

where $\Delta \tau$ is the shear stress range, $\tau_{cr}$ the critical load of the web given by linear buckling theory, and $N$ is the number of loading cycles to which the web is subjected.

All test results related to the initiation of the first fatigue crack, i.e. to the serviceability limit state, are plotted in Figure 3b, the two straight lines having the same meaning as above in Figure 3a.

Mathematically the S-N curve for the limit state of serviceability is given by this relationship:

$$\log(\Delta \tau/\tau_{cr} + 1) = -0.0756\log N + 0.5265$$  \hfill (2b)

all symbols having the same meaning as in the formula for the fatigue limit state.

From an analysis of formulae (2a, b) and Figures 3a, b a very important conclusion can be drawn, namely that for breathing webs the average shear stress range $\Delta \tau$ must be lower, and for large values of $N$ even substantially lower, than critical stress $\tau_{cr}$.

And what about the effect of various stress ranges?

If, during its lifetime, the web is subjected to various stress ranges $\Delta \tau_i$, Palmgren-Miner’s criterion can be used:

$$\sum \frac{n_i}{N_i} \leq 1$$  \hfill (3)

where $n_i$ is the actual number of loading cycles for the stress level $\Delta \tau_i$ and $N_i$ is the life, determined from the above S-N curves, of the web determined on the assumption that $\Delta \tau_i$ is the only loading to which the breathing web is subjected during its whole lifetime.

**FATIGUE ASSESSMENT OF BREATHING WEBS IN THE LIGHT OF THE S-N CURVES ESTABLISHED BY THE AUTHORS**

The fatigue assessment of breathing webs should then proceed as follows:

- The first limit state, connected with fatigue failure, shall not be reached before the whole planned life of the structure has been exploited.
- The other limit state, related to the (experimentally observable) initiation of the first-fatigue-through-crack, governs the maximum time before which the first inspection of the girder for potential fatigue cracks needs to be carried out.

If no fatigue fissures are found during the inspection, the useful life of the girder can be extended until another inspection is conducted after one half of the time period to the first...
inspection (this reflecting the fact that the degree of cumulative damage in the breathing web is then larger than during the first period). Failing to detect any fatigue cracks even then, the system of inspections can be extended in the same way. If, and when, a fatigue crack is detected, it shall be carefully measured – via frequent enough inspections – with the view to find out whether it (i) propagates or (ii) has stabilised.

The results of the two checks mentioned above will decide whether some retrofitting of the girder is necessary.

CONCLUSIONS

Based on their experimental results, the authors established S-N curves which can serve as a basis for the design of plate girder webs breathing under many times repeated combined shear and bending.

ACKNOWLEDGEMENT

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REFERENCES


STRUCTURAL ANALYSIS AND DESIGN OF THE THEME PAVILION OF WORLD EXPO.2010

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KEYWORDS

Theme pavilion, World Expo.2010, structural system, 126-span truss string structure, cable pretension force, support, column brace, joint

ABSTRACT

This paper presents a structural analysis and design case on the Theme Pavilion of World Expo.2010. The theme pavilion, which shows the theme of World Expo.2010 in Shanghai - ‘Better City, Better Life’, is in a cube format with a plane dimension of 217.8m×288m, where the west exhibition hall with a column-free space covering 126m×180m will be built as one of the largest exhibition halls in China. The structural system and arrangement of large-span roof structure and steel frame with staggered floor have been introduced in detail. After sufficient analysis and comparison of different structural schemes, a 126m-span truss string structure and mixed column braces scheme have been used in roof structure and steel frame structure, which represents innovative structural applications. At the same time, some special issues, involving cable pretension force, joint of cable-strut system, roof support, temperature action of steel frame, are discussed. Some suggestions about these issues are put forward, which can optimize roof and frame structural behaviors, make construction stage easier and reduce structural budget. The contents of structural analysis and design can present a valuable reference for similar structures.

INTRODUCTION

The theme pavilion of World Expo.2010 (hereinafter referred to as theme pavilion) is designed to show the theme of World Expo.2010 in Shanghai - ‘Better City, Better Life’ (figure 1). During the World Expo, the theme pavilion will hold opening and closing ceremonies, and other exhibitions. After the World Expo, it will be preserved as the permanent architecture with multi-functions including exhibition, conference, performance and so on.
The theme pavilion is in a cube format with a plane dimension of 217.8m×288m and 26.3m high. Its total area is 120000m² with a 80000m² above ground and 40000m² underground. According to architectural design, the theme pavilion includes west exhibition hall of one storey, middle hall and east exhibition hall of two storeys, and cornices in the north and south sides (figure 2). What’s worth mentioning, the west exhibition hall which has a column-free space covering 126m×180m, will be built as one of the largest exhibition halls in China.

Roof geometry of the theme pavilion is composed of six folded plate elements in the west-east direction, which form a wave shape. Each folded plate element is 288m long, 36m wide and 3m high. Under the folded plate element, there are structural elements. Above the folded plate elements, solar panels are arranged in a diamond-shaped pattern. Figure 3 shows the architectural system components.

**Figure 1: Perspective view of theme pavilion**

**Figure 2: Plan of theme pavilion**

**Figure 3: Architectural system components**

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**ROOF STRUCTURE**

**Roof structural system and arrangement**

The choice of roof structural system is mainly based on the following considerations:

1) Architectural and structural morphology should be in accordance with each other and unified;
2) Structural morphology should be regular and elegant;
3) The roof structure should be transparent to release oppression of the interior space.

However, due to the interior clear height requirement, the structural height of 126m-span roof of west exhibition hall has to be limited within 11.5m. The rise-span ratio is less than 1/11. So it is a critical issue for structural engineering to choose an appropriate roof structural system of west exhibition hall.

Comprehensively considering the three main factors of the choice of roof structural system as mentioned above, during scheme design stage, three feasible roof structural systems, including one-way truss string structure, two-way truss string structure and mega-frame structure, have been compared (figure 4). The mega-frame scheme has the best structural efficiency, but the cross braces of lattice column of mega-frame greatly influence architectural function of ancillary rooms, which cannot be accepted by architect. On the other hand, mega-frame will generate a huge thrust, which brings a great challenge to the design of column base and increases budget of supporting structure. The two-way truss string structure has the worst structural performance. As the height of roof structural is limited in 11.5m, the rise-span ratio of two-way truss string structural in the north-south direction(180m) is only 1/15.6, which cannot resist the dead load but increase the structural burden. At the same time, the configuration of two-way cable joint and the cable pretension stage are very complex. Finally, the one-way truss string structure has been used in roof structure.

The roof of west exhibition hall is carried by nine truss string structures (TSS), each sitting on two columns with a span of 126m center-to-center (figure 5). Each TSS includes two components. One is rigid elements on the top of TSS. The structural form of rigid elements is spatial truss, which section is equilateral triangle with a height of 3m and a width of 3m. The other is cable-strut system. Compared with many other large-span hybrid string structures, the roof structure in the theme pavilion has special characters as follows. Firstly, rigid elements of conventional TSS are reverse triangular truss, e.g. the roof of Guangzhou International Conference and Exhibition center. However, in the roof design of theme pavilion, the rigid element is equilateral triangular truss. Thus, one end of roof purlin can be supported by the top chord of equilateral triangular truss, and the other end can be supported by the bottom chord. This structural solution is easy to form architectural folded plate shape. Secondly, the cable-strut system adopts special V-shaped struts. Compared with conventional multi-struts system of TSS, V-shaped struts laid in the middle span of TSS can optimize internal force distribution and decrease bending and compressive stress intensity of rigid elements. At the same time, V-shaped struts of cable-strut system give people a new architectural vision.

The supporting spans of middle hall and east exhibition hall are 54m, 45m and 45m from west to east. Considering the roof architectural form, structural efficiency and continuous configuration, the spatial trusses of rigid elements of west exhibition hall are continued to the middle hall and east exhibition hall as the main roof structure, which forms four-span continuous girders with a length of 270m (figure 5b). In the north-south direction, five plane truss have been arranged at the ends of four-span continuous girders, which restrict the horizontal and torsional deformations of the end of continuous girder and ensure the out of plane stability of roof structure.
Cable pretension force

Cable pretension force of TSS of the theme pavilion is mainly based on the following considerations:

1) Active control of structural deformations. Comprehensive consideration of roof structural vertical deformation and supporting horizontal deformation, under the two extreme load cases including dead & live load case and dead & wind load case, Cable pretension force should make the difference between undeformed and deformed structure minimum. The purpose of active deformation control is to make structure having appropriate geometrical configuration and necessary shape stability, and to decrease the roof structural eccentric load action to supporting column, which lighten the P-Δ effect of the columns and improve their ultimate bearing capacity.

2) Active control of internal force of structural members. In TSS, cable pretension force makes struts in compression, which realizes unloading action to the top rigid elements and results in the decrease of bending or compressive stress of rigid elements.

3) Avoiding slack of cable under extreme case (mainly wind load). If cable repeatedly appears slack-tension phenomena under frequent wind action, the joints connecting cable and other members will be inclined to fatigue failure. Besides, slack will make cable lose its nonlinear effect, which maybe causes the out of plane instability of whole cable-strut system.

4) Integrating with construction scheme and improving construction stage efficiency. In China, most of construction schemes of large-span roof structure are assembling structure on scaffold. As for TSS, if the construction of rigid elements adopts assembling structure on scaffold, the cable pretension force should make sure TSS detaching from scaffold after cable-tension stage.

Comprehensive considering the influencing factors of cable pretension force as mentioned above, in the design of theme pavilion, the cable pretension force of TSS is 2225kN. In order to review the cable pretension force effect, nonlinear staged construction analysis is used in the SAP2000 model. After cable-tension stage, the mid-span of TSS uplifts 49.5mm, so the whole TSS detaches from scaffold. Under the dead & live load case, vertical deformation of mid-span of TSS is -274mm and horizontal deformation of the top of supporting column is -37.5mm. While under the dead & wind
load case, the vertical deformation is 101mm and the horizontal deformation is 37.2mm. The above analysis indicates the whole TSS can satisfy shape stability. Under the extreme wind action, the minimum internal force in cable is 1230kN, which indicates the cable will not appear slack.

**Roof support**

Upon architectural function request, the whole roof structure from west to east cannot arrange expansion joints. Therefore, the roof length of the theme pavilion in the west-east direction is up to 270m, and the whole roof structure form four-span continuous girders, which have a great impact on internal force of the supporting frame structure. In the design of supporting joints of the theme pavilion, seismic spherical bearings have been utilized to connect roof structure with supporting frame structure. Except that the supporting joint at the axial line 9 adopted rotary bearings that only restrict all of 3-dimensional translations, the other bearings would be sliding to form a self-balanced system during cable-tension and installation work. After roof accessory such as skylights, trellis louvers and gutters had been installation, all of the sliding bearings were welded to the base plates to form the rotary bearings.

The purpose of changing sliding bearing to rotary bearing during the construction stage can be accounted for as following:

1) Releasing the thrust of roof continuous girders to the supporting frame due to thermal expansion in construction stage, which will enormously decrease deflection and bending stresses to the supporting column. While in service state, indoor temperature change is almost constant, so the temperature action is relatively mild.

2) Releasing the unfavorable horizontal tension to the supporting column during cable-tension of TSS.

3) Making sure that the whole structure including roof structure and supporting frame structure can form an integral structure to counteract additional load, e.g. storm or earthquake action. It can decrease unbraced length of supporting columns and improving their ultimate bearing capacity.

**Joint of cable-strut system**

The design of cable-strut system of 126-span truss string structure is a great challenge to engineers. As the V-shaped struts form a geometrically stable system, the bottom end of V-shaped struts cannot move freely along the direction of cable internal force. If the joint of cable-strut system adopts the traditional single-strut joint style in which the bottom end of struts can move freely in the plane of truss string structure, the cable internal force will not be transferred along efficiently but generate an unfavorable behavior to the rigid elements of truss string structure. In joint design (figure 6), 2mm deep colloidal silica is used in the surface of cable hole which the cable is through, so the cable can move freely in the joint hole to transfer the internal force.
STEEL FRAME STRUCTURE

Structural system and arrangement of column braces

Steel frame structure of the theme pavilion is a staggered floor structure because of the architectural design. The mass and stiffness of the west and east exhibition halls are obviously inconsistent. This results in an inevitable overall torsional deformation of steel frame structure under horizontal earthquake action. On the other hand, as the steel frame structure is designed with no expansion joints and the structural lengths of west-east direction and north-south direction are 288m and 217m, temperature action is outstanding in this super-long structure.

To solve these problems, the design of column braces system of the theme pavilion has taken into account the following three points.

1) The column braceing system should improve the structural torsional stiffness effectively on the premise that structural lateral stiffness increases in a small range to avoid enhancing earthquake action.

2) The column braceing system should avoid enhancing temperature action obviously.

3) The arrangement of column braces should be consistent with architecture layout to avoid impacting the architectural functions.

Preliminary column braces plan was confirmed after collaborating with architect (figure 7a). However, the analysis of the original column braces plan showed that when most of column braces were arranged in the internal space of the theme pavilion, the structural lateral stiffness increased more than structural overall torsional stiffness. So the structural engineers optimized the original column braces plan. The internal column braces were taken out and the external column braces were kept, which not only improved the structural overall torsional stiffness, but also increased the architectural usable floor areas (figure 7b).

Furthermore, three styles of column braces schemes, which are total steel column braces scheme, total energy dissipation column braces scheme and mixed column braces scheme (figure 8), have been compared. The researches show that under horizontal earthquake action, total energy dissipation column braces scheme is difficult to satisfy the request of Chinese code of seismic design of buildings, and the budget of total energy dissipation column braces scheme is high. The total steel column braces scheme is favorable to seismic design, but in temperature action, the internal force of frame is high, which results in relative big cross sections of frame members. The mixed column braces scheme that has advantages of the above two column braces schemes shows a great structural performance and is used to the final scheme.
Temperature action analysis of super-long structure

As the frame of theme pavilion comprises steel columns and beams and concrete floor slabs, concrete creep of floor slabs is considered in the temperature action analysis of super-long structure. Elastic modulus reduced factor of concrete is used to simulate the favorable effect of concrete creep which reduce temperature action to the frame. By comparing some projects, the elastic modulus reduced factor of concrete is assumed as 0.3.

Temperature action analysis of steel column braces scheme and mixed column braces scheme are compared in the design. Table 1 and figure 9 show the results of internal force value and distribution of the two column braces schemes in temperature action. As the energy dissipation column braces don’t generate internal force in temperature action, the frame force values of mixed column braces scheme are less than the steel column braces scheme. From table 1, we can see that the maximum column axial force of mixed column braces scheme is 32% less than the steel column braces scheme, which shows the advantage in temperature action control of mixed column braces scheme.

### TABLE 1

INTERNAL FORCE VALUE OF STEEL FRAME IN TEMPERATURE ACTION

<table>
<thead>
<tr>
<th>Member</th>
<th>Mixed braces scheme (kN)</th>
<th>Steel braces scheme (kN)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel brace</td>
<td>2501</td>
<td>3108</td>
<td>20</td>
</tr>
<tr>
<td>Beam</td>
<td>1423</td>
<td>1582</td>
<td>10</td>
</tr>
<tr>
<td>Column</td>
<td>2375</td>
<td>3492</td>
<td>32</td>
</tr>
</tbody>
</table>
CONCLUSION

The theme pavilion of World Expo.2010 is a unique and innovative large-span structure which creates a dramatic architectural expression from both the interior and exterior of the building.

1) The 126m-span truss string structure satisfies the interior clear height requirement and realizes the unification of architectural and structural designs. The structural system and arrangement of TSS serve as material properties and improve structural efficiency.

2) In order to solve the overall torsional deformation of steel frame structure and reduce the unfavorable temperature action, mixed column braces scheme that has advantages of total steel column braces scheme and total energy dissipation column braces scheme is adopted.

3) Cable pretension force is one of the most important factors of structural optimization of HSS. Appropriate cable pretension force should be integrated with structural internal force, deformation, cable slack and construction scheme. In addition, together with sliding bearings, horizontal thrust force at the support could be reduced greatly, which lighten the burden of the supporting structures.

4) Structural design of the theme pavilion shows large-span structure should satisfy demands of architecture and function, and provide a rational layout, efficient and flexible column-free space for the owners. At the same time, as an important building, structural safety should be ensured. Based on this, structural system could be further optimized to improve structural efficiency.

In conclusion, meeting the requirements of function, attractiveness, safety, and economy is the goal of structural design of the theme pavilion of World Expo.2010.

REFERENCES

DESIGN AND ANALYSIS OF A FOLDABLE PROTECTIVE SHELTER

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KEYWORDS

Deployable shelter, Protective, Blast loading, Foldable, High strength steel plate

ABSTRACT

Deployable structures are conveniently stored and transported; it can be rapidly erected to realize its function. The engineering design and research on theories and experiments of this type of structures has been gradually developed. Due to human disasters such as wars, terrorist attacks and mass ethnical genocide, deployable shelters can be developed with protective ability and robustness to resist blasting loading and bullets. In this paper, an innovative and advanced deployable structural system is proposed, namely Foldable Protective Shelter to address the mentioned requirements. This shelter can be not only rapidly deployed and expanded, but also equipped with protection level. An engineering design concept for such a lightweight, foldable shelter to meet military requirements is introduced and investigated. The Foldable Protective Shelter can accommodate people for protection purpose, and can be folded into a compact shape to facilitate storage and transportation. In the field, it has a design option to be added with wheels to enhance its mobility. When it comes to deployment, a series of hinges and automation mechanical system can be applied to achieve rapid deployment. Structural materials and prototyping are further investigated. In addition, the numerical modeling for this foldable protective shelter under blast loading is presented, analyzed and discussed. This Foldable Protective Shelter shows a variety of advantages for both deployable and protective purposes.

INTRODUCTION

Deployable structures are structures which are capable of large configuration changes in an autonomous way. Most common is that the configuration changes from a packaged, compact state to a deployed, large state. Usually, these structures are used for easy storage and transportation. When required, they are deployed into their service configuration. This type of structures has been applied in various projects, and the engineering design concept and
research on its morphology study and application has been gradually developed (Vu et al. [1] [2]). The research involves the design and analysis of folding patterns and mechanisms for deployable structures, and investigating ways of creating a novel and proper folding/retractable pattern and mechanism that allows the deployment of a structure more convenient, reliable and smooth.

In human disasters such as local war, explosion and gunfight, a series of deployable shelters need to be designed to resist weaponry attacks which are unforeseen events. Several systems were developed and evaluated against their ability to resist blast pressure. The protective shelter termed Connectable Collapsible Container-based Shelter (3C Shelter) was developed by utilizing the shape and standard size of commercial standard container (Ma et al. [3] [4]). Container-based systems are collapsible and thus convenient for stacking and transportation. In this paper, an improved form of container-based shelter is introduced to minimize effect of blast overpressure by creating inclined side walls. This Foldable Protective Shelter is designed in medium size to resist blast loadings while providing protection to 12 to 15 people. The outer panels of the container are to be made of protective materials e.g. armor steel plates. The distinctive features of this medium-size foldable shelter are high stocking ratio, high level of automation, mobility, and ballasting with extendable beams to avoid time consuming of anchoring piles.

ENGINEERING DESIGN CONCEPT DEVELOPMENT

The initial concept for designing this shelter is shown in Figure 1, which is generated by basic origami. This folding pattern is further developed by detailed engineering design to actualize the design concept, the graphic modeling of which is shown in Figure 2. This foldable shelter comprises main frame system and covering high strength panels. The shelter size is about 6m (L) × 2.4m(W) × 2.4m(H), which may accommodate 12-15 persons. The main frame employs box steel beams and columns, while outer protective material uses 3mm thickness armor steel plate with high strength. The total weight of the structure can reach 3000kg. This type of shelter can be folded in a compact shape for convenient transportation. The stocking ratio is evaluated by comparing between a foldable shelter and a non-foldable one. It is found that the proposed form allows a stacking ratio of 5:1 as can be seen in the Figure 2.

Figure 1: Initial concept

Figure 3 illustrates that a prototype is made to demonstrate the whole deployment process and realize the engineering design concept. It also shows the practicality of this foldable protective shelter. On the other hand, making prototype also helps us to improve the engineering structural design. The wheels are added to improve mobility of the sheltering system.
BLAST ANALYSIS

This Foldable Protective Shelter takes advantages of both deployable structure and protective structure. It is imperative to conduct structural analysis, enabling the structure to exert good structural performance and possess high strength to resist blast loadings.

Finite Element Model

Half of the structure is modeled in ABAQUS as a replacement for modeling the whole structure due to the symmetry of the structure, loading, and boundary conditions. Finite Element model is shown in Figure 4. The structural members for frame utilize box steel, the dimension of which is 100mm ×100mm×6mm. All of the beams and columns are welded together, both of which apply mild steel as their material. The protective panels of the structure adopt a high strength material of 3mm thickness armor steel plate. In addition, the stress strain relationships for shell elements of both frame and plate are also shown in Figure 4. The stress-strain curve for plate is derived from experimental measurements. Pinned boundary conditions are simulated at two corners of the half structure. In the practical application, anchor systems can be used at the four corners to stabilize the whole structure. The current general finite element analysis software ABAQUS is applied to the time history method under blast loading. In the finite element model, material nonlinearity effect is included. It is described by von Mises yield criterion combined with an isotropic hardening rule.
(Note: prototype does not include armour covering material for clarity purpose during deployment demonstration)

Figure 3: Prototyping

Plate: high strength armor steel
Frame: normal mild steel

Note: mesh size=0.05m; Element Number=23305

Figure 4: Finite element model
Loadings

A uniform distribution of the blast wave on the panel is obtained from the software ATBLAST. The blast loading can adopt loading case of a 110kg weight explosion with a distance of 100m from the explosion source, the pressure history curve of which is shown in Figure 5. Two cases are applied on the structure. One (Case a) acts on the left side panels, the other (Case b) acts on the right side panels, which can also be clearly seen in Figure 5.
Results and discussions

Representative nodes are selected to observe the deformation of the whole structure under blast loading as shown in Figure 6. They are center points of the loaded side of two load cases respectively. Figure 6 also shows that the deformation time history curves of the center points under two load cases.

In Figure 6, from the deflection time histories, the maximum deformation is approximately 4 cm, which is equal to 1/150 span. Compared the deflection under two cases, it is found that the deflection under load case a is larger than that of load case b. In addition, compared to the results of 3C shelter (Ma et al. [4]) which has both vertical side panels, it can be concluded that, the bigger inclined angle of the side panel results in less structural deformation. Therefore, inclined side panels can improve the structural resistance performance under dynamic blast overpressure in contrast with vertical side panels.

<table>
<thead>
<tr>
<th>Load case</th>
<th>Structural member</th>
<th>Peak stress (MPa)</th>
<th>Yielding stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load case a</td>
<td>Frame 280.4</td>
<td>280.4</td>
<td>335</td>
</tr>
<tr>
<td></td>
<td>Plate 615.6</td>
<td>615.6</td>
<td>1380</td>
</tr>
<tr>
<td>Load case b</td>
<td>Frame 31</td>
<td>31</td>
<td>335</td>
</tr>
<tr>
<td></td>
<td>Plate 489</td>
<td>489</td>
<td>1380</td>
</tr>
</tbody>
</table>

The deformed geometry and stress for this shelter under two blast loading cases are shown in Figure 7. In addition, the stress distribution is also clearly illustrated. The maximum stress of the plates occurs along the connection of frame and plate. The deformation and stress do not affect structural performance. Therefore, this Foldable Protective Shelter offers excellent structural performance under blast loadings. In addition, this high strength armor steel plate is also qualified to resist bullet shooting, certified by shooting test.

Based on the stress time histories, the peak stress of the structure under two cases is shown in Table 1. Owing to high strength and yielding stress of the plate, obviously, the maximum stresses for both loading cases do not reach yielding stress 1380MPa. And the frame also remains in elastic range. The shelter possesses satisfied structural performance. The deformation and stress of the structure do not affect its structural performance, and then it can be utilized repeatedly under such blast loadings.
Figure 7: Deformation and stress
Case a: Support at loaded side

Case b: Support at loaded side

Case a: Support near nonloaded side
Anchoring systems are also significant for the stability of the whole protective system and its resistance to blast loadings. The base of protective shelter will be subject to high compressive force and tension force during blast loadings. The compression force is supported by base ground of the shelter while tension force needs to be resisted by anchoring systems. Therefore, the reaction forces shown in the Figure 8 are utilized to select suitable anchoring systems. In Figure 8, X direction is parallel to the blast loading direction, and Y direction is vertical to the ground, and Z direction is the longitudinal direction of the shelter, i.e. the direction perpendicular to the blast loading direction. It can be found that the reaction force in Z direction is higher than those in the other two directions at support at loaded side for both load cases, which may be ascribed to the inclined loading surface can increase the in-plane horizontal thrust. In contrast, for the support at non-loaded side, the reaction forces of X and Y direction are higher than Z direction.

In practice, if specific anchoring devices are unavailable at the site, anchoring of the shelter can be done by utilizing some counterweight which is commonly available on the site. Figure 9 clearly shows this structural design concept. The increased weight and the friction between the structure and ground surface is provided to stabilize the whole structure under blast loadings. In this design concept, expandable beams will facilitate the installation of the counterweight on the base part of the shelter. After the deployment of the whole structure, the beams can be protruded out from their original retracted position. Then it can hold the counterweight together with the shelter. The appropriate counterweight can provide required weight and friction to stabilize this foldable protective shelter, reduce the influence of dynamic blast pressure. Therefore, this design concept is a good alternative when the anchoring system is unavailable.

Figure 8: Reaction force time histories of two supports
CONCLUSIONS

A novel Foldable Protective Shelter is proposed and developed with high structural performance in terms of deflection, stress and reaction forces. This medium-size Foldable Protective Shelter takes the advantages of both deployable structure and protective structure. Through the numerical modeling, it also can be concluded that, inclined side panels has better performance under blast loadings. The distinctive features of this shelter are high stocking ratio, high level of automation, mobility, high level of protective ability and ballasting with extendable beams to avoid anchoring piles.

REFERENCES

EFFECTS OF PREBUCKLING LINEARIZATION ON BUCKLING ANALYSIS OF SHALLOW ARCHES

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KEYWORDS
Arches, bifurcation buckling, elastic, limit point buckling, linearization, nonlinear, shallow.

ABSTRACT

The buckling of elastic steel arches is usually described by nonlinear formulations. However, the difficulty of performing the nonlinear analysis may often be circumvented by linearizing the prebuckling equilibrium state. Some structures, such as straight steel columns subjected to central axial compression and steel plates subjected to in-plane loads exhibit a trivial state of stress, i.e. a state in which the bending moment is zero throughout prior to buckling. In this case, the equilibrium configuration prior to buckling can be determined from a linear analysis and the buckling analysis can be linearized. However, for arches that are subjected to transverse loading, the prebuckling linearization should be used with caution since the transverse loading may produce a bending moment in the arch and the arch may deform considerably prior to buckling. This paper investigates the effects of prebuckling linearization on the in-plane buckling of shallow steel circular arches by comparing the exact solutions with the solution of the corresponding linearized problem. It is found that the classical adjacent equilibrium method associated with the prebuckling linearization may overestimate the buckling loads of shallow steel arches, and so provides unsafe predictions of the buckling capacity of an arch.

INTRODUCTION

Buckling phenomena of elastic solids are usually described by nonlinear formulations. However, the difficulty of establishing and solving these nonlinear problems may often be circumvented and determination of a buckling load can be reduced to the solution of a linearized problem by applying the variation method. In the linearized buckling analysis, some simplifications are usually made. Firstly, the prebuckling structural behaviour is
assumed to be linear. Secondly, the effects of the prebuckling deformations on buckling are
ignored, while thirdly, the effects of the buckling deformations on the displacement and
geometric stiffness are ignored. From this point of view, the linear differential equations of
the classical buckling analysis for columns, beams and plates are the differential equations for
the lowest-order variations from the unbuckled state.

It should be noted, however, that although the resulting formulation of such a problem is
linear in the variations, the terms associated with the stress resultants are derived from the
variations of the nonlinear strains. Structures which prior to buckling exhibit a trivial state of
stress, i.e., there is no deformation in the buckling direction prior to buckling. Hence, the
bending moments are zero throughout the structure when it is subjected to compression such
as straight slender columns subjected to central axial compression, plates subjected to in-
plane loads, and the thin circular ring subjected to a constant radial pressure.

However, for shallow circular steel arches that are subjected to uniform radial loading (Figure
1), the prebuckling linearization should be used with caution since the transverse loading may
produce a bending moment in the arch and the arch may deform considerably prior to
buckling. The effects of prebuckling linearization on the in-plane buckling of shallow arches
may be significant. The classical adjacent equilibrium method associated with the
prebuckling linearization may overestimate the buckling loads of shallow arches, and so
provide unsafe predictions of the buckling capacity of the arch. This does not appear to be
widely realized. Because eigenvalue analysis is often used to obtain the buckling load of a
structure in many finite element codes, it may be used erroneously to predict the buckling
load of a shallow arch. The purpose of this paper is to study effects of the linearized
prebuckling analysis on the magnitude of the determined buckling loads of pin-ended shallow
circular steel arches (Figure 1).

![Figure 1 Arch subjected to uniform radial loading](image)

**BUCKLING ANALYSIS**

**Differential Equations of Buckling Equilibrium**

The dimensionless total potential energy of the arch and uniform radial load system (Figure 1)
can be written as [1]

\[
\Pi = \int_0^\theta \left( \frac{1}{2} \left[ \varepsilon_m^2 + \frac{\varepsilon_R^2 \nu^2}{R^2} \right] - \frac{qR\nu}{AE} \right) \, d\theta, \quad \text{with the membrane strain } \varepsilon_m = \tilde{w}' - \tilde{v} + \frac{1}{2} \tilde{v}'^2.
\]
The critical condition for buckling is that the second variation of the total potential energy of the system vanishes for any admissible variation of the displacements, which indicates a possible transition from a stable state to an unstable state. From Eqn. 1, the second variation of the total potential energy of the system can be obtained as

$$\delta^2 \Pi = \int_{\Theta}^{\Theta} \left[ \varepsilon_{mb}^2 + \varepsilon_m \tilde{\nu}_b^2 + \frac{r_s^2 \tilde{\nu}_b^2}{R^2} \right] d\theta,$$

with the buckling membrane strain

$$\varepsilon_{mb} = \tilde{\nu}_b' - \tilde{\nu}_b + \tilde{\nu}' \tilde{\nu}_b'. \tag{2}$$

The functions \(\tilde{\nu}_b\) and \(\tilde{\nu}_b\) that make the functional \(\delta^2 \Pi\) stationary satisfy the Euler-Lagrange equations of variational calculus, which lead to differential equations of equilibrium

$$\varepsilon_{mb}' = 0 \quad \text{and} \quad \frac{\tilde{\nu}''_b}{\mu^2} + \tilde{\nu}_b'' = \frac{R^2 \varepsilon_{mb}}{r_s^2 \mu^2} (1 + \tilde{\nu}')^2, \tag{3}$$

in the axial and radial directions respectively, where \(\mu\) is a dimensionless geometric parameter defined by \(\mu^2 = NR^2/EI_c\).

From the first equation of Eqn 3, the membrane strain \(\varepsilon_{mb}\) due to the buckling deformations is constant. The geometric and static boundary conditions are \(\tilde{\nu}_b = \tilde{\nu}_b = \tilde{\nu}_b = 0\) at \(\theta = \pm \Theta\).

**Bifurcation Buckling**

For antisymmetric bifurcation buckling, the dimensionless buckling displacement \(\tilde{\nu}_b\) is odd while the dimensionless prebuckling displacement \(\tilde{\nu}\) is even, so that the terms \(\tilde{\nu}_b\) and \(\tilde{\nu}' \tilde{\nu}_b\) are odd and their integrals within the interval \([-\Theta, \Theta]\) vanish. In addition, the boundary conditions require that \(\tilde{\nu}_b = 0\) at \(\theta = \pm \Theta\), so that the constant membrane strain \(\varepsilon_{mb}\) can be from \(\varepsilon_{mb}\) of Eqn 2 as

$$\varepsilon_{mb} = \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} \varepsilon_{mb} d\theta = \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} (\tilde{\nu}_b' - \tilde{\nu}_b + \tilde{\nu}' \tilde{\nu}_b') d\theta = 0. \tag{4}$$

Substituting this into Eqn 3 leads to the linear homogeneous differential equation for antisymmetric bifurcation buckling of shallow arches as

$$\frac{\tilde{\nu}''_b}{\mu^2} + \tilde{\nu}_b'' = 0. \tag{5}$$

Substituting the solution of Eqn 5 into boundary conditions \(\tilde{\nu}_b = \tilde{\nu}_b = 0\) at \(\theta = \pm \Theta\) leads to a group of homogeneous algebraic equations. The existence of nontrivial solutions of the homogeneous equations leads to \(\sin (\mu \Theta) = 0\), whose lowest solution is \(\mu \Theta = \pi\), from which and considering the definition of \(\mu\), it can be obtained that

$$N = \frac{\pi^2 EI_c}{(S/2)^2} = N_{p2}, \tag{7}$$

where \(N_{p2}\) is the second mode flexural buckling load of the pin-ended corresponding column about its major principal axis of the cross-section.

**LINEARIZED PREBUCKLING ANALYSIS**

To circumvent the complicated nonlinear analysis, the longitudinal normal strain at an arbitrary point of the cross-section can be linearized as
The dimensionless total potential energy of a shallow arch can then be rewritten as

$$\Pi = \int_{\theta=0}^{\theta=\Theta} \left\{ \frac{1}{2} \left[ (\tilde{w}' - \tilde{v})^2 + \frac{r^2 \tilde{v}^2}{R^2} \right] - \frac{q R \tilde{v}}{AE} \right\} d\theta.$$  \hspace{1cm} (9)

From the calculus of variations, the differential equations of equilibrium in the axial and radial directions can be obtained from Eqn 9 also by invoking Euler-Lagrange equations as

$$\varepsilon_m' = \tilde{w}' - \tilde{v}' = 0 \text{ and } \tilde{v}'' - \tilde{w}' = \frac{q R}{AE}.$$  \hspace{1cm} (10)

With the boundary conditions $\tilde{w} = \tilde{v} = \tilde{v}'' = 0$ at $\theta = \pm \Theta$, solutions of Eqn 10 can be obtained as

$$\tilde{v} = -\frac{5 q R \mu^2 (5 \Theta^2 - \Theta^2) (\Theta^2 - \Theta^2)}{8 N (15 + 2 \lambda^2)} \text{ and } \tilde{w} = \frac{q R \mu^2 \Theta (8 \Theta^2 - \Theta^2) (\Theta^2 - \Theta^2)}{8 N (15 + 2 \lambda^2)}$$

where $\lambda = S \sqrt{A / (4 R \sqrt{\lambda})}$.}

From the first equation of Eqn 10, the membrane strain is constant, which should be equal to the average membrane strain over the arch calculated from Eqn 8 as

$$-\frac{N}{AE} = \frac{1}{2 \Theta} \int_{\theta=0}^{\theta=\Theta} (\tilde{w}' - \tilde{v}) d\theta.$$  \hspace{1cm} (12)

Substituting Eqn 11 into Eqn 12 leads to the relationship between the radial load $q$ and the axial compressive force $N$ as

$$\frac{q R}{N} = \frac{15 + 2 \lambda^2}{2 \lambda^2}.$$  \hspace{1cm} (13)

**LINEARIZED BUCKLING**

**Antisymmetric Bifurcation Buckling**

From Eqn 13, considering the axial compressive force during bifurcation buckling given by Eqn 7 leads to the linearized bifurcation buckling loads as

$$\frac{q R}{N_{p2}} = \frac{15 + 2 \lambda^2}{2 \lambda^2}. \hspace{1cm} (14)$$

**Symmetric Limit Point Buckling**

For symmetric limit point buckling, the buckling membrane strain $\varepsilon_{mb}$ does not vanish. Substituting the prebuckling displacement given by Eqn 11 into Eqn 3 leads to the differential equation of buckling equilibrium as
\[
\frac{\dddot{v}_b^*}{\mu^2} + \dddot{v}_b^* = \frac{R^2\varepsilon_{mb}}{r_R^2 \mu^2} \left( 1 + \frac{15qR\mu^2 (\Theta^2 - \Theta^2)}{N (15 + 2\lambda^2)} \right).
\]

The solution of Eqn 15, which satisfies the boundary conditions \( \dddot{v}_b = \dddot{v}_b^* = 0 \) at \( \Theta = \pm\Theta \) is

\[
\dddot{v}_b = \frac{1}{\mu^2} \left[ 1 - \cos(\mu\Theta) + \frac{1}{2} (\mu\Theta)^2 \right] \left[ 1 - \frac{15qR}{N (15 + 2\lambda^2)} \right] - \frac{5qR\mu^2 (\Theta^2 - \Theta^2)}{8N (15 + 2\lambda^2)}.
\]

The average buckling membrane strain of \( \varepsilon_{mb} \) of Eqn 4 over the arch should be equal to the constant buckling membrane strain, which leads to

\[
A_1 \left( \frac{qR}{N} \right)^2 + B_1 \frac{qR}{N} + C_1 = 0,
\]

where

\[
A_1 = \frac{5}{7\mu^2\Theta^2 (15 + 2\lambda^2)} \left[ 315 \left( 1 - \frac{\tan(\mu\Theta)}{\mu\Theta} \right) + 17 (\mu\Theta)^6 + 42 (\mu\Theta)^4 + 105 (\mu\Theta)^2 \right],
\]

\[
B_1 = \frac{2}{\mu^2\Theta^2 (15 + 2\lambda^2)} \left[ 15 \tan(\mu\Theta) - 15 - 5 (\mu\Theta)^2 - 2 (\mu\Theta)^3 \right],
\]

\[
C_1 = \frac{1}{\mu^2\Theta^2} \left[ \frac{\mu\Theta}{\tan(\mu\Theta)} - 1 \right] \frac{2}{3}.
\]

Solving Eqns 13 and 17 simultaneously will yield the linearized limit point buckling load.

**NONLINEAR PREBUCKLING ANALYSIS**

The differential equations of equilibrium for the nonlinear prebuckling analysis can also be obtained from Eqn 1 by invoking the Euler-Lagrange equations, but with the nonlinear membrane strain given by Eqn 1. The differential equation of equilibrium in the axial direction is as in Eqn 10, and the differential equation of equilibrium in the radial direction can be obtained as

\[
\frac{\dddot{v}}{\mu^2} + \dddot{v} = P \quad \text{with} \quad P = \frac{qR - N}{N}.
\]

The solution of Eqn 20, which satisfies the boundary conditions \( \dddot{v} = \dddot{v}^* = 0 \) at \( \Theta = \pm\Theta \) is

\[
\dddot{v} = \frac{P}{\mu^2} \left[ \frac{\cos(\mu\Theta)}{\cos(\mu\Theta)} - 1 + \frac{1}{2} (\mu\Theta)^2 - (\mu\Theta)^3 \right].
\]

The nonlinear relationship between the load and displacement given by Eqn 21 is compared with the linear relationship given by Eqn 11 in Figure 2. It can be seen that when both the load and the displacement are small, the relationship becomes quite nonlinear. For the same load, the linear displacement is smaller than the nonlinear one, while for the same
displacement, the load from the linear analysis is much higher than that from the nonlinear analysis.

![Graph 2: Buckling load vs. displacement](image)

![Graph 3: Buckling load vs. axial compressive force](image)

The nonlinear equilibrium conditions can be established in the same way as in the case of the linearized analysis, but with the solution of Eqn 21 and the nonlinear membrane strain as

\[
- \frac{N}{AE} = \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} \left( \tilde{w}^\prime - \tilde{v}^\prime + \frac{\tilde{v}^\prime 2}{2} \right) d\Theta.
\]

(22)

Considering \( \tilde{w} = 0 \) at \( \theta = \pm \Theta \) and substituting Eqn 21 into Eqn 22 leads to the nonlinear equilibrium condition as

\[
A_2P^2 + B_2P + C_2 = 0,
\]

(23)

where

\[
A_2 = \frac{1}{4(\mu\Theta)^2} \left[ 5 - 5\tan\left(\frac{\mu\Theta}{\Theta}\right) + \tan^2\left(\frac{\mu\Theta}{\Theta}\right) \right] + \frac{1}{6},
\]

\[
B_2 = \frac{1}{(\mu\Theta)^2} \left[ 1 - \tan\left(\frac{\mu\Theta}{\Theta}\right) \right],
\]

\[
C_2 = \left(\frac{\mu\Theta}{\lambda}\right)^2.
\]

(24)

The nonlinear relationship between the external load and axial compressive force given by Eqn 23 is compared with the linear relationship given by Eqn 13 in Fig. 3. Again, it can be seen that when both the load and the axial compressive force are small, the relationship becomes nonlinear. For the same load, the axial compressive force from the linear analysis is smaller than that from the nonlinear analysis, while to reach the same axial compressive force, the required load from the linear analysis is higher than that from the nonlinear analysis.

**NONLINEAR BUCKLING**

**Antisymmetric Bifurcation Buckling**

Substituting the solutions given by Eqn 7 for bifurcation buckling into the nonlinear equilibrium condition 23 leads to the equations for bifurcation buckling

\[
\left(2\pi^2 + 15\right)P^2 + \left(4\pi^2 + 12\right)P + 12\pi^4/\lambda^2 = 0.
\]

(25)
The upper and lower bifurcation buckling loads can be obtained from Eqn 25. Typical bifurcation buckling loads are compared with linear buckling loads in Figures 2 and 3. It can be seen that the linear buckling load is higher than the upper nonlinear buckling load. In addition, the linear analysis cannot provide the lower buckling load. The nonlinear bifurcation buckling loads are compared with the linear counterparts also in Figure 4 as variations of the dimensionless buckling loads with the included angle $2\Theta$ of the arch. It can be seen that the linear buckling loads based the linearized prebuckling analysis are larger than the nonlinear results. This shows that the linearization of the prebuckling state should not be used for the bifurcation buckling analysis of shallow arches.

**Symmetric Buckling**

Substituting the solutions for the nonlinear prebuckling displacement given by Eqn 21 into the second equation of Eqn 3 leads to the differential equation of buckling equilibrium as

$$\frac{\ddot{v}_h}{\mu^2} + \ddot{v}_h = \frac{R^2 \varepsilon_{mb}}{r_s \mu^2} \left[ 1 + P \left( 1 - \frac{\cos (\mu \Theta)}{\cos (\mu \Theta)} \right) \right]. \quad (26)$$

The solution of Eqn 26 which satisfies the boundary conditions $\ddot{v}_h = v_h^r = 0$ at $\Theta = \pm \Theta$ is obtained as

$$\bar{v}_h = \frac{R^2 \varepsilon_{mb}}{r_s \mu^2} \left[ \frac{1}{2} \left( (\mu \Theta)^2 - (\mu \Theta)^2 \right) + (1 + 2P) \left( \frac{\cos (\mu \Theta)}{\cos (\mu \Theta)} - 1 \right) + \frac{P}{2} \left[ \mu \Theta \sin (\mu \Theta) - \mu \Theta \cos (\mu \Theta) \sin (\mu \Theta) \right] \right]. \quad (27)$$

The relationship between the dimensionless load $P$ and angle $\mu \Theta$ during symmetric snap-through buckling can be obtained in the same way as for the linearized buckling analysis as

$$A_3 P^2 + B_3 P + C_3 = 0, \quad (28)$$

where

$$A_3 = 2A_2 + D_3, \quad B_3 = 4A_2, \quad C_3 = B_2 - C_2, \quad (29)$$
\begin{equation}
D_3 = \frac{7\tan^2(\mu\Theta)}{8(\mu\Theta)^2} + \frac{15\tan(\mu\Theta)}{8(\mu\Theta)^3} - \frac{\tan(\mu\Theta)}{4\mu\Theta} \frac{\tan^3(\mu\Theta)}{\mu\Theta}.
\end{equation}

Solving Eqns 23 and 28 simultaneously will yield the nonlinear limit point buckling load. The nonlinear limit buckling loads are compared with the linearized counterparts in Figure 5. It can be seen that the linearized buckling loads based the linearized prebuckling analysis are larger than the nonlinear ones. This shows that the linearization of the prebuckling state should not be used for the limit point buckling analysis of shallow arches.

CONCLUSIONS

This paper has investigated the effects of linearization of the prebuckling state on the buckling load determination for pin-ended shallow steel circular arches that are subjected to a uniform radial load. It was found that the effects of prebuckling analyses are significant. It was realized that the usual adjacent equilibrium argument presented in the literature, according to which only the variation of the displacements are considered, is not applicable for determination of buckling loads of shallow arches. Proper formulation were presented and then used to derive solutions for the nonlinear in-plane antisymmetric bifurcation and symmetric limit point buckling loads for shallow arches. The relationship of the linearized buckling analysis to the exact solution of the corresponding nonlinear formulation was presented in order to clarify this rather important issue.

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CONSTRUCTION MECHANICS ANALYSIS & ERECTION MONITORING OF THE ROOF STEEL GIRDER FOR SZCEC

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KEYWORDS
Large span structure, construction stimulation, FBG sensor monitoring

ABSTRACT
The steel structure of Shenzhen Convention and Exhibition Center (SZCEC) was designed as a large span structure with special architectural requirements. Based on its complicated mechanical calculation and difficult erection procedure, the construction stimulation analysis of roof steel girder was needed and given. To test and verify the accuracy of the theoretical calculation, a whole course erection monitoring was arranged and completed. This paper presents the stimulation analysis calculation and the monitoring results of roof steel girders and rods during construction phase and the first three year service using Fibre Brugg Grating sensor. Comparison between the theoretical analysis and the field monitoring results proves that selected erection method was suitable and economical. The monitoring results are satisfying. Advance in construction stimulations and monitoring of large span structure in SZCEC can be referenced in further similar projects.

INTRODUCTION
Shenzhen Convention and Exhibition Center was a super-large building meeting international standards. The building height is 60m and the gross floor area is 255,615m². The exhibition halls hold around 6000 international standard size booths and the convention area seats 6400 people. The steel structures of the curved roof, consisting of girders, purlins and bracings, are supported at both sides of the center. With 30m intervals, pairs of steel box girders span 126m from edge of the exhibition hall to the reinforced concrete frame of the central part. Based on the mechanical analysis model, the roof steel girders can be classified into two types. One is girder with lower chord rods and the other is girder with column supports. Each type of the roof steel girder is composed of a double box-type beam with a uniformed width of 1m for a single box. The depth of the box girder at roof level is 2.6m. At corners, the depth has to be increased to 4.3m due to the bending force resulting from the frame effect as shown in Figure 1.
This steel girder is very different from those of other convention and exhibition centers. The cross section dimensions of the girders are large and the steel structure mass is heavy. In order to reduce the steel mass and the related deflection of girders, three parallel tension rods with a diameter of 140mm are formed the lower chords of girders. Because of the complexity and the required accuracy of the steel girder, construction stimulation calculation and erection monitoring should be considered and must be carefully studied before commencement of girder installation. The paper outlines the monitoring arrangements, procedures and results in the construction phase and the first three years of service. Based on the construction mechanics stimulation, the paper also gives comparison and analysis between the theoretical calculating values and the field testing records of the girder and rods.

Figure 1: Geometric diagram of a steel roof girder (front view)

ERECTION OF THE ROOF STEEL GIRDER SYSTEM

Based on actual conditions of the construction site and geometric dimensions of the double box steel girders, on-site segment by segment erection procedure was employed. In accordance with hoisting capacity of the caterpillar crane and the self-weight of a single roof steel girder, the whole roof steel girder was divided into seven segments. The length of each segment was depended on the capacity of the transportation. Each one was around 20 meters long. The longest one was 75 tons in weight. The segments were transported to the designated location on site. Firstly, the erection scaffolding tower should be carefully designed and formed, including its strength, stiffness, stability and the space requirements for installation of three tension rods. Then segments were lifted and laid on the scaffolding. Finally adjoining segments were welded together and a whole curved girder was formed.

On actual site practice, the trial lift was arranged. The girder segment was lifted 50cm above the ground level and suspended around 10 minutes to obtain static condition. Then, all parts of the caterpillar crane and the steel wire rope were checked for any abnormal phenomenon. After ensuring that every part was normal, the inclination of the girder segment was adjusted to the final requirements of orientation. Then, the girder segment was lifted again up to the level of 50cm or more above the height of the final designated position (referred as extra elevation value). It was very important that the extra elevation value of each girder was determined individually corresponding to their various loads. All extra elevation values must also be subjected to further calculations and stimulations. At this time, the side direction of the girder segment must be slowly adjusted and rotated subject to the need of final position.
Finally the girder segment was lowered onto the correct position of scaffolding tower platform. The final curved shape will only be achieved when the separate pieces of the girder were welded on the height of final designated position plus extra elevation values.

NUMERICAL SIMULATION OF ERECTION PROCESS

The final stress and deformation of the Beam-String structures depended on the sequence of erection procedures and steps. It was necessary to perform numerical simulation of erection process with finite element method. Various models and step by step loading process was used in simulation to select the best model which is consistent with the actual erection procedures. Computational stress and displacements served as reference values for erection monitoring. Erection process and the corresponding simulation steps were described as bellows:

1. According to the designed geometry \( S_0 \), the girder was divided into 7 segments with rough 18m interval and they were lifted to the scaffolding which was specially designed to match the height of the geometry \( S_0' \). By this method, each segment is simply supported at both ends and the deformation is \( S_0' \). Since the bending stiffness of the girder is much greater (with the cross section 1m \( \times \) 2.6m) relatively to its span(18m), it can be assumed that \( S_0 \approx S_0' \), which can be called the zero state;

2. After installing the lower chord rods and imposing a pre-tension force \( T_0 \), the girder geometry become \( S_T \);

3. When the scaffolding were gradually lowered, the axial force of lower chord rods was increasing because of the weight of girder and the Beam-String structure system come into being with the stable geometry was \( S_S \);

4. After installing purlins and other roof materials, the deformation of girder is \( S_P \). In order to precisely install roof system, \( S_P \) was required to approximately equal to the architectural geometry \( S_A \).

The general finite element analysis software ANSYS, which is a multi-stage nonlinear static analysis essentially, was adopted to perform the erection process simulation. The element stiffness matrix includes material linear property and its nonlinear geometric characteristic which is influenced by large displacements during the loading process. Updated Lagrange Method was applied to formulate the finite element method equation.

The corresponding numerical simulation process is as follows:

1. Applying the three-dimensional elastic beam element to build the finite element models of steel girder, with geometry \( S_0 \) and tension rods. The formulation of this zero state is:

\[
\begin{bmatrix}
K_L^0 + K_T^0
\end{bmatrix} \begin{bmatrix}
0
\end{bmatrix} = \begin{bmatrix}
F
\end{bmatrix}
\]

\( K_L^0 \) denotes linear element stiffness matrix of girder at \( S_0 \) state, \( K_T^0 \) refers to tension rods. Hereafter all matrix dimensions were expanded to the full number of nodes decided by grid mesh. No external force was applied, i.e. \( F = 0 \), \( \{0\} \);

2. Further applying the weight and tensile force \( T_0 \) to rods gradually. At any time interval \( t \) in the loading process, The finite element equation was:


\[
\left( \mathbf{K}_g + \mathbf{K}_{NL}^g + \mathbf{K}_L^r + \mathbf{K}_{NL}^r \right) \Delta \mathbf{u} = \mathbf{T}_0 + \Delta \mathbf{F} 
\]

Superscript \( t \) marks the time intervals, \( \mathbf{K}_g \) and \( \mathbf{K}_{NL} \) were nonlinear geometry stiffness matrix caused by node displacements of girder and rods respectively, \( \mathbf{T}_0 \) and \( \Delta \mathbf{F} \) were incremental loading step of tensile force and self-weight of rods at time \( t \). The sum of displacements \( \Delta \mathbf{u} \), structure deform expressed \( S_T \), and:

\[
S_T = S_0 + \sum_0^T \Delta \mathbf{u} 
\]

Scripts \( 0 \) and \( T \) denoted the starting and ending time points corresponding to state \( S_0 \) and \( S_T \).

(3) Now adding the weight to steel girder, the deformed geometry \( S_S \) could be obtained. The formulation resembles equation (2) except the right of equality sign was \( \Delta \mathbf{F}_g \) which was the incremental loading of self-weight of girder now. The following relationship was obtained:

\[
S_S = S_T + \sum_T^S \Delta \mathbf{u} 
\]

(4) Finally, add the purlins and other roof loadings and the geometry was \( S_P \). Similarly, the load was expressed as \( \Delta \mathbf{F}_p \) and the geometry relationship was:

\[
S_P = S_S + \sum_S^P \Delta \mathbf{u} 
\]

In the loading process, geometric nonlinear effect was considered, and the loads were carried out in several steps. Two parameters were required to be determined in the numerical simulation in advance: the pretension force of rods and the zero state of girder.

Since each rod was hinged at both ends and under the effect of its self-weight, the rod was curved and the axial force-deformation relation was non-linear. Until the tensile force was more than \( T_0 \), the rod axis became nearly straight line. Through on-site experiment in this project, the whole process force-strain curve of a sample rod, which was gradually loaded until yielding, was obtained, and \( T_0 = 30 \text{t} \) was determined[1-2].

After the tension-loading process, the zero state \( S_0 \) reached the specified architectural geometry \( S_A \), the relationship could be expressed with a function:

\[
S_A = S_0 + \sum_0^T \Delta \mathbf{u} + \sum_T^S \Delta \mathbf{u} + \sum_S^P \Delta \mathbf{u} = \phi(S_0) 
\]

The function \( \phi \) denote the simulation of erection process. \( S_0 \) needed to be inversely determined from \( S_A \), the iterative strategies for solving nonlinear system such as Newton-Raphson method was adopted and evolved to the inverse iteration method[3-4]. Assume a zero state \( S_k^0 \) and substitute to right of (6), the residual was:

\[
R_k = \phi(S_k^0) - S_A 
\]

The simple way to form iterative cycle was inversely super-imposing the residual \( R_k \) on \( S_k^0 \) to get new stimulated state:

\[
S_{k+1}^0 = S_k^0 - \left( \phi(S_k^0) - S_A \right) 
\]
The convergence condition was that residual became small enough at some specific key points whose number was \( n \), the sum of absolute residual values at them was an option to justify convergence:

\[
\sum_{1}^{n} \text{abs}(r_{k+1}^{m}) \leq \delta_{\text{min}}
\]

(9)

The norm of \( R_{k+1} \) would serve well also:

\[
\|R_{k+1}\| \leq \delta_{\text{min}}
\]

(10)

Steps were as follows [5]:

1. Assume an initial geometry \( S_{k}^{0} \);
2. After the numerical simulation of the erection process, it deformed to \( S_{k} \);
3. Inversely exert the difference between \( S_{k} \) and \( S_{A} \) to \( S_{k} \), new initial geometry \( S_{k+1}^{0} \) was obtained;
4. Redo above erection process simulation until \( S_{k+1} \) was approaching \( S_{A} \).

The iteration process can be seen in Figure 2.

![Figure 2: Iteration process](image)

MONITORING OF ROOF STEEL GIRDER SYSTEM

**Contents of monitoring**

According to the site conditions, the monitoring was divided into two stages, the construction phase and the building service time. Testing included displacement and stress monitoring. Displacement testing includes both angular deformation and linear deformation. The following parameters were tested.

1. Tension forces and performance of the lower chord steel rods;
2. Vertical deflection of the steel girder;
3. Stress and strain of the critical position;
4. Supporting movement of the steel girder end post at 30m high; and
5. Stress performance and angular deformation of steel girder ground joint at \( \pm 0.00 \).

**Monitoring points, location and arrangement**

1. Below the steel girder, all rods were tested. Testing points were arranged along the axis direction. Total number of testing points was 36.
2. At top surface and bottom surface of steel girder along axis direction, testing points...
were arranged. Total number of testing points was 22.

(3) On two lateral side of steel girder at ground hinge along axis direction, testing points were arranged during the period of disassemble of scaffolding tower.

(4) At top surface of steel girder along axis direction, testing points were arranged during the period of disassemble of scaffolding tower. Total number of testing points was \(3 \times 45\) (three directional).

**Special features of monitoring and records**

1. Long terms monitoring including the construction phase and the building service time, up to 3 years (36 months).
2. Large number of monitoring points with 350 FBG sensors for structural deformation, stress and tension force of the steel rods.
3. The structural status just before the tension of the steel rods was defined as the initial status, while all the testing records were referred to this model and FBG sensor was adopted.
4. Actual testing record collections in case of typhoon or storm or gale during whole course were required.

**Definition of key monitoring period**

1. After the structural erection of the steel girder, just before the lowering of the scaffolding tower, when the lower chord rods to be tensed step by step, monitoring of steel rods was carried out for 3 days continuously.
2. When the scaffolding tower was lowered, monitoring for entire structural system was carried out for 3 days continuously.
3. During roof materials erection, 227 days monitoring was carried out.
4. During the structural completion and approval, 5 days monitoring was carried out.
5. After the structural approval and on the date upon 3 months, monitoring was carried out for the entire structural system.
6. At the special time of the lower temperature and high temperature, 7 days monitoring was carried out respectively.
7. After the structural approval and on the date upon 6 months and 9 months, monitoring was carried out for the entire structural system respectively.
8. During typhoon or storm or gale, continuous monitoring was required.
9. After the structural approval and on the date upon 12 months, the last monitoring was carried out.

Comparison between theoretical calculating values and testing results for the steel girder system at axis directions.

1. Just after the scaffolding towers were lowered down and were loaded by self weight of the girder system, comparisons of performance for the structural system were shown in Figures 3-5.
2. Just after the erection of all steel girder system, comparisons of performance for the structural system were shown in Figures 6-8.

3. At the special time of the lower temperature and high temperature and the end date of testing period, comparisons of the performance for the structural system were shown in Figures 9-11.
Figure 9 Flange stress of the steel girder (Unit: MPa)

Notes:
① --- During lower temperature
② --- During high temperature
③ --- At the end of monitoring period
Monitoring result analysis

Tensile forces of lower chord steel rods

During the construction phase and building service time, maximum tensile force of the steel rods was 1763kN and 2209kN respectively. This means, under the normal construction loading and actual building service loading, maximum tensile stress of the steel rods was 114.5MPa and 143.5MPa respectively. Based on these testing results, ultimate design loads were given to study the behavior of the lower chord steel rods. Under the live loads, seismic loading, wind loading, superimposed loading variation and so on, the ultimate tensile force was estimated as 3247kN and 3669kN respectively for the period of the construction phase and the building service time. That means the ultimate tensile stress was 210.9MPa and 238.3MPa respectively. These values were less than one which is required by relevant National Steel Structural Design Code. Also, they were very much less than the actual tension capacity of steel rod materials.

The vertical deflection of the steel girder.

For steel girder with a span of 126m, vertical deflection was a key problem, which should be studied carefully before the commencement of girder erection. In accordance with site erection method and the calculating analysis, extra elevation value of 210mm at the centre of girder was set. That is to say, the tops of scaffolding towers were higher than the theoretically calculating values so that the expected deflection of the steel girder can be eliminated partly. During construction phase, when all roof loads were imposed and the scaffolding towers were lowered, maximum deflection at centre of the girder occurred and the value was 275mm. At the time of erection completion, it was 231mm. Then, during first one year monitoring of the building service time, the maximum deflection was 244.2mm. Site deflection survey records showed that deflection was decreasing with increasing of temperature. Comparing all the monitoring records, if eliminating the extra elevation value of 210mm, the actual maximum deflection was 65mm. If considering all other possible loads such as roof live loads, seismic loading, wind loading, loading variation and so on, the limit state design was employed to study the maximum deflection of the steel girder. The maximum value was 325.2mm. Taking away the extra elevation value of 210mm, this means the maximum deflection was 115.2mm. Comparing to 115.2mm or the appropriate Steel Design Code, the actual deflection of 65mm was very small.
Flange stress of the steel girder

Along top or bottom flange of the steel girder, flange stress was monitored. On the bottom flange, majority of testing points were for tensile stress, only a few points to be monitored as compressive stress. On the top flange, the arrangement was opposite to the bottom one. During the construction phase, maximum tensile stress was 78.13MPa which occurred at the bottom flange. Maximum compressive stress was 88.37MPa which occurred at the top flange. During first year monitoring of the building service time, maximum tensile stress was 59.58MPa at the bottom flange. Maximum compressive stress was 55.56MPa at the top flange. Compared to the Steel Design Code and the actual behavior of the double box-type steel beam, the actual stress was much less than the theoretical capacity of the steel girder.

CONCLUSIONS

Most large span buildings were designed as light steel structures. Although some field monitoring was carried out, the testing period was short and testing points were limited because design calculation and erection methods have been developed systematically. However, for this structure, any new problems need to be studied and addressed. Through the erection practice and the monitoring of the roof steel girder for SZCEC, a lot were learned. Especially, the erection method is crucial for such a large building. Comparing final solution to the original one, the change was big but the final erection method was successful and economical. It can be concluded as followings:

1. Using this erection method, the structure is safe and construction quality is reliable. Besides, according to the field testing records, actual stresses and strains of the roof steel girder were much less than the normal bearing capacity of the structural materials itself.

2. Before lifting the steel rods, pre-tensioning of the steel rods should be carried out so that errors from the steel material and its manufacturing process could be eliminated.

3. For such a large project about designing theory, construction mechanics and erection method, a new design method and approval standards should be developed as soon as possible.

4. Through the project practice, steel rods with the diameter of 140mm made in China, the material and the installation method can meet the designer’s requirements and international standards completely.

REFERENCES

SYSTEM RELIABILITY EVALUATION OF STEEL FRAMES

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KEYWORDS
System reliability, member reliability, steel frame, failure mode method, direct Monte Carlo simulation, CSSASM, random variables

ABSTRACT
Building structures serve as an integral part to perform their functions, so designers should pay more their attention to the system reliability than the member reliability of structures. Among the available approaches for system reliability evaluation of steel frames, failure mode method and direct Monte Carlo simulation (DMCS) are the two most popular methods. Failure mode methods were typically used for frame structures with rigid-plastic failure mechanisms. Previous investigation reported that the system reliabilities of steel frames based respectively on rigid-plastic analysis and second-order inelastic analysis depart evidently from each other. DMCS is exact but time-consuming which makes it not appropriate for the system reliability evaluation of steel frames. A composite sampling semi-analytical simulation method (CSSASM) is therefore proposed, which is based on basic structural performance function, combined sampling techniques, exponential polynomial method (EPM) used to fit the probability density function of the structural resistance, and FOSM (First Order Second Moment) approach to assess the system reliability of steel frames. The CSSASM can be easily connected with structural nonlinear analysis program and considers various random variables in the system. An example is studied to verify the proposed method. The comparison between the results from the CSSASM and that from DMCS shows that CSSASM is satisfactorily precise just using less samples.

In this paper, the main random variables and their statistics of steel frames are analyzed and determined, and the evaluations of the system reliability of steel frames under two basic loading cases are made through using the CSSASM. The comparison between CSSASM and the failure mode method is carried out through a steel frame example, which illustrates that failure mode method will overestimate the structural safety because of not considering the nonlinear factors influencing the structural resistance, while CSSASM can assess structural safety more exactly.
INTRODUCTION

One of the objectives of structural design is to ensure the safety and performance of a building structure for specified functions and loading conditions in a given period of time. Since building structures serve as an integral part to perform their functions, so designers should put more emphasis on the system reliability than the member reliability of structures. Reliability assessment of structural systems has received a great deal of attention by researchers in the past decades, the approaches of system reliability evaluation generally include the failure mode method, the response surface method, the stochastic finite element method (SFEM) and direct Monte Carlo simulation (DMCS). Among them, failure mode method and DMCS are the two most popular methods for system reliability evaluation regarding ultimate limit state of frame structures. Failure mode methods were typically used for frame structures with rigid-plastic failure mechanisms [1]. However, previous investigation reported that the failure mode method based on rigid-plastic analysis is will overestimate the system reliability index of steel frames [1~3]. DMCS is robust and widely considered to be exact. However, DMCS is expensive in computer time for even increasingly fast computers. A large number of deterministic runs are required to calculate the probability of structural failure, which is generally prescribed to be small in structural engineering (less than $10^{-3}$~$10^{-4}$). The efficiency of the simulation can be improved by reducing the simulation variance. Some variance-reduction techniques, such as importance sampling, systematic sampling, conditional expectation, antithetic variates, Latin hypercube sampling and directional sampling have been proposed and used in structural reliability estimation [4~7]. The variance reduction techniques can also be combined to further speed the simulation.

A composite sampling semi-analytical simulation method (CSSASM) is presented in this paper, the method proposed is conceptually straightforward and applicable, which is based on basic structural performance function, employs DMCS with combined variance-reduction techniques to obtain the samples of structural resistance, exponential polynomial method (EPM) to fit the probability density function (PDF) of the structural resistance, and first-order second-moment (FOSM) method to assess the system reliability of steel frames. Additionally, the CSSASM can be easily connected with structural nonlinear analysis program and considers various random variables in the system. Numerical examples are given to show the efficiency of the combined sampling techniques.

What’s more, the main random variables and their statistics of steel frames are analyzed and determined, and the evaluations of the system reliability of steel frames under two basic loading cases are made through using CSSASM. The comparison between CSSASM and the failure mode method is carried out through a steel frame example.

THEORY AND VERIFICATION OF CSSASM

General Remarks

The performance function of the reliability assessment of a structural system can be generally written as

$$Z = R - S$$  \hspace{1cm} (1)$$

where \( R \) and \( S \) represent the structural resistance and the corresponding load effect, respectively. Since the probabilistic statistics of the load effect, \( S \), can be found in the relevant load codes, FOSM methods can be applied to Eq. (1) and the failure probability or reliability
index can be obtained, provided that the statistics of structural resistance are determined. So, structural reliability assessments can be reduced to how to sufficiently simulate the probabilistic characteristics of structural resistance $R$ under the corresponding load effects $S$ applied.

Direct Monte Carlo simulation with variance-reduction techniques is simple to consider various randomness and is employed to obtain the samples of structural resistance in this paper. Variance-reduction techniques comprise updated systematic sampling and antithetic variates technique. When resistance samples and its first several moments obtained, EPM is used to fit the PDF of structural resistance, $R$, with the following strategy proposed.

Exponential polynomial method can provide good results for the approximate distribution and statistical characteristic of $R$ and then FOSM method can be carried out to calculate the structural system reliability.

**Monte Carlo Simulation**

**Systematic sampling (SS)**

Let $\{x_i\}, \quad i = 1,2,...,N$, denote a set of independent realization of random variable $X$. Fractile constraints employed as sampling rule in systematic sampling hold

$$F(x_i) = \frac{2i - 1}{2N} \quad (i = 1,2,...,N) \quad (2)$$

where $F(\cdot)$ is the cumulative distribution function of random variable $X$.[8]

For a random vector $\overline{X} = \{X_1, X_2, ..., X_K\}$, the components of which are independent, let $x_{jk}$ be the $j$th simulated value of $k$th component of $\overline{X}$, Define $\overline{F} = \{F_{jk}\}$ to be a $N \times K$ matrix, each column of which is an independent random permutation of $\{1,2,...,N\}$, then $x_{jk}$ is obtained by

$$F_k(x_{jk}) = \frac{2p_{jk} - 1}{2N} \quad (j = 1,2,...,N; \quad k = 1,2,...,K) \quad (3)$$

where $F_k(\cdot)$ is the cumulative distribution function of $X_k$.

For each time of simulation, there are $N$ observed values for each variable $\overline{X} = \{X_1, X_2, ..., X_K\}$, so it constitutes a matrix of $N \times K$, named $R$.

**Updated systematic sampling (USS)**

Florian[9] gave a method to reduce the statistic correlation using Spearman coefficient, which can be defined as

$$T_{ij} = 1 - \frac{6 \sum (R_{ki} - R_{sj})^2}{N(N-1)(N+1)}, \quad (k=1,2,...,N, \quad i,j=1,2,...,K) \quad (4)$$

where $T_{ij}$ is the Spearman coefficient between variable $i$ and $j$, the value of which is between $-1$ and $1$. $R_{ki}$ and $R_{sj}$ are random ordinal number of the $i$th and $j$th column of the $k$th row of
matrix $R$ respectively. Obviously, matrix $T$ is symmetrical and can be decomposed as

$$T = Q \cdot Q^T$$  \hspace{1cm} (5)$$

where matrix $Q$ is the lower triangular matrix, let $S = Q^{-1}$

Make a transformation, $R_s = R \cdot S^T$

The statistic correlation of matrix $R_s$ can be expressed by matrix $T_s$. It can be verified that matrix $T_s$ is close to identity matrix $I$, so the values of matrix $R$ is rearranged according to the order of matrix $R_s$, then the statistic correlation of matrix $R$ is reduced.

**Antithetic variates technique (AV)**

In antithetic variates technique, negative correlation between different cycles of simulation is induced in order to improve the simulation efficiency \[4\]. If $r$ is random number uniformly distributed in the interval $[0,1]$ and is used to determine the estimator $Z'$, the random number $1-r$ can be used in another run to obtain $Z''$ and an improved estimator for random variable $Z$ could be given in antithetic variates by

$$Z_d = \frac{1}{2}(z' + z'')$$  \hspace{1cm} (6)$$

Antithetic variates simulation is very easy to combine with DMCS or other variance-reduction techniques. In this paper, the Monte Carlo simulation by combining updated systematic sampling with antithetic variates technique is employed to obtain the sample of the resistance of structural systems. Numerical examples will show that this combination can improve the efficiency of Monte Carlo simulation.

**Exponential Polynomial Method**

Exponential polynomial method (EPM) \[10\] is employed in this paper to fit the PDF of the structural resistance $R$. EPM is in fact a method based on the principle of approximating a numerically specified probability distribution with maximum entropy probability distribution \[11\].

For a standardized random variable $X$, its PDF, $f(x)$, could be written by the truncated form of exponential polynomial

$$f(x) = c \cdot e^{Q_{\alpha}\cdot x}\cdot Q_{\beta}(x)$$  \hspace{1cm} (7)$$

Where

$$Q_{\alpha}(x) = \begin{cases} \sum_{i=1}^{n} a_i \cdot x^i & (x \in [\alpha, \beta]) \\ -\infty & (x \notin [\alpha, \beta]) \end{cases}$$

$$c = \frac{1}{\int_{\alpha}^{\beta} e^{Q_{\alpha}\cdot x} \cdot dx}$$

Normally, the values of $\alpha$ and $\beta$ can be set to $\alpha = -4\sigma$, $\beta = 4\sigma$, respectively.

In Eq. (7), $a_i$ ($i = 1, 2, \ldots, n$) are progressive coefficients and can be determined by using the first $2(n-1)$th moments of random variable by
\[
\begin{pmatrix}
\bar{\mu}_0 & 2\bar{\mu}_1 & \cdots & n\bar{\mu}_{n-1} \\
\bar{\mu}_1 & 2\bar{\mu}_2 & \cdots & n\bar{\mu}_n \\
\vdots & \vdots & \ddots & \vdots \\
\bar{\mu}_{n-1} & 2\bar{\mu}_n & \cdots & n\bar{\mu}_{2(n-1)}
\end{pmatrix}
\begin{pmatrix}
a_1 \\
a_2 \\
\vdots \\
\mu_{n-1}
\end{pmatrix} =
\begin{cases}
0 \\
-1 \\
\vdots \\
-(n-1)\bar{\mu}_{n-2}
\end{cases}
\]

(8)

where \(\bar{\mu}_k\) denotes the \(k\)th moment of \(X\) and \(\bar{\mu}_0 = 1\). Eq. (8) indicates that the number of moments of \(X\) needed for unique determination of the \(n\) unknown parameters \(a_i\) is 2\((n-1)\).

**Numerical Example**

Consider a single flexural steel beam. The performance function of the beam is given by

\[
Z = F_y S - M
\]

(9)

where \(F_y\) is the yield strength for steel, \(S\) is elastic section modulus of steel beam and \(M\) is the flexural moment. Assumptions of the uncertain variables are shown in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Coefficient of variation</th>
<th>Distribution type</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_y)</td>
<td>275.52(MPa)</td>
<td>0.125</td>
<td>Normal</td>
</tr>
<tr>
<td>(S)</td>
<td>(8.19 \times 10^{-4}) (m³)</td>
<td>0.05</td>
<td>Normal</td>
</tr>
<tr>
<td>(M)</td>
<td>113 (kN·m)</td>
<td>0.20</td>
<td>Normal</td>
</tr>
</tbody>
</table>

The reported probability of failure from DMCS with \(2 \times 10^5\) number of trials is \(1.183 \times 10^{-3}\), which is considered as the exact solution. Ten simulation analyses by the proposed approach, respectively, employing unique systematic sampling (SS), updated systematic sampling (USS) and updated systematic sampling with antithetic variates (USS+AV) are performed, each with 500 numbers of trials. The results of means and standard deviations of failure probability estimated are listed in Table 2. It can be seen from Table 2 that the proposed method can calculate the probability of failure satisfactorily compared with the result obtained by DMCS, and by combining USS with AV, the standard deviation of failure probability is significantly decreased.

<table>
<thead>
<tr>
<th>AVG</th>
<th>SS</th>
<th>USS</th>
<th>USS+AV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.191</td>
<td>1.086</td>
<td>1.184</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.220</td>
<td>0.117</td>
<td>0.068</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.184</td>
<td>0.108</td>
<td>0.057</td>
</tr>
</tbody>
</table>

The variation of mean and standard deviation of failure probability with the number of trials are shown in Figure 1, obtained by simulations of combining USS with AV.

It can be shown that when the number of trials equals 300, the probability of failure of steel beam has already been converged, which illustrates that the proposed method is satisfactorily precise just using a relatively small number of samples.
SYSTEM RELIABILITY ASSESSMENT OF STEEL FRAMES

The system limit state considered in this paper is that associated with the ultimate strength (limit load-bearing capacity) of steel frames under the applied loads. It should be noted that, however, the serviceability limit state of the structural system is as important as the ultimate limit state in structural design and will be considered in further research.

Uncertainty of Structural Capacity

The randomness of structural resistance of steel frames is reduced in this paper to the randomness of the sectional geometry of each component, the randomness of material yielding strength and the unrealistic modeling for calculating the capacity of the structure, and all the random variables reflecting the uncertainty of structural capacity are assumed to obey the normal distribution.

The ratio of mean to normal value (K) and the coefficients of variation (COV) of the fundamental random variables included in simulation of structural resistance are listed in Table 3[12,13].

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Ratio of mean to normal value(K)</th>
<th>Coefficient of variation(COV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sectional geometry</td>
<td>Length or width thickness 1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Material yield strength</td>
<td>steel 1.070</td>
<td></td>
</tr>
<tr>
<td>Calculation mode for</td>
<td>Vertical load</td>
<td>1.000</td>
</tr>
<tr>
<td>determining structural capacity</td>
<td>Horizontal and vertical load</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Figure 1: Mean and standard deviation of $P_f$
Uncertainty of Loads

Three loads are considered in the reliability evaluation of steel frames. They are dead load, live load and wind load, with corresponding statistics as tabulated in Table 4 [12,13].

Two load cases are investigated (see Figure 2a and 2b). Under loading case (a), only vertical loads including dead load (D) and live load (L) are considered. Under loading case (b), vertical dead load (D), live load (L) and wind load (W) are considered. In assessment of system reliability, dead load and live load are treated as random load in load case (a) while only wind load is treated as random load in load case (b) with the normal deterministic value of dead load and live load.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Ratio of mean to normal value(K)</th>
<th>Coefficient of variation(COV)</th>
<th>Distribution type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load</td>
<td>1.06</td>
<td>0.07</td>
<td>Normal</td>
</tr>
<tr>
<td>Live load</td>
<td>1.00</td>
<td>0.25</td>
<td>Gumbel</td>
</tr>
<tr>
<td>Wind load</td>
<td>0.999</td>
<td>0.193</td>
<td>Gumbel</td>
</tr>
</tbody>
</table>

System Reliability of Steel Frames under Vertical Loads

CSSASM is used to calculate the system reliability of the steel frame under vertical loads, as shown in Figure 3. In this example, the normal value of yield strength of steel is 235MPa and the elastic modulus is 206kN/mm². The normal value of the dead load and the live load on the first storey and the second storey are both 60kN/m, and the normal value of the dead load and the live load on the third storey are both 30kN/m. The spans of the two bays are both 5m and the height of each storey is 4m. The sections of all the beams are W16×50 and the sections of all the columns are W16×67. In the process of calculating ultimate strength of the structure, the dead load and the live load are assumed to increase until the failure of the structure.

Taking the summation of the normal values of dead load and live load as the vertical reference load, then the ultimate strength of the structure can be represented as the product between ultimate load coefficient and vertical reference load. In this case, the normal value of ultimate load coefficient of the structure is obtained to be 2.1000, the mean is 2.2209 and the standard deviation is 0.2567. The probability density function (PDF) curve of ultimate load coefficient is shown in Figure 5, the PDF of lognormal distribution with the same mean and standard deviation of ultimate load coefficient is shown also in Figure 4.
The performance function of the structure can be expressed as $Z = R - G - L$. The system reliability index result is 3.8069 given by the FOSM method and the corresponding failure probability is $7.0366 \times 10^{-5}$.

System Reliability of Steel Frames under Horizontal and Vertical Loads

A steel frame subjected to dead load, live load, and wind load is shown in Figure 5. In this example, the normal value of yield strength of steel is 235MPa. The wind load is assumed to apply on the joints of beams and columns, the normal value of the wind load on the first storey and the second storey are both 100kN, and the normal value of the wind load on the third storey is 50kN. The other data are the same as the example in Figure 3.

In the process of calculating ultimate strength of the structure, the summation of the dead load and the live load is applied on the structure constantly, and the wind load is assumed to increase until the failure of the overall structure.

Taking the normal value of wind load as the reference load, the normal value of ultimate load coefficient of the structure is obtained to be 2.5000, the mean is 2.7121 and the standard deviation is 0.2903. The PDF curve of ultimate load coefficient is shown in Figure 6, and the PDF of lognormal distribution with the same mean and standard deviation of ultimate load coefficient is shown also in Figure 6.
The performance function of the structure can be expressed as $Z = R - W$. The system reliability index can be obtained as $3.9813$ using CSSASM and the corresponding failure probability is $3.4272 \times 10^{-5}$.

**Comparison of Different Methods for Evaluating System Reliability**

The present failure mode method for calculating the system reliability of frame structures is compared with CSSASM proposed in this paper. An example frame, as shown in Figure 7, is used for this comparison[12]. The span of the frame is 10m and the height of the frame is 5m. A concentrated vertical load $V$ is applied on the middle span of the beam and a concentrated horizontal load $H$ is applied on the left joint of beam and column. The statistics of loads and ultimate yield moment of the frame elements are listed in Table 5.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Mean</th>
<th>Coefficient of variation</th>
<th>Distribution type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load $V$</td>
<td>80kN</td>
<td>0.2</td>
<td>normal</td>
</tr>
<tr>
<td>Load $H$</td>
<td>50kN</td>
<td>0.3</td>
<td>gumbel</td>
</tr>
<tr>
<td>Ultimate yield moment of elements</td>
<td>180kN·m 0.1</td>
<td>lognormal</td>
<td></td>
</tr>
</tbody>
</table>

The system reliability index of the example given by Li jihua[12] using failure mode method lies between 2.91 and 2.93 and the system reliability index obtained with CSSASM is 2.6980. Since the failure mode method can not consider the effects such as geometrical nonlinearity and plastic distribution on the ultimate strength of steel frames, the system reliability of structures is overestimated. So it is improper and unsafe to use the failure mode method to estimate the system reliability of steel frames in practice.

**CONCLUSIONS**

1) CSSASM proposed in this paper is verified to be efficient for obtaining satisfactory results of system reliability of steel frames.
2) The PDF of ultimate strength of steel frames may be expressed with the lognormal distribution with the same mean and standard deviation obtained from the statistics of sampling.
3) The failure mode method will overestimate the system reliability of steel frames and it is improper and unsafe to apply this method in practice.
REFERENCES


RESEARCH ON DAMAGE OF CONTINUOUS STEEL GIRDERS IDENTIFICATION BY WAVELET ANALYSIS OF THE CURVATURE MODE

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KEYWORDS
Curvature mode, wavelet analysis, continuous steel girders, damage identification, finite element analysis

ABSTRACT

Based on wavelet analysis theory, the damage identification method of the curvature modes for beams is presented. The model of finite element method for continuous steel girders is established. The vibration of beam may be obtained by means of finite element analysis. The curvature modes are analyzed using continuous wavelet transform by mexh wavelet (Mexican Hat wavelet) on multiple scales, and the position of the crack could be identified by the modulus maximum of the wavelet coefficients. Some useful conclusions for continuous steel girders have been drawn and it has been useful in real continuous steel girders bridge damage diagnosis.

INTRODUCTION

Beam is the basic and simplest structural member. The existence of damage in beams may cause local variation in stiffness and decrease its bearing capacity, which affect the service performance and durability of the whole structure, even threaten lives and assets. Based on the structural dynamics theory, the existence of structural damage may lead to changes in the dynamic characteristics such as the decline in stiffness, the increase in damping, boundary conditions changes, vibration frequency and vibration mode changes, which showed that the difference between the normal structure and the damage structure in the dynamic characteristics. In turn, it could be used to detect, locate and quantify the damage. So, how to identify the damage in beams effectively has become a very important research. Li, Y.Y.
etc.[1] carried out to obtain the modal strain curve by analyzing the modal strain of damaged shells and identified the position of the damage using the continuity condition and the residual strain mode curve method. Kim, J.-T. and Stubbs, N.[2] identified the damage in beams by a method of testing frequency and modal information. Pandey A.K. et al.[3] used modal curvature to identify damage. By using a cantilever and a simply supported analytical beam model, it was shown here that the absolute changes in the curvature mode shapes were localized in the region of damage and hence could be used to detect damage in a structure. Li Gongyu[4] studied on a curvature modal analysis of damage structures. The curvature modes of a cantilever beam with two types of notch were derived from its displacement results showed that the damage location and degree of the beam could be diagnosed effectively by means of the curvature modal analysis. Numerical simulation and test results have shown that the curvature mode was more effective than the method based on the natural frequency and the modal shape. The curvature mode of application has been of great importance.

Identification of the damaged beam was mostly based on the natural frequency and the modal shape changes. Because the damage was a typical local phenomenon, and modal parameters were the structure of the overall dynamic performance indicators. So, the natural frequencies and modal parameters on dynamic characteristics had a lack of sensitivity to the structural damage, which could be used to detect but not to locate[5]. Because the small damage had few influence on the beam natural frequency, so there was a great demand for more sensitive detection methods to identify the location for the small damage. In recent years, people have been successful in bringing the theory of wavelet analysis applied to the structure of the field of damage detection, and to obtain many meaningful results. Wavelet analysis method is regarded as "an mathematical microscope". Its quality of space localization (on low scale signal’s wavelet transform is decided by its local at certain point) can be used to analyze the singularity which is made by damage, so the damage in beams can be identified by wavelet analysis. Liew, K.M. et al.[6] carried out to identify the simple beam crack location based on discrete wavelet transform. Guan Deqing and Jiang Xin[7] presented a damage identification method of Timoshenko beam crack based on wavelet analysis of the angle displacement mode. Compared with the result of analysis of the mode shape, it could be concluded that the method using the wavelet analysis of angle displacement mode was more convenient and more effective. Ruzzene, M. et al.[8] identified the linear system of frequency and damping based on wavelet analysis. Chih-Chieh Chang and Lien-Wen Chen[9] identified the crack of a cantilever beam whose ratio of height to span is 1/80 based on an analysis of the mode function of the Timoshenko beam using wavelet transform on multiple scales. Quan Wang, et al.[10] compared deflection curve of the damaged beam under the concentrated load with that of non-destructive beam under concentrated load and identified the crack by wavelet transform on multiple scales. Douka, E. et al.[11] analyzed the fundamental vibration mode of a cracked cantilever beam using continuous wavelet transform and estimated both the location and size of the crack. Hera, A. et al.[12] identified the damage of a four-storey steel frame by calculating the nodes acceleration responses using the finite element method. In this paper, the curvature mode based on the wavelet transform has been applied to recognize the location of three-dimensional damaged beam. Taking a continuous steel girder of damaged as an object of study, we could obtain the curvature mode of damaged by the numerical calculation and identify the damage in beams by analyzing the curvature mode using continuous wavelet transform. The research provided an effective method to the identification of continuous steel girders.
THEORETICAL BASIS

Singularity Identification Theory of Wavelet

Set function $\psi(t) \in L^1(R) \cap L^2(R)$, and $\int_{-\infty}^{\infty} \psi(t)\,dt = 0$, where $\psi(t)$ is the basic wavelet or the mother wavelet. Translating and shifting mother wavelet can obtain Eqn. 1.

$$\psi_{a,b}(t) = \left(\frac{1}{\sqrt{|a|}}\right)^{1/2} \psi\left(\frac{t-b}{a}\right), \quad a, b \in R, \quad a \neq 0,$$

where $\psi_{a,b}(t)$ is the wavelet function, $a$ is the scale factor that reflects the signal frequency information, $b$ is the shift factor that reflect the signal time information.

In which mother wavelet should meet admissible condition,

$$c_\psi = \int_{-\infty}^{\infty} \left|\hat{\psi}(\omega)\right|^2 \,d\omega < +\infty$$

(2)

So, based on Eqn. 2., wavelet function requires not only has a definite concussion (it contains the characteristics of a certain frequency), but also has quality of space localization (it constants equal to 0 or converges to 0 quickly within a fixed range).

Assume that $\psi(t)$ is the basic wavelet, $\psi_{a,b}(t)$ is the continuous wavelet function. For arbitrary function or signal $f(t) \in L^2(R)$, continuous wavelet transform can be expressed by Eqn. 3.

$$WT_f(a,b) = \langle f(t), \psi_{a,b}(t) \rangle = \left|a\right|^{1/2} \int_{-\infty}^{\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)\,dt,$$  

(3)

where $a \neq 0$, $b$, $t$ are continuous variables, $\psi^*(t)$ denotes the complex-conjugate of $\psi(t)$.

Continuous wavelet transform can be written as Eqn. 4.

$$WT_f(a,b) = \left(\sqrt{|a|}\right)^{-1/2} \int_{-\infty}^{\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)\,dt = |a|^{1/2} f * \tilde{\psi}_{|a|}(b),$$

(4)

in which $\tilde{\psi}_{|a|}(t) = |a|^{-1/2} \psi^*\left(-t/a\right)$, so wavelet transform can also be considered as the signal and the filter convolution operation. From engineering significance of angle, $\tilde{\psi}_{|a|}(t)$ can be considered as the high pass filter.

Generally, singular signal can divided into two cases: (Ⅰ) Mutation in the signal amplitude at one point will lead to signal non-continuous. This type of mutation is called as discontinuity point of the first kind. (Ⅱ) Signal is very smooth in appearance and signal amplitude has no mutation, whereas First-order differential of a signal has suffered a mutation and non-continuous. This type of mutation is called as discontinuity point of the second kind.
Assume that $\theta(t)$ is a smooth function, $\int_{-\infty}^{\infty} \theta(s)ds = 1$, and $\theta(s)$ denotes $(1+x^2)^{-1}$'s infinitesimal of higher order. At the same time we know that $\theta_s(t) = \theta(1/s)/s$ and the wavelet function $\psi(t)$ denotes the first derivative of $\theta(t)$ ($\psi(t) = d\theta(t)/dt$), so $f(t)$ can be expressed by continuous wavelet transform as Eqn. 5.

$$Wf(s,u) = s^{1/2} \left( f \ast \psi_{s} \right)(u) = s^{1/2} \frac{d}{du} \left( f \ast \hat{\theta}_{s} \right)(u)$$  \hspace{1cm} (5)

The wavelet transformation module maximum $|Wf(s,u)|$ is the first derivative maximum of “$f$” polished by $\hat{\theta}_{s}$, which just correspond to point mutations of the signal “f”[13]. Analyzing the signal on scales and considering the wavelet function as the first or second derivative of a smooth function, the absolute value of the wavelet coefficients are a bit large in point mutations of the signal. So, we can detect the position of singular points by the module maximum. Detecting singular signals, the wavelet base should satisfy the compact support and enough vanishing moment at a certain interval. In this paper, the curvature mode of continuous steel girders is analyzed using continuous wavelet transform by mexh wavelet (Mexican Hat wavelet) and the function satisfies the wavelet admissible condition and vanishing moment.

**Curvature Mode Theory**

The curvature mode is deformation mode of structure neutral plane. The curvature mode reflects changes in the structure local characteristics and possess the orthogonality and the superposition. The curvature mode of the local structure have much greater sensitivity than that of the displacement mode.

According to mechanics of materials, we can see the curvature of a beam can be defined by Eqn. 6.

$$\rho = \frac{M}{EI(x)}, \hspace{1cm} (6)$$

where $M$ is bending moment of a beam, $E$ is the modulus of elasticity, $I(x)$ is the moment of area of the section.

The curvature of a beam can be expressed by Eqn. 7.

$$\rho(x,t) = \frac{\partial \theta}{\partial x} = \frac{\partial^2 y}{\partial x^2}, \hspace{1cm} (7)$$

where $\theta$ is the angle displacement of a beam, $y$ is the displacement mode of a beam.

Substituting Eqn. 6. into Eqn. 7. and using centered difference method can obtain Eqn. 8.
\[ \rho = \frac{M}{EI(x)} = \frac{y_{i+1} - 2y_i + y_{i-1}}{l^2} = \phi_n(x), \]  

(8)

where \( i \) is the \( i \)th test point, \( l \) is the distance between two test points, \( \phi_n(x) \) is the \( n \)th curvature mode of a beam.

The formula above showed that local damage in beam has led to the decline in local stiffness and the mutation in curvature mode shape. So, based on an analysis of the mutation in curvature mode shape, we could diagnose structural damage (the position and the extent of the damage). In this paper, the curvature mode of the damage beam was analyzed using continuous wavelet transform, and the position of the crack could be identified by the modulus maximum of the wavelet coefficients.

**COMPUTATIONAL ANALYSIS**

**The Geometric Model of a Continuous Steel Girder**

Taking a three-span continuous steel box girder as an example, and the span: 30m+40m+30m. It was 100 metres long. Calculating diagram and cross section diagram are given in Figure 1 and Figure 2.

![Figure 1: Calculating diagram (Unit: m)](image1.png)

![Figure 2: Cross section diagram (Unit: cm)](image2.png)

**The Finite Element Model of a Continuous Steel Girder**

The finite element method introduced a three-span continuous steel box girder, and the span: 30m+40m+30m. It was 100 metres long. The material introduces steel Q345, and the thickness of steel plate is 20m. The basic parameters of the structure are so...
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follows: \( E = 2.1 \times 10^{11} \, N/m \), \( \rho = 7800 \, kg/m^3 \), \( \mu = 0.3 \). When dynamic characteristics of a beam were analyzed using the finite element method, the beam was divided into 200 elements as viewed from portrait. Cross section was divided into 10 elements (Figure.2), and the bridge was divided into 2000 elements. The cracks were simulated using the decline in local stiffness. The average of 400 nodes mode shape that lie in mid axes of upper and lower surface of the bridge was analyzed using continuous wavelet transform. Calculation was divided into two conditions. ( I ) The base plate of side span and the base plate of middle span had both a crack. ( II ) The top plate of side span and the base plate of middle span had both a crack. The curvature mode was analyzed using continuous wavelet transform by mexh wavelet (Mexican hat wavelet), and the position of the crack could be identified by the modulus maximum of the wavelet coefficients.

THE NUMERICAL ANALYSIS

**Condition I**

Two damages (decline in local stiffness by 30%) separately lie in 10 meters of the base plate of side span and 40 meters of the middle plate of side span, including element 191,200,791 and 800. Based on analyzed the curvature mode of damage in beams using continuous wavelet transform by mexh wavelet, we could obtain the wavelet coefficients that are illustrated in Figure.3. The wavelet coefficients have two point mutations which lie in the position of two damages (20 and 80).

![Figure 3: the wavelet coefficients of curvature mode under condition I](image)

**Condition II**

Two damages (decline in local stiffness by 50%) separately lie in 20 meters of the top plate of side span and 60 meters of the base plate of middle span, including element 404,405,406,407,1191 and 1200. Based on analyzed the curvature mode of damage in beams using continuous wavelet transform by mexh wavelet, we could obtain the wavelet coefficients that are illustrated in Figure.4. The wavelet coefficients have two point mutations which lie in the position of two damages (40 and 120).
Computerized analysis above showed that the method in the paper have been used to identify the damage of continuous steel girders effectively.

CONCLUSIONS

Based on dynamic analysis of a continuous steel girder and the wavelet singularity detection theory, we have identified the damage in beams and deduced the following results:

( I ) The damage of continuous steel girders could be identified by wavelet analysis of the curvature mode. The curvature mode of damaged beams may be obtained using the finite element method, and then the position of the crack could be identified by modulus maximum of the wavelet coefficients. The numerical analysis showed that the effectiveness of this method.

( II ) Damage of continuous steel girders identification have been presented in this paper. Theoretically, this method could easily be extended to the damage identification (types of different materials and different types of beam damage). So, this method was a guide to identification of other types of beam damage.

(III) When this method was used to solve beam damage identification in practice engineering, we could obtain the vibration characteristics of damaged beams using the experiments. And then we analyzed the curvature mode of the damaged beams. So, this method may be useful in damage identification of beam structures.

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DAMAGE IDENTIFICATION RESEARCH OF PLATE-LIKE STRUCTURES BY MEANS OF THE WAVELET ANALYSIS

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KEYWORDS

Elastic plate structure, finite element method, damage identification, two-dimensional wavelet analysis, rotation mode

ABSTRACT

This paper is concerned about taking a rectangular damaged plate which is simply supported along its edge as the studying object, using equivalent stiffness to replace the stiffness of the damaged plate and established an elastic finite element model which has several damaged regions. Adopted with the finite element analysis to analyze the dynamic character of the plate and applied the Lanczos method for calculating the rotation mode along the plate’s two directions, the rotation mode is analyzed by Mexh wavelet, the location of the declining stiffness is identified by the place where maximum of wavelet coefficients appears. Therefore, this method is applied to the elastic damaged plate. Three types of damaged regions in rectangular simply supported plate are recognized, which indicates this method is effective and accurate. And this method has a certain value in the project application.

INTRODUCTION

The elastic plate may be damaged during the construction and using process. When the damage reaches a certain extent, it will affect the safety of the structure. You can timely deal with it and improve the life of the structure to avoid major security incidents by using an appropriate method to identify damaged plate. Therefore, the identification method of damaged research boards is not only having great theoretical significance, but also provides a valuable reference for engineering applications. A certain degree of damage in the board will cause the changes of physical parameters (quality, frequency, etc.), which also led to the changes of modal parameters (modal frequencies, modal vibration mode, etc.). Thus, to identify structural changes for damage has become a heated issue for scholars at home and abroad. Domestic and foreign scholars have done many researches on damaged...

In recent years, there have been successful in applying the theory of wavelet analysis to the field of structure damage detection, and made some meaningful results. Wavelet analyze method known as "mathematical microscope", it has characteristics of spatial localization. Signal at a certain point in the low-scale wavelet transform completely from the vicinity of the point determined by local information. Therefore, it is better to analyze the singularity of the signal location and the strength of singularity. The introduction of wavelet theory to the field of structural damage diagnosis by Liew.K.M and Wang [6], they analyzed the deflection of simply supported beam with a discrete wavelet and successfully identified the location of cracks in beam. By comparing with the deflection curve of cracks in simply supported beam and no damage one under concentrated load, Quanwang etc. [7] identified the location of cracks in beams with multi-scale wavelet transformation. Douka. E, etc. [8] identified damage location cracks in the vicinity of the horizontal openings of the rectangular cross-section cantilever beam, and estimated the depth of the crack in beam with the continuous wavelet transformation to fundamental vibration mode. Guan Deqing and Jiang Xin[9] presented a damage identification method of Timoshenko beam crack based on wavelet analysis of the rotation mode. Compared with the result of analysis of the mode shape, it could be concluded that the method using the wavelet analysis of rotation mode was more convenient and more effective. In this paper, the rotation modal wavelet transformation of the plate is applied to the damage identification of two-dimensional structure. With the four sides simply supported rectangular plate with damage as the research object, we can get rotation modal along the plate on both sides after damage by calculating the data, and then using two-dimensional wavelet analysis the rotation modal. In order to identify the damage in plate and having getting the satisfied results, the study of this paper provides an effective analytical method for the damage identification of plate.

THEORY

The wavelet transformation

A wavelet is an oscillatory, real or complex-valued function \( \psi(x) \in L^2(\mathbb{R}) \) of zero average and finite length. Function \( \psi(x) \) is called a mother wavelet and \( L^2(\mathbb{R}) \) denotes the Hilbert space of measurable, square-integrable one-dimensional functions. In this paper, apart from general definition, only the real wavelets and the space domain will be considered. The function \( \psi(x) \) localized in both space and frequency domains is used to create a family of wavelets \( \psi_{a,b}(x) \) formulated as

\[
\psi_{a,b}(x) = \left( |a| \right)^{-1/2} \psi \left( \frac{x-b}{a} \right), \quad a,b \in \mathbb{R}, \quad a \neq 0
\]
where the real numbers $a$ and $b$ are scale and translation parameters.

It can be seen that wavelet function not only requires a concussion, that it contains a certain frequency characteristics, but also requires a lo calized, that is, it is the Constant in a range which equals to 0 or quickly conver ges to 0. Set $\psi(t)$ as the ba sic wavelet, $\psi_{a,b}(t)$ is a continuous wavelet function. Then for any function or signal $f(t) \in L^2(\mathbb{R})$, Changes in its continuous wavelet is defined as:

$$
WT_f(a,b) = \langle f(t), \psi_{a,b}(t) \rangle = \left| a \right|^{-1/2} \int_{-\infty}^{\infty} f(t) \psi^{*} \left( \frac{t-b}{a} \right) dt
$$

which, $a \neq 0$, $b$, $t$ is continuous variables, $\psi^{*}(t)$ expressed by complex conjugation $\psi(t)$.

Generally, singularity of the signal is divided into two situations: (1) signal amplitude mutates at a given moment, and causes the non-continuous signal, this mutation is called the first type of discontinuity points; (2) The appearance of signal is very smooth, with no mutation amplitude, but the first-order differential of signal, which mutates and has no continuity. This mutation is called the second type of discontinuity points.

Suppose $\theta(t)$ is a smooth function, and $\theta(t)$ satisfies $\int_{-\infty}^{\infty} \theta(s)ds = 1$, and $\theta(s)$ is the higher-order infinitesimal of $(1+x^2)^{-1}$, at the same time let $\theta'_1(t) = 1/s \theta(1/s)$, that wavelet function $\psi(t)$ is its first derivative, that is $\psi(t) = d\theta(t)/dt$, the continuous wavelet transform of $f(t)$ is as follows:

$$
Wf(s,u) = s^{1/2} \left( f * \hat{\psi}_s \right)(u) = s^{1/2} {d \over du} \left( f * \hat{\theta}_s \right)(u)
$$

Wavelet transformation maxima modulus $|Wf(s,u)|$ is the maximum value of the first derivative of function after $f$ is polished by $\theta_s$, which just correspond to the mutative point of the signal, that is, we can find mutations in the signal by the wavelet transform modulus maxima point. In the multi-scale analysis of signals, when we use a wavelet function that is the first or second derivative of some smooth function, the absolute value of wavelet coefficients is very larger than the point mutation of the signal, so we can determine the location of singular points by detecting modulus maxima point.

The damage identification method of plate

Usually, the dam age of the plate only reduce its structural stiffness without changing its mass distribution, thus, in this paper the structural dam age is simulated only by the reducing of the elements’s flexural rigidity, and different $D$ values are represented different damage of elements.

$$
D = E\mu^2/[12 \times (1-\mu^2)]
$$
Where \( t \) is the plate thickness, \( \mu \) is Poisson's ratio of the material.

According to the conditions of plate deformation, it can be learned that the plate’s vertical displacement is the same at the section of stiffness change, but the rotation is different, therefore, when used the two-dimensional wavelet analysis the rotation mode of plate, the identification of the location of the singularity is corresponding to the location of the stiffness change. In this paper, wavelet mexh was selected as the mother wavelet function and two-dimensional wavelet transformation was made to the rotation mode of the plate. First of all, all the nodes of rotation mode were transformed by row, and then all the wavelet coefficients’ numerical value of the nodes were transformed again by list. Adopted with Matlab to draw the map of rotation mode wavelet coefficients of the plate and found that the wavelet coefficients will appear to modulus maxima at the section of stiffness change. Thus, the location of plate’s damage is identified.

![Figure 1: Four sides simply-supported rectangular plate with damage](image)

**NUMERICAL SIMULATION ANALYSIS**

**Analysis model**

Assuming that the section dimensions and material parameters of the plate are as follows: the length of the plate is \( L=1 m \), width is \( W=0.6 m \), thickness is \( t=0.006 m \), the materials of the plate is made up of steel Q235, Young’s Modulus \( E=1.92\times10^{11} N/m^2 \), Density \( \rho=7800 kg/m^3 \), Poisson’s Ratio \( \mu=0.3 \). When the finite element analysis is used for analyzing the dynamic characteristics of plates, the element type of the plate is shell163, and along the x direction and y direction of the plate is divided into \( 100 \times 60 \) elements, a total of 6161 nodes. Took the different locations and different damage types of plate into account and applied the Lanczos method for calculating rotation mode of four sides simply-supported plate on three conditions. Condition I: there is a damage in the center of plate; Condition II: there is a damage close to the edge; Condition III: damage zone and damage block are all existed.
Numerical calculation

Damage in the central of plate

Assuming there is a damage located at distance 0.3m from AB support and at distance from 0.5m AD support, damage region is $0.02 \times 0.02$ m, the degree of damage is 13.3%, and location of damage in board is as shown in figure 2. Finite element method is used for calculating rotation mode of plate with the damage along the x-axis and the vibration mode diagram draw in the help of Matlab is shown in figure 3. The research shows that all confirm damage region is not identified from rotation mode diagram of plate. Rotation mode of plate which contains the damage complete two-dimensional wavelet transform of scale 1 through wavelet Mexh as the mother wavelet in order to get the map of Wavelet coefficient shown in figure 4. From the map of wavelet coefficients, it is found that Wavelet coefficients rise significantly in the area of being away from AB side of the plate at 0.02m and AD side of it at 0.3m. It exactly corresponds to the location of damage, and thereby proves that wavelet transform of rotation mode can identify the location of the damage of central board.

![Figure 2: Damage in the central of the plate](image1)

![Figure 3: The rotation mode RX of the plate](image2)

![Figure 4: Wavelet analysis of RX](image3)

![Figure 5: Wavelet analysis of RY](image4)

Damage on the edge of plate

Assuming there is a damage located at distance 0.02m from AB support and at distance from 0.3m AD support, damage region is $0.02 \times 0.02$ m, the degree of damage is 13.3%, and location of damage in board is as shown in figure 6. Rotation mode of plate which contains the damage at the border complete two-dimensional wavelet transform of scale 1 through wavelet Mexh as the mother wavelet in order to get the map of Wavelet coefficient shown in figure 7. From the map of wavelet coefficients, it is found that Wavelet coefficients rise significantly in the area of being away from AB side of the plate at 0.02m and AD side of it at 0.3m. It exactly corresponds to the location of damage, and thereby...
that proves wavelet transform of rotation mode can identify the location of plate containing the damage at the border.

![Figure 6: Damage on the edge of the plate](image)

![Figure 7: Wavelet analysis of RX](image)

**Damage area and damage block in plate**

Considering the more general case, this article completes damage identification for the simply supported plate with damage area and damage block at the same time. In Condition 3, there is the assumption that at a distance of 0.3 meters from the AB side and 0.02 meters from the AD side there is damage area with $0.08 \times 0.02$ meters and degree of damage was 13.3%. At a distance of 0.3 meters from the AB side and 0.5 meters from the AD side there is the damage block with $0.02 \times 0.02$ meters and degree of damage was 13.3%. Location of damage in plate is shown in figure 8. Under the Condition 3, rotation mode of plate complete two-dimensional wavelet transform of scale 1 through wavelet Mexh as the mother wavelet in order to get the map of Wavelet coefficient shown in figure 9. From the map of wavelet coefficients, it is found that Wavelet coefficients raise significantly proves the effectiveness of the method.

![Figure 8: Location of damage area and damage block](image)
CONCLUSIONS

We can draw the following conclusions based on dynamic analysis of the four sides simply-supported plate with the damage and the principle of Wavelet Singularity Detection which effectively identifies the damage plate:

1. Two-dimensional wavelet transformation of rotation mode can basically identify location of the injury in the four sides simply supported plate.

2. In the same degree of damage, the different damage location of the wavelet coefficients corresponding to the maximum value is not the same, near the plate boundary damage is less than the wavelet coefficients around the central board of the wavelet coefficients injury, location of the injury to show that different wavelet coefficients the size of a certain degree of impact.

3. In this paper, the four sides simply-supported elastic plate of damage identification methods, from the principle can be extended to other types of different constraints and different types of elastic plate injury of the damage identification problems, and thus the method for other types of damage identification plate with guiding significance.

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REFERENCES


RESEARCH ON SPATIAL CRACK IDENTIFICATION OF STEEL BEAM USING WAVELET ANALYSIS

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KEYWORDS
spatial crack, damage detection, modal parameter, continuous wavelet transform, signal extension

ABSTRACT
This paper presents a method for spatial crack identification in beam based on wavelet singularity detection which is focused on transverse unsymmetrical and nonpenetrative spatial crack. A cracked beam has been simulated using three-dimensional finite element technique, and the modal parameter of its axes is extended as signal beyond the boundary by cubic spline extrapolation. Then the signal is analyzed using the continuous wavelet transform and the location of crack is identified by the maximum of wavelet coefficients at the site of the crack. The viability of the method is demonstrated by analysis of a simply supported beam which contains semi-elliptical surface crack and elliptical internal crack, respectively. The study have reference value in spatial crack identification and diagnosis in structures.

INTRODUCTION
The existence of cracks in structural members affects the performance of structures and presents a serious threat to the integrity of structures. The propagation of cracks might lead to a catastrophic failure. Beam is an important structural member. For this reason, research on identify the localization of cracks in beams has an important significance.

In recent years, damage identification methods have been studied by a number of researchers while the importance of nondestructive evaluation (NDE) of the civil infrastructure has been significantly increasing. Liew, K.M. and Wang[1] used the wavelet theory to identify the crack in a simply supported beam with a transverse on-edge nonpropagating open crack. Guan Deqing and Jiang Xin[2] presented a
method for crack identification of a Timoshenko beam by means of wavelet analysis of modal parameters, including vibration mode shape and slope mode. In contrast to the result of analysis of the mode shape, it is testified that the method using the wavelet analysis of slope mode is more convenient and more effective. Ren Yichun et al. [3] proposed a wavelet-based approach for crack identification in beam structure. The fundamental vibration mode of a cracked simply supported beam is analyzed using continuous wavelet transform and both the location and depth of the crack are estimated. Hong, J.C. et al. [4] used the Lipschitz exponent for the detection of singularities in beam modal data. The Mexican hat wavelet was used and the damage extent has been related to different values of the exponent. Loutridis, S. et al. [5] presented a method for crack identification in double-cracked beams based on wavelet analysis. The fundamental vibration mode of a double-cracked cantilever beam is analyzed using continuous wavelet transform and both the location and depth of the cracks are estimated. Rucka, M. et al. [6] present a method for estimating the damage location in beam and plate structures. The location of the damage is indicated by a peak in the spatial variation of the transformed response. Applications of Gaussian wavelet for one-dimensional problems and reverse biorthogonal wavelet for two-dimensional structures are presented.

This paper presents a method for spatial crack identification in beam based on wavelet singularity detection which is focused on semi-elliptical surface crack and elliptical internal crack.

CONTINUOUS WAVELET TRANSFORM IN DAMAGE DETECTION

Wavelet Transform Background

The wavelet is an oscillatory function $\psi \in L^2(R)$ having a zero average and finite length (compact support):

$$\int_{-\infty}^{+\infty} \psi(x) dx = 0$$  \hspace{1cm} (1)

The function is called the mother wavelet and it must satisfy the wavelet admissibility condition:

$$\int_{0}^{+\infty} \left| \hat{\psi}(\omega) \right|^2 d\omega < \infty$$  \hspace{1cm} (2)

Where $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(x)$.

The real or complex-valued function $\psi(x)$ localized in both time and frequency domains is used to create a family of wavelets $\psi_{a,b}(x)$, defined as:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi\left(\frac{x-b}{a}\right)$$  \hspace{1cm} (3)

Where the real numbers $a$ and $b$ denotes the scale and translation parameters respectively. The family of wavelet function is a dilated or stretched version of the mother wavelet $\psi(x)$.
For a given signal $f(x)$, where the variable $x$ is time or space, the continuous wavelet transform is obtained by integrating the product of the signal function and the wavelet function:

$$Wf(a,b) = \left\langle f, \psi_{a,b} \right\rangle = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(x) \psi^*\left(\frac{x-b}{a}\right) dx$$  \hspace{1cm} (4)

Where $\psi^*(x)$ is the complex conjugate of the wavelet function. In translating Eqn. (4) one might recognize the inner product of $f(x)$ with scaled and translated versions of the original wavelet function. Large values of scale $a$ correspond to big wavelets and thus coarse features of $f(x)$; while low values of $a$ correspond to small wavelets and fine details of $f(x)$. This inner product is carried out for all times so that the one-dimensional function $f(x)$ is transformed into a two-parameter function $Wf(a,b)$; so that useful information about the function analyzed will be revealed.

An important property of the wavelet transform is its ability to react to subtle changes of the signal structure. To point this out, suppose that the wavelet used is the derivative of a continuous function $f(x)$ usually called the scaling function, i.e., $\psi(x) = d\psi(x)/dx$: The wavelet transform can be written as

$$Wy(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x) \frac{d}{db} \phi\left(\frac{x-b}{a}\right) dx = \frac{d}{da} \left\{ \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x) \phi^*\left(\frac{x-b}{a}\right) dx \right\}$$ \hspace{1cm} (5)

The continuous wavelet transform is proportional to the first derivative of $f(x)$ smoothed by the function $\phi(x)$. In an analogous way, wavelets that are higher derivatives of a smoothing function can be constructed. The wavelet transform coefficients will be proportional to a smooth version of the second, third or higher derivatives of the signal, respectively. This means that it is possible to examine different rates of change of the signal for all scales of interest allowing a completely local or less local feature extraction procedure.

**Border Distortion Problem**

The CWT is defined as the integration of the product of a wavelet and the signal of infinite length, i.e. $x \in (-\infty, +\infty)$. Since the deflection of the beam $y(x)$, treated as a spatially distributed signal, has a finite length, i.e. $x \in (0, L)$, a border distortion problem appears. The wavelet coefficients achieve an extremely high value at the ends of a signal. Therefore the border of the signal should be treated independently from the rest of the signal. The influence of boundary effects can be reduced by extension from the signal beyond the boundary. It is obvious that the length of the extended signal depends on the scale of the wavelet used.

There are some possibilities to extend the signal, namely, extension by zeros, by reflection, by periodicity and by extrapolation [7]. Extension by zeros assumes that the signal is zero outside the domain; this method creates artificial discontinuities at the border. Another method is extension of the signal by reflection that assumes recovery of the signal outside its original support by symmetric boundary value replication. Reflection generally introduces discontinuities in the first derivative at the
border. It is also possible to recover the signal by its periodic extension. This method also creates discontinuities at the borders.

In this paper, to avoid large discrepancy at the boundaries, the signal is extended outside its original support by a cubic spline extrapolation based on three neighbouring points. The spline provides a technique for obtaining a smoother interpolation. Extrapolation is continuous, with continuous first and second derivatives. Only the third derivative is allowed to have jumps at the connection points.

SIMULATIONS ON A SIMPLY SUPPORTED BEAM

Vibration Model of a Cracked Simply Supported Beam

For a multi-degree of freedom system with damping, the equation of motion can be expressed as:

\[ [M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f(t)\} \]  

Where \([K]\) denotes stiffness matrix, \([M]\) denotes mass matrix, \([C]\) denotes damping matrix and \(\{f(t)\}\) denotes load vector.

If \(\{x\} = \{X\}e^{\imath \omega t}\), the equation of non damping free vibration can be expressed as:

\[ [K]\{\Phi_i\} = \omega_i^2 [M]\{\Phi_i\} \]  

Where \(\omega_i\) and \(\{\Phi_i\}\) denotes the order of modal vibration frequency and modal vibration vector, respectively. In this paper, this method is applied to calculate the displacement modal of the simply supported beam.

![Simply Supported Beam Model](image)

Figure 1: cracked simply supported beam model.

Due to the localized crack effect, the beam can be simulated by two segments connected by a massless spring, as is shown in Figure 1. For general loading, a local flexibility matrix relates displacements and forces. In our analysis, since only bending vibrations of thin beams are considered, the rotational spring constant is assumed to be dominant in the local flexibility matrix.

The displacement in each part of the beam is

\[ y_1(x) = A_1chx + A_2shx + A_3\cos kx + A_4\sin kx \]  

\[ y_2(x) = A_5chx + A_6shx + A_7\cos kx + A_8\sin kx \]
with \( k^i = \omega^2 \rho A_i/(EI) \), where \( A \) is the cross-sectional area, \( \omega \) is the vibration angular velocity, \( \rho \) is the material density and \( A_i (i=1,2,\ldots,8) \) are constants to be determined from the boundary conditions.

Consequently, the boundary conditions at the crack positions can be expressed as follows:

\[
\frac{dy_1(l)}{dx} + \frac{EI}{K_i} \frac{d^2 y_1(l)}{dx^2} = \frac{dy_2(l)}{dx} \tag{10}
\]

Where \( L \) is the length of simply supported beam, a crack located at distance \( l \) from the left support, and \( K_i \) is the bending spring constant in the vicinity of the cracked section.

**Determination of Crack Location**

In this section, to illustrate the feasibility of the wavelet-based crack identification approach, numerical simulations on simply supported cracked beams were performed. The simply supported beam, as shown in Figure 2(a), are studied in this paper. The length is \( L = 1000 \text{mm} \), the width and depth of the beam are \( b = 30 \text{mm} \) and \( h = 50 \text{mm} \), respectively. The numerically determined material properties are: Young’s modulus \( E = 2.07 \times 10^{11} \text{Pa} \), Poisson ratio \( \mu = 0.3 \) and mass density \( \rho = 7800 \text{kg/m}^3 \). The viability of this new technique is demonstrated with three examples: 1) single position semi-elliptical surface crack; 2) multiposition semi-elliptical surface cracks; 3) elliptical internal crack. The results are as follows:

**Single position semi-elliptical surface crack**

The simply supported beam above mentioned having one semi-elliptical surface crack located at distance \( l = 300 \text{mm} \) from the left support is considered, illustrated in Figure 2(b). The size of this crack in two direction are \( w = 15 \text{mm} \) and \( v = 5 \text{mm} \), respectively.

![Figure 2: The model of a simply supported beam with semi-elliptical surface crack](image)

The cracked beam which is divided into one thousand hexahedral elements is simulated by using three-dimensional finite element technique, and the displacement response of its axes is calculated with the Lanczos method, as shown in Figure 3. It is subsequently extended as signal beyond the boundary by cubic spline extrapolation. Then the signal were transformed which is implemented for scale15 with the “biorthogonal2.4” as the analyzing wavelet. The results of the wavelet analysis are presented in Figure 4. It is obvious that the wavelet transform coefficients exhibit a maximum at \( x = 300 \text{mm} \). This implies the presence of a singularity and the location of crack can be identified.
Assumed that the simply supported beam contains two semi-elliptical surface crack located at distance 300\( \text{mm} \) and 400\( \text{mm} \) from the left support, respectively. The size of both cracks in two direction are \( w=15 \text{mm} \) and \( v=5 \text{mm} \). The cracked beam is simulated by using three-dimensional finite element technique, and the displacement response of its axes is calculated with the Lanczos method. It is subsequently extended as signal beyond the boundary by cubic spline extrapolation. Then the signal were transformed which is implemented for scale15 with the “biorthogonal2.4” as the analyzing wavelet. The results of the wavelet analysis are presented in Figure 5. The location of crack is identified by the maximum of wavelet coefficients at the site of the cracks.

Assumed that the simply supported beam contains one elliptical internal crack located at distance 400\( \text{mm} \) from the left support. The size of both cracks in two direction are \( w=15 \text{mm} \) and \( v=10 \text{mm} \), as is shown in Figure 6. The cracked beam is simulated by using three-dimensional finite element technique, and the displacement response of its axes is calculated with the Lanczos method. It is subsequently extended as signal beyond the boundary by cubic spline extrapolation. Then the signal were transformed which is implemented for scale15 with the “biorthogonal2.4” as the analyzing wavelet. The results of the wavelet analysis are presented in Figure 7. The location of crack is identified by the maximum of wavelet coefficients at the site of the cracks.
CONCLUSIONS

A method for spatial crack identification in beam structures is presented, which is based on both FEM and wavelet singularity detection theory. The study applying theoretical analysis and numerical calculation leads to the following conclusions:

1) Based on Three-dimensional finite element technique, the fundamental vibration mode of cracked beam is analyzed using continuous wavelet transform. The location of the crack was determined by the maximum of wavelet coefficients at the site of the crack. The method is validated by numerical analysis.

2) As to damage detection of cracked beam, the method proposed in this paper can avoid the deviation in calculation result caused by applying simplification analyse and thereby achieve the desired identification effect.

3) The proposed method that the signal is extended outside its original support by a cubic spline extrapolation can effectively avoid large discrepancy at the boundaries, which lay the foundation for accurately determine the location of the crack.

4) In engineering practice, the mode parameters of cracked beam firstly obtained is analyzed using continuous wavelet transform, then the location of the crack can be determined by the maximum of wavelet coefficients at the site of the crack. The method has been proved to be effective for identify not only single position crack but also multiposition cracks. In the view of the results obtained, the study have reference value in spatial crack identification and diagnosis in structures.

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SYSTEM RELIABILITY OF STEEL SCAFFOLD SYSTEMS

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KEYWORDS
Probability-based limit state design, structural reliability, steel scaffold, advanced analysis, inelastic analysis, system reliability

ABSTRACT
Advanced analysis has the potential to design steel structures with direct system capacity check. This paper investigates the system reliability of steel support scaffold systems designed with advanced analysis. A series of 3-story 1 bay × 1 bay and 3-story 3 bay × 3 bay steel support scaffolds were considered. The joint stiffness, initial geometric imperfections, load eccentricity, yield stress and model uncertainty were modeled as random variables, and propagated through Monte Carlo simulations to develop the statistical data of system strength. The probabilistic models for construction dead and live loads were obtained from literatures and engineering judgment. With the statistics of the system strength and loads, the system reliability of the steel scaffold systems was estimated using the First-Order Reliability-Method (FORM).

INTRODUCTION
Current limit state design codes, e.g., the load and resistance factor design (LRFD), are based on a member level. In this design process, system effects are usually not considered directly. Studies have shown that the reliability of redundant structural systems is generally higher than the reliability of individual members (Ellingwood, [1]). To take advantage of beneficial system effects, it is necessary to design a structure as a system rather than a collection of members. With the recent increases in computing power and sophisticated structural analysis software, real behaviour of large-scale structures can be captured, including material nonlinearity (yielding), geometric nonlinearity ($P - \Delta$ and $P - \delta$), initial geometric imperfections, residual stresses etc. Analysis with this capability is referred as to “advanced analysis”. Advanced analysis makes it feasible to design a structure with direct system capacity check.
This paper investigates the system reliability of steel support scaffold systems designed with advanced analysis. Steel support scaffolds are commonly used in constructions. They are used as platforms while building the formwork for reinforced concrete, and are usually heavily loaded under the weight of fresh concrete, formworks, stacked material and workers. A steel support scaffold system normally consists of standards (column members), ledgers (beam members), braces and jacks, as shown in Fig. 1. Cold-formed circular hollow steel sections are widely used for standards and ledgers due to their high strength and reusability. The standards are connected to ledgers via various types of connections, such as wedge-type joints and cuplock joints. The bases of scaffold frames consist of jack bases which can be adjusted by a wing nut to accommodate irregularity of the ground. At the top adjustable shore extensions with U-head screw jacks are used to support timber bearers to ensure the levelling of the formwork.

Although steel scaffolds are temporary structures, their failure often has tragic consequence for the workers and the public, as well as legal and financial associated risks. The main causes of scaffold collapses are overloading (Hadipriono and Wang [2], Peng et al [3], Yu et al [4]). The current practice in designing steel scaffold system is to use the load capacity recommended by the manufacturers based on load tests, if such tests are available, and then apply a judgmental safety factor. A rational basis for the design of steel scaffolds by advanced analysis does not as yet exist. Moreover, steel scaffold systems are characterized by the large variations of geometric and mechanical parameters, notably joint stiffness, initial geometric imperfections, yield stress and load eccentricity. A statistical framework is required to consider the effects of these uncertainties. Aiming to develop a probability-based limit state design method, the researchers at the University of Sydney have investigated the load carrying capacity of steel scaffolds using experimental tests, advanced analysis and probabilistic study. This paper presents some of the research results and focuses on the system reliability of the steel scaffold systems designed with advanced analysis.

![Figure 1: A typical 1-story 1 bay × 1 bay steel scaffold system.](image)

**PROBABILITY-BASED DESIGN**

The current probability-based design codes such as LRFD are based on structural reliability theory. The basic structural reliability problem is to determine the probability of failure $P_f$ of the structure:

$$P_f = P(R - Q \leq 0) = P(G(R, Q) \leq 0) = \int F_R(x)f_Q(x)dx$$  \hspace{1cm} (1)
in which \( R \) is the structural resistance and \( Q \) the structural action due to the applied load. \( R \) and \( Q \) are modeled by random variables. \( G(R, Q) \) is termed the “limit state function” and \( G(R, Q) \leq 0 \) defines the unsafe domain. \( F_R(x) \) represents the cumulative distribution function of \( R \) and \( f_Q(x) \) is the probability density function of \( Q \). The integration in Eqn. (1) can be approximated using the First-Order Reliability Method (FORM). In the case of statistically independent normal \( R \) and \( Q \), \( P_f \) becomes

\[
P_f = \Phi(-\frac{\mu_R - \mu_Q}{\sqrt{\sigma^2_R + \sigma^2_Q}}) = \Phi(-\beta)
\]

where \( \Phi( ) \) is the standard normal distribution function, \( \mu \) and \( \sigma \) represent mean and standard deviation, respectively, and \( \beta \) is the so-called “reliability index” as an alternative to \( P_f \). A complete description of FORM can be found in Melchers [5]. For practical use in design, Eqn. (1) was replaced by the LRFD formula:

\[
\phi R_n \geq \sum \gamma_i Q_{ni}
\]

in which \( R_n \) is the nominal resistance, \( Q_{ni} \) are the nominal loads, \( \phi \) is the (member) resistance factor, and \( \gamma_i \) are the load factors. The partial factors \( \phi \) and \( \gamma_i \) are determined using reliability analysis to achieve a target reliability for structural members (Ellingwood and Galambos [6]). It must be noted that Eqn. (3) was initially developed for member capacity check. For design with advanced analysis, Eqn. (3) will be applied at the system level, with \( R \) representing the resistance of the system rather than of an individual member. However, developing such an LRFD type formula for system check on general structures proves to be very difficult. It involves the following challenges:

1. identify the appropriate system limit states and modes of failure;
2. select an appropriate target system reliability (index);
3. predict accurately the behaviour of the system;
4. estimate the statistics of system resistance;
5. estimate the statistics of applied loads.

Buonopane and Shafer [7] compared the system reliabilities of a series of two-story two-bay steel frames designed by elastic LRFD and advanced analysis. The load combination \( 1.05D_n + 2.05L_n \) was considered, in which \( D_n \) and \( L_n \) represent the nominal dead and live load. It was shown that for a target system reliability index of 3.0 on ultimate frame strength, the values of \( \phi \) range from 0.86 to 0.91. Li and Li [8] developed the resistance and load factors for advanced analysis of steel portal frames. A design formula was proposed as \( 0.8R_n = 1.05D_n + 2.05L_n \). A target system reliability index \( \beta = 3.7 \) was achieved. Design by advanced analysis is permitted in the Australian steel design standard AS4100 (Standards Australia [9]). However, no guidance was given on determining the value of system resistance factor \( \phi \).

Unlike steel buildings for which a wide variety of structural configurations and limit states need to be considered, the configurations and limit states for steel scaffold systems are relatively simple. In this work, the system limit state is violated when the applied load exceeds the ultimate system strength. The serviceability limit state is not considered. The ultimate strengths of steel scaffold systems were predicted using the advanced analysis. The inherent uncertainties and the model uncertainty are propagated through Monte Carlo simulations to obtain the statistics of system strength. The probabilistic models for construction dead and live loads were obtained from literatures and engineering judgment.
With the statistics of the system strength and loads, the reliabilities of the steel scaffold systems designed with different resistance factor $\phi$ were estimated using the FORM.

ADVANCED ANALYSIS FOR STEEL SCAFFOLD SYSTEMS

Researchers at the University of Sydney tested fifteen steel scaffold frames with cuplok joints (CASE [10]). The tests featured 3-stories and $3 \times 3$ bays with different story heights, jack extensions and bracing arrangement. Based on these experimental tests, a three dimensional second-order inelastic finite element (FE) model was developed using the commercial FE program Strand 7 [11] (Chandrangsu and Rasmussen [12]). Material and geometric nonlinear ($P - \Delta$ and $P - \delta$) effects are taken into account. The plastic zone beam-column element is employed to consider spread of plasticity through the cross-section and along the length of the member. The cuplok joints connecting standards and ledgers are modeled as semi-rigid connections. A tri-linear model as shown in Fig. 2 is used to approximate the relation between the moment and rotation of the cuplok joints. Initial geometric imperfections and load eccentricity were measured and the actual values were used in the analytical model. Based on the test set-up, the top of the scaffold frame is assumed to be restrained in the translational horizontal directions and a rotational spring with stiffness $k_{rt} = 40$ kNm/rad is applied to model the boundary condition between the U-head and the bearer. The value of $k_{rt}$ was determined through parametric study. The base plate connection is modeled by a pinned connection. A contact element is used to model the reaction eccentricity. The details of the full scale experimental tests and the advanced analysis can be found in CASE [10], Chandrangsu and Rasmussen [12]. The ratios of test capacity ($R_T$) and analytical prediction ($R_S$) for the fifteen frames are shown in Fig. 3. The mean of $R_T / R_S$ was found to be 1.0 with a COV (coefficient of variation) of 0.1, suggesting that the advanced nonlinear analysis used in this study is an “unbiased” and good predictor of the “true” strength of scaffold systems. Therefore the model uncertainty is assumed to be a lognormal with a mean of 1.0 and COV of 0.1.

![Figure 2: Tri-linear model for moment-rotation relationship for cuplok joints](image1)

![Figure 3: Test Capacity/Prediction of fifteen steel scaffold frames.](image2)
STATISTICS FOR THE STRENGTH OF STEEL SCAFFOLD SYSTEMS

In construction practice, members of steel scaffolding systems such as standards and cuplok joints are reused from one job to another and new members are mixed with old ones. Therefore steel scaffold systems are characterized by inherent high variations of geometric and material properties, notably joint stiffness, initial geometric imperfections, yield stress and load eccentricity. Those basic variables are modeled as random variables in this work. The statistics for the (static) yield stress is obtained from the literature (Galambos and Ravinda [20]). The mean yield stress is assumed to be $1.05 F_y$ with a COV of 0.1. Here $F_y$ represents the minimum specified yield stress for the grade of steel. The statistics data for the remaining random variables were acquired through field measurements and laboratory tests by researchers at the University of Sydney. Table 1 gives the statistical data for the joint stiffness in different joint configurations, i.e., the 4-way, 3-way and 2-way configurations, reflecting the number of ledgers connected at the joint. Table 2 presents the statistics for the random geometric properties, i.e., the out-of-straightness of the standards, out-of-plumb and load eccentricity which occurs at the top of scaffolds.

**TABLE 1**

<table>
<thead>
<tr>
<th>Joint Configuration</th>
<th>$K_1$ (kNm/rad) mean</th>
<th>COV</th>
<th>$k_2$ (kNm/rad) mean</th>
<th>COV</th>
<th>$k_3$ (kNm/rad) mean</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Way</td>
<td>38.6</td>
<td>0.22</td>
<td>102.4</td>
<td>0.18</td>
<td>5.3</td>
<td>0.30</td>
</tr>
<tr>
<td>3-Way</td>
<td>36.1</td>
<td>0.38</td>
<td>86.5</td>
<td>0.21</td>
<td>5.1</td>
<td>0.37</td>
</tr>
<tr>
<td>2-Way</td>
<td>40.9</td>
<td>0.35</td>
<td>77.6</td>
<td>0.20</td>
<td>4.6</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Distribution: normal distribution

**TABLE 2. STATISTICAL DATA FOR GEOMETRIC PARAMETERS**

<table>
<thead>
<tr>
<th>random variable</th>
<th>mean</th>
<th>nominal</th>
<th>COV</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>out-of-straightness</td>
<td>$L / 770$</td>
<td>$L / 1000^*$</td>
<td>0.62</td>
<td>lognormal</td>
</tr>
<tr>
<td>out-of-plumb</td>
<td>$L / 600$</td>
<td>$L / 500^*$</td>
<td>0.31</td>
<td>normal</td>
</tr>
<tr>
<td>load eccentricity</td>
<td>b/4 mm</td>
<td>b/4 mm**</td>
<td>0.59</td>
<td>lognormal</td>
</tr>
</tbody>
</table>

* Per AS4100 [9]. ** b: width of the bearer. Per AS3610 [15]

Two typical steel scaffolds were considered. The systems are 3 story 1 bay $\times$ 1 bay and 3 story 3 bay $\times$ 3 bay, designated as 3ST1B and 3ST3B, respectively. Fig. 4 shows the geometry of the frames. The jack extension at the bottom and top of the frames are equal. Three values of jack extensions are considered, i.e., 100 mm, 300 mm and 600 mm, covering the possible range encountered in construction practice. For each system, the nominal system strength was first evaluated based on the nominal material properties and the nominal (code specified) geometric properties. The nominal system strength is used in the practical design. For joint stiffness the nominal value is taken as its mean. The nominal values for initial geometric imperfections and load eccentricity are specified in design codes and presented in Table 2. The stress-strain relation is assumed to be elastic-plastic. The worst direction of load eccentricities is assumed, namely all load eccentricities are towards the same direction. This position of load eccentricities will cause the maximum instability effects.
Monte Carlo simulations were conducted to generate sample distributions of the system strength. 3,000 advanced analyses were performed for each system with randomly generated values for joint stiffness, initial geometric imperfections, yield stress and load eccentricity. It is assumed that no correlation exists between standard-to-standard, standard-to-joint, and joint-to-joint. The load eccentricity is assumed to be randomly positioned towards either side of each standard. The direction and magnitude of load eccentricities of all columns are assumed uncorrelated.

Figure 4: Models of steel scaffold systems.

The nominal \( R_n \), mean \( R_m \), and COV \( \nu_R \) for the system strength are summarized in Table 3. The mean strengths for the two systems with same jack extensions are similar. In all cases, the variabilities in the system strength are smaller than the variabilities in individual member properties such as the joint stiffness. This is a result of stochastic averaging of uncorrelated member properties. Table 3 shows that \( \nu_R \) for 3ST3B is noticeably smaller than those of 3ST1B. The reduction in variability of system strength with increasing system scale is to be expected. This is due to structural redundancy and stochastic averaging of uncorrelated member properties. From Table 3 it is evident that the jack extension has a substantial influence on the system strength. Using a higher jack extension results in a lower system strength. Moreover, \( \nu_R \) decreases as the jack extension increases. The discrepancy is especially pronounced when the jack increases from 300 mm to 600 mm. The mean-to-nominal value \( R_m/R_n \) is also dependant on the jack extension. In both systems, \( R_m/R_n \) is approximately 1.0 for 600 mm jack, and increases to about 1.8 for 100 mm jack. The discrepancy is mainly due to different modes of failure. While the failure of the systems with high jack extensions (600 mm) is characterized by large overall sway of the frame, the mode of failure of systems with short jack (100 mm) exhibits S-shape flexural buckling in the column members. The statistics for the “true” strength of the scaffolds can be obtained by combining the simulation results with the model uncertainty. Each simulated system strength is multiplied by a randomly generated value of model uncertainty which is a lognormal with a mean of 1.0 and COV of 0.1 as discussed in previous section. The total variability of the system strength, \( \nu_{RT} \) is listed in the last column of Table 3. It is evident that due to the high variability of the model uncertainty as compared with the variability of simulated system strength, the total uncertainty in the system strength mainly reflects the contribution of the model uncertainty. The resistance statistics for the 2 systems becomes remarkably similar after incorporating the model uncertainty. Therefore the same resistance statistics are used for the 2 systems. The following representative values will be used in the reliability analysis:
For the design formula \( \varphi R_n = 1.2D_n + 1.6L_n \), the system reliability index \( \beta \) corresponding to \( \varphi = 0.75, 0.85 \) and 0.9 is calculated using the FORM and plotted as a function of \( L_n/D_n \) in Fig. 5 for 100 mm and 600 mm jack extension. It can be seen that \( \beta \) reaches maximum when \( L_n/D_n \) is around 0.12, then \( \beta \) decreases as \( L_n/D_n \) increases. The nominal live-to-dead load ratio (\( L_n/D_n \)) for reinforced concrete structures is typically in the range of 0.5 – 1.5 (Ellingwood and Galambos [6]). For the design of scaffold system, the most critical stage of construction is usually during pouring of concrete. In the survey by Hadipriono and Wang [2], it was found that over 74% of the collapse of scaffolding occurred during concrete pouring operations. In this stage, the construction live load mainly includes the weight of workers and small equipments, together with an allowance for localized mounding during concrete
placing, $L_n/D_n$ is typically less than 1.0 during placing of concrete. The representative values for $\beta$ when $L_n/D_n$ is 1.0 are presented in Table 4. For example, in the case of 300 mm jack, $\beta$ takes the value of 1.8 and 2.4 for $\varphi = 0.9$ and $\varphi = 0.75$, respectively. As expected, systems with 100 mm jack have the highest reliability due to their high value of $R_m/R_n$. This result suggests that the resistance factor may be dependent on the jack extension to achieve a consistent level of safety.

If a target system reliability index can be selected, the system resistance factor can be readily determined. In the current LRFD specification, the target reliability indices for different members are in the range of about 2.5 – 3.0 (Ellingwood and Galambos [6]). The reliability of redundant structural systems would be generally higher than the reliability of individual members. The difference, however, is unknown and depends on the type of structural system (Ellingwood [1]). The selection of target system reliability index should also consider the failure causes and modes, as well as failure consequences. Compared with buildings and bridges, the consequence of collapse of a scaffold system may be seemed as less severe. Moreover, because of its temporary nature, it is not practical and economical to design a scaffold system using the same reliability index for a building with a service life of 50 years. Further work is needed to develop acceptable safety criteria for steel scaffold systems.

### Table 4

<table>
<thead>
<tr>
<th>Jack extension</th>
<th>$\varphi = 0.9$</th>
<th>$\varphi = 0.85$</th>
<th>$\varphi = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 mm</td>
<td>2.2</td>
<td>2.3</td>
<td>2.7</td>
</tr>
<tr>
<td>300 mm</td>
<td>1.8</td>
<td>2.0</td>
<td>2.4</td>
</tr>
<tr>
<td>600 mm</td>
<td>1.7</td>
<td>1.9</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Figure 5: System reliability index: (a) 100 mm jack extension; (b) 600 mm jack extensions

**CONCLUSION**

Advances in computation and system analysis make it possible to design steel scaffold systems as a system rather than a collection of members. An LRFD type of check formula, however, is not as yet available for direct design on the system level. The challenges in developing such a formula include developing the statistical data for system resistance and construction loads, and selecting an appropriate target system reliability index. In this paper, advanced analysis was used to predict the ultimate strength of steel support scaffold systems. A series of 3-story 1 bay × 1 bay and 3-story 3 bay × 3 bay steel support scaffold systems...
were considered. The random member properties and the model uncertainty were propagated through Monte Carlo simulations to obtain the system’s resistance statistics. It was found that the system resistance statistics is dependent on the jack extension. The model uncertainty is a main contributor to the overall variabilities in the (predicted) system strength. The reliability of the scaffold systems designed with advanced analysis was estimated using the FORM. For example, in the case of $\phi = 0.85$, $\beta$ is 2.3 and 1.9, respectively, for the systems with 100 mm and 600 mm jack extensions. To achieve a consistent reliability the system resistance factor $\phi$ may be dependent on the jack extensions.

The work presented in this paper is part of an ongoing effort to develop a system reliability based design method with advanced analysis for steel scaffold systems. Further work will include establishing an appropriate target reliability for steel scaffold systems.

ACKNOWLEDGEMENTS

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REFERENCE


GEOMETRIC IMPERFECTION MEASUREMENTS AND JOINT STIFFNESS OF SUPPORT SCAFFOLD SYSTEMS

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KEYWORDS
Geometric imperfections, Out-of-straightness, Out-of-plumb, Loading eccentricity, Joint tests, Joint stiffness, Support scaffold systems, Falsework

ABSTRACT
The paper describes the findings from various site measurements of geometric imperfections of support scaffold systems, also known as falsework in industry. The measurements consist of out-of-straightness of the standards (uprights), out-of-plumb of the frame and loading eccentricity between the timber bearer and the U-head. A special-made tool instrumented with a dial gauge was used to measure the out-of-straightness of standards at the mid-height of each lift. A theodolite was employed to measure the angle difference between top and bottom of the frame in order to compute storey out-of-plumb, and a vernier calliper was used to measure the loading eccentricity at the top. The measurements were taken from different support scaffold construction sites before the pouring of concrete, representing actual initial imperfections and loading eccentricities encountered in practice. The paper also summarises the results of support scaffold joint tests. The tests were performed on randomly chosen used components to investigate the joint stiffness for rotations about vertical and horizontal axes. Tests were performed for various joint configurations, bending axes, and loading directions. The statistical analysis of the data is presented in the paper for practical application in modelling.

GEOMETRIC IMPERFECTION AND LOADING ECCENTRICITY MEASUREMENTS
Support scaffold systems are used to temporarily support heavy loads such as the weight of fresh concrete, formwork, rebar, equipment and workers. Due to heavy loads and geometric imperfections of the systems, the strength and behaviour of support scaffolds are susceptible to adverse 2nd order member P-δ and frame P-Δ effects. To be able to consider the effects of
geometric imperfections and loading eccentricity of scaffold systems, actual data is needed. At present, no available data is published for the geometric imperfections and loading eccentricity of scaffold systems. This paper presents the methods and results of the procurement of geometric imperfection data including measurements of the loading eccentricity between the timber bearer and the U-head for Cuplok support scaffold systems. A statistical analysis on the data is also presented.

**Method of procurement**

A special-made device instrumented with a dial gauge was used to measure the out-of-straightness (crookedness) of the standards (uprights) at the mid-height of each lift in two perpendicular directions aligned with the ledgers and referred to as the N-S and E-W directions. The device was made in two different lengths that fit into 1 m and 1.5 m lift heights of the scaffold systems for out-of-straightness measurement. The dial gauge attached to the device was calibrated with a perfectly flat surface so that it read directly the imperfection value once aligned with the standard. Figure 1(a) shows the devices used to measure the out-of-straightness at mid-height of the standard and Figure 1(b) exemplifies the measurement of the out-of-straightness of a 1 m lift standard on one of the sites. A theodolite was set up to measure the angle difference between the top and bottom of the frame in order to determine storey out-of-plumb. The horizontal distance between the standard (upright) and the theodolite was also measured together with the angle difference so that a simple trigonometric calculation could be carried out to calculate the storey out-of-plumb. Measurements were made for both the N-S and E-W axes of the construction plans. A vernier calliper was used to measure the loading eccentricity at the top between the centrelines of the timber bearer and the U-head. All measurements were taken before the pouring of concrete on to the formwork, representing actual initial geometric imperfections and loading eccentricity encountered in practice.

![Figure 1: (a) Device used to measure out-of-straightness (b) Actual measurement](image)

**Survey results of geometric imperfections and loading eccentricity**

A total of 302 on-site measurements of out-of-straightness of the standard were taken and 80 measurements of storey out-of-plumb were acquired. In addition, 74 measurements of loading eccentricity were obtained from various construction sites. From data observation, it was found that the directions (axes) of these geometric imperfections were random. Also the
directions of the loading eccentricity were shown to be random and occurred on either side perpendicular to the bearer. The results of out-of-straightness of the standards were normalised with the lift height and the results of storey out-of-plumb were normalised with the storey height of the scaffold systems measuring from the base plate up to the U-head. Figures 2-4 show the histograms of normalised out-of-straightness of the standards ($\delta/L_h$ where $\delta$ is the deflection at mid-height of the lift and $L_h$ is the lift height), normalised storey out-of-plumb ($\Delta/H$ where $\Delta$ is the sway and $H$ is the height of the scaffold systems) and loading eccentricity, respectively. The complete set of survey data is available in [1].

Figure 2: Histogram of normalised out-of-straightness of the standards ($\delta/L_h$)

Figure 3: Histogram of normalised storey out-of-plumb ($\Delta/H$)

Figure 4: Histogram of loading eccentricity
The mean normalised out-of-straightness of the standards including standards with and without spigot joints is 0.00048 (L_h/2080) with standard deviation of 0.00042; however, it is computed that the mean normalised out-of-straightness of the standards with spigot joints is 0.0013 (L_h/770) with standard deviation of 0.0008 and the mean normalised out-of-straightness of the standards without spigot joints is 0.0004 (L_h/2500) with standard deviation of 0.0003. The mean normalised storey out-of-plumb is 0.0016 (H/625) with standard deviation of 0.0005 whereas the mean loading eccentricity is 18.1 mm with standard deviation of 10.7 mm. Ultimately, these uncertainties must be studied by statistical methods to find appropriate distributions so that probabilistic assessment of the strength of the support scaffold systems can be carried out. Table 1 shows fitted statistical functions for the normalised out-of-straightness of the standards (δ/L_h), the normalised storey out-of-plumb (Δ/H) and the loading eccentricity of the support scaffold systems. More information regarding the statistical analysis is available in [1].

TABLE 1

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Probability distribution</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised out-of-straightness of the</td>
<td>Lognormal</td>
<td>0.0004</td>
<td>0.0003</td>
</tr>
<tr>
<td>standards without spigot joints (δ/L_h)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normalised out-of-straightness of the</td>
<td>Lognormal</td>
<td>0.0013</td>
<td>0.0008</td>
</tr>
<tr>
<td>standards with spigot joints (δ/L_h)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normalised storey out-of-plumb (Δ/H)</td>
<td>Normal</td>
<td>0.0016</td>
<td>0.0005</td>
</tr>
<tr>
<td>Loading eccentricity</td>
<td>Lognormal</td>
<td>18.1 mm</td>
<td>10.7 mm</td>
</tr>
</tbody>
</table>

JOINT STIFFNESS OF SUPPORT SCAFFOLD SYSTEMS

Support scaffold systems are fast to erect and easy to disassemble with the well-known Cuplok joint providing flexibility and ease of construction. Cuplok components consist of a lower cup, which is welded on to the vertical (standard) at 500 mm intervals, and a sliding upper cup. The locking mechanism of Cuplok joints uses no nuts, bolts or wedges; instead the method of connecting the ledger (horizontal) to the standard (vertical) is by simply positioning the end blades of up to 4 ledgers at desired angles into the lower cup, moving the upper cup down and rotating it clockwise by hammer blows until it locks up against a locking bar to achieve a tight connection. Conversely, disassembling the Cuplok joint is done by applying hammer blows in the counter clockwise direction until the upper cup can be lifted up to allow the ledgers to be taken out. Cuplok joints can be assembled into 2-, 3- and 4-way connections at any angle providing versatility of application. Cuplok parts are made of steel, and depending on the manufacturer they are usually either galvanised or painted for corrosion resistance, durability, and better handling.

Little information on Cuplok joint tests is available in the literature; only Godley and Beale provide some details and results of the Cuplok joint tests performed in 1990 [2]. Due to the lack of available test data on Cuplok joints, this paper presents tests on Cuplok joints and
discusses the semi-rigid joint behaviour observed from the tests as well as the joint stiffness derived from the test data. Particular attention is paid to the nonlinear moment-rotation curve. The slope of this curve is a direct measure of the stiffness of the joint, which can be incorporated in the numerical modelling of Cuplok scaffold system. Furthermore, since the Cuplok joint tests exhibit considerable variability in joint stiffness, statistical analysis of joint test data has been carried out in preparation for numerical modelling.

Test setup and materials

The test rig was specifically designed and built for the Cuplok joint tests, consisting of a 310UC158 column mounted on to a strong floor and a pivoted U-shaped clamp made from three lengths of 150x50x5 RHS section, attached to the column by M20 bolts and a rotatable pin. The clamp was designed to be able to rotate via the pin so that the specimen can be tested in any loading direction. Two 50 mm thick plates were attached to the inside of the top and bottom tips of the U-shaped clamp to grip the test specimen. Each plate featured a hole slightly smaller than the nominal diameter of the standard and two M12 bolts for tightening the grip. A hydraulic jack, capable of producing up to 32-kN of load and attached with a load cell and a half-circular loading plate, was mounted on to the rail beams of the strong floor to apply loading. Five LVDTs, clamped on external posts, were used to read the displacement at different locations along the specimen. Flat and smooth plates were attached with pipe clamps to the specimen at those locations to ensure accurate readings of the LVDTs. Figure 5 shows a schematic of a typical Cuplok joint test setup.

![Figure 5: Schematic of typical Cuplok joint test setup](image_url)

The materials consisted of 12 Cuplok open ended 2.80 m standards and 27 Cuplok 1.83 m ledgers. The nominal tubular cross-section dimensions and yield stress of the Cuplok standards were 48 mm x 4 mm and 450 MPa respectively. Also, the Cuplok ledgers were of nominal tubular dimensions of 48 mm x 3.2 mm with a nominal yield stress of 350 MPa. These materials were cut into specific lengths to fit in the test rig. Each standard was cut into four usable Cuplok joint specimens with an approximate length of 500 mm and each ledger was cut into 2 usable ledger specimens with an end blade at one end. The lengths of the
ledger specimens were 300 mm for connection elements and 600 mm for loading elements (Figure 6).

**Test configurations**

The experiments were conducted to investigate the stiffness and strength of Cuplok joints in the vertical direction (rotation about the z-axis) and horizontal direction (rotation about the y-axis), see Figure 6. Types A, B and C, and D represent 4-way, 3-way, and 2-way connections (number of ledgers joining into the Cuplok) respectively. The loading directions consist of up or down for bending about the z-axis and left or right for bending about the y-axis. To be able to differentiate the test results, different labels for each test configuration are used, as shown in Table 2.

![Figure 6: Cuplok joint stiffness axes and typical 4-way (Type A) connection](image)

<table>
<thead>
<tr>
<th>Label</th>
<th>Bending axis</th>
<th>Joint type</th>
<th>Loading direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>KzA1</td>
<td>z</td>
<td>A (4-way)</td>
<td>1(down)</td>
</tr>
<tr>
<td>KzB1</td>
<td>z</td>
<td>B (3-way)</td>
<td>1(down)</td>
</tr>
<tr>
<td>KzC1</td>
<td>z</td>
<td>C (3-way)</td>
<td>1(down)</td>
</tr>
<tr>
<td>KzD1</td>
<td>z</td>
<td>D (2-way)</td>
<td>1(down)</td>
</tr>
<tr>
<td>KzA2</td>
<td>z</td>
<td>A (4-way)</td>
<td>2(up)</td>
</tr>
<tr>
<td>KzB2</td>
<td>z</td>
<td>B (3-way)</td>
<td>2(up)</td>
</tr>
<tr>
<td>KzC2</td>
<td>z</td>
<td>C (3-way)</td>
<td>2(up)</td>
</tr>
<tr>
<td>KzD2</td>
<td>z</td>
<td>D (2-way)</td>
<td>2(up)</td>
</tr>
<tr>
<td>KyA1</td>
<td>y</td>
<td>A (4-way)</td>
<td>1(right)</td>
</tr>
<tr>
<td>KyB1</td>
<td>y</td>
<td>B (3-way)</td>
<td>1(right)</td>
</tr>
<tr>
<td>KyC1</td>
<td>y</td>
<td>C (3-way)</td>
<td>1(right)</td>
</tr>
<tr>
<td>KyD1</td>
<td>y</td>
<td>D (2-way)</td>
<td>1(right)</td>
</tr>
<tr>
<td>KyA2</td>
<td>y</td>
<td>A (4-way)</td>
<td>2(left)</td>
</tr>
<tr>
<td>KyB2</td>
<td>y</td>
<td>B (3-way)</td>
<td>2(left)</td>
</tr>
<tr>
<td>KyC2</td>
<td>y</td>
<td>C (3-way)</td>
<td>2(left)</td>
</tr>
</tbody>
</table>

From the data recorded, the joint stiffness was investigated by determining relationship between moment and rotation. The moment, \( M \), is calculated as

\[
M = F \times L
\]  

where \( F \) is the applied load and \( L \) is the perpendicular distance between load application point and the centre of the joint. The rotation, \( \theta \), is given as
where $\Delta_2$, $\Delta_3$, $\Delta_4$, and $\Delta_5$ are the displacement of LVDT2, LVDT3, LVDT4 and LVDT5 respectively, $d_{2-3}$ is the distance between LVDT2 and LVDT3, $d_4$ is the perpendicular distance between LVDT4 and the centre of the joint, and $d_5$ is the perpendicular distance between LVDT5 and centre of the joint. The LVDT numbering system is shown in Figure 5. LVDT1 is used only for calibration.

**Test results**

A total of 172 joint tests were carried out. The Cuplok joint stiffness was determined from the initial slope of the moment-rotation curves [3], as exemplified in Figure 7. The tests show that approximately 30% of the joints exhibit looseness at the beginning of loading, notably among the joints loaded in vertical bending (rotation about the z-axis). A statistical analysis of the coordinates of the moment-rotation curves corresponding to the end of the initial looseness range for those joints exhibiting looseness shows that looseness is overcome when the moment reaches 0.4 kNm with a corresponding rotation of 0.01 rad, on average. The ultimate moment capacity of the joints bent about the z-axis is about 3.5 kNm, on average. In contrast, joints bent about the y-axis have a very small moment capacity of 0.4 kNm, on average, and much smaller joint stiffness than the joints bent about the z-axis. Most of the Cuplok joints have a large plastic range associated with large rotation, except a few of the joints that failed abruptly resulting from fracture at the end blade of the ledger. A statistical analysis is presented for each test configuration in Table 3. The complete set of experimental data is available in [3], which also shows that the joint stiffness follows a normal distribution.

![Figure 7: Typical moment-rotation curve for Cuplok joints bent about the z-axis](image)

<table>
<thead>
<tr>
<th>Initial Cuplok joint stiffness (kNm/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{zA1}$</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Coefficient of variation</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
</tbody>
</table>

**Initial Cuplok joint stiffness (kNm/rad)**

<table>
<thead>
<tr>
<th>$K_{yA1}$</th>
<th>$K_{yB1}$</th>
<th>$K_{yC1}$</th>
<th>$K_{yD1}$</th>
<th>$K_{yC2}$</th>
<th>$K_{yD2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>19.50</td>
<td>12.65</td>
<td>11.38</td>
<td>4.77</td>
<td>15.95</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>6.09</td>
<td>2.06</td>
<td>2.71</td>
<td>2.91</td>
<td>4.28</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>31%</td>
<td>16%</td>
<td>24%</td>
<td>61%</td>
<td>27%</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

In this paper, measurements of geometric imperfections and loading eccentricity of Cuplok support scaffold systems have been obtained from various construction sites in the Sydney area, and the results are presented as histograms and fitted statistical functions. The probability distributions for geometric imperfections and loading eccentricity are presented for the numerical modelling of support scaffold systems. The paper also presents a study on the joint stiffness of Cuplok scaffold systems. Using secondhand Cuplok components, Cuplok joints were assembled and tested in a specially designed rig in the structures laboratory of the School of Civil Engineering at the University of Sydney. The joint tests featured bending about two distinct axes, different loading directions, and four types of joint configuration. The statistical results of the joint stiffness for each test configuration were tabulated. The vertical bending stiffness was found to be much higher than the horizontal bending stiffness, and 4-way connections provided greater stiffness than other joint configurations. The joint stiffness data was fitted to normal distributions for probabilistic modelling. The results are useful for modelling and performing probabilistic analyses of the strength of Cuplok scaffold systems.

**ACKNOWLEDGEMENTS**

The authors would like to thank Boral Formwork & Scaffolding Pty Ltd for allowing site access as well as providing testing materials for the joint tests. The first author also wishes to thank Boral staff, Mr. Ranji Premaratne, and student colleagues who helped out on sites.

**REFERENCES**


FULL-SCALE TESTS AND ADVANCED STRUCTURAL ANALYSIS OF FORMWORK SUBASSEMBLIES

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KEYWORDS
Advanced analysis, Formwork subassemblies, Support scaffold systems, Steel scaffolds, Falsework, Subassembly tests, Structural models, Calibrations

ABSTRACT
In this paper, accurate three-dimensional advanced analysis models are developed to capture the behaviour of support scaffold systems, as observed in full-scale subassembly tests consisting of three-by-three bay scaffold systems with combinations of various lift heights, number of lifts and jack extensions. The paper proposes methods for modelling spigot joints, semi-rigid upright-to-beam connections and base plate eccentricities. Material nonlinearity is taken into account in the models based on the Ramberg-Osgood expression fitted to available experimental data. Actual initial geometric imperfections including member out-of-straightness and storey out-of-plumb are also incorporated in the models. The ultimate loads from the nonlinear analyses were calibrated against failure loads and load-deflection responses obtained from full-scale subassembly tests. The numerical results show very good agreement with tests, indicating that it is possible to accurately predict the behaviour and strength of highly complex support scaffold systems using material and geometric nonlinear analysis. The paper forms part of the ongoing development of a design methodology for support scaffold systems based on advanced analysis currently undertaken at the University of Sydney.

INTRODUCTION
Support scaffolds normally consist of standards (vertical members), ledgers (horizontal members), and braces. The scaffold standards are connected to each other to create a lift via couplers, also known as spigot joints (Figure 1). In order to connect ledgers to standards, wedge-type or Cuplok joints (Figure 2) are usually preferred for the connection because no bolting or welding is required; though, in some systems manually adjusted pin-jointed couplers are still being used. The connections for diagonal brace members are usually made of hooks for easy assembling. The base of scaffolds consists of adjustable threaded jacks, which can be extended up to typically 600 mm by a wing nut to accommodate irregularity of the ground. The top of scaffolds consists of adjustable threaded jacks with U-heads which support timber bearers and ensure the levelling of the formwork.
FULL-SCALE SUBASSEMBLY TESTS

Test setup and procedures

A total of 18 support scaffold subassembly tests were conducted at the University of Sydney to study the behaviour and ultimate load-carrying capacities of such systems. In all tests, the formwork subassembly, also known as Cuplok scaffold system, was constructed as a grid frame of three-by-three bays with a constant nominal bay width of 1829 mm in both directions. All testing components were taken from stocks of used material representing those encountered in practice. Details of all components are available in the test report [1].

A test frame was constructed for the subassembly tests consisting of four loading beams at both the top and bottom running in the North-South direction. A total of sixteen hydraulic jacks attached to the top loading beams (four hydraulic jacks per loading beam) were used to load the timber bearers running in the East-West direction that applied loading to the top of each standard via U-heads. In all tests except Test no. 6, the loads were applied at an eccentricity of 25 mm in the North-South direction to the top adjustable jacks (Figure 3(a)) along the second row in the East-West direction while for the rest of the standards the loads were applied concentrically. In addition, the base plates in the row of eccentrically loaded standards were placed on 3 mm diameter circular steel rods at a nominal eccentricity of 15 mm, as shown in Figure 3(b). The load eccentricities applied at the top and bottom of the system were arranged such that the standards were bent in single curvature. The loads were applied equally by hydraulic jacks on each standard through primary bearers, except in Test No. 14 where the loads were applied to the corner, perimeter, and centre jacks in the ratio of 1:2:4 respectively. The results of Test No. 1 were discarded due to excessive lateral displacements of the loading jacks. The results of Test No. 7 were also discarded since the hydraulic jacks moved out of positions during the test [1].

Test configurations and results

A summary of the test configurations and results which includes test number, lift height, number of lifts, top and bottom jack extension length, position of spigot, bracing arrangement, type of loading,
loading eccentricity and ultimate load at three jack locations is presented in Table 1. As an example, Figure 4 shows the actual Test No. 8 setup consisting of a 3-lift, 1.5 m lift height and 300 mm jack extension subassembly test with full bracing arrangement [1].

FINITE ELEMENT MODELS

Three-dimensional finite element models have been developed for analysing support scaffold systems. The analyses include geometric and material nonlinearities and are performed using the commercial finite element software package Strand7 [2]. The models are compared with the tests [1] and calibrated against the ultimate loads and displacement responses.

Spigot joints

In the studied systems, the spigot joint consists of an insert made from a circular hollow steel tube with 38.2 mm outside diameter, 3.2 mm in thickness, and 300 mm in total length. The insert feeds into the abutting top and bottom standards to create a required lift height, as shown in Figure 1. The spigot modelling suggested by Enright et al. [3] is adopted, as shown in Figure 5.

The top and bottom standards are modelled as nonlinear beam element connected to the spigot via pinned connections. The spigot beam element is connected to the standards by three pinned stiff links capable of only transferring lateral forces from the standards to the spigot. The degree of bending of the spigot depends on the amount of initial geometric imperfection of the standard and vertical force. In three-dimensional analyses, the spigot model is arranged in the direction perpendicular to the primary bearers which is in the same direction as that of the load eccentricity. For simplicity, the spigot model is applied at mid height of the lift, even though the spigot is often located at little below or above mid height in actuality.
<table>
<thead>
<tr>
<th>Test</th>
<th>Lift height (m)</th>
<th>No. of lifts</th>
<th>Jack extension (mm)</th>
<th>Spigot (lifts)</th>
<th>Bracing</th>
<th>Loading</th>
<th>Eccentricity in 2nd row standards</th>
<th>Ultimate load at corner/perimeter/centre jacks (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.5</td>
<td>3</td>
<td>600</td>
<td>2nd &amp; 3rd</td>
<td>full</td>
<td>uniform</td>
<td>25 mm top &amp; 15 mm bottom</td>
<td>87 86 89</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>3</td>
<td>600</td>
<td>2nd &amp; 3rd</td>
<td>full</td>
<td>uniform</td>
<td>25 mm top &amp; 15 mm bottom</td>
<td>91 90 91</td>
</tr>
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<td>uniform</td>
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</tr>
<tr>
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<td>3</td>
<td>600</td>
<td>2nd &amp; 3rd</td>
<td>perimeter</td>
<td>uniform</td>
<td>25 mm top &amp; 15 mm bottom</td>
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</tr>
<tr>
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<td>600</td>
<td>2nd &amp; 3rd</td>
<td>perimeter</td>
<td>uniform</td>
<td>15 mm bottom</td>
<td>60 60 60</td>
</tr>
<tr>
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<td>1.5</td>
<td>3</td>
<td>300</td>
<td>2nd &amp; 3rd</td>
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<td>uniform</td>
<td>25 mm top &amp; 15 mm bottom</td>
<td>13 0 0</td>
</tr>
<tr>
<td>9</td>
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<td>3</td>
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<td>uniform</td>
<td>25 mm top &amp; 15 mm bottom</td>
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<tr>
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<td>3</td>
<td>300</td>
<td>2nd &amp; 3rd</td>
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<td>uniform</td>
<td>25 mm top &amp; 15 mm bottom</td>
<td>70 70 70</td>
</tr>
<tr>
<td>11</td>
<td>1.5</td>
<td>3</td>
<td>300</td>
<td>2nd &amp; 3rd</td>
<td>full</td>
<td>uniform</td>
<td>25 mm top &amp; 15 mm bottom</td>
<td>12 0 0</td>
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<td>uniform</td>
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<tr>
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<td>1</td>
<td>4</td>
<td>300</td>
<td>3rd</td>
<td>full</td>
<td>uniform</td>
<td>25 mm top &amp; 15 mm bottom</td>
<td>15 0 0</td>
</tr>
</tbody>
</table>

Figure 5: Schematic of spigot joint model
Semi-rigid standard-to-ledger connections

The relation between the moment and rotation of the Cuplok connections is modelled by a tri-linear curve, as illustrated in Figure 6. The parameters that describe the tri-linear curve \((k_1, k_2, k_3, \beta_1, \beta_2, \beta_3)\) were obtained from laboratory tests [4]. Three different joint configurations were tested in bending about vertical and horizontal axes (Figure 2), i.e. the 4-way, 3-way and 2-way configurations, reflecting the number of ledgers connected at the joint. The average values for \(k_1, k_2, k_3, \beta_1, \beta_2\) and \(\beta_3\) for different joint configurations and bending axes are presented in Table 2. The connection element in Strand7 [2] is used to model the relation between moment and rotation. A multi-linear moment-rotation table is specified for bending about vertical and horizontal axes; other degrees of freedom are assumed to be rigid.

**TABLE 2**

<table>
<thead>
<tr>
<th>Joint configuration</th>
<th>Bending about horizontal axis</th>
<th>Bending about vertical axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(k_1)</td>
<td>(\beta_1)</td>
</tr>
<tr>
<td>4-way</td>
<td>80</td>
<td>0.014</td>
</tr>
<tr>
<td>3-way</td>
<td>75</td>
<td>0.012</td>
</tr>
<tr>
<td>2-way</td>
<td>70</td>
<td>0.007</td>
</tr>
</tbody>
</table>

![Figure 6: Tri-linear moment-rotation for the Cuplok joints](image)

Brace connections

The braces are made of telescopic members with hooks at the ends. They are modelled using two rigidly connected elements with different cross-sections. Connection elements with only axial stiffness form the connection between the brace member elements and the ledgers. The axial spring stiffness is taken as 1.8 kN/mm as obtained from test calibrations of braced scaffold systems. The braces are offset 60 mm from the nodal points between the standards and ledgers, representing actual practice.

Base plate eccentricity

The placement of the base plate of scaffold systems on an uneven or sloped ground can create eccentricity. The base eccentricity model proposed is illustrated in Figure 7. The base eccentricity is labelled as “e” in the figure. The standard and the base plate are modelled using nonlinear beam elements with their corresponding cross-sectional and material properties. A contact element is used to model a gap between the base plate and the ground. The contact element is set to provide stiffness only in compression, and only when the nodes to which it is connected come into contact, that is when the gap closes. The stiffness is specified as “infinity,” implying that when the load transfers from the standard to the base plate causing the far end of the base plate to rotate and touch the ground, the point of contact becomes infinitely stiff representing solid ground or other hard surface.
A load eccentricity can occur between the timber bearer and the U-Head since the bearer is not always positioned such that its centre line coincides with the centre line of the jack. In order to model the eccentricity, a rigid link with length equal to the load eccentricity is applied to the top of the jack in the direction perpendicular to the bearer, and the vertical point load is applied at the far end of the link. The rigid link behaves as a short, stiff cantilever that introduces vertical force and additional moment into the jack.

**Geometric imperfections**

The magnitudes of the member out-of-straightness and the frame out-of-plumb are implemented directly at the nodes of the finite element models from acquired initial imperfection measurements taken as part of the subassembly tests [1]. The member out-of-straightness ($\delta$) is applied at mid height of the standard in each scaffold lift and the frame out-of-plumb ($\Delta$) is applied at each ledger-standard connection point and at the U-head at the top of the scaffold. Full details of the initial geometric imperfections of the subassembly tests are available in the test report [1].

**Material nonlinearities**

In material nonlinear analysis, the nonlinear relationship between stress and strain is applied. The stress-strain relations for the scaffold components used in the models are based on the Ramberg-Osgood expression [5] fitted to experimental data obtained from supplementary tests on components of the subassembly tests [1]. The Ramberg-Osgood parameters ($E_0$, $\sigma_{0.2}$, $n$) for each scaffold component are summarised in Table 3. The Ramberg-Osgood stress-strain relations are applied to the beam elements of each scaffold component by the method referred to as plastic-zone analysis [6].

<table>
<thead>
<tr>
<th>Component</th>
<th>$E_0$ (GPa)</th>
<th>$\sigma_{0.2}$ (MPa)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>200</td>
<td>530</td>
<td>38.2</td>
</tr>
<tr>
<td>Ledger</td>
<td>200</td>
<td>380</td>
<td>38.2</td>
</tr>
<tr>
<td>Jack</td>
<td>200</td>
<td>495</td>
<td>16.0</td>
</tr>
<tr>
<td>Base plate</td>
<td>200</td>
<td>260</td>
<td>25.0</td>
</tr>
<tr>
<td>Brace</td>
<td>200</td>
<td>430</td>
<td>38.2</td>
</tr>
<tr>
<td>Spigot</td>
<td>200</td>
<td>430</td>
<td>38.2</td>
</tr>
</tbody>
</table>

**Calibrations**

The commercial software package Strand7 [2] was used to create a finite element model for each of the full-scale subassembly tests using the actual frame dimensions and measured values of imperfections. The mean of the measured dimensions of components were used for cross-sectional properties in the finite element models. The ultimate loads and displacements obtained from the nonlinear analyses accounting for both material and geometric nonlinearities were calibrated against
failure loads and load-deflection responses obtained from the full-scale subassembly tests [1]. The calibrations were achieved by changing the stiffness of the elastic restraints applied at the U-head and base plate, as well as the axial spring stiffness of the brace connections; the latter was changed after the calibrations were performed on unbraced systems for the top and bottom rotational stiffness. Table 4 shows the results of the stiffness parameters obtained from the calibrations. In the table, K represents translational stiffness with subscript showing its direction, and R represents rotational stiffness with subscript showing the axis of bending according to Figure 8. Table 5 shows the calibration results for the failure loads and their statistics.

![Finite element model of Test No. 3 showing axes](image)

**Figure 8: Finite element model of Test No. 3 showing axes**

<table>
<thead>
<tr>
<th>TABLE 4</th>
<th>PARAMETRIC CALIBRATION RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bottom boundary conditions</strong></td>
<td></td>
</tr>
<tr>
<td>$K_x$ (kN/mm)</td>
<td>$K_y$ (kN/mm)</td>
</tr>
<tr>
<td>Rigid</td>
<td>Rigid</td>
</tr>
<tr>
<td><strong>Top boundary conditions</strong></td>
<td></td>
</tr>
<tr>
<td>$K_x$ (kN/mm)</td>
<td>$K_y$ (kN/mm)</td>
</tr>
<tr>
<td>Rigid</td>
<td>Rigid</td>
</tr>
<tr>
<td><strong>Brace end connections</strong></td>
<td></td>
</tr>
<tr>
<td>Axial stiffness (kN/mm)</td>
<td>1.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 5</th>
<th>LOAD CALIBRATION RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>Ultimate load from advanced analysis (kN)</td>
</tr>
<tr>
<td>2</td>
<td>96</td>
</tr>
<tr>
<td>3</td>
<td>91</td>
</tr>
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<td>16</td>
<td>100</td>
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<tr>
<td>18</td>
<td>147</td>
</tr>
<tr>
<td><strong>Average</strong></td>
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</tr>
<tr>
<td><strong>STD</strong></td>
<td></td>
</tr>
<tr>
<td><strong>COV</strong></td>
<td></td>
</tr>
</tbody>
</table>
DISCUSSION

The calibrations show that advanced analysis using geometric and material nonlinear finite element models gives very good predictions of the ultimate loads of the systems. Most of the predictions are within 10% of the actual failure loads. In fact, the average of the ratios between failure test load and predicted ultimate load is very close to 1 (1.014) with a relatively small COV of 0.0966. In addition, advanced analysis gives good results in predicting deformation responses of support scaffold systems. The finite element analysis results of the load-deflection responses fit the test results [1] reasonably closely with most of the values within 20% of one another [7]. Two distinct failure modes are observed from the advanced analysis, as one exhibiting an S-shape member buckle and the other a lateral frame buckle with large lateral displacements at the top story. These failure modes were also observed in the tests [1] suggesting that advanced analysis is capable of accurately predicting the behaviour of support scaffold systems.

CONCLUSIONS

In this paper, nonlinear finite element analysis models for support scaffold systems have been developed. Calibrations of these models to the full-scale subassembly tests [1] consisting of three-by-three bay formwork systems with the combinations of different numbers of lifts, jack extension, and lift height are achieved by adjusting the top and bottom boundary conditions as well as the brace connection stiffness. The ultimate loads obtained from advanced analysis are in close agreement with the failure loads of the tests; moreover, comparisons of load-deflection responses also show close agreement, demonstrating that advanced analysis is able to accurately predict the behaviour and strength of highly complex support scaffold systems.

ACKNOWLEDGEMENT

The authors would like to thank Boral Formwork & Scaffolding Pty Ltd for providing subassembly test data and support of this research project.

REFERENCES

WIND LOADS ON NETTED METAL ACCESS SCAFFOLDS

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KEYWORDS
Scaffold Structures, wind loads, non-linear analysis, computational fluid dynamics, porous media

ABSTRACT
Most of the previous research into scaffold structures has concentrated on the analysis procedures for the scaffolding with particular reference to non-linear force-displacement effects (both $P - \delta$ and $P - \Delta$) and the influence of the semi-rigid standard-ledger and standard-transom connections. The loads used in the analyses are often taken from codes of practice. However, recent research by Beale and Godley in 2006 showed that when scaffolds are sheeted or netted that the wind loads assumed from the codes can be of the same magnitude as the dead and imposed loads. These loads are derived from experiments conducted on permanent structures and make no allowance for any changes in load due to the proximity of the façade of the building to which the scaffold is attached or due to any wind effects on the inside of the sheet.

The authors have developed models of netted scaffolds using the Computational Fluid Dynamics package, “Fluent” treating the net as a porous media. From these models it can be seen that the pressure distributions on sheeted scaffolds exhibit high values at the corners of the scaffold with mean pressure coefficients on the scaffold of the order of 60% of the wind pressure coefficients acting on the walls of an unsheeted scaffold. For an impermeable sheeted scaffold the wind pressure coefficients on the scaffold are approximately 50% higher than those acting on an unsheeted structure.

INTRODUCTION
Tubular steel scaffolds are widely used in the building construction in the UK. They are light in weight, easy to maintain, install, and dismantle. They are mostly fabricated from hot rolled steel tubes. Scaffold structures are often slender and constructed of elements with semi-rigid connections and are prone to fail by buckling. A review of the extensive research into the analysis and design of scaffold structures has been given by Beale [1], reviewing research undertaken between 1970 and 2006 and citing 145 references. This review shows that analysis procedures are well developed and usually require second-order geometric and material non-linear properties to be considered.
However, scaffold failures regularly occur, often being attributed to wind action. Beale and Godley [2] analysed tube and fitting scaffold structures and showed that wind loads could be of the same order as dead and live imposed loads. A conference held in 1994 under the auspices of the UK Health and Safety Executive [3] at Buxton, UK on wind loads stated that wind loads at that time could only be determined through the use of wind tunnel procedures. A paper by Hoxey et al [4] pointed out that for scaffold structures the maximum wind pressure could arise for the end wall of a long scaffold by wind acting between $30^\circ$ – $40^\circ$ from the plane of the façade. Schanbel [5] reported on wind tunnel experiments on models of dimensions $0.6 \times 0.3 \times 0.6$ and $0.6 \times 0.6 \times 0.6$. The experiments determined the force coefficients on a cl added scaffold. Effects of permeability of the debris netting were included. This was shown to reduce the total force on the netting used by $20\%$ over that acting on an impermeable cladding.

Limited applications of Computational Fluid Dynamics (CFD) to the analyses of wind loads on structures have been reported. Huang et al [6] used the Large Eddy Simulation (LES) and Reynolds Averaged Navier-Stokes Equations (RANS) techniques to study wind effects on the Commonwealth Advisory Aeronautical Council Building in 2007 and Yue et al analysed the Integral Lift scaffolds [7] used in China and gave guidelines for safe practice. The authors in 2007 used the Fluent program to determine wind loads on bare-poles access scaffolds and have analysed sheeted scaffolds, comparing theoretical results with experimental wind-tunnel data obtained using an open circuit atmospheric boundary layer type wind tunnel at Red Consultants, Hong Kong, in collaboration with Hong Kong Polytechnic University [8, 9]. Figure 1 shows a photograph of the wind-tunnel model. Good agreement was obtained between the CFD simulation, the wind-tunnel experiments and experiments on an unsheeted full scale building erected at Silsoe in the UK. Full details of all the analyses undertaken in this research project are found in the thesis by Irtaza [10]. This paper extends the previous research on fully sheeted scaffolds into the analysis of pressures and flows on debris-netted scaffolds where the debris net is considered as a porous media.

**Figure 1: Wind-tunnel model of sheeted scaffold**

**POROUS MEDIA**

Porous media are modelled by the addition of a momentum source term to the standard fluid flow equations [11].

\[
S_i = -\left( \frac{\mu}{\alpha} v_i + C_2 \frac{1}{2} \rho v_{\text{mag}} v_i \right)
\]  

(1)

where $S_i$ is the source term for the $i^{\text{th}}$ momentum equation, $\mu$ the dynamic viscous velocity, $\alpha$ the permeability of the media, $v_i$ the velocity in the $i^{\text{th}}$ coordinate direction, $v_{\text{mag}}$ the magnitude of the velocity, $\rho$ the density of air and $C_2$ an inertial resistance factor. This momentum sink contributes to the pressure gradient in the porous cell, creating a pressure drop that is proportional to the fluid velocity (or velocity squared) in the cell. The first term on the right hand side of Eqn. 1 corresponds to the viscous loss occurred in passing through the media (for laminar flows this is Darcy’s law). The second term provides a correction due to the inertial loss, allowing the pressure drop to be specified as a function of the dynamic head. For debris nets the coefficient $C_2$ and the permeability $\alpha$ were determined from wind-tunnel experiments.
Two types of net (called Type A and Type B) were used here for the simulation as porous media and are shown in Figure 2. The Type A net is a commonly used debris net for cladding scaffold structures during construction, for the improvement of protection of both public and workforce from falling debris, and also to shield the workforce from weather in the United Kingdom. The Type B net was made by double folding the Type A net. The two nets were tested for drop in pressure versus velocity in the small wind-tunnel of the School of Technology, Oxford Brookes University of cross-section 305 mm × 305 mm. This data is required for CFD simulations but has not been published for netting before. This is a non-boundary-layer wind-tunnel, of the open-circuit type, constructed mainly in aluminium and supported by a tubular steel framework. The air enters the tunnel through a carefully shaped effuse, the entrance being covered by a protective screen. The working section is of Perspex, giving full visibility and the various models are supported from one of the side walls or by means of the three component balance, when provided. At the upstream end of the working section is a static tapping and a total head tube which may be traversed over the full height of the working section, whilst at the downstream end is a pitot static tube which may be similarly traversed.

![Type A net](image1.png)  ![Type B net](image2.png)

**Figure 2: Tested Debris nets**

After the working section a diffuser leads to the axial flow fan unit and the air velocity is controlled by means of a double butterfly valve on the fan outlet. The fan discharges by way of a silencer. The maximum air velocity is such that pressure differences of the order of 30 cm of water are developed and these may be read with suitable accuracy by the simple manometer provided.

The mean thicknesses of the nets were measured with the help of a digital micrometer screw gauge. Their average approximate thicknesses were measured to be 0.42 mm and 0.65 mm for Type A and B nets respectively. The thickness of the Type B net was not double that of Type A, because of the interlocking of the fibres between adjacent weaves. The experimental setup for the wind-tunnel test of the nets is shown in Figure 3 and the plots of free stream velocities versus pressure drops across the two nets are shown in Figure 4. In Figure 3 U/S and D/S are the pitot positions upstream and downstream of the net.

The regression equations relating pressure drop ∆p to velocity v are:

For the Type A Net:  

\[ \Delta p = 0.524v^2 + 1.082v \]  

(2)
and for the Type B Net \[ \Delta p = 1.238v^2 + 2.249v \] (3)

A simplified version of the momentum equation, relating the pressure drop to the source term, can be expressed as:

\[ \Delta p = -S_n \Delta n \] (4)

where \( \Delta n \) is the width of the region of porous media.

Hence, for Type A Net \[ 0.524 = C_2 \frac{1}{2} \rho \Delta n \] (5)

and for Type B Net \[ 1.238 = C_2 \frac{1}{2} \rho \Delta n \] (6)

With \( \rho = 1.225 \text{ kg/m}^3 \), and a porous media thickness \( \Delta n \) equal to 0.42 mm and 0.65 mm for Type A and Type B nets respectively, the inertial resistance factors are \( C_2 = 2037 \) for Type A and \( C_2 = 3110 \) for Type B. Likewise,

For Type A Net \[ 1.082 = \frac{\mu}{\alpha} \Delta n \] (7)

For Type B Net \[ 2.249 = \frac{\mu}{\alpha} \Delta n \] (8)

with \( \mu = 1.7894 \times 10^{-5} \text{ m}^2 \), the viscous inertial resistance factor (1/permeability) \( \alpha = 6.946 \times 10^{-9} \) and \( 5.172 \times 10^{-9} \) for Type A and Type B nets respectively.

THEORETICAL SIMULATION OF THE NET IN CFD

The thickness of the debris net is very small. It is difficult to simulate the same thickness in CFD because it will lead to the size of the mesh being very small. Keeping this in mind the thickness of the net for CFD simulation was increased from 0.42 mm for Type A Net and 0.65 mm for Type B Net to 4 mm for both nets.

The values of \( \alpha \) and \( C_2 \) were modified in proportion to the ratio of the actual thicknesses of the nets to those of the simulated nets developed for the CFD, so as to make the drop in pressure be the same for both the wind-tunnel experiments and the theoretical equations developed for the CFD. The value of \( \alpha \) was increased in proportion by \( 4/0.43 \) for the Type A Net and \( 4/0.65 \) for the Type B Net respectively. The value of the inertial resistance \( C_2 \) was decreased to maintain the same drop in pressure in proportion to \( 0.43/4 \) and \( 0.65/4 \) for Type A Net and Type B Net respectively.

The computational domain in Figure 5, covers 29 \( B \) (\( B \) is the outer dimension of the net clad scaffold) in the stream (X) direction (-6.5 < \( x/B < 22.5 \)) and 13 \( B \) in the lateral or normal (Y) direction (-6.5 < \( y/B < 6.5 \)), using the centre of the building as the origin of coordinates. For models with thinner membranes a simplification called a “porous jump” was used to model the velocity/pressure drop characteristics and was applied to the faces of the media. The unsteady RNG \( k-\varepsilon \) method was used for computations over a period of 4 seconds as it gives reasonable results on the windward and side faces; previous computations having shown that for this method suction on the leeward face was not as accurate and the values were lower. The time step was taken to be 0.001s and 4000 time steps performed. These were iterated to obtain the time averaged results for each time step. The porous jump boundary condition was used for the net in all four directions. The inlet velocity for the analysis was kept constant at 5 m/s. This generated approximately the same wind speeds and pressures that occur at 2/3 height in a 3D simulation; this height being approximately the stagnation height where
there is little vertical fluid flow and hence the flow is horizontal. A turbulence intensity of 15% and a length scale of 0.3 were kept constant for all the trials.

![Diagram of Computational domain and boundary conditions](image)

**Note:** 0, 1, 2 and 3 are the corners of the outside of the net, B is the width of the net

**Figure 5:** Computational domain and boundary conditions

The pressure coefficients on the outer and inner face of net clad scaffold surrounding a cubical building are given in Figures 6 and 7. Figure 8 gives the difference of pressure coefficients of the outer and inner face of the nets. Note that the distances are given using the notation in Figure 5 where distance ratio 0-1 is along the windward side, distance ratio 1-2 is along a side wall and distance ratio 2-3 is along the leeward side. Figure 9 gives the pressure coefficients on the walls of the building.

From the pressure coefficient differences between the inner and outer faces in Figure 8 it can be seen that the maximum pressure differences occur near to the corners on the windward side of the scaffold being 0.52 for the Type A net and 0.75 for the Type B net. The mean pressure coefficients on the windward side are 0.30 for net Type A and 0.54 for net Type B. Both these coefficients are less than the 0.6 used in the new Eurocode EN12811 [12]. However, these values are less than those reported by Schnabel [5] who found reductions of 0.2 in his wind tunnel experiments. This difference could be attributed to either a more dense net mesh which would have higher pressures or to the difficulties in reducing nets consistently with respect to model dimensions in wind-tunnels (the dimensions of the fibres at the wind-tunnel scale of 1:30 would be 0.01 mm diameter). Along the side walls both nets show suction along the first half of the wall followed by a zone of nearly zero pressure. The mean suction are 0.07 and 0.16 for Type A and Type B nets respectively. On the leeward side the mean suction are respectively 0.03 and 0.04. The values of suction on the sides and the leeward are well below those used in the current codes.

![Graph of Net outer face pressure coefficients](image)

![Graph of Net inner face pressure coefficients](image)

**Figure 6:** Net outer face pressure coefficients

**Figure 7:** Net inner face pressure coefficients
To see if these results were fully representative, a parametric study was undertaken with nets having permeabilities varying from $1.0 \times 10^{-6}$ m$^{-1}$ to $1.0 \times 10^{-10}$ m$^{-1}$ and having the inertial constant $C_2$ (inertial resistance constant) equal to zero. The inertial constant $C_2$ was also simulated with all other data kept the same as that used for the Type A and Type B nets. A permeability of $1.0 \times 10^{-10}$ m$^{-1}$ corresponds to a nearly impermeable sheet and a permeability of $1.0 \times 10^{-6}$ m$^{-1}$ corresponds to a very permeable sheet (almost non-existing). The resulting pressure coefficients are shown in Figures 10-12. Figure 13 gives the effects of different permeability on the pressure coefficients on the faces of the building. The pressure distribution for the net with permeability $1.0 \times 10^{-10}$ m$^{-1}$ corresponds to that found by the authors when analysing a completely impermeable sheet [9]. Similarly the pressure distribution for the very permeable net, $1.0 \times 10^{-6}$ m$^{-1}$, corresponded to the distribution around an unsheeted building. The CFD analyses for a building surrounded by an impermeable sheet and around a building with no scaffolding attached had good agreement with wind-tunnel experiments; hence showing that in the limits the permeable CFD model was able to agree with both extremes, giving confidence in its ability to predict pressure distributions. The mean value of the pressure difference on the impermeable sheeting on the windward face of the building is 1.46, slightly higher than the 1.3 used in the German code of practice [13].

Figure 8: Pressure coefficient differences and between inner faces

Figure 9: Pressure coefficients on walls for outer Type A and Type B nets

Figure 10: Pressure coefficients on the outer face

Figure 11: Pressure coefficients on the inner face

Figure 12: Pressure coefficient differences

Figure 13: Pressure Coefficients on the outer wall
A plan view showing the static pressure contours around the two nets is shown in Figures 15 and 16 and velocity contours in the stream (X) direction are shown in Figure 17 and 18.

DISCUSSION AND CONCLUSIONS

Whilst performing the wind-tunnel experiments on the nets, the nets stretched because at high wind speed the nets took a curved shape creating a problem in placing the pitot tubes. The permeability of Net Type B is possibly more representative of the permeability of actual nets used in construction industries because although it was stretched, the double folding means that it offers more resistance to wind. This would happen if the sheet was looser and overlapping. Figure 8 shows that on the windward face there is a change in the net pressure coefficient for Type A netting from about 0.15 to 0.48 and from 0.35 to 0.75 for the Type B netting. The average net pressure coefficient is approximately 0.3 for the Type A material and 0.6 for the Type B material. The drop in the pressure coefficient is directly proportional to the inverse of permeability.

Hence it can be concluded that when designing nets clad scaffolds, the wind load on scaffolds clad with Type B netting should be considered to be 60% of the total wind load on the covered area of the scaffold (which is defined as the product of the area of sheeting presented to the wind and the dynamic wind pressure). Note that these results are provisional and do not allow for a possible increase in wind pressure when a building is inclined at 30° to the wind direction as reported by Hoxey [4]. When the permeability decreases the inside pressure coefficient decreases and consequently becomes negative because of suction both on the inner face of the net and on the walls of the building.
Similar patterns of results have been found for simulated nets with permeabilities varying from $1.0 \times 10^{-6}$ to $1.0 \times 10^{-18}$ m$^2$ corresponding to sheeting with properties ranging from nearly impermeable to almost none existent.

REFERENCES

STRUCTURAL ANALYSIS AND MODELING OF SYSTEM SCAFFOLDS USED IN CONSTRUCTION

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KEYWORDS
Critical load, Load-carrying capacity, Scaffold, Structural analysis, System scaffold

ABSTRACT
This research focuses on the structural analysis and modelling of system scaffolds during construction. The study indicates that the critical load of a system scaffold unfastened by inner diagonal braces is close to that of a modular steel scaffold. The joint stiffness among vertical posts in the system scaffolds can be defined based on the comparison between analytical and experimental results. The critical loads of system scaffolds reduce while the scaffold stories increase. The inner diagonal braces evidently increase the critical loads of system scaffolds. The coupling joint positions among vertical posts are supposed to be kept away from the story-to-story joints to prevent a reduction in critical loads of system scaffolds. When the quantity of the extended vertical posts at the bottom of a system scaffold structure increases, the critical load of the system scaffold structure decreases. An example of a large Christmas tree setup by system scaffolds is analyzed in this study.

INTRODUCTION
The modular falsework is always used in the construction field. Traditional modular scaffolds have commonly used as a temporary structure. However, modular scaffolds use cross-braces to connect each modular scaffold unit. This installation causes modular scaffolds to have two axes, a strong axis and a weak axis. To avoid the weak axis in scaffolds, a new modular support material, “system scaffold,” has been introduced into the construction fields. In comparison with conventional modular scaffolds, the system scaffold has some superior features: no distinction between the strong axis and weak axis, freely-adjustable heights of the jack base, no need to use other shores in the headroom between the scaffold and the formwork, and fast and easy installation based on environmental conditions in construction sites. As designers lack verified design parameters for adequate bearing capacity, designers may generate incorrect data that would cause the collapse of scaffold structures. Figure 1 shows the collapsed system scaffolds in Taiwan.
Most research has focused on strength and failure of modular scaffolds. Godley and Beale [1] investigated the load-carrying capacities and failure behaviours of reduced scaffolding models. Yu [2] examined column curves based on experimental tests of modular scaffolds. Peng [3][4][5] investigated the bearing capacities and failure of various falsework including one-layer shoring, double-layer shoring and modular scaffolds. Weesner and Jones [6] explored experimental load-carrying capacities of system scaffolds. Their study focused on system scaffolds used when finishing a building façade. Many studies of modular scaffolds have conducted; however, the failure behaviours of modular scaffolds differ from those of system scaffolds. Due to the lack of analysis and design data for system scaffolds, identifying failure behaviours and bearing capacities of system scaffolds will contribute to the safety of these temporary structures in construction sites.

SETUP AND MATERIAL PROPERTIES

A system scaffold comprises three steel tubes, i.e., the vertical prop, the horizontal bar and diagonal brace. Figure 2 displays a joint connected with a specific coupling between two vertical props. A circular plate with predrilled holes is welded onto the vertical prop. Horizontal bars and diagonal braces are then inserted into these holes. A jack base 35 cm high in a system scaffold is setup on the ground. This jack base also connects horizontal bars inserted into the holes on the welded circular plate. Thus, the system scaffold is extended to become a large-scale structural system via vertical props, horizontal bars, diagonal braces and coupling joints.

The system scaffold material properties are cross-sectional area of steel tube \( A \), moment of area \( I \) and elastic modulus \( E \). Cross-sectional area \( A \) for the vertical prop is 3.982 cm\(^2\), moment of area \( I_y (=I_z) \) is 10.747 cm\(^4\), and elastic modulus \( E \) is 2012.4 kN/cm\(^2\) (2040 tonne/cm\(^2\)). For the horizontal bar and diagonal brace, both cross-sectional areas \( A \) are 3.794 cm\(^2\), moment of area \( I_y (=I_z) \) is 10.668 cm\(^4\), and elastic modulus \( E \) is 2012.4 kN/cm\(^2\).

TEST OF 2-STORY SYSTEM SCAFFOLD

The experimental test for the 2-story system scaffold generates necessary reference data for analysis. The diagonal braces are removed from the system scaffold structure to obtain the lower bound of the critical load. The tested critical load can be adopted as a reference for designing bearing capacity of a 2-story system scaffold structure in construction sites. In addition, exclusion of diagonal braces reduces joint stiffness of diagonal braces in the system scaffold, which is conducive to verifying the joint stiffness between vertical props in analysis.

Figure 3 presents the setup and dimensions of the 2-story system scaffold. This setup, except for diagonal braces, was based on real installations in construction sites in Taiwan. In accordance with the headroom of buildings during construction, an extra shoring length must be considered since the headroom is not just a multiple of system height (180 cm). In this study, four additional vertical props, each 60 cm high, are added to the 2-story system scaffold. Figure 4 shows the tested deformation shape of the system scaffold after loading. The system’s failure behavior differs from that of conventional modular scaffolds. The tested critical load of the 2-story system scaffold without diagonal braces was 131.9 kN, which is not superior to that of a 2-story simple door-type steel scaffold with jack bases.

This study calculates bearing capacities of system scaffolds based on 3D structural analysis. A second-order elastic semi-rigid-joint analysis is utilized in this study. To simulate initial imperfections within the system scaffold, the notional lateral forces applied on various system scaffolds are 0.1–0.5% of total vertical load. This study uses the computer program GMNAF developed by Chan [7].

ANALYTICAL RESULTS AND COMPARISONS

Boundary Conditions
Boundary conditions based on the system scaffold setup are of two types: fixed end and hinged end. Changing joint stiffness of the system scaffold alters the critical load. To simplify analyses, joint stiffness of vertical props and pedestals is assumed identical since these two positions have the same coupling. The top and bottom boundary conditions of the system scaffold are also identical because their base-plates are almost the same. Joint stiffness of horizontal bars is considered as a rigid joint.

Figure 5 shows the relationships between joint stiffness and critical load for the 2-story system scaffold without diagonal braces based on different boundary conditions. The trend of curves for the two boundary conditions is approximately linearly ascending as joint stiffness increases. Little difference and a parallel tendency between two curves indicate that effects on the system scaffold with different boundary conditions are minimal in Figure 5. The major reason for these minimal effects is good lateral supporting effects from the structure base and top with four horizontal bars comprising a rectangle adjacent to the boundary ends of the structure.

Joint Stiffness

The joints in the system scaffold are of three types: joints for vertical props, joints for horizontal bars, and joints for diagonal braces. The first joint type is connected by a specific curl-pin coupling two vertical props. The second joint type is a specific connection on a thick plate welded onto a vertical prop. A pin is inserted into the hole of the thick plate to connect a horizontal bar. This solid coupling connection makes joint stiffness relatively high in addition to the welded part. The third joint type has a hinged-joint mechanism linking to the second joint type. For the joint stiffness where vertical props connect, a semi-rigid joint is utilized to simulate the real setup. The joint stiffness of the horizontal bar is simulated as that of a rigid joint since its special coupling style significantly increases stiffness. Joint stiffness of the diagonal brace is assumed a hinged joint due to its special hinged mechanism.

Joint stiffness of vertical props can be derived according to comparison of test results and analyzed critical loads of the system scaffold. The critical load of the test result for the 2-story system scaffold is 131.9 kN. As the critical load is 131.9 kN in Figure 5, a horizontal line drawn from this point that crosses the curve can obtain the corresponding joint stiffness, which is 687 kN-cm/rad (70 tonne-cm/rad) for the fixed-end boundary condition, and 785 kN-cm/rad (80 tonne-cm/rad) for the hinged-end. Since Jack bases are seldom used in real construction sites, the boundary conditions at ends of this system scaffold are considered hinged-ends. Thus, joint stiffness of the vertical props is 785 kN-cm/rad.

Different Base Heights

The heights of the extended vertical props added at the bottom are 60 cm and 120 cm, respectively. The top and bottom boundary conditions are hinges; joint stiffness is 785 kN-cm/rad in the analyses. With different positions of vertical props located on the bottom, Figure 6 shows that the different base heights on the ground vary. Figure 6 shows the relationships between critical loads of the system scaffolds and different heights of extended vertical props at the bottom. As the quantity of extended vertical props increases, the critical load of the system scaffold decreases. As shown in Figure 6, the critical load for Case C1 with extended vertical props forming a diagonal line is higher than that of C2 with single-side vertical props. The critical loads of Case C1 and Case B are similar. By adding three and four extended vertical props to the bottom, as in Cases D and E, the critical loads of the system scaffolds decrease by 50%. Thus, these two configurations should be avoided.

Scaffold Height

The analysis of critical load is based on a 2-story system scaffold with its height extended to a 10-story structure. To simplify analyses, an integration of boundary conditions and joint stiffness values coincides with the test value of 131.9 kN, which is the data for analyses. The analytical models are two cases. Top and bottom boundary conditions are both fixed ends and joint stiffness is 687 kN-
cm/rad and the top and bottom boundary conditions are both hinged ends and joint stiffness is 785 kN-cm/rad.

Figure 7 shows the relationships between critical loads and different numbers of stories. The similarity between critical loads under the two analytical models indicates that both models can be taken as a basic model for theoretical analyses. The critical loads of the system scaffolds decreased as the number of stories increased; a fixed value is seen during a convergence of the curves in Figure 7. This characteristic is extremely similar to the column curve in stability analysis.

**Diagonal Brace Installations**

The analytical model is based on a 2-story system scaffold with four diagonal braces added to each story. The scaffold structures in the analysis range from 2 to 10 stories. Hinges are considered at the top and bottom boundary conditions, and the joint stiffness is 785 kN-cm/rad. Figure 8 illustrates three configurations for the system scaffolds with added diagonal braces. Figure 9 shows a comparison of critical loads of the system scaffolds with various diagonal braces. The critical loads are markedly increased with diagonal braces added to the structure. For the setup of diagonal braces, Case B has the highest critical load for these three cases as the setup of diagonal braces in Case B bears horizontal forces. Thus, the setup of Case B is recommended to use in construction sites.

**Joint Positions**

Figure 10 is a drawing of different joint positions in the system scaffold. As in Case A, the joint positions are on identical planes with a height of 240 cm from the joints on the bottom story and base height is 35 cm. In Case B1, two joints at symmetrical diagonal positions are lowered to a position 180 cm from joints on the bottom story. In Case B2, two joints located on the same side of the scaffold are lowered to 180 cm from joints on the bottom story. In Case C, four joints are on an identical plane 180 cm from the joints on the bottom story.

Figure 11 shows the relationships between critical loads and various joint positions without diagonal braces based on different stories of the system scaffolds. The critical load of the system scaffold in Case A is superior to that for other cases; notably, Case C is weakest. The configurations of joint positions in Cases B1 and B2 are combinations of configurations in Cases A and C. Thus, the critical loads for Cases B1 and B2 are between those for Cases A and C.

The evidence that joint positions in Case B1 are symmetrical may contribute to a better critical load than that for Case B2. Inefficiency in transmission of horizontal forces and worse critical load can occur when joints on vertical props are near the story-to-story joints. Since joint positions in Case C are adjacent to story-to-story joints (i.e., joints of horizontal bars), its critical load is low. For coupling of vertical props in the system scaffold, joint positions should be kept away from story-to-story joints.

**Real Examples**

A giant Christmas tree setup by 14-story system scaffolds was erected in the Encore Garden in Taiwan. The height of each story of the scaffold is 180 cm, and the height of the top story is 60 cm. The vertical load is from artificial leaves, ornaments, and pre-stressed steel cords on the Christmas tree. The horizontal load is from lateral wind forces. For this system scaffold, a joint stiffness 785 kN-cm/rad is selected for the second-order/elastic/semi-rigid joint analysis.

When the vertically uniform load is applied to the Christmas tree structure, the critical load of the system scaffold is 3982 kN. Figure 12 presents the failure shape of the system scaffold of the Christmas tree under vertically uniform loading.
Lateral wind force

Figure 13 shows four types of lateral wind loads: lateral load concentrated at the top, inverse triangle-shaped lateral load, uniform lateral load, and lateral load based on Code of Construction Technology for Taiwan. The concentrated load simulates the lower bound of bearing capacity and the uniform load simulates the upper bound of bearing capacity. The inverse triangle-shaped lateral load is located between the upper and lower bounds. The fourth type load is utilized for analyses using specifications for wind load in the Code of Construction Technology for Taiwan. Analytical results are as follows.

1. The concentrated load at the top results in a 43.2-kN failure load on the Christmas tree structure.
2. The inverse triangle-shaped lateral load generates a 239.4-kN failure load on the Christmas tree structure, which is roughly 5.5 times higher than the concentrated lateral load.
3. The uniform lateral load causes a 596.4-kN failure load on the Christmas tree structure, which is approximately 14 times higher than that for the concentrated lateral load.
4. According to specifications for wind loads in the Code of Construction Technology for Taiwan, this Christmas tree is positioned at a 150-class wind area. Wind pressure in this area is 110 kg/m² at heights < 9 m, 150 kg/m² for heights of 9–15 m, and 190 kg/m² for heights of 15–30 m. Based on this lateral load, the failure load for the Christmas tree structure is 262.9 kN, roughly 6 times higher than the concentrated lateral load and between the inverse triangle-shaped lateral load and uniform lateral load. After the load is applied to the structure, deformation-inducing failure is located on the top bulge in Figure 14.

Based on these analyses, bearing capacity under vertically uniform loading is extremely high, and actually vertical design loads should not destroy the scaffold structure. When lateral wind loads vary, failure loads for the tree structure differ. As the lateral load acting on the structure height increases, the failure load of the entire structure decreases. This analytical result can be used as a reference when designing a Christmas tree installed by system scaffolds.

CONCLUSIONS

The critical load of a system scaffold without diagonal braces is similar to that of a door-shaped steel tube scaffold with the same number of stories and jack bases. The four horizontal bars adjacent to structure boundaries may weaken the effect of hinged and fixed boundary conditions on critical load of system scaffolds. Joint stiffness between vertical props in system scaffolds is 785 kN-cm/rad (80 tonne-cm/rad) in this study. As the quantity of extended vertical props at the structure bottom increases, the critical load of the system scaffold decreases gradually. As the number of stories of a system scaffold increases, the critical loads of the structure decline. The diagonal braces substantially enhance the critical load of a system scaffold. Thus, we recommend that Case B—"the setup of the reversed parallel of story-to-story diagonal braces and parallel of story-to-story coplanar diagonal braces"—should be installed to markedly increase the bearing capacity of a system scaffold. The coupling joint positions between vertical props should be kept away from story-to-story joints (i.e., horizontal bar joints) to prevent the weakening of critical loads of system scaffolds. According to wind specifications in the Code of Construction Technology for Taiwan, the failure load of the large-sized Christmas tree installed using system scaffolds in the Encore Garden of Taichung, Taiwan, is 262.9 kN.

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Figure 1: Collapse of system scaffolds at construction sites in Taiwan

Figure 2: Setup and accessories of system scaffold
Figure 3: Test setup of 2-story system scaffold without diagonal braces

Figure 4: Deformation shape of 2-story system scaffold without diagonal braces

Figure 5: Critical loads vs. joint stiffnesses of 2-story system scaffold without diagonal braces
Figure 6: Comparison of critical loads of system scaffolds with different base heights

Figure 7: Critical loads of system scaffolds with various stories based on different JS & BC

Figure 8: Different bracing installations of 2-story system scaffold
Figure 9: Critical loads vs. No. of stories of system scaffolds with different installed diagonal braces

Figure 10: Different joint positions of system scaffolds used in construction sites

Figure 11: Critical loads vs. No. of stories of system scaffolds with different joint positions
Figure 12: Deformed shape of Christmas tree setup by system scaffolds under vertically uniform load

Figure 13: Different lateral wind forces applied on Christmas tree

Figure 14: Deformed shape of Christmas tree setup by system scaffolds under wind load by specifications
STABILITY DESIGN OF MIXED BAMBOO-STEEL SCAFFOLDING SYSTEMS

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KEYWORDS
Steel tube, Bamboo, Second-order analysis, Effective length, Buckling, Eurocode 3

ABSTRACT:
New scaffold for ms and m aterials are regularly developed in d i ferent parts of the world. Design codes like the Eurocode -3 and EN12811-3(2003) cover mainly the typical steel scaffolds that the design of unconventional new scaffold systems is d ificult. This paper proposes the use of stability and plastic–element collapse analysis of a new scaffold system made of steel and bamboo materials. The calculation shows the safety of the system when properly designed, fabricated and used.

INTRODUCTION
Extensive work has been carried out on design, buckling analysis and testing of steel scaffolding. Chu and Chan

Not many studies on design of bamboo scaffold systems have been carried out and verified by full scale experiments and simulation, due to the relatively large variation in the mechanical properties of bamboo compared to those of steel. Other factors such as differences in species, non-uniform cross sections and variation of moisture content make potential investigations even more complicated. The following is a summary of the studies carried out in Hong Kong on design of bamboo members used as structural elements.

Chan and Chung \cite{1} and Chung, Chan and Yu \cite{2} conducted full-scale testing and simulation using a nonlinear integrated design and analysis adopted by software NIDA \cite{3}. Chan et al. \cite{4} presented an empirical design for bamboo scaffolding that used a computational method, and
tested it on a double-layer scaffolding system with dimensions of 9 m high x 9 m long x 0.7 m wide for verification.

The double-layer Metal Bamboo Matrix Scaffolding System (MBMSS)

In this paper, the **Metal Bamboo Matrix Scaffolding System** (MBMSS) is selected for studies and design. MBMSS is a double layer scaffold that consists of steel pipes and bamboo poles for the main structure. It attempts to make use of the advantages of steel and bamboo materials in the aspects of strength, weight, cost, environmental sustainability and erection difficulty. The steel pipes are used as the main posts and ledgers to support the loading. This structure can provide a much more stable, secure and reliable scaffold for workers than a bamboo-only scaffold.

Bamboo poles are used in the MBMSS to reduce the loading of the scaffold and to increase its flexibility. The MBMSS provides flexibility and adaptability to various building structures with different requirements, resulting in enhanced productivity and efficiency in comparison to that offered by metal scaffolds.

The double layer MBMSS consists of an outer layer and inner layer. The outer layer provides suitable guard rails during the initial construction period. It is connected to inner layer of the scaffold to provide a work platform for workers.

The design of the double layer MBMSS is given in Figure 1. The main layer of the MBMSS has approximately the same design as the external layer. At the two ends are two vertical poles. To maintain the same load bearing strength, these poles have to be metal tubes of 48.6 mm in diameter and 2.3 mm in wall thickness. The two inner tubes are connected to each other by a metal tube ledger. The connection between the tubes is achieved using metal couplers, each of which bears a loading of 2 kN. The inner fold of the MBMSS is connected to the external fold by means of two pieces of bamboo transom of similar size in diameter and wall thickness at the two ends of the scaffold. The length of the transom will depend on the required width of the internal passageway.

As there is an external bracing layer, there is no need to erect a bracer for the internal layer. Some site managers or safety officers who do not understand the concept behind the design may insist on the installation of internal bracers, but that would increase the self load of the scaffold, and may hinder rendering workers at later stages of the construction project. Just like the internal guard rail for the inner layer scaffold, when the rendering workers find that these members are obstructing their work, they will dismantle them. To avoid infringing on safety regulations, the distance between the external façade scaffold and the reinforced concrete face should be no more than 150 mm. The scaffold is anchored to the reinforced concrete wall by a L-shape steel bar anchor specifically designed for this purpose. The whole set-up is illustrated in Figure 1. The solid circles in the figure represent bamboo and the open circles represent metal tubing. The spacing of the metal tubes and bamboo is optimised for the loading capacity and to ensure adequate safety.
The buckling strength and resistance of the system is determined using a plastic-element second-order analysis that the buckled member will be assumed to receive no increase in its load components namely as moments and axial force. The result shows the system is able to resist vertical permanent and imposed loads according to the light duty requirement. The computer model for the analysis is shown in Figure 2. In the design, no effective length has been assumed for finding the load resistance of compression members and the initial imperfection for steel follows that in the steel code and the imperfection for bamboo is taken as 7% which is obtained from curve fitting of buckling curves of bamboo struts.

CONCLUSIONS

A plastic-element second-order analysis is proposed. Imperfections at element and frame system levels are allowed in the system. The designed system was found to be adequate in resisting loads from workers and accessories under the catalogue of light to medium duty. It is recommended that new and unconventional scaffolding systems should be designed using the new method instead of the conventional linear analysis and effective length approach which carries uncertainty in the assumed value of effective length.
REFERENCES


SEISMIC BEHAVIOR OF STEEL REINFORCED CONCRETE COLUMN–STEEL TRUSS BEAM COMPOSITE JOINTS

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KEYWORDS

Steel reinforced concrete column, steel truss beam, composite joint, seismic behavior, experimental study, failure mode, analytical study, shear strength, finite element analysis

ABSTRACT

Experimental and theoretical studies were conducted for the seismic behavior of steel reinforced concrete (SRC) column-steel truss beam composite joints based on the prototype of New China Science & Technology Museum built at Beijing City. The seismic behavior of SRC column-steel truss beam composite joints was investigated through reversed cyclic loading tests on four specimens divided into two groups. The strength superposition method was used to evaluate the shear strength of SRC column-steel truss beam joints, and the applicability of the formulae in several current design codes was discussed. The comparison between the analytical predictions and the test results indicated that all the formulae discussed were conservative, and the formula proposed by ACI Code and AISC Specification could most precisely predict the shear strength of the joints. Furthermore, an analytical method for controlling the failure mode was proposed and verified by the experimental observations. Finally, a nonlinear numerical simulation of these joints using the commercial FE package MSC.MARC (2005r2) gave a satisfactory result.

INTRODUCTION

Various types of structural systems are often used simultaneously to meet the functional requirements of the large scale buildings. The design of the joints transforming one structural system to another is critical for the seismic behavior of the whole structure. Two types of structural systems are used in New China Science & Technology Museum at Beijing City: one is the large-span space steel truss system; and the other is the concrete frame-shear wall system. The seismic behavior of the transforming joint, i.e., the steel reinforced concrete
(SRC) column-steel truss beam joint, which is not covered in the current design codes [1-5], directly influences the seismic performance of the whole structure. In this paper, reversed cyclic loading tests of four specimens divided into two groups were carried out first to investigate the failure mode, energy dissipation capacity and seismic behavior of this new type of joints. The strength superposition method was used to evaluate the shear strength of SRC column-steel truss beam joints, and the applicability of the formulae in several current design codes [1-5] was discussed. An analytical method for controlling the failure mode of the joints was also proposed for a design practice. Finally, a nonlinear numerical study using the commercial FE package MSC.MARC (2005r2) was carried out to study more thoroughly the performance of SRC column-steel truss beam joints under cyclic loading conditions.

EXPERIMENTAL PROGRAM

Test Specimens

The prototype structure of the specimens is located at the 2nd floor of the west gate of New China Science & Technology Museum. Two representative joints (designated as SRCTJ1 and SRCTJ2, respectively) were selected for the 1/3 scale model tests. The heights of the truss beams were the same for all the specimens, but the height of the SRCTJ1’s column section was twice larger than the SRCTJ2’s. The SRCTJ1 and SRCTJ2 each have two specimens with different shear stud arrangements but same dimensions. Shear studs were arranged along the whole SRC columns for the specimens SRCTJ1-1 and SRCTJ2-1 whereas only in the joint regions for the specimens SRCTJ1-2 and SRCTJ2-2. Assuming that the positions of the inflection points were determined based on the results of the global structural analysis, the tee-shaped subassemblies along with boundary and loading conditions could simulate part of a structure subjected to an earthquake-induced moment. The dimensions of the specimens are shown in Figure 1. The mean yield strengths of 10mm, 12mm, and 14mm-thick steel plates are 341.3MPa, 339.6MPa, and 348.6MPa, respectively. The mean yield strengths of Φ8 (used as the column hoop reinforcements) and Φ16 (used as the column longitudinal reinforcements) steel bars are 323.1MPa and 372.3MPa, respectively. The cubic strengths of concrete are 42.8MPa for the SRCTJ1-1 and SRCTJ1-2, 44.1MPa for the SRCTJ2-1, and 46.8MPa for the SRCTJ2-2.

Test Setup and Procedure

The test setup scheme is shown in Figure 2. The SRC column was in the horizontal direction and the truss beam was in the vertical direction. The top end of the beam was connected to an actuator while the other end of the actuator was connected to the reaction wall. Four triangular supports were designed as lateral support devices to prevent the out-of-plane global buckling of the specimens. The column was anchored to the laboratory base at the inflection points. Horizontal loads were imposed using force-control scheme before the specimen yielded, and then using displacement-control scheme after the specimen yielded. Loads were repeated only once at each control point before the specimen yielded and twice after the specimen yielded.
Experimental Phenomenon

The shear failure was observed at the joint core of the SRCTJ1. The crushing of concrete occurred due to the shear force at the joint core, resulting in the loss of loading capacity (Figure 3). The flexural failure was observed at the beam of the SRCTJ2. A plastic hinge was formed at the beam near the beam-column surface, where the compressive steel flange underwent local buckling and the steel brace fractured near the weld heat-affected zone (Figure 4).

Experimental Results

The load-displacement hysteretic hoops of the specimens are shown in Figure 5. The hysteretic hoops of all the specimens are in a saturated shuttle type, and the curves of the SRCTJ2 demonstrate stronger loading capacity, higher stiffness and better energy dissipation capacity than those of the SRCTJ1, indicating that the strong column-weak beam joints which failed at the beam possess superior seismic behavior. The superposition of the curves of the two SRCTJ1 (or SRCTJ2) specimens shows that the composite action can be achieved through the natural cohesion between the steel and concrete for the SRC columns whether there exists the shear studs or not.
The $Q_j\gamma_j$ hysteretic hoops of the specimens are shown in Figure 6, where $Q_j$ and $\gamma_j$ are the shear force and the average shear deformation of the joint core, respectively. The curves of the SRCTJ1 are plentiful, stable, and energy consumptive, indicating the shear failure of the joint core. The curves of the SRCTJ2 demonstrate that the shear deformation developed insufficiently, as the failure mode of these joints was the flexural failure of the beam.

Figure 5: Load-displacement hysteretic hoops

Figure 6: $Q_j\gamma_j$ hysteretic hoops

The $E_n$-$n_h$, $E_c$-$n_h$, and $h_e$-$n_c$ curves of the specimens are indicated in Figure 7, where $E_n$ = the energy dissipated per hemicycles of the hysteretic loops, $n_h$ = the number of hemicycles, $E_c$ = the cumulative dissipated energy, $h_e$ = the equivalent damping ratio, and $n_c$ = the number of cycles. These curves show that the SRCTJ2 which failed at the beam under the bending moment possessed stronger energy dissipation capacity than the SRCTJ1 which failed at the joint core under the shear force.

Figure 7: Comparison of the energy dissipation capacity of the specimens

In the tests, the center point of the steel panel zone was equipped with a 3-axis rosette to measure the shear strain. Figure 8 gives the comparison between the shear force-average shear deformation curve and the shear force-center point shear strain curve of the specimen SRCTJ1-2. A good correlation of these two curves (which can also be observed for the specimen SRCTJ1-1) indicates that the shear deformation at the center point of the steel panel zone could represent the average shear deformation of the whole joint core and the steel web of the panel zone developed shear deformation plentifully and evenly when the shear failure of the joint core occurred.

ANYLITICAL STUDY

**Beam-Column Joint Shear Strength**

The beam-column joint shear strength can be evaluated using the strength superposition method [6]. Namely, the shear strength contributed by the concrete $V_c$, the hoop reinforcements $V_r$, and the steel $V_s$ are first calculated separately, and then the three strengths are superposed to obtain the joint strength $V_{ju}$. The formulae for calculating the shear strength of SRC column-solid web steel beam joints proposed by the current Japanese (AIJ Standard [1]), American (ACI Code [2] and AISC Specification [3]), and Chinese (JGJ Specification [4]...
and YB Specification [5]) design codes are used for reference, and their applicability for evaluating the joints investigated in this paper is discussed here. These formulae can be written in a unified form:

\[
V_{ju} = V_c + V_s = \phi \delta f_c^2 A_j + f_{yr} A_w \left( h_0 - a_s' \right) / s + \psi f_{yw} t_w h_w / \sqrt{3}
\]

(1)

where \( \phi \) = the coefficient reflecting the constraint applied on the concrete core by the steel flanges; \( \delta \) = the coefficient for the joint type; \( f_c \) = the compressive strength of concrete; \( a \) = the index based on the statistic approach; \( A_j \) = the effective area of the joint; \( f_{yr} \) = the yield strength of the hoop reinforcement; \( A_w \) = the total area of the hoop reinforcements at one section; \( h_0 \) = the effective height of the column; \( a_s' \) = the cover thickness of the compressive bars; \( s \) = the spacing of the hoop bars; \( \psi \) = the coefficient for the increased strength contributed by the steel flange and strain hardening of the web; \( f_{yw} \) = the yield strength of the steel web; \( t_w \) = the thickness of the steel web; and \( h_w \) = the height of the steel web. The coefficients \( \phi \) and \( \psi \) reflect the composite effect between the steel and concrete.

Table 1 gives the different values of \( A_j, \phi, \) and \( \psi \) proposed by different design codes. How much the composite effect is considered in different formulae can be clearly demonstrated. Table 2 shows the comparison between the analytical and test results, where \( V_{bu,Vj} \) represents the ultimate load at the loading point when the shear failure of the joint core occurs. In the tests, only the SRCTJ1 had the shear failure of the joint. The comparison of these two specimens shows that all the formulae proposed by the current design codes are conservative for predicting the shear strength of the joints discussed in this paper since the composite effect and the strength contribution of concrete is not fully taken account of by these formulae (Table 1), and the predictions of ACI Code [2] and AISC Specification [3] are the closest to the test results because the contributions of the steel flanges and strain hardening of the web are fully considered in the design formula, and the effective area of the joint is selected as the total cross-sectional area of the column, which is reasonable according to the test observations.

### Table 1

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
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<tr>
<td>( A_j )</td>
<td>0.5b_c(h_{j0}-a_s)</td>
<td>b_c h_c</td>
<td>b_c h_c</td>
<td>0.5b_c(h_{j0}-a_s)</td>
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<td>( \phi )</td>
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<td>( \psi )</td>
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</table>

Note: \( b_c \) = the width of the column; \( h_c \) = the height of the column; \( a_s \) = the cover thickness of the tensile bars; \( b_c \) = the width of the column steel flange; \( t_c \) = the thickness of the column steel flange; and \( d_b \) = the steel beam depth.

### Discussion on Failure Mode of Joints

In addition to the shear failure of the joints (represented as SJ in Table 2), the flexural failure of the beam (represented as FB in Table 2) was also observed in the tests. The last part has given the loading capacity of the joint when the shear failure of the joint core occurs. The loading capacity of the joint when the flexural failure of the beam occurs \( V_{bu,Mb} \) can also be obtained by assuming that the plasticity is fully developed at the weakest section of the beam near the beam-column surface, i.e., the section A-A in Figure 1. The actual loading capacity of the joint \( V_{bu} \) can be selected as the minimum value of \( V_{bu,Vj} \) and \( V_{bu,Mb} \), and the
corresponding failure mode is the actual failure mode the joint undergoes subjected to the earthquake action. This analytical method has been applied to the tested specimens as indicated in Table 2. The predicted failure modes agree with the experimental results, and the loading capacity of the joint can be predicted accurately and conservatively. We can conclude that the proposed analytical method can be used to control the failure mode of the joint effectively in order to avoid that the shear failure of the joint core occurs earlier than the flexural failure of the beam.

**TABLE 2**

**ANALYTICAL RESULTS OF LOADING CAPACITY ANALYSIS**

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<tr>
<th></th>
<th>$V_c$ (kN)</th>
<th>$V_r$ (kN)</th>
<th>$V_s$ (kN)</th>
<th>$V_{ju}$ (kN)</th>
<th>$Q_{bu,Vj}$ (kN)</th>
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Note: The superscript * is used for the experimental results and no superscript is used for the analytical results.

**NUMERICAL STUDY**

**Finite Element Model**

A numerical study was conducted using the commercial FE package MSC.MARC (2005r2). The concrete, steel plates and reinforcements are simulated using the No.7 3D-SOLID, No.75 3D-SHELL, and No.9 3D-TRUSS elements, respectively. The bond-slip effects at the steel-concrete and reinforcement-concrete interfaces are excluded in the current numerical model. All the material properties are specified with the elasto-plastic constitutive model. A tri-linear hardening model is used to define the uniaxial stress-strain relationship of the steel and reinforcement material. The Rüsch curve [7] is adopted for the compressive behavior of...
concrete. The initial yield point of concrete is defined as \(1/3f_c\). The smeared fixed-crack concrete model is adopted in the FE model. Figures 9 and 10 show the FE models of the tested specimens.

Verification and Discussion

Figures 11 and 12 compare the load-deflection hysteretic hoops and skeleton curves between the numerical and test results, respectively. Good correlations can be observed and the slightly overestimated loading capacity of the SRCTJ1 can be observed since a large area of concrete crushing and spalling at the joint core can not be accurately simulated using the current commercial FE package.

Figure 13 shows the numerical results of the equivalent cracking strains at the maximum positive displacement. A large area of cracking at and near the joint core is observed for the SRCTJ1 while only a slight cracking appears for the SRCTJ2. Figure 14 shows the numerical results of the equivalent plastic strains at the maximum positive displacement. The plasticity is developed at the steel panel zone for the SRCTJ1 while a plastic hinge is formed at the beam for the SRCTJ2. These numerical results correlate well with the test observations.

CONCLUSION

Experimental and theoretical studies were conducted for the seismic behavior of SRC column-steel truss beam composite joints based on the prototype of New China Science & Technology Museum built at Beijing City. The experimental program demonstrated that the
investigated joints possessed superior seismic behaviors. The joints having the flexural failure mode at the beam showed stronger loading capacity, higher stiffness and better energy dissipation capacity than those having the shear failure mode at the joint core. The formulae proposed by the current design codes [1-5] are conservative for evaluating the shear strength since the composite effect is not fully taken account of, and the predictions of ACI Code [2] and AISC Specification [3] are the closest to the test results because the effective area of the joint is selected as the total cross-sectional area of the column and the contributions of the steel flanges and strain hardening of the web are fully considered. An analytical method was proposed to effectively control the failure mode of the joint. A nonlinear numerical study using the commercial FE package MSC.MARC (2005r2) was conducted to study more thoroughly the performance of SRC column-steel truss beam joints under cyclic loading conditions. A good correlation between the numerical and test results was observed and the complicated behavior of the joints could be studied intensively through the proposed numerical simulation.

ACKNOWLEDGEMENT

The writers gratefully acknowledge the financial support provided by the National Science Fund of China (#90815006), Changjiang Scholars and Innovative Research Team in University (IRT00736).

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EFFECT OF BI-DIRECTIONAL CYCLIC LOADING ON SEISMIC CAPACITY AND BUCKLING BEHAVIOR OF THIN-WALLED CIRCULAR STEEL BRIDGE PIERS

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KEYWORDS

bi-directional cyclic loading, circular steel pier, seismic capacity, ductility, buckling behavior, numerical analysis

ABSTRACT

This study is concerned with ductility of circular steel bridge piers under bi-directional cyclic loading. The numerical analyses are carried out using finite element models with implementing modified two surface material constitutive law (developed at Nagoya University) to predict precise cyclic behavior of the pier. The parametric study is conducted to observe the effects of radius-thickness ratio, slenderness ratio and axial load ratio on ductility of the pier and simultaneously statistical formulas are derived. Based on these ultimate ductility formulas, the seismic verification method is proposed for the steel piers subjected to uni and bi-directional earthquake motions. The effect of uni and bi-directional loading on buckling behavior of the pier is discussed at the end.

INTRODUCTION

In the urban area of Japan where land prices remain high, the thin-walled steel bridge piers have been designed and constructed commonly due to their smaller section size and rapid erection. The seismic design of these piers has great importance as they directly affect transportation in post earthquake period. Although, the severe damage observed in the steel piers during 1995 Hyogoken-Nanbu earthquake (Bruneau [1]) pointed out the drawbacks of former design specifications, hence the recently revised Japanese seismic design code (Japan Road Association [2]) introduced new performance-based design concept in which failure of pier is controlled by keeping seismic response of pier within the allowable limits. However this concept follows the conventional seismic verification method which confirms response of structure in individual longitudinal and transverse direction, whereas earthquake motion in
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fact generates waves in three dimensions and their coupling results into complex seismic response of the structure (Okazaki [3]). Therefore it is needed to investigate the ultimate capacity of steel bridge piers under bi-directional loading which is closer to the coupled seismic waves.

During past few years, research concerned with effect of bi-directional loading on reinforced concrete columns has been carried out experimentally (Qie [4], Takizawa [5]) whereas ultimate capacity of steel pipes and box columns subjected to uni-directional loading were tried to evaluate analytically and confirmed against experimental results by Gao [6] and Usami [7]. Since it has proved that bi-directional cyclic loading reduces strength of column significantly than uni-directional loading, the researchers started focusing on this subject by performing experimental and numerical analysis (Watanabe [8], Goto [9],Obata [10]) and predicted capacity of steel columns. As the uni-directional seismic response of steel pier is easier to check against allowable ductility however, the bi-directional earthquake displacement response of pier top has found to be complex and extended into two orthogonal directions (Okazaki [3]). In this case the seismic verification method has not been understandable enough though the allowable ductility formulas are predicted in previous studies for circular as well as box steel piers. Hence the objective of this study is to propose a bi-directional ductility based seismic verification method for circular steel bridge piers.

In this work, the finite element (FE) model of steel circular piers are prepared by using shell elements with employing modified two surface constitutive law (2SM) (developed at Nagoya University, Shen [11], Mambahani [12]) for material no-linearity in cyclic loading and applying uni and bi-directional displacement to the top of the piers. Further by performing parametric study the formulas are proposed for ductility. Based on these formulas attempts are made to generate a seismic verification procedure which confirms uni as well as bi-directional earthquake response of the pier. In the last section the effect of loading on the buckling behavior in the bottom part of the pier is discussed.

NUMERICAL ANALYTICAL METHOD

Analytical Models and Loading Pattern

The Figure 1 shows the model of hollow steel circular steel bridge pier having uniform cross section all over the height and intermediate diaphragms, which is considered for the numerical analysis. For solving the finite element problem of the pier, the ABAQUS [13] program is used. From the evidences of past few studies it is accepted that the hollow steel piers buckle near the base part hence in FE model the bottom part is made up of four node shell elements S4R and upper part where bending moments are negligible is with beam element. The shell element part is divided into three sections as shown in the Figure 1a. All sections are meshed circumferentially into thirty divisions where in heightwise direction bottom most section 1 has thirty segments and remaining two are divided into five segments. The upper part of beam element is made up of ten elements. The other details are as shown in the Figure 1a, b, c. The radius thickness ratio \( R_t \), slenderness ratio \( \bar{\lambda} \) and the axial force ratio \( P/P_x \) are the design parameters of the circular steel pier which are given by the Eqn. 1, 2, 3.

\[
R_t = \sqrt{3(1-\nu^2)} \frac{\sigma_y}{E} \frac{d}{2t}
\]  

(1)
Where, 
\[ \nu = \text{Poisson's ratio}, \quad \sigma_y = \text{yield stress}, \quad E = \text{modulus of elasticity,} \quad d = \text{diameter of column (D-t),} \quad t = \text{thickness of plate,} \quad h = \text{height of column,} \quad r = \text{radius of gyration of the cross section,} \quad A = \text{cross section area,} \quad P_y = \text{yield load}. \]

The Table 1 and 2 gives the range of parameters considered for creating the FE model and the material properties of the SM490 grade (\( \sigma_y = 315 \text{MPa} \)) steel respectively.

**Figure 1:** The details of analytical FE model of circular steel pier and loading pattern.

**TABLE 1**

<table>
<thead>
<tr>
<th>Parameter Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_t )</td>
</tr>
<tr>
<td>( \bar{\lambda} )</td>
</tr>
<tr>
<td>( P/P_y )</td>
</tr>
<tr>
<td>( t )</td>
</tr>
</tbody>
</table>

**TABEL 2**

<table>
<thead>
<tr>
<th>Material Properties of SM490 Grade Steel</th>
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</thead>
<tbody>
<tr>
<td>( E ) (GPa)</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>200 315</td>
</tr>
</tbody>
</table>

Where, \( \varepsilon_y = \text{yield strain}, \quad E_{st} = \text{initial hardening modulus,} \quad \varepsilon_{st} = \text{initial hardening strain,} \quad \sigma_u = \text{ultimate stress.} \)
Two types of cyclic displacement loading patterns as cyclic uni and bi-directional circular are applied to the top of the pier with constant vertical load and which are shown in Figure 1d, e respectively. It has been proved that circular bi-directional loading is the most severe for circular steel piers (Goto [9]) hence the similar pattern is selected in this work. The displacement amplitude in each cycle is in multiple of yield displacement \( \delta_y \) of the respective pier and which is given by Eqn. 4.

\[
\delta_y = \frac{H_y h^3}{3EI} + \frac{H_y h}{\kappa GA} \quad (4)
\]

Where, \( H_y \)=yield strength of column, \( I \)=moment of inertia, \( \kappa \)=transverse shear stiffness, \( G \)=shear modulus. To insert the value of \( H_y \) in Eqn. 4, choose smaller one from the Eqn. 5,6 bellow.

\[
H_y = \frac{M_y}{0.85h} \left( 1 - \frac{P}{P_E} \right) \left( 1 - \frac{P}{P_u} \right) \quad (5)
\]

\[
H_y = \frac{M_y}{h} \left( 1 - \frac{P}{P_y} \right) \quad (6)
\]

Where, \( M_y \)=yield moment, \( P_E \)=Euler’s buckling load (\( Py/\lambda^2 \)), \( P_u \)=ultimate load (\( Py(1.109-0.545\lambda) \)). The results obtained from the large displacement nonlinear static analyses are used to generate the ultimate strength and ductility formulas which is explained in following section.

**RESULTS AND DISCUSSION**

**Strength and Ductility Formulation**

To confirm the seismic performance of the steel pier it is necessary to predict its capacity under worst condition of loading hence cyclic circular bi-directional pattern is adopted in such a way that it can be compared with conventional cyclic uni-directional pattern. Here, the strength factors \( H_{max}/H_y \)=normalized maximum restoring strength (\( \sqrt{H^2_x + H^2_y} \)), and ductility factors \( \delta_{max}/\delta_y \)=normalized horizontal displacement at \( H_{max} \) which is also called as conventional ductility and \( \delta_{95}/\delta_y \)=normalized horizontal displacement in the post peak curve when the restoring force reduces to 95% of \( H_{max} \) (Gao [6]), of the pier are defined in the Figure 2.

![Figure 2: \( \delta_{max} \) and \( \delta_{95} \) defined on envelop curve.](image)
The results are observed by applying horizontal cyclic uni and bi-directional displacement with vertical constant load on twelve FE models constructed with various values of $R_t$ and $\lambda$ and five cases of $P / P_y$ (from Table 1). As these three are the main parameters which affect the performance of the pier, the combination of them is derived by nonlinear regression analysis to plot the best fitted curves with respect to ductility factor $\delta_95 / \delta_y$ as shown in the Figure 3. The curve equations of dashed lines are given in Eqn. 7, 8.

$$\frac{\delta_{95}^{UNI}}{\delta_y} = \frac{1}{R_t^{0.95} \lambda^{0.28} (2.22 + P / P_y)^{1.40}} - 1.17 \quad (7)$$

$$\frac{\delta_{95}^{CIR}}{\delta_y} = \frac{\lambda^{0.1}}{R_t^{0.94} (2.31 + P / P_y)^{1.25}} - 0.28 \quad (8)$$

### Displacement based Seismic Verification Method

In the seismic verification procedure, it is required to confirm the dynamic response of the structure within allowable limits. Hence, the verification of displacement response for one directional earthquake motion is easy to check however; generally earthquake produces multi-directional waves which give random displacement response of the pier top where conventional method fails to relate. Therefore here a new procedure is designed for the same purpose. At first, the relationship between $\delta_{95}^{CIR} / \delta_{95}^{UNI}$ and combined parameter of $R_t, \lambda$ and $P / P_y$ is established (see Figure 4a) and which can be written as in Eqn. 9. If the response of the pier top gives elliptical shape then its ultimate ductility can be calculated by using Eqn. 10 (see Figure 4b).

$$\frac{\delta_{95}^{CIR}}{\delta_{95}^{UNI}} = R_t^{0.038} \lambda^{0.37} (2.07 + P / P_y)^{0.2} - 0.06 \quad (9)$$
The step by step procedure of verification is explained in the Figure 4c. In first two steps apply dynamic motions independently in longitudinal and transverse directions and check its response like conventional method by using Eqn. 7. After that, apply both components of earthquake motions simultaneously which gives displacement response similar as shown in Figure 4b. In next step, draw a maximum response ellipse by making center of ellipse coinciding to the crossing of axes (EW, NS) and covering distant points of displacement response, then find out the lengths of major and minor axis ($a$, $b$) of this ellipse. By using the Eqn. 9 and substituting $b/a$ ratio in Eqn. 10, determine the major axis length of ultimate state ellipse ($b/a$) and draw it concentrically to the maximum response ellipse. If the plotted ultimate state ellipse falls outside the maximum response ellipse then it is said to safe condition and if it falls inside the maximum response ellipse then it’s a failed case which needs further upgradation or redesign from beginning.

Effect of Loading Pattern on Buckling Behavior

When the steel pier is subjected to the cyclic loading it shows outward buckling at the bottom part which is typically called as ‘elephant-foot buckling’ in case of circular piers. The Figure
5 indicates the contour graphs of outward bulge in the bottom section of pier up to height, \(D\) (pier type: \(R_p = 0.075\), \(\lambda = 0.4\), cycles number: 3.5). The contour lines are plotted for \(\Delta d/t\) (\(\Delta d\) = surface deformation, \(t\) = thickness of plate) at interval of 0.2. The dashed lines are for inside buckling whereas solid lines show outward deformation. The solid thick line indicates the maximum outward buckling. Comparing Figure 5a and 5b, it is clearly noticed that bi-directional cyclic loading pattern generates large buckling than uni-directional cyclic loading. In other sense, circular cyclic loading reduces pier strength significantly due to every point of cross section passes through tension-compression cycles, whereas in uni-directional cyclic loading, only two opposite points of the cross section experience extreme tension-compression cycles than remaining part hence uni-directional loading may not be considered as severe case for circular steel piers.

![Contour graphs of buckling at bottom of steel circular pier](image)

**Figure 5: Contour graphs of buckling at bottom of steel circular pier**

**SUMMARY AND CONCLUDING REMARKS**

Typically, seismic excitation generates multi-directional earthquake waves and combination of two horizontal components expects more severe effect on ultimate behavior of steel pier than independent components. Most of the studies carried out so far related to observing ultimate capacity of steel piers with cyclic uni-directional loading failed to capture this coupling effect hence different patterns of bi-directional cyclic loading were adopted. However, application of the ultimate ductility formulas from bi-directional loading analysis in the seismic verification was not tackled until the present work.

In this work, the ultimate ductility formulas for the circular steel bridge piers are derived numerically for uni and bi-directional loading patterns. Further, based on the relationship between ultimate displacement due to circular bi-directional loading over uni-directional loading and design parameters, a seismic verification method is developed which is useful to check coupling effect of earthquake components on the pier response. In the last part of this paper, an effect of loading pattern on the buckling behavior at the bottom of the pier is discussed. The pattern of contour lines indicates that bi-directional cyclic loading generates large buckling than uni-directional cyclic loading which again proves severity of bi-directional loading. The concept of verification method for bi-directional dynamic analysis which explained in this paper is expected to be helpful for researches and practical engineers.
References


USE OF CRESCENT SHAPED BRACES FOR CONTROLLED SEISMIC DESIGN OF DUCTILE STRUCTURES

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KEYWORDS
Horizontal-resisting element, horizontal-resisting system, stiffness, strength, ductility, crescent-shaped braces, performance objectives.

ABSTRACT

In general, the lateral-resisting system of a given building structure can be seen as composed of a series of single lateral-resisting elements working together. The mechanical characterization of each component necessarily requires to capture both its elastic and inelastic behaviour and can be assumed to be an elastic-perfectly plastic one, univocally identified by three independent parameters: stiffness (secant at yield point), strength, and ductility. The mechanical characterization of the whole lateral-resisting system, as composed of the n lateral-resisting components working in parallel, can be directly obtained from the mechanical characterization of each single component and is therefore controlled by 3n parameters. Consequently, depending upon the type of lateral-resisting used, the structural designer is allowed to control up to 3n parameters upon which he can “act” to reach the desired seismic performance objectives. In this paper, this concept will be applied on a specific application, characterized by the separation between the lateral resisting system and the vertical resisting system. The lateral resisting system is calibrated, within a Performance-Based Seismic Design approach, to satisfy a multiplicity of performance objectives through the identification of a “objectives curve”, in the Force-Displacements diagram, of the mechanical characteristics of the structure (“target push-over”). The calibration is obtained by methods borrowed from Displacement Based Design. From a practical point of view, in order to apply this innovative design approach, the lateral resisting system is thought as composed of special bracing elements capable of realizing a sort of properly-calibrated seismic isolation (“revised/tuned soft-storey”) called crescent-shaped braces.
DESIGNER POSSIBILITY TO CONTROL THE HORIZONTAL RESISTING SYSTEM

In general, the horizontal-resisting system (hereafter referred to as HRS) of a given building structure can be seen as composed of a series of single horizontal resisting elements (hereafter referred to as “horizontal resisting components”, HRC), working together.

THE SINGLE HORIZONTAL RESISTING ELEMENT (ELEMENTARY COMPONENT)

The mechanical characterization of each component (being either a shear wall, a bracing system or other) of the horizontal resisting system necessarily requires to capture both its elastic and inelastic behaviour. Without loss of generality, the mechanical characterization of each elementary component can be assumed to be an elastic-perfectly plastic one or a bilinear one with hardening, as represented in Figure 1 (a) and (b) (Paulay [1]), respectively. The independent parameters, which are necessary only four and they are stiffness, strength, ductility and strain hardening ratio. For sake of simplicity, in order to present the basic ideas of the design approach here proposed, in the following parts of this paper, the elastic-perfectly plastic model has been assumed.

![Figure 1: $F - \delta$ constitutive law of the i-th HRC](image)

THE HORIZONTAL RESISTING SYSTEM OF A BUILDING STRUCTURE

Without loss of generality, let us consider plane single-degree-of-freedom (SDOF) structural systems equipped with $n$ elementary horizontal resisting components working in parallel (which is a correct schematization under the common assumption of building structures characterized by rigid in-plane floor systems), as the illustrative ones represented in Figure 2. The mechanical characterization of the whole horizontal resisting system, as composed of the $n$ horizontal-resisting components working in parallel, can be directly obtained from the mechanical characterization of each single component. Without loss of generality, let us focus on a HRS composed of three HRCs (i.e. let us assume $n = 3$). The following equations provide the relationships between the mechanical characterization of the HRS and those of the three HRCs which make up the whole system following the magnitude of the yield displacement: the system with the smallest yield displacement is numbered as system 1, the second smallest yield displacement is numbered as system 2, and so on. Consequently, the constitutive law of the HRS is given by:
As an illustrative example of the above equations, Figure 3 represents the mechanical characterization of the HRS composed of three HRCs. It can be observed that the overall mechanical behaviour of a HRS composed of \( n \) HRCs is controlled by 3\( n \) parameters, given that the mechanical behaviour of a single HRC is controlled by 3 parameters. However, note that, in Figure 3, the force-displacement curve of the HRS for \( \delta \geq \delta_{u1} \) is represented as a dashed line. This is because the structural system should reach collapse for \( \delta = \delta_{u1} = \delta_{u,HRS} = \min(\delta_{u1}, \delta_{u2}, \delta_{u3}) \), thus diminishing the number of parameters controlling the overall mechanical behaviour from 3\( n \) to 2\( n \); nonetheless also parameters \( \delta_{ui} > \delta_{u,HRS} \), \( \forall i \), (i.e., in the considered case: \( \delta_{u2} \) and \( \delta_{u3} \)) can still be the object of the characterization of the system (e.g., if it is imposed to have \( \delta_{ui} \equiv \delta_{u,HRS}, \forall i \), in order not to waste the ductility capacities of the single HRCs).

Figure 2: (a) Different SDOF structural systems, each one composed of three homogenous horizontal-resisting components; (b) \( F - \delta \) constitutive law of a HRS composed of three HRCs, as obtained combining the constitutive laws of the single HRCs.

**THE PROPOSED DESIGN APPROACH**

The proposed design approach can be formalised in a three-step procedure which is aimed at identifying the characteristics of the structural system resisting to horizontal loads which enables to directly satisfy given seismic performance objectives (without recurring to any trial-error processes). It is composed of the following steps:

- **Step 1**: calibration of the fundamental characteristics which should be possessed by the structural system resisting to the horizontal loads to satisfy given performance objectives,
- **Step 2**: identification of the physical and geometrical characteristics of the single structural elements which constitute the horizontal resisting system,
- **Step 3**: verification, by means of appropriate time-history analyses, of the seismic performances achieved.

The procedure is illustrated in detail in the following part of the paper with reference to an applicative example.
THE STRUCTURE CONSIDERED AND THE GENERAL SEISMIC DESIGN STRATEGY

The applicative example is carried out with reference to a building structure composed of five-storey pendular steel. The total building mass is $2.7 \times 10^6$ kg. The building is assumed to be located in Bologna (Italy) on D.M. 14/01/2008 soil type C and on topographic surface S1. It is designed to meet the D.M. 14/01/2008 provisions (D.M. 14/01/2008 [2]). The structure is characterized by the separation between the vertical resisting system (beams and columns) and the horizontal resisting system (special bracing system). The vertical-resisting system is sized to support just the vertical loads. The horizontal resisting system is designed to display a controlled inelastic behaviour at the ground level and to behave elastically from the second storey up. It is composed of: (1) special bracing elements, named “crescent-shaped braces” placed between the ground storey and the first storey, enable to realize a sort of “revised/tuned soft-storey”, (2) traditional diagonal bracing elements (following an “X” arrangement, as represented in Figure 3) from the second storey up. The horizontal resisting system, placed between the ground and the first storey, is calibrated, within a Performance-Based Seismic Design approach, to satisfy a multiplicity of performance objectives through the identification of a “objective curve”, in the Force-Displacement diagram, of the mechanical characteristics of the structure. The horizontal resisting elements from the second storey up can be designed through a capacity design approach and will not be considered in the following analyses. Figure 3 shows the geometry of one of the perimeter pendular steel frames in both the North-South (NS) and the East-West (EW) directions. The seismic behaviour of the building along each direction may be schematised as the one of a SDOF system characterized by a mass corresponding to that of the whole superstructure (second storey up) and by the lateral force-displacement relationship controlled by the HRSs composed of 12 HRCs and 8 HRCs along the NS and the EW directions, respectively, together with the little contribution to the lateral resistance provided by the vertical resisting systems (which will be simply dimensioned in order to carry the vertical loads). In the following part of the paper, for sake of conciseness, only the seismic behaviour of the building along the NS direction will be considered.

Figure 3: Schematic representation of the building structure considered

IDENTIFICATION OF THE F-A OBJECTIVE CURVE (STEP 1)

The selected performance objectives

With the aim of designing the horizontal resisting system, we refer to the Basic Objectives defined in the Vision 2000 (Bertero [3]) document. In particular, for the design of the horizontal-resisting system we impose on the structure the following Basic Objectives:

1. First Performance Objective (PO1): defined as a coupling of the Fully Operational performance level with the Frequent Earthquake Design Level;
2. Second Performance Objective (PO2): defined as a coupling of the Operational performance level with the Occasional Earthquake Design Level;

3. Third Performance Objective (PO3): defined as a coupling of the Life Safe performance level with the Rare Earthquake Design Level.

"Objectives Curve" in the force-displacements diagram

Imposing on the considered structure, the previous performance objectives, we have found the “objectives curve” in the Force-Displacements diagram, for the city of Bologna, for D.M. 14/01/2008 soil type C and for topographic surface S1 (Figure 4). The "objectives curve" provides the target points for an optimised/controlled seismic behaviour of the structure (performance objectives). It represents a designing tool, because it may allow to calibrate the horizontal resisting system to satisfy a multiplicity of performance objectives (PO1, PO2, PO3).

![Objectives curve in the Force-Displacements diagram](image)

Figure 4: "Objectives curve" in the Force-Displacements diagram.

IDENTIFICATION OF THE HORIZONTAL RESISTING COMPONENTS CHARACTERISTIC (STEP 2)

Lateral stiffness of the vertical-resisting system

For the calibration of an horizontal resisting system capable of satisfying the imposed performance objectives, we have to consider the columns contribution on the lateral stiffness. For the case-study at hand, the lateral stiffness (initial inclination of the force-displacement relationship) of the vertical-resisting system, as composed by 28 equal columns, is computed as:

\[
K_{VRS} = \sum_{i=1}^{28} k_i = \sum_{i=1}^{28} 1.7 \frac{EJ}{h^3} = 19960 \text{ kN/m}
\]

where \( E = 210000 \text{ MPa} \) (Young modulus), \( J = 8563 \text{ cm}^4 \) (moment of inertia along the considered direction), and \( h = 3.5 \text{ m} \) (inter-storey height). Figure 5 shows the contribution to the lateral resistance provided by the vertical resisting system, together with the “objectives curve”, in the Force-Displacements diagram. It is possible to note that the vertical-resisting system alone is not able to satisfy the performance objects imposed.
Without modifying the lateral stiffness of the vertical resisting system, we assign the part of the lateral stiffness, required for satisfying the performance objectives, to the horizontal resisting system placed between the ground and the first storey. Figure 6 shows the “objective curve” of each HRC. It is obtained subtracting from the structure “objective curve” the lateral contribution of the vertical resisting system and dividing by the total numbers of horizontal resisting components which compose the horizontal resisting system along the considered direction. Let us indicate the lateral stiffness of the horizontal resisting system with \( \Delta K \) or \( K_{HRS} \):

\[
\Delta K = K_{HRS} = K_{Framed} - K_{VRS} = 131110 \, \text{kN/m} \quad (3)
\]

The lateral stiffness of each horizontal resisting component is 1/12 of the lateral stiffness of the horizontal resisting system (there are six Crescent Shaped Braces on each face of the building in North-South direction):

\[
K_{HRC} = \frac{K_{HRS}}{12} = 10930 \, \text{kN/m} \quad (4)
\]

The single Crescent Shaped Brace (Figure 7 (a)) can be sized through the following two equations (the first equation can be found imposing the equality between the expression of the lateral stiffness of the single Crescent Shaped Brace and the value of the lateral stiffness of the single HRC; the second equation can be found imposing that the bending moment in the most stressed section reach the first yielding moment):
\[
\frac{J}{d^2} = \frac{K_{\text{HRC}} \cdot l}{3E \cos^2 \alpha}
\]
\[
\frac{J}{d \cdot h} = \frac{1}{2} \cdot \frac{F_{y1,\text{HRC}}}{\cos \alpha} \cdot \frac{1}{f_y}
\]

where:
- \(E\) is the steel Young Modulus;
- \(J\) is the inertia moment of the HRC’s cross section;
- \(\alpha\) is the inclination of the portal’s diagonal;
- \(d\) is the distance of the knee point, \(P\), from the portal diagonal;
- \(l\) is the length of the portal diagonal;
- \(h\) is the height of the cross section
- \(F_{y1,\text{HRC}}\) is the first yielding force of each HRC.

Fixing \(d = 1\) m, we obtain \(J = 16255\) cm\(^4\) and \(h = 35\) cm. Once \(J\) and \(h\) are known, the width \(b\) of the single HRC cross section can be obtained (\(b = 4.5\) cm). The constitutive law of each HRC has been obtained with a non-linear static push-over analysis (displacement control). Figure 7 (b) shows the constitutive law and the “objectives curve” of each HRC. It is clear that the single horizontal resisting component is enabled to satisfy the performance objective, consequently, also the structure is able to satisfy the imposed performance objectives.

Figure 7. (a) Generic Crescent Shaped Brace; (b) The constitutive law and the “objectives curve” of each HRC

ANALYSIS AND VERIFICATION (STEP 3)

A plane model of the structure has been realized. Each Crescent Shaped Braces have been modelled with non-linear elements. Non-linear time-history dynamic analyses are developed on the structure model. The earthquake ground motions used for the time history analyses are seven Frequent Earthquake spectra-compatible accelerograms and seven Rare Earthquake spectra-compatible accelerograms (obtained with REXEL v 2.2 (beta) (Iervolino, Galasso and Cosenza 2008)). Table 1 (a) and Table 1 (b) shows the maximum displacement of the first storey caused by the seven accelerograms spectrum-compatible with the D.M. 14/01/2008 Frequent Earthquake spectrum and by the seven accelerograms spectrum-compatible with the D.M. 14/01/2008 Rare Earthquake spectrum, respectively. In these tables there are also the average value of the previous displacements. The average value of the displacements of the first storey caused by the seven accelerograms spectrum-compatible with the D.M. 14/01/2008 Frequent Earthquake spectrum and by the seven accelerograms spectrum-compatible with the D.M. 14/01/2008 Rare Earthquake spectrum -
compatible with the D.M. 14/01/2008 Rare Earthquake spectrum represents the displacement demand (average value) required by the earthquakes. Comparing the average value of the displacement demand with the value of the displacement demand imposed in Step 1 for each performance objective, it is possible to note that the value are almost the same.

### TABLE 1

Maximum Displacement and Average Value of the First Storey Caused by the Seven AcceleroGrams Spectrum-Compatible with the (A) D.M. 14/01/2008 Frequent Earthquake Spectrum and the (B) D.M. 14/01/2008 Rare Earthquake Spectrum

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**AVERAGE VALUE** 1.82

**CONCLUSIONS**

This paper presents a procedure for the identification of the characteristics of the structural system resisting to horizontal loads which enables to satisfy given seismic performance objectives. This is achieved by considering a total conceptual separation between the structural systems resisting to vertical and horizontal loads.

The original aspect of the proposed procedure mainly relies in the combined use of the following recent contributions in the field of seismic engineering:

- active role of the engineering process;
- satisfaction of a multiplicity of performance objectives within a PBSD approach;
- methods/tools of both FBD and DBD approaches (depending on the performance to be imposed at hand);
- conceptual separation between the structural systems resisting to vertical and horizontal loads;
- design proposal of peculiar crescent shaped steel.

**REFERENCES**


SEISMIC RESPONSE CONTROL OF TRANSMISSION TOWER-LINE SYSTEM WITH FRICTION DAMPERS

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3 Briline Corporation, Tokyo, Japan

KEYWORDS
Transmission tower-line system, coupled vibration, seismic response, friction damper, passive control, control equation.

ABSTRACT
The seismic response control of a transmission tower-line system by using passive friction dampers is actively investigated in this study. The multi-degree-of-freedom (MDOF) model of transmission line is developed. The mechanical model of friction damper is presented by involving the effects of damper's axial stiffness. The dynamic model and analytical approach of transmission tower-line system incorporated with friction dampers are developed for both in-plane/out-of-plane vibration. A real transmission tower-line system constructed in China is taken as the example to examine the feasibility and reliability of proposed control approach. The observations demonstrate that the friction damper can be utilized in the earthquake-induced vibration control of a transmission tower-line system because of its simple configuration as well as satisfactory energy-dissipating capacity.

INTRODUCTION
To be a typical kind of high-rise structures with small damping, a transmission tower-line system is prone to the strong external excitations. Much attention has been paid by researchers and engineers across the world for examining the performance of transmission tower-line system subjected to external excitations. To examine the properties of a transmission tower-line system, many theoretical and experimental investigations have been carried out during the past three decades (Kempner et al 1984; Ozono et al. 1988). It is a practical task for engineer and researcher to guarantee the structural safety under seismic loading and avoid the structural damage (Housner et al 1997). Dynamic absorbers, such as the tuned mass dampers, tuned spring dampers and tuned liquid dampers, can be utilized for
the earthquake-induced vibration mitigation of a transmission tower-line system. Although these devices can reduce dynamic responses to some extent, they require much space for installing many large masses. Energy-dissipating dampers may be an alternative approach to overcome the shortcomings of dynamic absorbers. The dampers can be manufactured as axial members to replace common members of the truss tower so as not to occupy additional structural space. In addition, energy-dissipating dampers can reduce dynamic responses of all the mode shapes for obtaining satisfactory control performance. Modern versions of friction damper have already been implemented in a number of civil engineering structures around the world. To this end, the earthquake vibration control of a transmission tower-line system by using passive friction damper is actively carried out in this study. The analytical model of transmission tower-line system is developed. The mechanical model of passive damper is presented by involving the effects of damper's axial stiffness. The dynamic model and analytical approach of transmission tower-line system incorporated with friction dampers are developed for both in-plane/out-of-plane vibration. A real transmission tower-line system constructed in China is taken as the example to examine the feasibility and reliability of proposed control approach. The made observations demonstrate that the passive friction damper can be utilized in the seismic response control of a transmission tower-line system because of its simple configuration as well as satisfactory energy-dissipating capacity.

**DYNAMIC RESPONSES OF TRANSMISSION TOWER-LINE SYSTEM**

The transmission line can be modeled as several lumped masses connected with elastic elements and the transmission tower is simplified as a two-dimensional lumped mass model. After establishing the kinetic energy and potential energy of coupled tower-line system, the mass and stiffness matrices of coupled system can be determined through differential coefficients with respect to generalized velocity and generalized displacement, respectively. The analytical models of transmission tower-line system for in-plane/out-of-plane vibration are displayed in Figure 1. The transmission tower-line system consists of an elastic-gravity coupled vibration system when it vibrates in in-plane direction subjected to earthquake. The kinetic energy and potential energy of coupled system for in-plane vibration are

\[
T = \sum_{j=1}^{n} \frac{1}{2} m_i (\dot{u}_i^2 + \dot{v}_i^2) + T_{tower} = T_{line} (\dot{\xi}_2, \dot{\xi}_3, \ldots, \dot{\xi}_{n-1}, \delta l_1, \delta l_2, \ldots, \delta l_n) + T_{tower}
\]

\[
U = \sum_{i=1}^{n-1} m_i g v_i + \sum_{j=1}^{n} \frac{E A}{2} \left( \frac{l_j + \delta l_j}{l_j} \right)^2 \left( \frac{l_j}{l_j} \right)^2 + U_{tower}
\]

**Figure 1: Analytical model of a transmission tower-line system**
Where $n$ is the number of discretized elements for transmission line. where $u_i$ and $v_i$ are the horizontal and vertical displacement of the $i$th lumped mass. By substituting Eqs. (1) and (2) into Lagrange equation, the mass and stiffness matrices can be determined by computing the partial differential of kinetic energy and potential energy with respect to general coordinates. Transmission tower-line system is a continuum with any towers and transmission lines, which make it impossible to establish an analytical model involving all the towers and lines. The development of simplified computational model with satisfactory accuracy is essential for dynamic analysis and vibration mitigation. For convenience, the coupled system with one transmission tower and two lines is adopted in this study. For in-plane vibration, the stiffness and mass matrices of transmission tower-line system are expressed as

$$K_{in} = \text{diag}[K_{in}^{l}, K_{in}^{r}, K_{in}^{c}]$$

$$M_{in} = \begin{bmatrix} M_{in}^{l} & M_{leftcouple}^{T} \\ M_{leftcouple} & M_{in}^{r} & M_{rightcouple} \\ M_{rightcouple}^{T} & M_{in}^{l} & M_{rightcouple}^{T} \end{bmatrix}$$

Where $M_{leftcouple}$ and $M_{rightcouple}$ are the left and right coupled matrices respectively. Similarly, the equivalent mass increment of towers can also be determined through differential analysis on kinetic energy of coupled system to simulate the coupled effects of tower due to transmission lines. Based on above analysis, the mass and stiffness matrices of tower-line system for out-of-plane vibration can be expressed as

$$M_{out} = \text{diag}[M_{out}^{l}, M_{out}^{r}, M_{out}^{c}]$$

$$K_{out} = \begin{bmatrix} K_{out}^{l} & K_{couple}^{T} \\ K_{couple} & K_{out}^{r} & K_{couple} \\ K_{couple}^{T} & K_{out}^{l} & K_{couple}^{T} \end{bmatrix}$$

Where $K_{couple}$ is the coupled stiffness matrix

**VIBRATION CONTROL OF TOWER-LINE SYSTEM**

Commonly, the force produced by friction damper to the structures is transmitted through elastic braces. The configuration of passive friction damper with brace is displayed in Figure 2. The magnitude and direction of friction force are closely related to the deformation and velocity between the two ends of the damper as well as the damper slippage

$$u = u(d, \dot{d}, e)$$

Where $d$ is the deformation between the two ends of the damper, which includes the axial deformation of the damper; $e$ denotes the damper slippage. If the damper is sticking, it behaves like a pure brace.
\[ u = (d - e) EA/L; \dot{e} = 0 \]  

(8)

If the force in the damper is larger than the slipping force, the damper begin to slip

\[ u = \mu N \text{sgn}(d); \]  

(9)

If the velocity between the two ends of the dampers is in different direction to the damper force, the friction damper is sticking again. The equation of motion of a controlled tower-line system for in-plane vibration subjected to seismic excitations can be expressed as

\[ (\begin{pmatrix} M^\text{in} & \mathbf{0} \\ \mathbf{0} & M^\text{out} \end{pmatrix} \ddot{x}(t) + (\begin{pmatrix} C^\text{in} & \mathbf{0} \\ \mathbf{0} & C^\text{out} \end{pmatrix} \dot{x}(t) + (\begin{pmatrix} K^\text{in} & \mathbf{0} \\ \mathbf{0} & K^\text{out} \end{pmatrix} x(t) = (\begin{pmatrix} P^\text{in}(t) \end{pmatrix} + (\begin{pmatrix} H^\text{in}^\text{T} & H^\text{out}^\text{T} \end{pmatrix} u^\text{in}(t) \]

(10)

where \( M^\text{in}, C^\text{in}, K^\text{in} \) are mass, damping and stiffness matrices of the coupled system for in-plane vibration, respectively; \( x(t), \dot{x}(t) \) and \( \ddot{x}(t) \) are the displacement, velocity and acceleration responses for in-plane vibration, respectively; \( P^\text{in}(t) \) is the seismic excitations for in-plane vibration; \( u^\text{in}(t) \) is control force provided by friction damper for suppressing in-plane vibration; \( H^\text{in} \) is the influence matrix for \( u^\text{in}(t) \). Similarly, the equation of motion of a controlled tower-line system for out-of-plane vibration is

\[ (\begin{pmatrix} M^\text{out} & \mathbf{0} \\ \mathbf{0} & M^\text{in} \end{pmatrix} \ddot{x}(t) + (\begin{pmatrix} C^\text{out} & \mathbf{0} \\ \mathbf{0} & C^\text{in} \end{pmatrix} \dot{x}(t) + (\begin{pmatrix} K^\text{out} & \mathbf{0} \\ \mathbf{0} & K^\text{in} \end{pmatrix} x(t) = (\begin{pmatrix} P^\text{out}(t) \end{pmatrix} + (\begin{pmatrix} H^\text{out}^\text{T} & H^\text{in}^\text{T} \end{pmatrix} u^\text{out}(t) \]

(11)

The meanings of symbols in Eq.(11) are similar to those in Eq.(10).

**CASE STUDY**

To examine the performance of the proposed control approach based on passive friction dampers, a transmission tower-line system, located in China, is selected as analytical example. The tower has a height of 107.8m and the span of transmission line is 832m (see Figure 3.). A two dimensional lumped mass model is constructed based on original three dimensional model. Twenty friction dampers are evenly distributed along the main body of the tower with 10 placed in-plane and the other 10 placed out-of-plane. The initial friction force for each damper is set as 10 kN.

To compare the mitigation effects of the original system with the controlled transmission tower-line system installed with passive friction dampers, two cases are investigated. The first one is the original transmission tower-line system without control. The second one is the tower-line system incorporated with passive

\[ u = (d - e) EA/L; \dot{e} = 0 \]  

(8)

If the force in the damper is larger than the slipping force, the damper begin to slip

\[ u = \mu N \text{sgn}(d); \]  

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If the velocity between the two ends of the dampers is in different direction to the damper force, the friction damper is sticking again. The equation of motion of a controlled tower-line system for in-plane vibration subjected to seismic excitations can be expressed as

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(10)

where \( M^\text{in}, C^\text{in}, K^\text{in} \) are mass, damping and stiffness matrices of the coupled system for in-plane vibration, respectively; \( x(t), \dot{x}(t) \) and \( \ddot{x}(t) \) are the displacement, velocity and acceleration responses for in-plane vibration, respectively; \( P^\text{in}(t) \) is the seismic excitations for in-plane vibration; \( u^\text{in}(t) \) is control force provided by friction damper for suppressing in-plane vibration; \( H^\text{in} \) is the influence matrix for \( u^\text{in}(t) \). Similarly, the equation of motion of a controlled tower-line system for out-of-plane vibration is

\[ (\begin{pmatrix} M^\text{out} & \mathbf{0} \\ \mathbf{0} & M^\text{in} \end{pmatrix} \ddot{x}(t) + (\begin{pmatrix} C^\text{out} & \mathbf{0} \\ \mathbf{0} & C^\text{in} \end{pmatrix} \dot{x}(t) + (\begin{pmatrix} K^\text{out} & \mathbf{0} \\ \mathbf{0} & K^\text{in} \end{pmatrix} x(t) = (\begin{pmatrix} P^\text{out}(t) \end{pmatrix} + (\begin{pmatrix} H^\text{out}^\text{T} & H^\text{in}^\text{T} \end{pmatrix} u^\text{out}(t) \]

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**CASE STUDY**

To examine the performance of the proposed control approach based on passive friction dampers, a transmission tower-line system, located in China, is selected as analytical example. The tower has a height of 107.8m and the span of transmission line is 832m (see Figure 3.). A two dimensional lumped mass model is constructed based on original three dimensional model. Twenty friction dampers are evenly distributed along the main body of the tower with 10 placed in-plane and the other 10 placed out-of-plane. The initial friction force for each damper is set as 10 kN.

To compare the mitigation effects of the original system with the controlled transmission tower-line system installed with passive friction dampers, two cases are investigated. The first one is the original transmission tower-line system without control. The second one is the tower-line system incorporated with passive...
friction dampers. The peak displacement responses of the transmission tower-line system with/without control for in-plane and out-of-plane vibration are displayed in Figure 4, respectively. Displayed in Figure 5 are the time histories of the displacement responses of the top floor of the transmission tower. It is observed that the dynamic responses of the transmission tower for in-plane and out-of-plane vibration can be suppressed. The dynamic responses of all the individual floors of the tower can be reduced by only installing twenty dampers.

In order to examine the control performance of the transmission tower-line system, the Vibration Reduction Factor (VRF) is defined as (Chen and Xu 2008):

\[ VRF = \frac{Z_{nc} - Z_{co}}{Z_{nc}} \]  

(12)

Figure 4: Control performance of earthquake-excited transmission tower

Figure 5: Time histories of displacement responses on top of tower
Where: $Z_{nc}$ is earthquake-induced peak response of original structure; $Z_{co}$ is peak response with control. The parameter investigation is also carried to examine the effects of axial stiffness of the friction damper on the control performance. The obtained observations demonstrate that the VRFs of all the three kinds of dynamic responses gradually increase with the increasing axial stiffness until the axial stiffness reaches a certain value. After that, the VRFs remain almost constant and additional increment of damper's axial stiffness cannot remarkably improve the control performance. Therefore, it is unnecessary to increase the axial stiffness of friction dampers unrestrainedly. To approach an optimal value, the control performance can reach its optimal value without wasting damper cost. It should be pointed out that the above parameter investigation is carried out only for the chosen example structure in this study. For other transmission tower-line systems with different parameters, the optimal parameters may be different. However, the procedure of the parameter study demonstrated here is still valid.

**CONCLUDING REMARKS**

The feasibility of using passive friction dampers to suppress the seismic responses of a transmission tower-line system is actively carried out in this study. The transmission line is modeled as several lumped masses connected with elastic elements and the transmission tower is simplified as a two-dimensional lumped mass model. A real transmission tower-line system constructed in China is taken as the example to examine the feasibility and reliability of proposed control approach. The made observations demonstrate that the friction damper can be utilized in the seismic response control of a transmission tower-line system because of its simple configuration as well as satisfactory energy-dissipating capacity.

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**REFERENCES**


EFFECTS OF HORIZONTAL RESTRAINTER ON SEISMIC PERFORMANCE OF STEEL PLATE SHEAR WALL

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KEYWORDS

steel plate shear wall, restrained steel plate shear wall, restrainer, seismic design, capacity design, boundary element,

ABSTRACT

Steel plate shear wall (SPSW) can effectively resist horizontal earthquake forces by allowing the development of diagonal tension field action after the thin infill plates buckle in shear, and then dissipate earthquake energy through the cyclic yielding of the infill plate in tension. This research proposes a new type SPSW named as restrained steel plate shear wall (R-SPSW). A horizontal restrainer is made from a pair of steel tube member sandwiching over the infill steel panels from two sides using through bolts and pin-connected to the boundary columns of an SPSW. This research conducted extensive numerical studies on the seismic behavior of R-SPSW using the nonlinear FEM method and simplified strip model. The design method for the R-SPSW was developed based on the numerical studies. Four 2-story 2.2-meter wide by 6.5-meter tall SPSWs were constructed and cyclically tested to a roof drift of 0.05 radians at National Center for Research on Earthquake Engineering (NCREE) in 2007. The low yield strength steel plates of 2.6 mm were adopted for all four specimens. Two out of four SPSWs are R-SPSW. The effectiveness of the horizontal restrainers on the responses of the SPSW was investigated. Analytical and experimental results show that horizontal restrainers effectively reduced the force demands on the boundary elements, which are the beam and column surrounding the infill panels. The paper concludes with recommendations on the design and analysis of the SPSW constructed with horizontal restrainers.

INTRODUCTION

Steel plate shear wall (SPSW) has seen increased usage in North America and Asia in recent years. As shown in Fig. 1a, an SPSW is composed of a structural frame and infill steel plates. The beams and columns surrounding the infill panels are named as boundary elements.
SPSW can effectively resist horizontal earthquake forces by allowing the development of diagonal tension field action after the infill plate buckles in shear. The energy is then dissipated through the cyclic yielding of the infill plates in tension. The concept of using the post-buckling strength of the steel plate was first proposed by Thorburn et al. in 1983 [1]. In their research, a strip model (Fig. 1a) and an equivalent brace (EB) model (Fig. 1b) were developed. The strip model uses at least 10 inclined truss members pin-connected to the columns and beams to represent the tension field action of a steel plate. When a tension field action is developed in a typical SPSW, the large out-of-plane deformation and the unpleasant sounds accompanying the buckling of the steel plates present major issues. In order to alleviate these phenomena, the idea of a restrained SPSW (Fig. 1d), bolting pairs of steel structural member over the two sides of the panel (Fig. 1e), has also been proposed by Lin and Tsai [2]. These horizontal steel restrainers can be conveniently connected to the two columns using slotted or pin end condition (Fig. 1f). In order to prevent the interference of the “sliding” restrainers on the tension stretching of the panel in the tension field direction, enlarged holes for the bolts through the steel panel have been adopted on the steel panels or restrainers. In this research, the effectiveness of the pin-end horizontal restrainers, as shown in Fig. 1f, in reducing the force demands imposed on the boundary elements has been investigated extensively using analytical and experimental methods. Seismic design methods for this kind of restrained SPSW (R-SPSW) are proposed in this paper.

Figure 1: The illustration of (a) a steel plate shear wall; (b) a strip model; (c) an equivalent brace model; (d) a restrained SPSW; (e) and (f) the details of the restrainer connections

RESTRAINED STEEL PLATE SHEAR WALL

General Introduction

As stated earlier, the interference of the restrainers on the stretching of steel panel can be prevented by using enlarged bolt holes on restrainers. Thus, in a restrained steel plate shear wall (R-SPSW) (Figs. 1d, 1e and 1f), the pin-end horizontal restrainers can effectively restrain two kinds of deformations: (1) the out-of-plane deformation of the buckled infill plates and (2) the inward flexural deformation of the boundary columns induced from the panel forces. When an infill panel is constructed with the proposed restrainers, the out-of-plane buckling deformation could be significantly reduced as the buckling length is effectively reduced [2]. This kind of restraining effect is similar to the two-way stiffeners in a stiffened SPSW (S-SPSW). However, there are some differences between the R-SPSWs and the S-SPSWs:

(1) In an S-SPSW, the infill panels are stiffened sufficiently such that the panels can develop shear yield strength prior to the shear buckling. In the R-SPSW, the panels are not required to be stiffened to become “compact” plate. The R-SPSW panels are permitted to buckle in shear at a very low lateral force like those in a typical thin SPSW.

(2) The main purpose of utilizing restrainers is to reduce the boundary element demands due to the tension field action. In a typical SPSW, the horizontal component of tension field forces would pull the boundary columns inward and induce significant boundary element
demands such as beam axial forces, column flexural forces. In an R-SPSW, the restrainers would help the boundary elements to resist the horizontal tension field forces. Thus, the stated boundary element demands could be reduced. In a typical S-SPSW, the stiffeners are not designed to reduce the force demands on the boundary elements.

Seismic Design for R-SPSW

Design Consideration for Restrainers and the Restrainer Effects on Beam Design

Restrainers must be designed to resist compressive forces to prevent the columns from being pulled inward. Figure 2 shows that the effects of restrainers on the columns are akin to lateral supports for the columns. The restrainers are expected to remain elastic even if the R-SPSW has developed a plastic mechanism and the plastic hinges have strain-hardened. With the transverse panel forces applied on the fix-ended columns, the restrainer compressive forces can be estimated from the reaction forces at the lateral supports. If a panel is constructed with \( N_R \) pairs of restrainers which are uniformly distributed, each restrainer’s axial force can be estimated as \( \Omega_{h_p} \omega_{c_h} h_c / (N_R + 1) \), where the \( \Omega_{h_p} \) is the overstrength factor accounting for the panel strain hardening effect, the \( \omega_{c_h} \) is the horizontal component of the tension field forces of the steel panels acting on the boundary column, the \( h_c \) is the clear panel height. When a typical SPSW is subjected to the lateral forces, the boundary beams support the boundary columns to resist the horizontal component of panel forces and an axial compressive force will develop at the beams. When the adjacent panels develop fully yielding tension field forces, the \( J^{th} \) floor beam axial force \( P_b \) can be determined from:

\[
P_{b,J} = \frac{\omega_{c_h,J-1} h_{J-1} + \omega_{c_h,J} h_J}{2} / 2
\]

As shown in Fig. 2, if each panel in an R-SPSW are restrained with \( N_R \) pairs of uniformly distributed restrainers, the beam axial force \( P_{b,J} \) will be reduced to:

\[
P_{b,J} = \frac{\omega_{c_h,J-1} h_{J-1} + \omega_{c_h,J} h_J}{2(N_R + 1)} / 2(N_R + 1)
\]

Effects on Column Design

The action on the boundary elements when an SPSW is subjected to lateral forces can be considered as the superposition of two parts [3]. One part is only due to frame sway while the other part is due to panel forces. As shown in Fig. 3, the column bending moment due to the frame sway action is linearly distributed and that due to the panel forces is quadratic distributed. Figures 3a and 3b illustrates that the plastic hinge would form above the bottom end the compressed boundary column when an SPSW with weak column develop its plastic mechanism. Figure 3c demonstrates that the restrainers will reduce the column bending moment induced from panel forces. The restrainer effect is akin to the lateral pin support for columns. Thus, the restrainers would reduce the column flexural demand. This research [4] has proposed a column capacity design method to assure the plastic hinge forming at the column bottom end for typical and restrained SPSW.
ANALYTICAL EXAMPLE

Example Designs and Finite Element Analysis

The ABAQUS finite element analyses of four different designs of a single-story single-bay SPSWs were compared with strip and EB models to investigate the effectiveness of the restrainers and the proposed capacity design method. The overall dimension of the example designs is 2-meters wide by 3-meters tall. The boundary elements in the four designs are identical. The H400×200×9×12 beam and two H300×300×12×20 columns are employed. The differences in the four designs are infill panel thickness and the use of restrainers. The four SPSW designs are designated as T2, R2, T3 and R3, where the “T” or “R” represents “typical SPSW” or “R-SPSW”, respectively. The number of “2” or “3” implies that the panel thickness is 2.0 mm or 3.4 mm, respectively. Designs R2 and R3 are constructed with two pairs of uniformly distributed restrainers made from steel hollow structural sections (HSSs). The steel material for the panels and the boundary elements are all assumed as elastic-perfect-plastic (EPP) model with the yield stress $f_y = 350$ MPa. For each example design, a nonlinear finite element model was constructed using the ABAQUS program. In addition, a strip model and an equivalent brace (EB) model have been made using the PISA3D/GISA3D programs [5]. Pushover analyses for a story drift up to 2.5% radians have been carried out for each model.

Analytical Results

Lateral force versus displacement responses

Figure 4 presents the total shear versus displacement relationships for all analytical models. The figures also provide the panel shear forces obtained from the FE models. In addition, the fully-yielding panel strengths, $V_{yp}$, calculated from $0.5f_{yp}t_pL_{cf}sin2\alpha$ [1] are also marked in Fig. 4. Figure 5 shows the Von Mises stress contours at the 2.5% radians story drift. In these contours, the white regions represent the plastic zones. Figure 5 also shows the extent of the plastic deformations in all strips and the boundary elements at the end of the pushover. From Fig. 4, it can be found that except for Designs T3, the total shear versus story drift responses predicted by three different kinds of analytical models (strip, EB and FE) are rather similar. In addition, the infill panels in all the Designs, except T3, are found capable of developing the fully-yielding panel strength $V_{yp}$ marked in each corresponding figure. Comparing strips’ deformation marks in Figs. 5c with 5a (or 5d with 5b), it can be found that the restrainers can effectively help in promoting a more uniform tension field action in the steel panels.

Figure 3: Schematics of superposition for computing the compressed 1F column’s moments for (a) proper design; (b) improper design; and for (c) R-SPSW.
shown in Figs. 5a, 5c and 5d for T2, R2 and R3, almost the entire infill panel in each figure has yielded and most of the plastic zones on the columns concentrate at the bottom ends at the 2.5% radians story drift.

For Design T3, the total and the panel shear versus displacement responses obtained from the strip model is very similar to that from the FE model (Fig. 4c). However, the infill panel can not develop its fully yielding strength $V_{yp}$. As shown in Fig. 5b-left for Design T3, the FE stress contours (darkened zones) and the strips’ axial plastic deformations indicate that the bottom right corner region of the infill panel near the compressed column bottom end could not be fully stretched to yield at the end of the pushover. Furthermore, from the stress contours (Fig. 5b-left), it can be found that the plastic zones in the compressed column in Design T3 have spread over the bottom half of the column. The range of this kind of plastic zones (white regions) is much greater than those observed in the other three Designs. It can be found these plastic zones in the FE stress contours spread from the column bottom over about 40% of the entire column height. The widest plastic zone (Fig. 5b-left) appears to be located at about 500mm measured from the column bottom end. In the strip model shown in Fig. 5b-middle, it can be found that a flexural plastic hinge (shown by a circle) also forms above the compressed column bottom end. The location of this circle is very close to the widest plastic zone stated above for the FE stress contours for Design T3. By judging the location of the plastic hinges, the extent and distribution of the strip deformations, and the total shear versus lateral displacements of the structure shown in Figs. 4 and 5, it is evident that the strip model can satisfactorily predict the FE nonlinear responses of all four Designs regardless whether restrainers or weak column exist or not. From Figs. 5b-left and 5b-middle, it is confirmed that if the column strength is insufficient, the plastic hinge would form near the mid-height of the compression column. In addition, it can be found that if a plastic hinge formed near, or plastic zones too far spread out into, the mid-height of the compressed column, the yielding tension field action could not be fully developed by the weak boundary columns as evidenced in the Design T3.

On the contrary in Design R3, the plastic hinge indeed formed at the compressed column bottom end (Figs. 5d-left and 5-middle). The infill panel could develop its fully yield strength $V_{yp}$ (Fig. 4d). Comparing Design R3 with T3, it is confirmed that the restrainers could reduce the panel force effects on the boundary columns, and help the weak boundary columns to anchor and develop the fully yielding of the tension field action. Thus, the lateral load-carrying capacity of Design R3 (Fig. 4d) is greater than that of T3 (Fig. 4c). However, it is worthy noting in Figs. 4a and 4b that the ultimate strengths of Designs R2 and T2 are similar. This is because that the same boundary elements themselves are sufficient to anchor the thin steel panel (2.0mm) adopted in Design T2 in developing the panel strength $V_{yp}$ (Fig. 4a) and stretching the entire panel as expected (Fig. 5a-right).
EXPERIMENTAL INVESTIGATION

Test Program

Four 2-story 2.14-meter wide by 6.5-meter tall narrow SPSW were constructed and cyclically tested to a roof drift of 0.05 radians in NCREE. The test setup is shown in Fig. 6. The low yield strength steel plates of 2.6mm were adopted for all four specimens. Two out of four specimens were constructed with horizontal restrainers. The key parameter of this series of tests is the size of the boundary elements of the specimens and the use of restrainers. All the boundary elements are A572 grade 50 steel. Three of the four Specimens are introduced in this paper: Specimen N is normal, named from the fact that the columns complied with the proposed capacity design method [4]. Specimen RS is an R-SPSW designed considering proposed capacity design. Because of the effectiveness of the restrainers, the size of boundary elements in Specimen RS is smaller than that of Specimen N. The boundary elements in Specimen S are identical to Specimen RS.

Test Results

Plastic hinges near the column bases

After the tests, as shown in Fig. 7, the plastic hinges developed at the 1F column bottom ends can be recognized from the flaking of the whitewashes. Two types of plastic hinges can be observed in the columns. One is the plastic hinge developed when the column is compressed. This kind of plastic hinge can be observed on the column web shown in the photo (Fig. 7c) as a ring shape. This is due to the occurrence of web local buckling at the center of the ring. The other type of plastic zone develops when the column is tensioned. In this case, uniform web yielding would be evident as shown in the photo (Fig. 7c) at the column bottom end. It consists of many vertical and horizontal yield lines. As there is no plastic hinge zone forming at the mid-height of column for Specimens N and RS, these two types of plastic zones overlap at the column bottom ends (Figs. 7a and 7b). On the other hand, the ring-shape zone in both columns of Specimen S was evidently above the column bottom end where uniform yielding zone developed (Fig. 7c). The distance from the center of this ring to the column
bottom is about two times the column depth. It clearly indicates that the plastic hinges form away from the bottom ends when the columns were compressed in Specimen S. Comparing the test results between Specimens S and RS, it confirms the effectiveness of the restrainers in reducing the column flexural demands.

![Figure 6: Test setup](image1)

![Figure 7: Plastic zones near the column bottom ends](image2)

**Local buckling of the middle beam web**

2005 AISC Seismic provisions recommend that width-to-thickness slenderness ratios (b/t ratios) of the boundary elements in SPSWs must be lower than the slenderness ratio limits, $\lambda_{ps}$, prescribed for the seismically compact section. This requirement is checked herein for the middle beam webs at the RBSs in the four specimens using the following equations:

\[
C_u = \frac{P_u}{P_y} = \frac{P_u}{(F_{yw} A_{RBS})} \\
\text{for } C_u \leq 0.125, \quad \lambda_{ps} = 3.14 \sqrt{E/F_{yw}} \left[1 - 1.54 C_u \right] \\
\text{for } C_u > 0.125, \quad \lambda_{ps} = 1.12 \sqrt{E/F_{yw}} \left[2.33 - C_u \right] \geq 1.49 \sqrt{E/F_{yw}}
\]

where $F_{yw}$ is the mean yield stress of the beam web and flange plates; $A_{RBS}$ is the area of the most reduced section of the RBS; $E$ is the modulus of elasticity and $F_{yw}$ is the yield stress of the web plate. Noting that these equations consider the axial force, $P_u$, in the member, the axial forces developed in the middle beams at the 2.5% radians roof drift level are estimated from the following equation:

\[
P_u = \frac{\Omega_{hp}(\omega_{h1} h_{11} + \omega_{h2} h_{12})}{2(N_R + 1)}
\]

where $\Omega_{hp}$ factor accounts for the strain hardening effect of the infill panel at the 2.5% radians roof drift. Table 1 contains the evaluation details of the web local buckling of the middle beams incorporating the stated axial forces at a roof drift of 2.5% radians. For Specimen S, the middle beam web slenderness ratio is higher than $\lambda_{ps}$ and the web local buckling occurred before the 2.5% radians roof drift. For the other three specimens, the b/t slenderness ratios of the middle beam webs are lower than $\lambda_{ps}$ and the web local buckling was further delayed and occurred after the 2.5% radians roof drift. The evaluation agrees with the test results, confirming the effectiveness of the stated design specifications. Alternatively, assuming $C_u=1$, a more compact section can be chosen for the practical design of the middle beam. However, this evaluation illustrates an example of how to incorporate the actual axial load effects to achieve an economical selection of the middle beam.
TABLE 1

EVALUATION OF WEB LOCAL BUCKLING (WLB) OF MIDDLE BEAM RBS

<table>
<thead>
<tr>
<th>Middle beam RBS section</th>
<th>$P_u$ (kN)</th>
<th>$P_y$ (kN)</th>
<th>$C_u$</th>
<th>$\lambda_{ps}$</th>
<th>$h_c/t_w$</th>
<th>Instability check</th>
<th>Experimental responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>H350×150×8×12 (Specimen N)</td>
<td>750</td>
<td>2348</td>
<td>0.319</td>
<td>52.42</td>
<td>40.75</td>
<td>Seismically Compact</td>
<td>WLB occurred at 3% Rads. roof drift</td>
</tr>
<tr>
<td>H350×105×7×11 (Specimen S)</td>
<td>633</td>
<td>2056</td>
<td>0.322</td>
<td>45.33</td>
<td>46.86</td>
<td>Non-Seismically Compact</td>
<td>WLB occurred at 2% Rads. roof drift</td>
</tr>
<tr>
<td>H350×105×7×11 (Specimen RS)</td>
<td>305</td>
<td>2056</td>
<td>0.119</td>
<td>49.93</td>
<td>46.86</td>
<td>Seismically Compact</td>
<td>WLB occurred at 3% Rads. roof drift</td>
</tr>
<tr>
<td>H350×105×7×11 (Specimen CY)</td>
<td>300</td>
<td>2056</td>
<td>0.119</td>
<td>49.93</td>
<td>46.86</td>
<td>Seismically Compact</td>
<td>WLB occurred at 3% Rads. roof drift</td>
</tr>
</tbody>
</table>

CONCLUSION

Based on the stated studies, conclusions can be drawn as follows:

- When a multi-story SPSW is subjected to the lateral forces, the compressed 1F column is the critical column. If the column strength is insufficient, the plastic hinge on the compressed 1F column would form above the bottom end. This kind of inelastic column deformation would cause the 1F infill panel to fail in developing the full yielding tension field action.

- The analytical examples and test results confirm that the proposed sliding restrainers can effectively help in promoting a more uniform tension field action in the steel panels. In addition, the restrainers can reduce the force demands on the boundary elements.

- All boundary elements of Specimens N and RS satisfied the proposed capacity design requirements. In these two specimens, the plastic hinges formed at the ends of the boundary elements. The total weight of Specimen RS including the restrainers (about 2.7 tons) is less than that of Specimen N (about 3.3 tons), suggests that the restrained SPSW could potentially save more material than a typical SPSW.

REFERENCES


THE APPLICATIONS OF PERFORMANCE BASED SEISMIC DESIGN FOR STRUCTURES IN MAINLAND CHINA

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ABSTRACT

“Performance Based Seismic Design” (PBSD) is extremely popular in seismic research community, this terminology, however, for most engineers is still “hypothetical” rather than “reality”. With the 2008 Beijing Olympics; development of “Informal” engineering approach 1 and contemporary architectural design as catalyst, PBSD approach is becoming an essential tool in designing structures in seismic region. Couple of complete projects will be presented in this paper to further demonstrate the importance and merits of adoptability performance based approach in seismic design over the traditional code base approach.

KEYWORD

Performance Based, Seismic, Nonlinear Analysis, Severe Earthquake

INTRODUCTION

Design Codes and Practice are typically prescribe design and construction rules developed largely based on past performance and experience. They were believed by most engineers and approval authorities that structures designed according to the codes are capable of attaining desired safety level and performance. This is usually correct for regular structures. Nevertheless, if the structures are somehow fall outside the coverage of the codes, e.g. new structural system, the rules set based on past experience is not longer valid or applicable to the new structural system. In most of these cases, authorities will instruct the designers or engineers them selves to increase the design forces in the structures. It is believed that such “increase” will therefore increase member stiffness and strength and hence the safety level and performance of the overall structures. This assumption is not always true. Cheng-Meng Lin et al had pointed out that such increases could have significant impacts on the design and also decrease the performance of the structures.
We understand that if codes are prepared based on past experience or research experiments, there are definitely areas which are not covered by the codes. Furthermore, most codes are developed to satisfy one objective – the safety, this single objective approach is now unable to satisfy the current development on the society demand. Therefore, a systematic methodology was developed from end 90’s to overcome the lead of code guidance design. This is the Performance Based Seismic Design (PBSD) approach which this paper is focused on. The following sections will briefly explain the background of PBSD approach; the theory. The use of PBSD in Beijing Olympic projects will also be presented to demonstrate the success of this approach in solving engineering problems which are outside the coverage of any Chinese and National Codes.

BACKGROUND

Performance Based Seismic Design (PBSD) concept was developed by late 90’s. Unlike most of the current single level target codes of practice, PBSD will design the structures that satisfy multi-level performance objectives at say minor, medium and severe level of earthquake. PBSD allows the design team (client, engineer, architect, insurance company etc.) to work together to determine the appropriate level of performance objectives against various levels of ground motion shaking.

Therefore, PBSD is not just to provide the ultimate safety requirement but also the control in downtime and damage. The following describes the different between current building design codes and PBSD approach.

Current Building Code Requirement
• Only one objective – life safety for the design earthquake intensity level;
• By means of seismic detailing to increase enhance the structural performance but the resulting level of performance is not explicitly defined;
• In Chinese Code, the design seismic load is a function of the material/damping and structural system while UBC codes is a function of the ductility factor, “R”
• Chinese code allows different seismic detailing for different structural members which creates unpredictable structural behavior in severe earthquake;
• UBC code use R-factor which the reduction of seismic load applies to the whole building.

PBSD approach
• Allows the owner, architect, engineer, insurance company, operator to select the appropriate level of protection at various level of ground motion;
• Multi-level of ground motion can be evaluated against various level of performance objectives;
• With the understanding of the behavior of the structures at various earthquake intensities, we can determine the actual seismic load that should be applied to the structures and members instead of single damping value or single R-factor.

PBSD DESIGN PROCEDURE

Generally speaking, three levels of performance are defined as follow:

<table>
<thead>
<tr>
<th>Serviceability Level</th>
<th>Negligible structural and nonstructural damage; nearly no downtime for the use of the building; and repair costs are minimal to nil.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Immediate Occupancy</td>
<td>Negligible structural damage and minor non-structural damage; structure may not function but with limited interruption of operations; building requires repairing but cost is limited.</td>
</tr>
<tr>
<td>Level</td>
<td></td>
</tr>
</tbody>
</table>
**Collapse Prevention Level**

| Extensive structural and non-structural damage; building is unable to be re-use; repair cost might be too high and re-build is recommended. |

Because of the above, we sometimes may also call PBSD as “3D” design. “3D” in PBSD refers to Downtime, Damage and Deaths which are all a factor in risk analysis.

\[
\text{RISK} = P\left\{\begin{array}{l}
\text{Deaths (safety)} \\
\text{Dollars (damage)} \\
\text{Downtime (loss of use)}
\end{array}\right\}
\]

PBSD is a series of design and evaluation process on a structural system to enforce certain behavior of structural as well as nonstructural system and/or components under application of simultaneous ground motions at various probabilities of exceedance.

The design process of PBSD has 4 steps namely

1. Setup Performance Objectives in various Hazard Level;
2. Conceptual Design;
3. Design Evaluation and
4. Modification (if required)

**Setup Performance Objectives in various Hazard Level**

Performance Objectives should be selected based on the hazard level and the building’s occupancy; the importance of functions occurring within the buildings; economic consideration including costs related to building damage repair and business interruption.

Hazard levels are based on the probability of exceedance of earthquake. In Chinese code, it refers to 63%, 10% and 2% in 50 years as minor (Level 1), moderate (Level 2, or design intensity) and severe (Level 3) earthquake respectively.

The following is an example extracted from FEMA 356 on setting-up the Performance Objectives:

![Building Performance Levels](image)
Conceptual Design

Once the performance objectives were established with the design team, the engineer should then develop the concept on how to achieve the performance objectives. It could be achieved by means of strength, stiffness, ductility, damping device (base isolators, unbonded braces) and control on the locations of structural fuses etc.

Design Evaluation

As PBSD involves the design of structure at various earthquake hazard levels, various tools are generally required to carry out the design evaluation work. For example, in level 1 earthquake, the structure is supposed to be no damage and therefore it should basically remain elastic. Therefore, elastic static and elastic dynamic (time-history) should be able to handle this evaluation process.

Moreover, if the designer wants to capture the behavior of the structures in level 2 and 3 earthquakes where minor or major plasticity had been occurs in the structures, nonlinear analysis software will be an essential tool at this stage of work. With the current development on computer software, push-over and nonlinear time-history analysis is becoming more popular.

Modification

Because PBSD requires the structure to satisfy the performance objectives at multiple hazard levels, convergence and cost to achieve the performance objectives may sometime requires the design team to review and modify either the structures or the original performance objectives.

PBSD APPROACH FOR PROJECTS IN CHINA

Because the rapid growth of economic in China and BJ Olympics, starting from millennium, there are many engineering challenge projects in China. However, at the same time, the limitation of Chinese code had created barriers for these projects. To go through the hurdle, PBSD is used to demonstrate to the Chinese approval authorities on not just the safety but also the performance of the structures at multiple hazard levels.

EXAMPLES

Example 1: CCTV

CCTV (photo 1) new headquarters is located the new Central Business District (CBD) of Beijing City. The main “twins” tower is named as CCTV consists of 450,000m² GFA which GFA is equivalent to 2.5 IFC2 in Hong Kong. Both towers are tilted by 6 degrees in both directions. The maximum height of the tower is 234m and both towers are connected by means of a 9 to 13 storey overhang cantilevers of 67 to 70m. By flipping the links vertically, the tower becomes two 300m tall towers. At the base of the towers, there are 3 levels of basement and a 9 storey podium.

The engineering challenge for CCTV is how to design the building such that it can satisfy the basic performance objective set in Chinese Code – no damage in Minor EQ; repairable damage in Design Intensity EQ Level and finally no collapse in Severe EQ. Because the building geometry, general core plus frame lateral system would not work. Therefore, external mega frame system was selected. Each structural member had to meet the Chinese Code as the basic requirement.
Furthermore, specific performance objectives for CCTV and for various structural elements at various zones were established with the client and the approval authority based on FEMA356 are listed as follow:

<table>
<thead>
<tr>
<th>Performance Objective</th>
<th>Service Level Earthquake (SLE) (50 year)</th>
<th>Design Bases Earthquake (DBE) (475 year)</th>
<th>Maximum Credited Earthquake (MCE) (2500 year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Damage (Structure remains in Elastic)</td>
<td>Repairable Damage</td>
<td>Collapse Prevention</td>
</tr>
<tr>
<td>Allowable Drift Ratio</td>
<td>h/300</td>
<td>h/100</td>
<td>h/50</td>
</tr>
<tr>
<td>Inter-storey Ductility Requirement</td>
<td>&lt; 1(Elastic)</td>
<td>&lt; 2</td>
<td>&lt; 4</td>
</tr>
<tr>
<td>Steel Beam Performance Objective</td>
<td>Elastic</td>
<td>$\theta &lt; 0.01$ radians</td>
<td>$\theta &lt; 0.04$ radians</td>
</tr>
<tr>
<td>Steel Bracing Performance Objective</td>
<td>Elastic</td>
<td>Majority Remain Elastic</td>
<td>Comp. Strain 7Dc Tensile Strain 9Dt</td>
</tr>
<tr>
<td>Columns Performance Objective</td>
<td>Elastic</td>
<td>Elastic</td>
<td>SRC Comp. Strain 0.02 Steel Comp. Strain 7Dc Tensile Strain 5Dt</td>
</tr>
<tr>
<td>Transfer Structure Performance Objective</td>
<td>Elastic</td>
<td>Elastic</td>
<td>Elastic</td>
</tr>
</tbody>
</table>

Nonlinear time-history analysis was adopted to justify the performance had met the objective. The approval of the seismic expert panel review was obtained in 9 months after the project was started. Such period of design and approval time is similar to those building structures which exceeds the code limit.

**Example 2: National Stadium**

National Stadium is one of the most importance stadiums for 2008 Beijing Olympics. The National Stadium is 260m (wide) x360m (long)x 60m (high) which houses 96,000 audiences in Olympics period. The structures were divided into two parts namely the bowl and the roof. The bowl was made of...
reinforced concrete. The roof is made of structural steel with the majority of the sections being 1.2m square boxes. For area which does not affect the appearance, 800mm square box sections were used.

Similar to CCTV, the performance objectives were first determined and agreed with the authorities. Because of the nature (geometry and function) of National Stadium is different from “buildings” only secondary members are designed as structural fused in level 3 EQ in order to protect the preliminary structures to behave elastically in all levels of EQ.

CONCLUSIONS

The concept of performance was briefly described. With the success of the applications in projects like BJ CCTV, National Stadium and others such Water Cube, China World Trade Centre phase 3A, Beijing T3 airport etc. This new design tool is becoming more and more popular for engineers in designing structures.

REFERENCES

THEORETICAL AND EXPERIMENTAL ANALYSIS OF DISSIPATIVE T-STUBS

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KEYWORDS

Damper, Connections, T-stubs, Cyclic, ADAS, Dissipation, Fatigue, Modelling, Experimental.

ABSTRACT

In this work, with the goal to enhance the energy dissipation capacity of classical T-stubs by working on the shape of the flange, a new type of axial damper is developed. In particular, a preliminary mechanical modelling of the device and a FEM modelling carried out in ABAQUS code is proposed and validated by comparison with experimental results. Hence, an experimental program devoted to the comparison of the energy dissipation capacity of classical T-stubs with respect to innovative T-stubs is presented. Finally, the comparison is carried out in terms of low-cycle fatigue curves exploiting the results obtained by constant amplitude cyclic tests.

INTRODUCTION

Basing on the energy methods, in the past decades, two new approaches have been individualized to design earthquake resistant buildings. The first approach is related to the possibility of filtering the input earthquake energy employing base isolation systems (Aiken et al., 1993b; Griffith et al., 1998). The other approach is based on the improvement of the energy dissipation capacity of the structure. Such enhancement is usually obtained by adding specific devices, increasing, in the energetic balance of the structure, the amount of dissipation provided by viscous and/or hysteretic damping. Many works have been carried out in the last decades on such a subject, leading to relevant results and to the development of a large number of high dissipation capacity dampers, either of friction or of yielding type (Aiken et al., 1993a; Christopoulos and Filiatrault, 2000; Kim et al., 2007). The basic idea of the use of energy dampers is to increase the hysteretic/viscous dissipative term of the energetic balance of the structure under seismic loading conditions, so that, for a certain level
of input energy, a larger hysteretic/viscous damped energy allows a lower demand of elastic strain energy, which means fewer damages in the structure. In other words the application of dissipative devices makes possible to concentrate the damage in specific parts of the structure, i.e. the dampers, and to avoid the failure or to limit the damages, of the structural and non structural parts. In this work, basing on the past experience of the authors dealing with the knowledge of the monotonic and cyclic behaviour of T-stubs (Piluso and Rizzano, 2008) a new type of axial damper is developed starting from the concepts usually applied to ADAS devices.

In the past, many scientific efforts have been devoted to the understanding and modelling of the cyclic behaviour of simple rectangular T-stubs pointing out, in many experimental studies, its behaviour under cyclic reversals. Such behaviour, as well known in scientific literature, is affected by significant pinching resulting from a lot of factors, such as the plastic deformation of bolts and the unavoidable contact phenomena. In addition, the plastic mechanism of rectangular T-stub is characterized by the concentration of plastic strain in two finite regions corresponding to the flange-to-web connection and to the flange section in correspondence of the bolt. Such plastic mechanism, concentrating the deformation demand only in two zones, under cyclic loadings leads to the quick deterioration of the stiffness, strength and energy dissipation capacity and to the collapse of the detail with a relatively small amount of dissipated energy.

MODELLING OF T-STUB CONNECTIONS

The energy dissipation capacity of T-stub connections can be significantly increased adopting the concepts governing the design of the steel hysteretic dampers working in double curvature, such as the ADAS devices (Added Damping And Stiffness plate elements) (Whittaker et al., 1989). The advantages of the use of X-shaped elements with respect to the rectangular ones can be well understood considering the bending moment diagram developed along the T-stub flange. The classic rectangular shape of the T-stub flange doesn’t allow engaging in plastic range the whole plate because it yields only at the ends concentrating the plastic deformations in finite regions. Therefore, the resulting curvature and strain demand is extremely high. Conversely, an ideal X-shaped element subjected to linear bending moment uniformly yields along the plate so that the plastic deformation and strain demand is ideally distributed over the plate length. Due to the unavoidable shear and axial stress present in the plate and aiming to avoid stress concentrations which may affect the ductility, the resistance and the fatigue life of the element (Whittaker et al., 1989), usually the shape of the ADAS devices must differ from the ideal hourglass shape.

The concepts governing ADAS devices have been already applied to beam-to-column connections in (Fleischman & Hoskisson, 2000) where modular connectors produced as cast piece have been proposed and analyzed. In this paper, in order to simplify the manufacturing of the dissipative T-stubs, the ADAS concepts are applied to design proper cuts along the T-stub flange which are executed by means of the oxy-fuel cutting method. Usually the shape of the ADAS devices is given by an hourglass tape red shape, which is defined by means of exponential functions. The definition of stiffness, resistance and ductility of this detail is untrivial in a closed form solution, because it should start from the integration of curvatures on the real shape of the device, which as showed in the following is described by means of exponential functions. Therefore, generally in the modelling a simplified linear X-shape is
considered. In such a case, as already proposed by (Whittaker et al., 1989), the largest X-shape inscribed within the real shape of the device is assumed.

Hence, in order to model the hourglass T-stub behaviour, for each assigned moment distribution along the plate, the corresponding rotations and displacements of the idealized X-shape can be evaluated by integrating the steel moment-curvature. The mathematical expression of the ADAS devices tested at the University of Berkeley (Whittaker et al., 1989) has been computed by means of a regression analysis in (Tena-Colunga, 1996). With reference to the geometrical parameters specified in Figure 1, this shape is given by the following functions:

\[
\begin{align*}
    b(z) &= Be^{-\alpha z} & 0 \leq z \leq m/2 \\
    b(z) &= s \cdot e^{\alpha(z-m/2)} & m/2 \leq z \leq m
\end{align*}
\]

where \( \alpha = \frac{2}{m} \ln \left( \frac{B}{s} \right) \) (1)

![Diagram of geometry, curvature and bending moment diagrams and scheme for the stiffness definition](image)

Eqs. 1-2 allow to define the effective width of the simplified X-shape, which is given by:

\[
B_{\text{eff}} = \frac{\alpha B m}{2} e^{-\alpha m} \left( \frac{1-\alpha m}{2} \right)
\]

In order to describe the elastic behaviour, the ideal cantilever of Figure 1 loaded by the bolt force has been considered. The initial stiffness of the X-shaped T-stub \((K_{0,\text{XS}})\) is given by the following relation:

\[
K_{0,\text{XS}} = \zeta \frac{EBt^3}{m^4}
\]

(4)
The value of $\zeta$ corresponding to $s/B$ equal to 1 provides the well known solution $\zeta = 0.5$ available for the T-stub with rectangular flange (Faella et al., 2000). Besides, considering the usual $s/B$ ratios, which, considering the technical literature, are contained in the range 0.1-0.2, it appears reasonable to assume for design purposes $\zeta = 0.25$. Regarding the inelastic behaviour of the T-stub, the prediction of the whole monotonic force-deflection curve can be performed increasing step-by-step the bending moment level on the T-stub and assuming the position of the zero point of the moment diagram constant and equal to $m/2$. This procedure already applied in (Piluso et al., 2001) results significantly simplified in the case of X-shaped T-stub because, for each assigned moment distribution, the curvatures are constant along the plate (Figure 1).

Therefore, taking into account that the curvature, for a compact rectangular section is $\chi_i=2\varepsilon_i/t$, with reference to single non preloaded T-stubs, the following expressions have been obtained:

$$\delta_i = \frac{F_i}{K_{0,\text{XS}}} \quad 0 \leq \delta \leq \delta_y$$

$$\delta_i = \frac{F_i}{K_{0,\text{XS}}} + \frac{(\varepsilon_i - \varepsilon_y)m^2}{t} \quad \delta_y \leq \delta \leq \delta_u$$

where $\delta_y$ and $\delta_u$ are given from eq.(7) for $\varepsilon_i$ equal to $\varepsilon_y$ and $\varepsilon_u$ respectively. $F_i$ is defined by the following relation:

$$F_i = \frac{4M_i}{m} = \frac{2B_{\text{eff}}t^2}{3m} \frac{M_i}{M_y} f_y$$

In addition, the accuracy of the mechanical model has been preliminary verified by means of FEM modelling carried out in ABAQUS code with eight-node solid elements.

**EXPERIMENTAL PROGRAM**

The whole experimental program consists of 16 welded T-stub specimens. T-stubs have been connected through the flanges by means of four M18 bolts of 8.8 class axially preloaded with a preloading level leading to a bolt tensile stress equal to 80% of the yield one. Steel mechanical characteristics have been determined by means of coupon tensile tests. Specimens are divided into two series. The first one is constituted by eight specimens which have been realized connecting the T-stubs to a rigid and strong flange (specimens TS01-M, TS01-C15, TS01-XS-M, TS01-XS-C15, TS02-M, TS02-C15, TS02-XS-M, TS02-XS-C15). Specimens of type TS-M and TS-C15, tested under monotonic and constant amplitude cyclic loads respectively, have been composed starting from elements characterized by rectangular flanges. Specimens of type TS-XS-M and TS-XS-C15 are the corresponding X-
shaped version designed according to the criteria previously described and also subjected to monotonic and constant amplitude cyclic loads. The second series is composed by 8 specimens, constituted by two equal T-elements, which are divided in two groups of four specimens: the first one is characterized by rectangular flanges (T SD-M, TSD-C10, TSD-C20, TSD-C30) and the second one by X-shaped flanges (TSD-XS-M, TSD-XS-C10, TSD-XS-C20, TSD-XS-C30). For each group of specimens one monotonic test and three constant amplitude cyclic tests have been performed. With reference to the symbols of Figure 2, the values of the geometrical properties of the specimens are given in Table 1.

The goal of the cyclic tests under different values of constant amplitude is the evaluation of the low cycle fatigue curves. The experimental tests have been carried out at the Materials and Structures Laboratory of the Department of Civil Engineering of Salerno University. Under displacement control, all the specimens have been subjected to a tensile axial force, applied to the webs tightened by the jaws of the testing machine, a Schenck Hydropuls S56 (maximum test load 630 kN, piston stroke ±125 mm).

In addition, coupon tensile tests have been performed to establish the mechanical properties of the material. The values of the mechanical properties are given in Table 1.

### TEST RESULTS

**T-stubs on rigid support (TS01, TS02, TS01-XS, TS02-XS)**

The experimental tests (Figure 3) of the T-stubs connected to the rigid support evidenced a good reliability of the design procedure previously described. The monotonic tests show that the experimental curve of the X-shaped T-stub is very close to the rectangular one (Figure 4). In addition, it can be observed that both tests have been influenced by the axial behaviour, due to the presence of the rigid support which represents a shear constraint for the bolts. The
collapse of specimens TS01-M, TS02-M, TS01-XS-M and TS02-XS-M was caused by the axial stress in the flange: in the case of TS-M specimens arising the bolt shear failure and in the case of TS-XS-M specimens the collapse of the hourglass mid-section occurred. Figures 5-6 show the comparisons between the monotonic experimental curves and the predicted ones.

Figure 3: TS-XS and TS series

Figure 4. TS01-M/TS02-M vs TS01-XS-M/TS02-XS-M: monotonic tests comparison

In particular, for the rectangular T-stub, the prediction of the F-δ curve has been computed by means of the model proposed by (Piluso et al., 2001), while for the X-shaped T-stub the model derived in section 2 has been applied.

Figure 5: TS01-M, TS-02-M, TS01-C15 and TS02-C15 test results and model comparisons
The corresponding cyclic tests, TS-C15 and TS-XS-C15, were led under constant amplitude displacement of 15 mm. In order to establish the attainment of the cyclic collapse, the Energy Degrade has been checked. The collapse has been identified by the condition $\frac{E_i}{E_1} = 0.5$ where $E_i$ is the energy dissipated during the $i$-th cycle and $E_1$ is the corresponding value recorded during the first cycle.

The cyclic behaviours of the two T-stubs topologies are very different evidencing the significant increase of energy dissipation capacity when the reduced section strategy is adopted. The rectangular T-stub exhibits the classic failure caused by the cracking propagation in correspondence of the flange-to-web plastic hinge.

Conversely, diffused plasticization of the steel over the plate length has been shown by X-shaped T-stub, which attained the collapse after a number of cycles much higher than that provided by TS-C15 tests (Figures 5-6-7).

**Coupled T-stubs (TSD, TSD-XS)**

The experimental analysis on coupled T-stubs is constituted by 8 tests, two monotonic and six cyclic under constant cyclic amplitude of 10, 20 and 30 mm, in order to obtain low-cycle fatigue curves (Figure 8).
The two monotonic tests TSD-M and TSD-XS-M show a very ductile behaviour (Figure 9-10). Because of geometric non-linearity for high displacement values the monotonic behaviour is significantly influenced by the plates' axial stress. In Figs. 9-10 the comparison among the experimental F-\(\delta\) curves, the mechanical modeling and FEM modeling previously described is also shown.

In order to completely point out the different plastic behaviour under cyclic loads of the two details the low cycle fatigue curves have been carried out. As well known these can be represented by means of straight lines in bi-logarithmic diagrams. The number of plastic reversals to failure \(N_f\) is related to the total displacement amplitude \(\Delta v\) by means of the following relationship:

\[
\Delta v = aN_f^b
\] (9)

The low-cycle fatigue curves have been defined evaluating the parameter \(a\) and \(b\) by means of cyclic tests under different values of constant amplitude displacement (Figure 10).

In Figure 10 low-cycle fatigue curves of rectangular and X-shaped T-Stubs are depicted. In the examined case, the monotonic points aren’t well aligned with the curves obtained by means of regression analysis of the three cyclic points. This could be justified considering that cyclic amplitudes are contained in a displacement range in which second order effects don’t have relevance, so that the failure modes under cyclic and monotonic conditions are different. The comparison between the low-cycle fatigue curve of the rectangular T-stub and
that of the X-shaped one shows the significant increase of the energy dissipation capacity obtained by properly cutting the T-stub flanges.

![Figure 10: Cyclic test of the TSD-XS and TSD series and low cycle fatigue curves](image)

**CONCLUSIONS**

In this paper, the possibility to enhance the energy dissipation capacity of bolted T-stubs under cyclic loads has been analyzed. Starting from the concepts usually applied to ADAS devices, a design of X-shaped T-stubs has been performed by means of a FEM model and of a proposed mechanical model. Experimental monotonic tests have been executed in order to validate the predicting models. The energy dissipation capacity of the proposed X-shaped T-stubs has been investigated through cyclic tests and the improvement of the joint behaviour has been verified by means of the comparison between the T-stub with reduced section and the rectangular T-stub in terms of low-cycle fatigue curves.

**REFERENCES**


THE "GENERAL METHOD" FOR ASSESSING THE OUT-OF-PLANE STABILITY OF STRUCTURAL MEMBERS AND FRAMES AND COMPARISON WITH ALTERNATIVE RULES IN EN 1993 - EUROCODE 3 - PART 1-1

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KEYWORDS

Steel Structures, Stability, Lateral torsional stability, members, frames, Eurocode 3.

ABSTRACT

This paper describes the "General Method" for assessing the stability of members and frames including the torsional and lateral torsional buckling. The method is suitable for members and frames with uniform as well as for non-uniform cross sections. The basis of the "General Method" is formed by the method developed by Maquoi and Rondal to treat flexural buckling of columns. Extending this method accurate lateral torsional buckling curves are derived. In the "General Method" consideration is given to the effects of out-of-plane loading. In the paper attention is paid to the justification of the rules of the "General Method" for lateral torsional buckling by statistical evaluation against test results.

INTRODUCTION

EN 1993 - Eurocode 3 - Part 1-1 [1] gives in its section 6.3.4 a "general method" [2] for assessing the out-of-plane stability of structural members and frames, which is based on the following assumptions:

1. The design values for the in-plane action effects, for which the out-of-plane stability shall be checked are $E_d$. 

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2. The main parameters for the assessment are the following:

a. The characteristic value of the in-plane resistance $R_k$, which can be determined from

$$\alpha_{ult,k} \cdot E_d = R_k$$

(1)

where $\alpha_{ult,k}$ is the minimum load amplifier to reach the characteristic resistance at the most critical cross section of the member or frame considering the in-plane behaviour without taking the flexural or flectural torsional buckling into account.

b. The elastic critical out-of-plane buckling resistance $R_{cri}$ can be determined from

$$R_{cri} = \alpha_{cri} \cdot E_d$$

(2)

where $\alpha_{cri}$ is the minimum amplifier to reach the elastic critical resistance of the member or frame with regard to flexural or lateral torsional buckling.

c. The main parameters can be summarised in a global slenderness $\overline{\lambda}_{op}$ for out-of-plane stability, which can be obtained from

$$\overline{\lambda}_{op} = \sqrt{\frac{R_k}{R_{cri}}} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cri,op}}}$$

(3)

d. Given a suitable buckling curve for out-of-plane buckling, the assessment can be performed by either

$$\frac{E_d}{R_d} = \frac{E_d}{\alpha_{op,d} \cdot E_d} \leq 1$$

(4)

where

$$\alpha_{op,d} = \frac{\alpha_{ult,k} \cdot \lambda_{op}}{\gamma_{M1}}$$

(5)

or

$$\frac{\lambda_{op} \cdot \alpha_{ult,k}}{\gamma_{M1}} \geq 1$$

(6)

At the stage of working out EN 1993 - Eurocode 3 - Part 1-1 this "general method" could not yet be specified with all details, in particular the way how to determine the out-of-plane buckling curve $\chi(\overline{\lambda}_{op})$ accurately and how to take additional out-of-plane loading into account, so that opening notes with some safe-sided recommendations have been included that allow National Choices in the framework of National Annexes.

In the mean time research projects have been carried out that make the rules for the general method complete and applicable for all conditions of out-of-plane stability. This paper gives an outline of the complete set of rules.
BASIS OF THE GENERAL METHOD

The basis of the "General Method" is the method of Maquoi and Rondal [3] to treat flexural buckling of columns. This method models a uniform column in compression by a member with imperfections, see figure 1, that are all summarized as equivalent geometrical imperfections, see figure 2.

Figure 1: Sources of imperfections

Figure 2: Shapes of equivalent geometrical imperfections

The shape of the equivalent geometrical imperfection is defined by the critical flexural buckling mode $\eta_{cu}$ and takes for uniform compression the form

$$\eta_{mu} = e_0 \cdot \frac{N_{cu}}{EI(x) \cdot \eta_{cu}(x)_{\text{max}}} \cdot \eta_{cu}(x)$$  \hspace{1cm} (9)

which in case of a column with hinged ends gives

$$\eta_{mu} = e_0 \cdot \sin \frac{\pi x}{l}$$  \hspace{1cm} (10)

Figure 3 shows how on this basis and with the assumption for the amplitude of the equivalent geometrical imperfection:

$$e_0 \frac{M_{cr}}{N_e} \alpha (\bar{\lambda} - 0.2)$$  \hspace{1cm} (11)

the European flexural buckling curves $\chi(\bar{\lambda})$ have been derived.
The amplitude of the equivalent geometrical imperfection in equation (11) contains an imperfection parameter $\alpha$, that has been used as calibration factor for the evaluation of column tests according to EN 1990 - Eurocode - Basis of Design [4] to reach the characteristic values from the formulae for column resistance:

$$R_k = \chi \cdot A \cdot f_y$$

(12)

Hence the classes of $\alpha$ used in Eurocode 3 expressed by the buckling curves $a_0, a, b, c, d$ do not only cover imperfections, but also on the level of characteristic resistances the model uncertainty and the scatter of yield strength.

**EQUIVALENT GEOMETRICAL IMPERFECTION IN CASE OF NON-UNIFORM MEMBERS AND COMPRESSION FORCES**

Whereas for uniform members and compression forces the reference value for the amplitude of the geometrical imperfection in equation (7) is at the point $x = x_d$ where $\eta_{cru}$ has its maximum $\eta_{cru,max}(x)$, the design point $x = x_d$ is not a priori known in case of non-uniform members and compression forces, see figure 4. Often this point is between the values $x\left(\alpha_{ult,k,\text{min}}\right)$ where the value $\alpha_{ult,k}$ takes the minimum and $x\left(\eta_{cru,max}\right)$, where the curvature $\eta_{cru,max}(x)$ takes the maximum.

Therefore the assumption for the equivalent geometrical imperfection is modified:

$$\eta_{init} = \left[ e_0 \cdot \frac{\alpha_{cru} \cdot N_f(x)}{E I(x) \cdot \eta_{cru}(x)} \right]_{x=x_d} \cdot \eta_{cru}(x)$$

(13)

It needs an iterative process to determine $x = x_d$. 

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Figure 4: Example for a non-uniform buckling compression member and buckling mode $\eta_{\text{crii}}$

EQUIVALENT GEOMETRICAL IMPERFECTION FOR LATERAL TORSIONAL BUCKLING

In case of lateral torsional buckling the buckling mode is characterized by a deflection $\eta_{\text{crii}}$ and a twist $\varphi_{\text{crii}}$, see figure 5, that gives for the upper compression flange

$$\eta_{\text{crii}}^* = \eta_{\text{crii}} + z_{M,FI} \cdot \varphi_{\text{crii}}$$ \hspace{1cm} (14)

This gives a formula for the equivalent geometrical imperfection of the compression flange similar to formula (13):

$$\eta_{\text{crii}}^* = \left[ e^* \cdot \frac{\alpha_{\text{crii}} \cdot N_{E,FI}(x)}{EI_{FI}[\eta_{\text{crii}} + z_{M,FI} \cdot \varphi_{\text{crii}}]} \right] \cdot \left( \eta_{\text{crii}} + z_{M,FI} \cdot \varphi_{\text{crii}} \right) \hspace{1cm} (15)$$

Figure 5: Deflection $\eta$ and twist $\varphi$ from lateral torsional buckling

In order to determine the amplitude $e^*$ from the test evaluation of columns in compression it is necessary to make the conditions for lateral torsional buckling "column-like". To this end, formula (15) can be written in the form

$$\eta_{\text{crii}}^* = \left[ e^* \cdot \frac{\alpha_{\text{crii}} \cdot N_{E,FI}(x)}{EI_{FI}[\eta_{\text{crii}} + z_{M,FI} \cdot \varphi_{\text{crii}}]} \right] \cdot \left( \eta_{\text{crii}} + z_{M,FI} \cdot \varphi_{\text{crii}} \right) \hspace{1cm} (16)$$
where $e_0$ is the equivalent geometrical imperfection for columns, see equation (11) and $\alpha_{\text{cri}}$ is the amplifier of $E_d$ for a member or frame, for which the torsional stiffness $I_t$ is neglected.

A comparison of equations (15) and (16) gives

$$e^* = \frac{M_{R,FI}}{N_{R,FI}} \cdot \alpha^* \cdot (\bar{\lambda} - 2)$$

(17)

where $\alpha^* = \frac{\alpha_{\text{cri}}}{\alpha_{\text{cri}}}$ is the imperfection factor modified for the case of lateral torsional buckling.

This clarifies that the flexural buckling curve as derived in figure 3 is also applicable to lateral torsional buckling, if it is applied to the design point $x = x_d$, and if the imperfection factor $\alpha$ is modified accordingly to the effect of torsional rigidity $I_t$, [5].

DERIVATION OF ACCURATE LATERAL TORSIONAL BUCKLING CURVES

Out-of-plane buckling may be flexural or lateral torsional or a mixture of both. The elastic critical value $\alpha_{\text{cri}}$ may in general be determined from the eigenvalue equation [6]:

$$\begin{bmatrix} E_{1} & 0 \\ 0 & E_{2} \end{bmatrix} \begin{bmatrix} \eta^* \\ \varphi^* \end{bmatrix} - \alpha_{\text{cri}} \cdot \begin{bmatrix} E \left( \eta, \varphi, \eta', \varphi' \right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(18)

As in equation (18) the loading term $E_d$ is in equilibrium with the reaction term $R_k$. The effects of $E_d$ on the flange in compression can be written with taking 2nd order effects into account:

$$E = \frac{\alpha_{E}}{\alpha_{\text{cri}}} \cdot R_k \cdot \frac{1}{1 - \frac{\alpha_{E}}{\alpha_{\text{cri}}}}$$

(19)

This gives the additional moment in the flange:

$$M_{E,FI} = \frac{\alpha_{E}}{\alpha_{\text{cri}}} \cdot E_{FI} \cdot e_0^* \cdot \left[ \frac{\alpha_{\text{cri}} \cdot N_{E,FI} (x)}{E_{FI} \cdot \left( \eta_{\text{cri}} + z_{M,FI} \cdot \varphi_{\text{cri}} \right)} \right]_{x=x_d} \cdot \frac{1}{1 - \frac{\alpha_{E}}{\alpha_{\text{cri}}}} \cdot \left( \eta_{\text{cri}} + z_{M,FI} \cdot \varphi_{\text{cri}} \right)$$

(20)

In using the interaction formula for the resistance of the flange
\[
\frac{N_{E,i} + M_{E,i}}{N_{R,i} + M_{R,i}} = 1
\]

and

\[
N_{R,i} = \alpha_{\text{ult}, \beta} \cdot N_{E,i}
\]

the basic equation

\[
\frac{\alpha_E}{\alpha_{\text{ult}, k}} + \frac{\alpha_E}{\alpha_{\text{ult}, k}} \cdot (\bar{\lambda} - 0,2) \cdot \alpha^* \cdot \frac{1}{1 - \frac{\alpha_E}{\alpha_{\text{crit}}} \cdot \eta_{\text{crit}}(x) + z_M \cdot \varphi_{\text{crit}}(x)} = 1
\]

is obtained which gives for

\[
\chi = \frac{\alpha_E}{\alpha_{\text{ult}, k}}
\]

and at the location \( x = x_d \):

\[
\chi + \chi \cdot \alpha^* \cdot (\bar{\lambda} - 0,2) \cdot \frac{1}{1 - \chi \cdot \bar{\lambda}} = 1
\]

which is the same as the one given in figure 3 for flexural buckling except for the use of \( \alpha^* \) instead of \( \alpha \).

Figure 6 shows how a lateral torsional buckling curve differs from the flexural buckling curves by the use of \( \alpha^* \) instead of \( \alpha \), and figure 7 demonstrates the accuracy of the lateral torsional buckling curve in relation to test results.

**Figure 6:** Shape of lateral torsional buckling curve with imperfections acc. equ. (15) and (17) in relation to the flexural buckling curves KSL a and KSL b  

**Figure 7:** Reliable enveloping of test results with a lateral torsional buckling curve computed with equ. (15)

**DERIVATION OF A MODIFIED LATERAL TORSIONAL BUCKLING CURVE**

Often it looks easier to perform the out-of-plane stability assessment at the location \( x = x \left( \alpha_{\text{ult}, \text{min}} \right) \), where the in-plane loads give the maximum action effects instead of at the location \( x = x_d \) as assumed in the previous section.
in this case the reference value \( \bar{\lambda} = \sqrt{\frac{\alpha_{ul,k}(x_d)}{\alpha_{crit}}} \) would be substituted as follows:

\[
\bar{\lambda} = \sqrt{\frac{\alpha_{ul,k}(x_d)}{\alpha_{crit}}} \cdot \sqrt{\frac{\alpha_{ul,k,min}}{\alpha_{crit}}} \cdot \sqrt{\bar{\lambda}_{mod} \cdot \sqrt{\beta}}
\] (26)

Instead of equation (25) the basic equation then reads

\[
\chi_{mod} \cdot \frac{1}{\beta} + \chi_{mod} \cdot \frac{1}{\beta} \cdot \alpha' \cdot (\bar{\lambda}_{mod} \cdot \sqrt{\beta} - 0.2) \cdot \frac{1}{1 - \frac{\chi_{mod}}{\beta} \cdot \bar{\lambda}_{mod}^2 \cdot \beta} = 1
\] (27)

and gives the following modified lateral torsional buckling curve:

\[
\chi_{mod} = \frac{\beta}{\phi + \sqrt{\phi^2 - \beta \cdot \bar{\lambda}_{mod}^2}}
\] (28)

with \( \phi = 0.5 \left[ 1 + \alpha' \cdot (\sqrt{\beta \cdot \bar{\lambda}_{mod}^2 - 0.2} + \beta \cdot \bar{\lambda}_{mod}^2) \right] \)

\[
\bar{\lambda}_{mod} = \sqrt{\frac{\alpha_{ul,k,min}}{\alpha_{crit}}}
\]

\[
\beta = \frac{\alpha_{ul,k}(x_d)}{\alpha_{ul,k,min}} \geq 1
\]

The results from this modified lateral buckling curve are identical with those from the lateral buckling curve applicable to \( x = x_d \).

Figure 8 gives an example for the shape of the modified lateral torsional buckling curve, and figure 9 gives some indications for \( x = x_d \) and \( \beta = \frac{1}{f} \).

![Figure 8: Comparison of lateral buckling curves applicable to \( x_d \) or \( x(\alpha_{ul,k,min}) \)](image-url)
CONSIDERATION OF ADDITIONAL OUT-OF-PLANE LOADING

For flexural buckling a possibility to take additional out-of-plane loading proportional to the buckling eigenmode \(\eta_{\text{eq}}\) into account has been derived by Roik and Kindmann [7]. It gives the assessment equation

\[
\Delta n_E \leq \Delta n_R
\]

(29)

Where

\[
\Delta n_E = \frac{N_E}{\lambda \cdot N_R} + \frac{M_{x,0}}{M_{y,R}}
\]

(30)
The first term of equation (30) represents the portion from imperfections and the second term represents the additional transverse moment from out-of-plane loads.

\[ \Delta n_R = 1 - \frac{N_E}{N_R} \left( 1 - \frac{N_E}{N_R} \right) \gamma^2 \beta^2 \leq 0.90 \]  

(31)

This method can be extended to lateral torsional buckling using [6]

\[ \Delta n_E = \frac{1}{\chi \alpha_{alt,k}} + \frac{M_{E,FL}}{M_{R,FL}} (1 - q_M) + \frac{B_{E,FL}}{B_{R,FL}} (1 - q_B) \]  

\[ \Delta n_E = 1 - \frac{1}{\chi \alpha_{alt,k}} \left( 1 - \frac{1}{\chi \alpha_{alt,k}} \right) \gamma^2 \beta^2 \]  

(32)

where \( M_{E,FL}, M_{R,FL} \) are the out-of-plane bending moments in the cross section, \( B_{E,FL}, B_{R,FL} \) are the warping bi-moments in the cross section and \( q_M, q_B \) are factors to account for moment distributions not proportional to \( \eta_{cr}, \phi_{cr} \). Some \( q_M \) values are given in figure 10.

<table>
<thead>
<tr>
<th>Moment distribution ( M_z )</th>
<th>( q_{M_z} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ q_{M_z} = 0.21 \cdot (1 - \psi_z) + 0.36 \cdot (0.33 - \psi_z) \cdot \frac{1}{\alpha_{crit}} ]</td>
<td>( \leq \frac{1}{\alpha_{crit}} )</td>
</tr>
</tbody>
</table>
| \[ q_{M_z} = \frac{1}{\alpha_{crit}} \left( \frac{\pi^2 EI_z \cdot \max|\delta_z|}{f^3 \cdot \max|M_{z,ED}|} \right) \] | \[ \max|\delta_z| = \text{maximum lateral deflection along the member} \]  
| \[ \max|M_{z,ED}| = \text{maximum lateral bending moment along the member} \] |
| \[ q_{M_z} = 0.18 \cdot \frac{1}{\alpha_{crit}} \] | \[ q_{M_z} = 0.03 \cdot \frac{1}{\alpha_{crit}} \] |

Figure 10: Moment coefficient \( q_{M_z} \) for lateral torsional buckling [9]

JUSTIFICATION OF THE RULES FOR LATERAL TORSIONAL BUCKLING BY TESTS

General

The complete set of rules as specified above has been evaluated has been evaluated with test results according to EN 1990 -Annex D, to verify that they give characteristic values of resistance and what magnitude of partial factor \( \gamma_M \) is necessary.

In the following examples from test evaluations are given

Lateral torsional buckling tests

Tests published in the years 1969-1984 were checked [10] to select 144 tests with rolled sections and 71 tests with welded sections for test evaluation, because they were sufficient documented to be useable for evaluation, see figure 11.
The ratios of experimental results and results from the rules for roled sections and welded sections calculated with $\alpha^*$-values based on $\alpha$-values for buckling about the weak axis taken from table 6.2 of EN 1993-1-1 are given in figure 12.

The $\gamma_M^*$ -value resulting from log-normal distributions of the data, see figure 13, is $\gamma_M^* = 1.048$ for rolled sections and $\gamma_M^* = 1.087$ for welded sections.

Figure 11: Loading conditions and support conditions for lateral torsional buckling tests

Figure 12: Test evaluation of lateral torsional buckling tests for rolled sections and welded sections

Figure 13: Longitudinal distribution of test data for rolled sections and welded sections
Lateral torsional buckling tests with channels

Tests according to figure 14 with variations of the angle $\rho$ and of the eccentricities $y_p$ and $z_p$ of the lateral load $F$ [8], tests according to figure 15 with bending and torsion from a lateral force $F$ and with compression from a force $N$ [8] and also tests according to figure 16 with eccentric compression were used to evaluate the rules for lateral torsional buckling with additional out-of-plane loading.

Figure 17 gives the ratio of experimental and calculative results and figure 18 shows the log-normal distribution of results that gave $\gamma_M^* = 1,095$.

![Figure 14: Tests with beams with bi-axial bending and torsion](image1)

![Figure 15: Tests with beams with bending, torsion and compression](image2)

![Figure 16: Tests with beams in compression with load eccentricities](image3)

![Figure 17: Test evaluation using the "plastic warping resistance"](image4)

![Figure 18: Log-normal distribution of results](image5)

Lateral torsional buckling tests with rolled sections under bending, torsion and compression

For beams with channel sections loaded in bending according to figure 19 and in bending and compression according to figure 20. Figure 21 gives the ratio of experimental and calculative results and figure 22 gives the log-normal distributions of results yielding a $\gamma_M^*$-value $\gamma_M^* = 1,083$ with using plastic warping resistances.
Haunched beams

Lateral torsional buckling tests were carried out for a haunched beam as given in figure 23 [11], [12] where the following parameters were varied: \( k_v = 1.5 \), \( k_L = 2 / 6 ; 1 / 3 ; 1 / 3 \) and \( f_0 = \frac{M_F}{M_S} = 1 / 2 ; 1 / 3 \).

The assumptions for the calculations may be taken from figure 24. Figure 25 gives the ratios of the experimental and calculative results. The log-normal distribution of the results, see figure 26, gives the \( \gamma_M \)-value \( \gamma_M^* = 0.998 \).

Figure 23: Test beams with haunches with parameters for variation and details of haunches
Beams with fin-plate connections
For tests with beams in bending as given in figure 27 [13] with fin-plate connections with a variation of the dimensions of the fin-plates and of the cope cuts as given in figure 28 evaluations were undertaken by using FEM-calculations. Figure 29 gives the ratio of experimental results and calculative results taking account of the position of the load in the axis of the fin-plates. The log-normal distribution of test results may be taken from figure 30. It leads to $\gamma_{mL} = 0.953$.
EXAMINATION OF ACCURACY OF THE ALTERNATIVE RULES FOR OUT-OF-PLANE STABILITY GIVEN IN EN 1993 - EUROCODE 3 - PART 1-1

**Lateral torsional buckling curves according to EN 1993-1-1, section 6.3.2.3**

In the following the "General Method", as specified above, has been used to examine the accuracy of the alternative methods for assessing the out-of-plane stability given in EN 1993-Eurocode 3 - Part 1-1. Figures 31 up to 36 give comparisons of the results obtained from the lateral torsional buckling curve in EN 1993-1-1, section 6.3.2.2 (→ $\chi \cdot LT$) and section 6.3.2.3 (→ $\chi \cdot LT \cdot mod$) and those from the "General Method" (→ $\chi \cdot LT \cdot GM$). The particular differences of curves for a linear moment distribution may be taken from figures 37 and 38, which result in effects given in figure 39.
Figure 31: LT-buckling curves for simple beams under uniform load

Figure 32: LT-buckling curves for simple beams under concentrated load at midspan

Figure 33: LT-buckling curves for fixed-end beam under uniform load

Figure 34: LT-buckling curves for fixed-end beams under concentrated load at midspan

Figure 35: LT-buckling curves for beams with one fixed end under uniform load

Figure 36: LT-buckling curves for beams with one fixed end under uniform load

Figure 37: LT-buckling curve $\chi_{LT,mod}$ acc. to [1] for simple beams with end moments

Figure 38: LT-buckling curve $\chi_{LT,GM}$ of General Method for simple beams with end moments

Figure 39: Deviation of LT-buckling curve $\chi_{LT,GM}$ acc. to General Method from $\chi_{LT,mod}$ curve acc. to [1] for simple beams with end moments
Lateral torsional buckling with additional out-of-plane loading according to EN 1993-1-1, section 6.3.3

For lateral torsional buckling with additional out-of-plane loading figure 40 gives an example for which the utilisation rates for different scaling values $\beta$ have been determined, see figure 41. The utilisation rates demonstrate the large differences of results from the methods in Annex A and Annex B of EN 1993-1-1. Figure 42 compares the utilisation rates from both methods with the results from the "General Method" and the numerical method. For the example in figure 43 the values $\alpha_{Ed}$ applied to the loading are plotted versus the span length in figure 44. In conclusion the alternative design rules in EN 1993-1-1 give results that differ from the results of the "General Method" by about 10%.

Figure 40: Calculation example 1 - Simple beam with in- and out-of-plane loading

Figure 41: Utilisation rates of calculation example 1 for different scaling factors $\beta$ acc. to EN 1993-1-1 section 6.3.3(4)

Figure 42: Calculation example 1 - Comparison of utilisation rates from EN 1993-1-1 using Annex A and B with results from the "General Method" and a numerical method

Figure 43: Calculation example 2 - Beam with one fixed end with in- and out-of-plane loading

Figure 44: Calculation example 2 - Comparison of utilisation rates from EN 1993-1-1 using Annex A and B with results from the "General Method" and a numerical method
Stable lengths according to EN 1993-1-1, section 6.3.5.3

Figures 45 and 46 give "stable length" for a member with end-moments according to the "General Method" and rules in EN 1993-1-1, section 6.3.5.3. It is evident that the rules in 6.3.5.3 are safe-sided because effects of torsional stiffness are neglected. The rules for stable lengths in EN 1993-1-1, Annex BB3 are based on approximate assumptions yielding to safe-sided or non safe-sided results depending on the case at hand.

![Figure 45](image1.png)  
![Figure 46](image2.png)

Figure 45: Stable length of a simple beam (IPE 600) with lateral supports at its ends and linear moment distribution

Figure 46: Stable length of a simple beam (HE300B) with lateral supports at its ends and linear moment distribution

CONCLUSIONS

The "General Method" in section 6.3.4 of EN 1993 - Part 1-1 has not yet been specified sufficiently in this code to use its full potential for the stability assessment of beams for flexural and lateral torsional buckling.

This paper presents an extension of the basic rules given in EN 1993, so that it is applicable to any out-of-plane stability problem.

The basis of this extension is a generalisation of the rules for equivalent geometric imperfections for flexural and lateral torsional buckling that is considered as the basis both for numerical assessments (e.g. by FEM) and for formulae for hand-calculation (e.g. with the use of buckling curves).

The derivation of the buckling curves for flexural buckling and for lateral torsional buckling makes clear that both buckling phenomena have a common origin and differences arise from the torsional stiffness $GJ$.

There are two alternative approaches for buckling curves. One for the case that the design location along the beam is known, and the other for the case that the location with the highest stresses from in-plane loads is taken as a reference. Both approaches give identical results.

Additional loading transverse to the main load carrying plane of the beam (bending and torsion) can be easily integrated in the assessment procedure of the "General Method".
The reliability of the method has been checked by test evaluations according to EN 1990-Anex D.

Comparisons of the results of the "General Method" with the results of the alternative methods given in EN 1993-1-1 reveal certain differences. The results of alternative methods are often in the range of results of the "General Method" and depending on the case sometimes less economical and sometimes less safe.

REFERENCES

NON-LINEAR REFINED FINITE ELEMENT ELASTIC ANALYSIS OF STEEL FRAMES WITH GENERALISED TRANSVERSE MEMBER LOADS

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KEYWORDS
Elastic, finite element, member load effect, minimal discretisation, non-linear, steel frames, superposition, transverse loading.

ABSTRACT

In the finite element modelling of steel frames, external loads usually act along the members rather than at the nodes only. Conventionally, when a member is subjected to these transverse loads, they are converted to nodal forces which act at the ends of the elements into which the member is discretised by either lumping or consistent nodal load approaches. For a contemporary geometrically non-linear analysis in which the axial force in the member is large, accurate solutions are achieved by discretising the member into many elements, which can produce unfavourable consequences on the efficacy of the method for analysing large steel frames. Herein, a numerical technique to include the transverse loading in the non-linear stiffness formulation for a single element is proposed, and which is able to predict the structural responses of steel frames involving the effects of first-order member loads as well as the second-order coupling effect between the transverse load and the axial force in the member. This allows for a minimal discretisation of a frame for second-order analysis. For those conventional analyses which do include transverse member loading, prescribed stiffness matrices must be used for the plethora of specific loading patterns encountered. This paper shows, however, that the principle of superposition can be applied to the equilibrium condition, so that the form of the stiffness matrix remains unchanged with only the magnitude of the loading being needed to be changed in the stiffness formulation. This novelty allows for a very useful generalised stiffness formulation for a single higher-order element with arbitrary transverse loading patterns to be formulated. The results are verified using analytical stability function studies, as well as with numerical results reported by independent researchers on several simple structural frames.
INTRODUCTION

Generalised load cases for steel framed structures, such as permanent loads, live loads, wind loads and their combination, usually involve patterns of loading which act transversely along the members of the frame. It is usual in second-order finite element modelling to convert these loads to nodal loads, and to discretise the member into several elements, with the transverse loads taken account of as nodal forces in order to capture the structural response accurately. Because of the coupling of the member loading with the axial loading, incorporating transverse loads has been a barrier for formulations which claim to represent proper second-order structural behaviour using one element per member. Kondoh and Atluri [1] presented a simplified procedure for the finite deformation analysis of space frames using one beam element to model each member, which involved the non-linear coupling of bending and stretching. However, the use of a few elements was required for a single member in some of the examples in order to obtain accurate solutions. Al-Bermani and Kitipornchai [2] addressed a method which is capable of the accurate modelling of large deflections in frame structures using one element per member, and applied it to the analysis of transmission line structures. Later, Chan and Zhou [3] developed a fifth-order finite element to simulate the second order effect on a member using one element with an initial geometric imperfection. Izzuddin [4] subsequently formulated a fourth-order displacement-based finite element for structures under thermal loads, while Liew et al. [5] made use of a stability function formulation in their stiffness matrices so that geometric non-linearity in a member could be incorporated using one element.

Despite the advocacy of using a second-order analysis with a single element discretisation for each member, it appears that an efficient element of this type which accounts for member loading has not been presented in the open literature, and either consistent or lumped load methods are used in lieu of incorporating transverse loading into the element formulation. The main drawback of using lumped loads is its inaccuracy, since it takes the form of a first-order member loading response when using one element per member. The consistent load method is also unable to reproduce second-order member loading throughout the entire loading range using one cubic element per member, because it ignores the effect of coupling of the bowing deformations of the member and axial loads; particularly under moderate and high axial loads. Because of this, most reported research has hitherto accounted for the coupling effect at the system level by merely dividing the member into a few elements, with the consequence that using several elements to correctly capture the member load effect increases computational time, as well as effort in data preparation and output interpretation.

In order to eliminate the need to use many elements per member to conduct an accurate second-order structural analysis with transverse loads, Zhou and Chan [6] presented a second-order analysis that is capable of modelling the effects of member loads in the element stiffness formulation, as an alternative to doing so by a system analysis. However, each member loading pattern or regime requires a specific element stiffness matrix, which is limiting because of the usual multiplicity of loading scenarios met in practice. To overcome this difficulty, a single sophisticated element is developed in the present paper which facilitates the modelling of second-order loading effects covering a wide range of loading regimes using a generalised element formulation, which is founded on the principle of superposition of simple loading cases within a second-order analysis framework. It allows for complex loading regimes to be decomposed into these simple or fundamental loading cases, each of which is characterised by one representative bending moment coefficient, and for which the load effect for the regime in question is defined by the sum of these moment
coefficients only at the advent of the non-linear analysis, with the element stiffness matrix itself remaining unchanged. As such, the method is a trade-off between simplicity in the formulation and accuracy in describing the member load effect, since the expression for the element stiffness matrix is more concise for the simpler fundamental load cases, with drift-off error being eliminated in the non-linear solution procedure through a predictor-corrector method. The ranges of the validity of the proposed non-linear analysis which incorporates member loading are illustrated through examples.

ASSUMPTIONS AND INTERPOLATION FUNCTIONS

The following assumptions are made in the formulation: (1) the beam is prismatic and slender, with the Euler-Bernoulli hypothesis being valid; (2) warping deformations, shear deformations and the Wagner effect are neglected, so that lateral buckling is not considered; (3) the loads increase and decrease incrementally and proportionally; (4) the loading is conservative, with both nodal and member loading being admissible; and (5) the strains are small but large deformations are included. The transverse loading is not restricted as can occur in conventional finite element formulations, insofar as lumped and consistent nodal approaches are not merely used to treat the transverse load in the formulation.

![Equilibrium conditions for element under general loading (about z-axis)](image)

The vector of deformations along an element are taken as \( \mathbf{u}(x) = \{u(x), v(x), w(x), \phi(x)\}^T \), which comprise of the deformations \( u \) in the longitudinal \( x \)-direction, \( v \) in the \( y \)-direction, \( w \) in the \( z \)-direction and the twist \( \phi \) about the \( x \)-axis. Because the displacement functions for the element representation herein are referred to a co-rotational coordinate formulation, the dependent variables for the transverse displacements \( v \) and \( w \) are replaced by the nodal rotations \( \Theta_1 \) and \( \Theta_2 \) about the \( z \) and \( y \)-axes respectively. External transverse loads on a member generate additional non-linear effects that are manifested in the second-order member response. To this end, the effect of transverse loading in the member is taken into account in the element stiffness formulation, in which a relationship between the deflections and the loading under transverse member loads is modelled accurately and adequately using a single element. As a result, apart from satisfying the primary kinematic boundary conditions, the displacement function proposed which includes the general transverse member distributed loading \( q \) and concentrated loading \( Q \) shown in Figure 1 can be derived by satisfying the secondary static boundary condition of force equilibrium. Without loss of generality, the mid-span moment \( M_0 \) obtained by superimposing the loading effects using elementary force statics is used in the equilibrium condition for the moments about the \( z \) and \( y \)-axes; this superposition being valid prior to the commencement of the non-linear analysis. Further, the second-order moments \( P_v \) and \( P_w \) due to the member P-\( \delta \) effects are also introduced into the equilibrium equation when equilibrium is formulated along the element instead of at the end nodes of the element. It is therefore helpful to incorporate the member bowing and member load effects into the element stiffness formulation based on a single element, whose higher-order elastic displacement function is derived in the following.
Concentrated moment

\[ 8M \quad (a \leq L/2) \]
\[ 8M \quad (a \geq L/2) \]

Point load

\[ 8Q_o \quad (a \leq L/2) \]
\[ 8d(L-a) \quad (a \geq L/2) \]

General n point loads

\[ \frac{L}{EI} \sum_{i=1}^{n} 8Q_i \alpha_i \quad (a \leq L/2) \]
\[ \sum_{i=1}^{n} 8Q_i(L-a_i) \quad (a \geq L/2) \]

Uniformly distributed load over entire length

\[ 2qL \]

Uniformly distributed load over partial length

\[ 8qb \left( a + \frac{b}{2} \right) \quad (a+b \leq L/2) \]
\[ 8qb \left( L - \left( a + \frac{b}{2} \right) \right) \frac{a+b}{2} \quad (a \leq L/2 \leq a+b) \]
\[ 8qb \left( L - \left( a + \frac{b}{2} \right) \right) \quad (a \geq L/2) \]

General n uniform loads

\[ \sum_{i=1}^{n} 8q_i \left( a + \frac{b_i}{2} \right) \quad (a+b \leq L/2) \]
\[ 8q_i \left( L - \left( a + \frac{b_i}{2} \right) \right) \left( \frac{L}{2} - a_i \right)^2 \quad (a \leq L/2 \leq a+b) \]
\[ \sum_{i=1}^{n} q_i \left( L - \left( a + \frac{b_i}{2} \right) \right) \quad (a \geq L/2) \]

Hydrostatic loading

\[ 4q\left( a + \frac{2b}{3} \right) \quad (a+b \leq L/2) \]
\[ 8q \left( \frac{b}{3} \left( L - \left( a + \frac{2b}{3} \right) \right) - \left( L/2 - a \right)^2 \right) \frac{3b}{3b} \quad (a \leq L/2 \leq a+b) \]
\[ 4q \left( L - \left( a + \frac{2b}{3} \right) \right) \quad (a \geq L/2) \]

\[ 4q\left( a + \frac{b}{3} \right) \quad (a+b \leq L/2) \]
\[ 8q \left( \frac{b}{3} \left( L - \left( a + \frac{b}{3} \right) \right) \left( L/2 - a \right) \left( L/2 - a + b \right) \right) \frac{3b}{3b} \quad (a \leq L/2 \leq a+b) \]
\[ 4q \left( L - \left( a + \frac{b}{3} \right) \right) \quad (a \geq L/2) \]

Figure 2: Values of the moment factor \( \bar{M} \_o/EL \) for various loading cases

Linear functions are assumed for the axial deformation and twist; pure axial deformation and twist are assumed as being independent of the member load effect, so that

\[ u = (1-\xi)u_1 + \xi u_2 \quad \text{and} \quad \phi = (1-\xi)\theta_1 + \xi \theta_2, \quad (1) \]

in which \( u = u_1 \) at \( x = 0 \), \( u = u_2 \) at \( x = L \) are the axial nodal deformations, \( \phi = \theta_1 \) at \( x = 0 \), \( \phi = \theta_2 \) at \( x = L \) are the twist nodal deformations, and where \( \xi = x/L \). In order to include the member bowing effect and transverse member loading in a single element, the kinematic boundary conditions about the \( y \)-direction are \( w = 0 \) and \( \partial w/\partial x = 0 \) at \( x = 0 \) and \( L \), while the equation of bending given by

\[ EI \frac{\partial^2 w}{\partial x^2} = Pw - M_1 (1-\xi) + M_2 \xi + M_0 \quad (2) \]

leads to the deflection

1189
\[
\begin{align*}
\omega &= \frac{4(24+q)\xi^2}{48+\omega} + \frac{(48+5q)\xi^3}{48+\omega} - \frac{2q\xi^4}{48+\omega} \quad \text{for} \\
&= \left[ \frac{(48-q)\xi^2}{48+\omega} + \frac{3(16-q)\xi^3}{48+\omega} + \frac{2q\xi^4}{48+\omega} \right] \omega_{12} + \omega_{11},
\end{align*}
\]

in which \(\omega = PL^2/EI\) is a dimensionless axial load parameter and \(N_1, N_2\) and \(N_q\) are displacement functions with respect to the first and second node rotations, and member loads respectively. The equivalent mid-span moment \(\bar{M}_0\) for a variety of member loads is given in Figure 2. The transverse displacement \(w\) in the \(z\) direction can be defined similarly. The different member load responses for different loading regimes is merely incorporated into the equivalent moment \(\bar{M}_0\) given in Figure 2 which does not depend on the independent variable \(x\), but only on the magnitude of the loading and the point of application of the load with respect to the mid-span location. This is significant, because it allows the principle of superposition for the general load cases, and so only the fundamental load cases listed in Figure 2 are needed to customise complex loading regimes in the second-order analysis.

**STIFFNESS FORMULATION FOR 4TH ORDER BEAM-COLUMN ELEMENT**

The internal strain energy \(U\) caused by the axial strain and twist strain can be expressed as

\[
U = \int \left\{ \frac{EA}{2} \left( \frac{d\varepsilon}{dx} \right)^2 + \frac{P}{2} \left( \frac{d\varepsilon}{dx} \right)^2 + \frac{EI_z}{2} \left( \frac{d^2\varepsilon}{dx^2} \right)^2 + \frac{EI_y}{2} \left( \frac{d^2\varepsilon}{dx^2} \right)^2 + \frac{GJ}{2} \left( \frac{d\phi}{dx} \right)^2 \right\} dx,
\]

in which \(EA\) is the axial rigidity, \(EI_z\) and \(EI_y\) the flexural rigidities about the \(y\) and \(z\)-axes respectively, \(GJ\) the torsional rigidity, \(P\) the axial force; and \(E\) is the elastic modulus and \(G\) the shear modulus.

In this study, external loads are produced by nodal force vectors \(f\) and member load vectors \(\Phi\), so that the external work done \(V\) comprises of two components. The first is the work done by the nodal forces \(f\) in moving through nodal displacements \(u\), while the second is the work done by the transverse member load \(\Phi\) moving through the assumed transverse displacement field associated with the element displacement function vector \(N\) over the element length, in which \(u = [\Delta u, \theta_1, \theta_2, \theta_3, \theta_4]^T\) with \(\Delta u = u_1 - u_2\) and \(\Delta \theta = \phi_1 - \phi_2\). The principle of superposition can be applied to simplify the effect of the member load \(\Phi\) on the external work \(V\), for which in accordance with the assumption of conservative loading the work done \(V\) caused by the member load vector \(\Phi\) moving through the element deflections represented by \(N\) is independent of the axial load \(P\) (and thus axial load parameter \(\omega\)) throughout; hence setting \(\omega = 0\) gives

\[
V = \int_u [u^T N^T \Phi dx + u^T f] dx.
\]
order analysis, it is obtained from the internal strain energy in Eqn. (5) and external work done in Eqn. (6) as

$$\Pi = U - V.$$  \hspace{1cm} (7)

The strain energy functional in Eqn. (5) depends not only on the variables $u_k$ but also on the axial load parameter $\omega$. Hence from Castigliano’s first theorem of strain energy, the secant stiffness matrix is obtained from

$$K_s = \frac{\partial U}{\partial u_k} + \frac{\partial U}{\partial \omega} \frac{\partial \omega}{\partial u_k}.$$  \hspace{1cm} (8)

This then leads to

$$M_{a1} = \frac{\partial U}{\partial \theta_{a1}} = \frac{E l_0}{L} \left( C_1 \theta_{a1} + C_2 \theta_{a2} + C_3 \ddot{M}_0 \right),$$  \hspace{1cm} (9)

in which

$$C_1 = \frac{9216}{5(48 + \omega)^2} \frac{3456}{5} \frac{68\omega^2}{5} \frac{8\omega^3}{105}; C_2 = \frac{4668}{5(48 + \omega)^2} \frac{576\omega}{42}; C_3 = \frac{\omega^2}{210(48 + \omega)^2}$$  \hspace{1cm} (10)

and to

$$M_{a2} = \frac{\partial U}{\partial \theta_{a2}} = \frac{E l_0}{L} \left( C_1 \theta_{a1} + C_2 \theta_{a2} - C_3 \ddot{M}_0 \right),$$  \hspace{1cm} (11)

in which $(\cdot)_a = (\cdot)_y$ or $(\cdot)_z$ as appropriate. Eqn. (8) also leads to

$$P = P_1 - P_2 = \frac{\partial U}{\partial e} + \frac{\partial U}{\partial \omega} \frac{\partial \omega}{\partial e} = E A \left[ \frac{e}{L} + \sum_{a=y,z} \left( C_3 + b_{a1} \left( \theta_{a1} - \theta_{a2} \right) \ddot{M}_0 + b_{a2} \dddot{M}_0 \right) \right]$$  \hspace{1cm} (12)

and

$$C_p = b_1 \left( \theta_{a1} + \theta_{a2} \right)^2 + b_2 \left( \theta_{a1} - \theta_{a2} \right)^2$$  \hspace{1cm} (13)

$$b_1 = \frac{6(48)^2}{5} + \frac{18(48)\omega}{5} + \frac{126\omega^2}{35} + \frac{29\omega^3}{40}; b_2 = \frac{2(48)^2}{5} + \frac{14(48)\omega}{5} + \frac{66\omega^2}{35} + \frac{11\omega^3}{840}; b_{a1} = \frac{16\omega}{35(48 + \omega)^3}; b_{a2} = \frac{-12 + \omega}{35(48 + \omega)^3}$$  \hspace{1cm} (14)

It can be seen that the internal strain energy $U$ is load-dependent, so that coupling of the external member load and the member deformations is inherent in the stiffness formulation of Eqs. (9) to (12).

The tangent stiffness matrix is obtained by taking the second derivative of the total potential functional in Eq. (7) with respect to the variables $u_k$ and axial load parameter $\omega$. When the work done $V$ is linear, this differentiation results in
\[ K_i = \frac{\partial^2 \Pi}{\partial \mathbf{u}_i \partial \mathbf{u}_i} = \frac{\partial}{\partial \mathbf{u}_j} \left( \frac{\partial U}{\partial \mathbf{u}_i} \right) + \frac{\partial}{\partial \omega} \left( \frac{\partial U}{\partial \mathbf{u}_i} \cdot \frac{\partial \omega}{\partial \mathbf{u}_j} \right). \]  \hfill (15)

The tangent stiffness matrix needed for assembly and transformations in global coordinates \( K_T \) is

\[ K_T = \sum_{\text{elements}} L K_e L^T = \sum_{\text{elements}} L \left( T^T K_e T + M_e \right) L^T, \]  \hfill (16)

in which \( T \) is the transformation matrix relating the member forces to the element forces in local coordinates, \( L \) is the transformation matrix from the local coordinates to the global coordinates and \( M_e \) is a stability matrix to allow for the work done by rigid body motions or the change of geometry of the structures. Because of the nature of the non-linearity in Eq. (7), an incremental-iterative solution procedure is needed to trace the non-linear equilibrium path.

The non-linear solution procedures then follow those of the well-known arc length procedure [7] or the minimum residual displacement method of Chan [8]. The standard Newton-Raphson representation is implemented from the total potential energy equation (7) by converting it into a linear incremental relationship from its Taylor expansion, so that equilibrium at the solution point is represented as

\[ \Delta \mathbf{f} = \mathbf{f} - K_S \mathbf{u} = K_T \Delta \mathbf{u}, \]  \hfill (17)

in which \( K_S \) is the secant stiffness matrix in global coordinates, \( K_T \) the tangent stiffness matrix in Eq. (16), \( \mathbf{u} \) the vector of displacements, \( \Delta \mathbf{u} \) the vector of incremental displacements, \( \mathbf{f} \) the load vector and \( \Delta \mathbf{f} \) the prescriptive load increment or unbalanced force.

![Figure 3: Load-deformation response of right-angle frame.](image)

**NUMERICAL VERIFICATIONS**

**Postbuckling of Right-Angled Frame**

Roorda [9] and Koiter [10] provided the first experimental and analytical results respectively for the right-angled frame shown in Figure 3, with the analytical formulation accounting for member bowing and for postbuckling. This structure was later studied by Chan and Zhou [3], and others. The frame in with pin supports was analysed herein with a member point
load $P$ at an eccentricity of $e = 254$ mm, applied directly to the beam without resolving it as an eccentric moment and point loads at the member nodes. Figure 3 plots the joint rotation $\theta$ against the dimensionless load $P/P_E$, where $P_E$ is the Euler load. The proposed non-linear modelling using only one element produces results which are in close coincidence with those of Chan and Zhou [3], as well as approaching the postbuckling response of the perfect frame ($e = 0$) given by Koiter. This example validates that the present approach is capable of accounting for the member load effect of a simple framed structure in which load transformation is needed between member and global coordinates. It should be noted that the present analysis produces slightly larger joint rotations than those obtained by Lu and Bradford [11] which utilise lumped loading, as the load produced in the beam due to its restraint induces a minor second-order member load effect.

![Figure 4: Two-storey steel frame](image)

Two-Storey Rigid Steel Sway Frame

Applied forces in practical steel frames commonly act along the members, and these contribute to sway deflections in rigid frames. It is important, therefore, to be able to use an element permitting member load effects in second-order frame analysis to predict these sway deflections. The two-storey frame shown in Figure 4 has been studied; this frame was also analysed by Zhou and Chan [6] and its geometry, member cross-sections and material properties are given in Figure 4. The frame is subjected to a uniformly distributed gravity loading $q$ on both beams and to a lateral loading $\alpha q$, where $\alpha$ is taken as $10^{-3}$, $10^{-2}$ and $10^{-1}$. Figure 4 shows the sway behaviour in terms of the lateral drift of the roof of the building, which is effected by both $P-\delta$ and $P-\Delta$ responses. The lateral force parameter $\alpha$ has a large influence on the frame behaviour, with the structural responses being different for the three
values of $\alpha$ considered. It can be seen, however, that the discrepancies between the results using the lumped loads and the present member load approach for each value of $\alpha$ are not overly large. This is because the most significant contribution to the P-$\Delta$ sway effect is the lateral force rather than nodal moments, so that the load lumping method retains the important lateral force effects, and its effect especially for low axial forces is very minor. With larger loads the discrepancy increases owing to the coupling effect between the member load effect and the element stiffness.

CONCLUDING REMARKS

The implication of the superimposition principle being applied in the derivation of the element stiffness formulation is that the large number of general member loading scenarios can be simply and succinctly unified from a few standard individual load cases. As a result, the element stiffness matrix with the member load does not need to be reformulated for these numerous member load cases. The present numerical analysis is therefore versatile and adaptive to a steel structure with diverse member load cases without loss of accuracy.

In this novel approach of including the member load effect, the first-order member load response is successfully implemented in a simple manner. Moreover, the present higher-order element formulation is also capable of capturing the second-order member load effect accurately, which is the member bowing effect being magnified by the transverse member load, since the coupling between the member load effect and the element stiffness are taken into account by the present approach. In particular, when a member is subjected to a high level of axial load, this second-order member load effect is of much significance. In short, the key feature of this paper is to augment a higher-order element formulation for a single element with the member load effect to include the first- and second-order member load response. It consequently leads to a less computational demanding and user-friendly numerical analysis with the member load effect for general applications for steel frames.

REFERENCES


FATIGUE CLASSIFICATION OF WELDED JOINTS IN ORTHOTROPIC STEEL BRIDGE DECKS

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KEYWORDS

Bridges, orthotropic decks, welded joints, fatigue.

ABSTRACT

The objective of the study in this paper is to provide the code writers of the Eurocode on Steel Structures with additional information on the fatigue classification of welded details in orthotropic steel bridge decks. The current Eurocode 3 – Design of steel structures – Part 1.9 Fatigue, still needs some modification with respect to the design ∆S-N curves and there is a strong need for considering the mean ∆S-N curves for the fatigue evaluation of the existing orthotropic bridge decks. A review of all available data on orthotropic decks and related fatigue test results (762 tests) is included. The statistical analysis procedure used to derive the weld classes for Eurocode 3 is used to place all of the main joints in the appropriate class, and rank different designs of each joint. For a risk-based maintenance it is necessary to guarantee the safety of the structure with a required safety level described as the reliability index β. Target values for the reliability index β are given in Eurocode 1 – Basis of structural design. Depending on degree of possible inspection, repair and damage tolerance, the target reliability index β for fatigue varies from 1.5 to 3.8. In the fatigue lifetime calculations different safety factors are used considering the detail classification, the number of stress cycles or stress ranges. These safety factors all have to be used in combination with the mean value of the fatigue strength and not with the characteristic value of these curves. In this paper also the mean values at two million cycles of the different data sets are given.

INTRODUCTION

Orthotropic steel bridge decks

The orthotropic steel deck (see figure 1) consists of a deck plate supported in two mutually perpendicular directions by a system of transverse and longitudinal stiffeners (AISC, 1963). It may therefore be likened to a plate with dissimilar elastic properties in two directions, in other words it is effectively an ORTHOgonal anisoTROPIC plate or, for brevity, an
‘orthotropic plate’. In this form of construction the deck itself can form the top flange of the main load-carrying member making up the bridge, in some instances it has, for example, formed the top flange of a box girder. The use of orthotropic decks was provided by their relatively low weight, which made them attractive where a high degree of pre-fabrication or rapid erection was required. In addition the development of weldable high strength steels enabled further desirable reductions in deck weight to be obtained.

![Diagram of orthotropic bridge deck structure](image)

Figure 1: Typical orthotropic bridge deck structure (AISC, 1963)

**Analysis of fatigue data**

Generally, the fatigue strength curves are evaluated from series of fatigue tests performed on specimens which typically reproduce the detail to be studied. Fatigue strength curves (Δσ-N curves or S-N curves) can be most accurately determined when a group of fatigue specimens is tested for each series corresponding to different stress range levels. The S-N values are presented in a graph showing log N (endurance in cycles) as the abscissa and log S (range of fatigue actions) as the ordinate. Experimental data are scattered; the extent of scatter tends to be greatest in the low stress regime. For a statistical evaluation, a Gaussian log-normal distribution is assumed. Preferably in the statistical analysis the number of failed test specimens should be at least 10. The most common approach for analyzing fatigue data is to fit S-N curves by regression analysis, taking log (S) as the independent variable. Then, characteristic values at 2 million cycles are established by adopting curves lying k standard deviations of the dependent variable below the mean curve. The characteristic values are calculated for 95% survival properties on a two-sided confidence level of 75% of the mean. These values are the same as those on which the detail classifications in theEurocode 3 (EN 1993-1-9, 2005) are based. The fatigue data in Kolstein (2007) have been analyzed using a value of 2 for the parameter k. In the analysis the slope of the S-N line (m) was forced to -3.0 for comparison with weld classes in the Eurocode 3 (EN 1993-1-9, 2005). For a risk-based maintenance it is necessary to guarantee the safety of the structure with a required safety level described as the reliability index β. De Jong (2006) describes the need for this approach for the maintenance of existing bridges. Target values for the reliability index β are given in Eurocode 1 (EN 1990, 2002). Depending on degree of possible inspection, repair and damage tolerance, the target reliability index β for fatigue varies from 1.5 to 3.8. In the fatigue life calculations three safety factors are used:

\[
\begin{align*}
\gamma_{S-N} & \quad \text{safety factor on the detail classification} \\
\gamma_{n} & \quad \text{safety factor for the number of stress cycles} \\
\gamma_{\Delta\sigma} & \quad \text{safety factor for the stress ranges}
\end{align*}
\]
Safety factors depending on the reliability index $\beta$ have been to be used in combination with the mean value of the fatigue strength curves ($S_{\text{mean,c}}$) and not with the characteristic value of these curves ($S_{\text{char,c}}$). In this paper also the mean values at two million cycles of the different data sets are given.

TROUGH TO DECK PLATE JOINT

The total length of welds required in the fabrication of a deck with closed longitudinal stiffeners (troughs) is about half of that for a deck with open stiffeners (bulbs). However, welding of troughs to the deck plate raises technological problems, as the welds can only be made from the outside of the trough web. Under traffic loading, in particular the effect of local wheel loads, the trough to deck plate welds are subjected to local transverse bending moments and are susceptible to fatigue cracking. Therefore, for design and fabrication of these joints, specifications are required. However, too severe requirements increase the fabrication costs and reduce the advantage of troughs in orthotropic decks. With respect to the deck plate joint, over the period 1974 – 2000, 245 fatigue tests have been carried out within 9 research programs. Typical results compared to the current detail category given in Eurocode 3 (EN 1993-1-9, 2005) are shown in figure 2. Based on the results of the regression analyses (see table 1) the given detail categories for detail 7 and 8 in Table 8.8 of Eurocode 3 Part 1-9 (EN 1993-1-9, 2005) should be replaced by the values given in Table 1. At trough splice joints the trough to deck plate weld will always be manually and overhead welded. It is recommended to shape the edges of the trough webs as shown in figure 3. Special attention should be paid to a smooth transition between the weld and the deck plate. As this detail in the trough splice joint is a critical connection in the orthotropic deck plate it must be regularly inspected.

![Figure 2: Test results compared to current categories given in EN 1993-1-9](image_url)

![Figure 3: Trough web preparations at the trough splice joint](image_url)
TABLE 1

RESULTS OF THE REGRESSION ANALYSES OF TROUGH TO DECK PLATE JOINTS

Stress range $\Delta S$ at 2 million cycles using a fixed slope of -3 (N/mm$^2$)

<table>
<thead>
<tr>
<th>Description $^*$)</th>
<th>$S_{mean,c}$</th>
<th>$S_{max,c}$</th>
<th>Recommended Design category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) For cracking in the deck plate $^{**}$</td>
<td>183</td>
<td>140</td>
<td>125</td>
</tr>
<tr>
<td>2) For cracking in the weld or trough web $^{***}$:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2a) Fillet weld, gap $\leq$ 0.5mm Trough web shaped parallel to deck plate</td>
<td>92</td>
<td>70</td>
<td>63</td>
</tr>
<tr>
<td>2b) Partial penetration weld $^{****}$ 0.5mm $&lt; \text{gap} \leq$ 2mm</td>
<td>73</td>
<td>52</td>
<td>50</td>
</tr>
<tr>
<td>2c) Partial penetration weld $^{****}$ Gap $\leq$ 0.5mm</td>
<td>155</td>
<td>110</td>
<td>100</td>
</tr>
<tr>
<td>2d) Partial penetration weld $^{****}$ Gap $\leq$ 0.5mm Overhead welding</td>
<td>-</td>
<td>-</td>
<td>90</td>
</tr>
</tbody>
</table>

Note: 
$^*$) Weld throat $>$ web thickness
$^{**}$) Assessment based on the nominal stress range $\Delta S$ in the deck plate at the location of the trough web
$^{***}$) Assessment based on the nominal stress range $\Delta S$ in the trough web at the deck plate
$^{****}$) Nominal penetration 80% with two-sided scatter band of 20%

TROUGH TO DECK PLATE AND CROSSBEAM JOINT

When using troughs passing through crossbeams, the locations where trough web, crossbeam web and deck plate meet (see figure 4) is a location that behaves as a rather hard point in the deck plate structure. Between the trough webs of one single trough, the deck at the crossbeam is not supported by the web. For years, the fatigue behaviour of this typical joint has not been recognized and not included in Eurocode 3 Part 1-9 (EN 1993-1-9, 2005). With respect to the deck plate joint over the period 1995 – 2004, 21 fatigue tests have been carried out within 2 research programs. Typical results are shown in figure 5. Based on the results of the regression analysis (see table 2.) it is recommended to include the trough to deck plate and crossbeam joint in Table 8.8 of EN 1993-1-9 (2005). The proposed detail category including the description of the detail and requirements with respect to the stress to be used is indicated in this table.

Figure 4: Typical crack location

Figure 5: Test results trough to deck plate and crossbeam joint
TABLE 2

RESULTS OF THE REGRESSION ANALYSES
OF TROUGH TO DECK PLATE CROSSBEAM JOINTS

<table>
<thead>
<tr>
<th>Description *)</th>
<th>$S_{\text{mean,c}}$</th>
<th>$S_{\text{char,c}}$</th>
<th>Recommended Design category</th>
</tr>
</thead>
<tbody>
<tr>
<td>For cracking in the deck plate **)</td>
<td>215</td>
<td>158</td>
<td>125</td>
</tr>
</tbody>
</table>

Note:
*) Weld throat $\geq$ web thickness, gap between deck plate and trough web $\leq 0.5$ mm, nominal penetration 80% with a two-sided scatter band of 20%
**) Assessment based on the geometric stress range $\Delta S$ at the weld root in the deck plate using the direct stress at a distance of 0.4 times and 1.4 times the thickness of the deck plate

TROUGH TO CROSSBEAM JOINT

The trough to crossbeam joint is the most complex joint in the orthotropic deck structure as two highly stressed members cross each other and it is impossible to provide a continuous load path for both. Over the years, different designs of the joint between the longitudinal closed stiffeners (troughs) and the transverse stiffener (crossbeam) have been used. Basically this joint can be made in two ways. In some bridges the troughs were butted up to the crossbeam. The joints of later designs have troughs passing through cut-outs in the crossbeam. In addition to the cut-outs, in many cases a cope hole has been provided in the crossbeam at the soffit of the trough. With respect to the trough to crossbeam joint over the period 1961 – 2005, 147 fatigue tests have been carried out within 15 test series.

Discontinuous trough to crossbeam joint

Several researchers studied the fatigue strength of the discontinuous trough to crossbeam joint by testing specimens in a 3-point bending tests rig or using a test rig simulating the ‘out of plane rotation’ of the crossbeam. In all cases single trough specimens have been used. For the analysis of the 58 fatigue test data in terms of S-N curves the tested details have been grouped considering fillet welded joints respectively butt welded joints. Typical results compared to the current detail category given in Eurocode 3 Part 1-9 (EN 1993-1-9, 2005) are shown in figure 6. Based on the results of the regression analyses (see Table 3) the given detail categories for detail 7 and 8 in Table 8.8 of Eurocode 3 Part 1-9 (EN 1993-1-9, 2005) should be replaced by those given in table 3.

Figure 6: Test results discontinuous trough to crossbeam joint
### TABLE 3

RESULTS OF THE REGRESSION ANALYSES OF DISCONTINUOUS TROUGH TO CROSSBEAM JOINTS

| Description†† || **S** † †m | **S** † †h | Recommended Design category |
|---|---|---|---|
| Fillet weld** † ‡ || 72 | 44 | 40 |
| Partial penetration weld** † ‡ || 92 | 61 | 56 |

Note:
* For both types of weld the assessment is based on the direct stress range ΔS at the soffit of the trough at the crossbeam. Misalignment must not exceed 15% of the thickness of the intermediate plate
** W ild throat ≥ trough web, weld to be checked according to EN 1090
*** Not recommended for new design

---

**Continuous trough to crossbeam joint welded all around**

Two researchers studied the fatigue strength of the continuous trough to crossbeam joint by testing specimens in a test rig simulating the “out of plane rotation” of the crossbeam and one test program has been carried out including ‘out of plane rotation’ of the trough in combination with ‘in plane bending’ of the crossbeam. In all cases fillet welding was used. Typical results compared to the current detail category given in Eurocode 3 Part 1-9 (EN 1993-1-9, 2005) are shown in figure 7. The results of the regression analysis based on the measured stress 15mm from the crossbeam are shown in table 4. Detailed analysis of the measured stresses showed that the hot spot stress approach seems to be more appropriate for this joint. Extrapolation of the measured strains results in a stress level at the weld root that is 28% higher than the measured stresses 15mm from the crossbeam. It is clear that the detail category for the application of the geometric stress method for this joint exceeds the current maximum detail category of 112 given in EN 1993-1-9 (2005). Compared to tubular structures, for a plate thickness of 16mm the design category is 112. Including the thickness effect for a plate thickness of 12mm the design category will be 124 (Zhao, 2001).

![Welded All Round Diagram](image)

Figure 7: Test results discontinuous trough to crossbeam joint
TABLE 4

RESULTS OF THE REGRESSION ANALYSES
OF CONTINUOUS TROUGH TO CROSSBEAM JOINTS WELDED ALL AROUND

Stress range ΔS at 2 million cycles using a fixed slope of -3 (N/mm²)

<table>
<thead>
<tr>
<th>Description</th>
<th>$S_{\text{peak},a}$</th>
<th>$S_{\text{char},a}$</th>
<th>Recommended Design category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis based on measured stress 15mm from the crossbeam</td>
<td>139</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td>Analysis based on geometric stress at the weld toe</td>
<td>-</td>
<td>-</td>
<td>124</td>
</tr>
</tbody>
</table>

Continuous trough to crossbeam joint with cope holes

Several researchers studied the fatigue strength of the continuous trough to crossbeam joint with a cope hole at the soffit of the trough. Several types of specimens and load arrangements have been used. The fatigue data has been grouped in two main data bases depending on the location of the cracks in the crossbeam web respectively the trough web. As shown in figure 8 cracks in the crossbeam can initiate at the free edge of the cope hole or a crack can initiate in the cope hole at the weld toe of the weld connecting the trough web to the crossbeam.

As shown in figure 9 cracks in the trough web can initiate at the weld toe along the vertical weld connecting the crossbeam to the trough web or a crack can initiate at the lower end of this weld.

![Figure 8: Cracks at the cope hole in the crossbeam web](image1)

![Figure 9: Cracks locations in the trough web](image2)

Typical results for the continuous trough to crossbeam joint with cracks initiating at the free edge of the cope hole and for cracks in the cope hole initiating at the weld toe, compared to the current detail category given in Eurocode 3 Part 1-9 (EN 1993-1-9, 2005) are shown in figure 10.
Figure 10: Results of cracks at the cope hole in the crossbeam

Typical results of cracks in the trough web at the weld toe and for cracks at the lower weld end are shown in figure 11.

Figure 11: Results of cracks in the trough web

The results of the regression analysis on the test data of the different test series and the recommended design category are shown in table 5. For cracks initiating at the free edge of the cope hole category 125 is recommended and for cracks initiating in the cope hole at the weld connecting the trough web to the crossbeam the recommended detail category is 71. In both cases the geometric stress approach must be used. The same stress approach has to be used for cracks initiating at the lower end of the weld in the trough web. In this case design category 125 must be used. For cracks in the trough web along the vertical weld the detail category depends on the thickness of the crossbeam web and the assessment is based on the direct stress range in the trough web at then crossbeam.
TABLE 5

RESULTS OF THE REGRESSION ANALYSES
OF CONTINUOUS TROUGHS TO CROSSBEAM JOINTS WITH COPE HOLES

<table>
<thead>
<tr>
<th>Description*</th>
<th>$S_{mean,c}$</th>
<th>$S_{char,c}$</th>
<th>Recommended Design category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Trough web failures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1a) Failure of the trough web along the vertical weld</td>
<td>184</td>
<td>107</td>
<td>71 or 80 **</td>
</tr>
<tr>
<td>1b) Failure of the trough web at the lower end of the weld</td>
<td>188</td>
<td>143</td>
<td>125</td>
</tr>
<tr>
<td>2) Crossbeam failures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2a) Cracks starting at the free edge of the cope hole</td>
<td>305</td>
<td>170</td>
<td>125</td>
</tr>
<tr>
<td>2b) Cracks starting at the weld toe in the cope hole</td>
<td>119</td>
<td>81</td>
<td>71</td>
</tr>
</tbody>
</table>

Note:
* The fatigue data have been grouped in two main databases depending on the location of the cracks, in the crossbeam and the trough web respectively.
** For $t<12$mm Cat 80, for $t>12$mm Cat 71 ($t$ = thickness of crossbeam). Assessment based on the direct stress range $\Delta S$ in the trough web at the crossbeam.

Trough splice joint

With respect to the welded splice joint over the period 1972 – 1995, 302 fatigue tests have been carried out within 10 research programs. From the experimental data obtained, it could be concluded that full-scale test specimens should be used to define the fatigue strength of the trough splice joint because;

- in the complete splice joint, only welding defects in the region of high stress will cause cracking, but in strips every welding defect is in the region of high stresses and will initiate a crack;
- the crack growth in the strip specimens very quickly led to complete fracture but in the complete splice joints most cracks started at the inside and did not propagate so quickly and were not observed until they reached the outside.

Butt weld with steel backing plate

Most fatigue tests have been carried out on the splice joint using butt welds on a steel backing plate. Depending on parameters like the root gap or misalignment (fit) between the splice plate and the connecting stiffener, the results of the fatigue tests have been analyzed. Typical examples are shown in figure 12.

Butt weld, welded from both sides

One researcher tested a splice joint using a double V- weld. For this type of joint it must be kept in mind that after welding the double V-weld connecting the troughs, the deck plate must be placed in position and welded afterwards. Typical results of the fatigue tests on this type of joint are shown in figure 13.
**Butt splice joint with sealing plates**

The butt splice joint with a sealing plate consists of a 19mm thick infill piece of the same cross section of the trough which was cut to fit between ends of the adjacent troughs leaving a rectangular weld preparation at each end. The gap between the ends of the adjacent troughs amounts 12mm. Typical results of the fatigue tests on this type of joint are shown in figure 14.

![Butt splice joint with backing strip](image)

**Figure 12:** Results of butt splice joints with a steel backing plate

![Double V-butt welded splice joint](image)

**Figure 13:** Double V-butt welded splice joints

![Butt splice joint with sealing plate](image)

**Figure 14:** Butt splice joint with sealing plates

The results of the regression analysis on the test data of the different test series and the recommended design category are shown in Table 6.
TABLE 6
RESULTS OF THE REGRESSION ANALYSES
OF SPLICE JOINTS IN LONGITUDINAL STIFFENERS

<table>
<thead>
<tr>
<th>Description</th>
<th>$S_{\text{mean}}$</th>
<th>$S_{\text{char}}$</th>
<th>Recommended Design category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Butt weld with steel backing plates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1a) V-joint with root gap ≥ 6mm; weld convexity ≤ 10% of weld width</td>
<td>140</td>
<td>104</td>
<td>100</td>
</tr>
<tr>
<td>1b) 4mm ≤ root gap &lt; 6mm</td>
<td>103</td>
<td>84</td>
<td>71</td>
</tr>
<tr>
<td>1c) root gap &lt; 4mm **</td>
<td>65</td>
<td>44</td>
<td>-</td>
</tr>
<tr>
<td>2) Butt weld, welded from both sides; root gap ≥ 4mm; weld convexity ≤ 20% of weld width</td>
<td>145</td>
<td>114</td>
<td>80</td>
</tr>
<tr>
<td>3) Butt weld with sealing plates ≥ 20mm; root gap ≥ 10mm; weld convexity ≤ 10% of weld width</td>
<td>184</td>
<td>147</td>
<td>112</td>
</tr>
<tr>
<td>4) Single lap splice with fillet welds **</td>
<td>74</td>
<td>56</td>
<td>-</td>
</tr>
</tbody>
</table>

Note:
*) Assessment based on the direct stress range $\Delta S$ in the trough. Welds (except type 1c and 4) to be checked by NDT. Tack weld inside shape of the butt weld. Misalignment between the end of the trough stiffener and the splice plates should be less than 1.5 mm for a 6mm thick trough stiffener.

**) Not recommended for new design.

The recommended design of the splice joint in longitudinal stiffeners (see figure 15) comprises a connection in section of trough (the splice) at least ≥ 150mm long, attached to the existing troughs with two butt welds made on backing strips (6mm thick, width ≥ 30mm). Misalignment between the stiffener and the splice plate and fit-up of the gap with the backing should be ≤ 1mm. At the splice joint the edge of the total splice plate (at the deck plate side) and the weld length of the connecting stiffener (100 – 200mm) should be shaped to perform a good weld between the deck plate and the splice plate (see figure 3). Tack welds should be of the same quality as the butt weld and contained within it (i.e. no external tack welds). There is clear evidence that a root gap of at least 6mm is essential and it is recommended that the gap be equal to or greater than the stiffener plate thickness.

Figure 15: Recommended trough splice joint

SUMMARY AND CONCLUSIONS

This paper reports the conclusions and recommendations with respect to the fatigue strength of typical welded joints in orthotropic bridge decks with closed longitudinal stiffeners. Proposed detail categories to be used in EN 1993-1-9 and recommendations for structural detailing are presented.
REFERENCES


SECOND-ORDER AND ADVANCED ANALYSIS OF STRUCTURES ALLOWING FOR LOAD AND CONSTRUCTION SEQUENCES

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KEYWORDS

Advanced analysis, second-order analysis, load sequence, construction sequence, initial imperfections

ABSTRACT

The advantage of second-order nonlinear analysis lies on its capacity to model actual structural response instead of making assumption on the structural behavior such as the arbitrary assumption of effective length factor. Considering the construction process of a structure, it may become a sway frame from a non-sway frame when a story is added or, on the contrary, to convert to a non-sway frame from a sway frame when bracings or infill walls are constructed and this makes the first-order linear analysis extremely inconvenient to use. Further, the deformation and load distributions of the structures considering the construction sequence can be much different from those obtained by assuming all structural elements share loads simultaneously. This paper proposes an advanced simulation-based design using an improved second-order analysis for structures accounting for both local and global imperfections, sequential loads and construction process. The technique for such an analysis is described and the error for ignoring the effects of sequential loads and construction procedure is quantified. Both the elastic and the plastic second-order analyses are used for comparison.

INTRODUCTION

Second-order analysis is a type of simulation-based nonlinear analysis and design method and it is also a rigorous design for steel structures. The method has been extensively researched for decades. Its application to practical structures becomes extensive in Hong Kong and Macau in recent years after the introduction of Eurocode3 [1] and CoPHK [2]. In the past, most second-order analyses were only a secondary check on the stability of a structure rather
than a primary design with member sizing which was mostly carried out by the first-order linear analysis used in conjunction with effective length method. One of the drawbacks of the previous second-order analysis is its ignorance of P-δ effect as well as initial imperfections. As a result, the traditional second-order analysis cannot be applied directly to practical design.

Advanced analysis combines the effects of material yielding and geometrical change with sufficient accuracy for practical uses so that isolated member capacity checks can be avoided. Advanced analysis can be defined as any analysis method that does not require specification member bucking resistance checks and it requires the consideration of frame, member P-δ effects as well as their associated imperfections along the most critical modes so that the structural behavior can be captured conservatively in the analysis. Advanced analysis is deemed to be the new generation design method in line with the popular performance-based design concept. Second-order elastic analysis can be considered as a subset of advanced analysis considering only the elastic stage of a structure and it has been used quite widely in many steel structure projects in Hong Kong. This paper discusses and proposes a design method for structural response under construction that some members are "pre-loaded" and their load sharing is affected by the construction sequence rather than simple sharing of loads according to their stiffness.

The influence of construction sequence has been aware in engineering practice but it is sometimes ignored in design because of complexity in its consideration. In conventional design, the structural response is based on the complete structure under factored loads without the consideration of construction sequence. In many cases, the ignorance of construction sequence may bring difficulties in the process of erection and installation of steelwork, fail to meet the architecture requirements, under-design some critical members and finally inaccurate assessment of the safety of the structure.

In this paper, a beam-column element allowing for member initial imperfection is employed for the one-dimensional structural elements so that the P-δ effect can be taken in account at element formulation level. Slabs, floors and walls are considered by shell elements. The first buckling mode is used to determine the direction of local member imperfection and the global frame imperfection due to out-of-plumbness. Furthermore, the load and construction sequences are considered by the robust incremental-iterative nonlinear procedure. The new approach represents a modern and practical second-order P-Δ-δ analysis and design method of steel and composite structures.

SECTION CAPACITY CHECK

One basic requirement for advanced analysis is to take initial imperfection into consideration with direction following the buckling mode. Real structural members contain geometrical and material imperfections so design codes require implicit consideration via use of different buckling curves in the first-order linear analysis or explicitly via the element formulation for initial imperfection at element level in a second-order or advanced analysis. Because of this requirement, the straight cubic element or the conventional stability function cannot be used directly and a new curved stability function element derived by Chan and Gu [3] is used in the present studies for fulfillment of the code requirements and realistic structural modeling. Once the P-δ effect is taken into account, the P-Δ effect can be automatically considered in the nonlinear incremental-iterative procedure. Thus, the real structural response can be captured and the exact internal forces and moments in each section along the member can be
determined. As a result, only the section capacity check is needed in the proposed second-order analysis and the tedious member design based on traditional linear analysis can be avoided.

From above, one only needs to identify a condition for failure of a section which is the yield function of the cross section when adopting the proposed second-order analysis. To deal with the strength and stability design, the geometric non-linear equilibrium path is traced by incrementing the load in Eq. (1) until the sectional capacity factor \( \varphi \) given below is equal to 1 as,

\[
\frac{P}{p_y A} + \frac{\bar{M}_y + P(\Delta_y + \Delta_{0y}) + P(\delta_y + \delta_{0y})}{M_{cy}} + \frac{\bar{M}_x + P(\Delta_x + \Delta_{0x}) + P(\delta_x + \delta_{0x})}{M_{cx}} = \varphi = 1 \quad (1)
\]

in which, \( P \) is the axial force in member, \( p_y \) is the design strength, \( A \) is the cross sectional area, \( \bar{M}_y \) and \( \bar{M}_x \) are the moments about the minor and major axes obtained from a first order analysis, \( M_{cy} \) and \( M_{cx} \) are the moment capacities about the minor and major axes, \( \Delta_y \) and \( \Delta_x \) are the nodal displacements due to out-of-plumbness of frame sway induced by loads, \( \Delta_{0y} \) and \( \Delta_{0x} \) are the nodal displacements due to out-of-plumbness of frame imperfections, \( \delta_y \) and \( \delta_x \) are the member deformations due to loads on the member, \( \delta_{0y} \) and \( \delta_{0x} \) are the member deformations due to member initial bow, \( \varphi \) is the section capacity factor. If \( \varphi > 1 \), a member fails in design strength check and if \( \varphi << 1 \), the member section can be reduced in size.

In normal design, the load resistance is taken as the load causing the first plastic hinge and thus the section strength check for value of \( \varphi \) is applied to every section of all members in the structure. In second-order elastic analysis, any member attaining a section strength factor equal to 1.0 is considered as the failure of the structure with the load at this instance taken as the load resistance. In advanced analysis, more than one plastic hinges are allowed and the load resistance is taken as the peak load in the load vs deflection path of the structure.

**PLASTIC HINGE BY SECTION SPRING MODEL**

A structure will be loaded to the inelastic range with progressive stiffness degradation in an advanced analysis, and the post-yield response is monitored. Therefore, a simple, accurate and efficient method for determining the plastic hinges and robust numerical procedure are needed in an advanced analysis.

The lumped plasticity approach, also named as the plastic hinge method, rather than plastic zone approach is used in this paper. While both methods are allowed in CoPHK [2], the plastic zone approach is more accurate than the plastic hinge method as it considers spread of plasticity. However, owing to the huge amount of computational effort required, it is only suitable for simple structures but not for most practical design. The plastic hinge approach is significantly more efficient and thus adopted more widely in practice. Furthermore, the moment and force interactive equation on various design codes can be directly incorporated into the analysis program so that the method satisfies the design code requirements directly and remains consistent with the conventional design using code formulae.
The basis of the plastic hinge method is cross-section plastification. Material yielding is accounted for by zero-length plastic hinges at one or both ends of each element. Plasticity is assumed to be lumped only to the ends of an element, while the portion within the element is assumed to remain elastic throughout the analysis.

Several methods of modeling the cross-section plastification at the ends of an element have been proposed, such as Liew et al. [4], Liew et al. [5], King et al. [6], Chan and Chui [7]. In this paper, the refined-plastic hinge method proposed by Liu et al. [8] is used due to its simplicity and efficiency. In this method, two predefined section springs will be set at the two ends of each beam-column element. The end section springs will be formulated in the element stiffness matrix of the curved stability function beam-column element [3] and therefore a new hybrid element (see Figure 1) is formulated. More detailed element formulation can be referred to Liu [9].

Eliminating the internal degrees of freedom by a standard static condense procedure, the bending equilibrium equations in an incremental form can be expressed as,

\[
\begin{bmatrix}
\Delta M_{s1} \\
\Delta M_{s2}
\end{bmatrix} =
\begin{bmatrix}
S_{s1} - S_{s2}^2 (K_{22} + S_{s2}) / \beta_s & S_{s1} S_{s2} K_{22} / \beta_s \\
S_{s1} S_{s2} K_{21} / \beta_s & S_{s2} - S_{s1}^2 (K_{11} + S_{s1}) / \beta_s
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_{s1} \\
\Delta \theta_{s2}
\end{bmatrix}
\]  

(2)

where

\[
\beta_s = \begin{bmatrix} K_{11} + S_{s1} & K_{12} \\ K_{21} & K_{22} + S_{s2} \end{bmatrix} > 0
\]  

(3)

and, \( S_s \) is the stiffness of section spring, \( \Delta M_{si} \) is the incremental nodal, \( \Delta \theta_s \) is the incremental nodal rotations, \( K_{ij} \) is the stiffness coefficients of the stability function element.

![Diagram of internal forces of the hybrid element with end springs](image)

**Figure 1: Internal forces of the hybrid element with end springs**

To consider the progressive cross-section yielding, the section spring stiffness \( S_s \) can be defined as,

\[
S_s = \frac{6EI}{L} \left| \frac{M_{pr} - M}{M - M_{cr}} \right| \quad (M_{cr} < M < M_{pr})
\]  

(4)

where \( EI \) is the flexural constant, \( L \) is the member length, \( M \) is bending moment due to external forces, and \( M_{cr} \) and \( M_{pr} \) are the first yield and plastic moments respectively. In computer analysis, the section spring is taken as \( 10^{-10} \, EI / L \) and \( 10^{-10} \, EI / L \) for the elastic case (i.e. \( M < M_{cr} \)) and the plastic case (i.e. \( M > M_{pr} \)) respectively. In case of a force point outside the full yield surface (i.e. \( M > M_{pr} \)), it should be moved back onto the surface to avoid the violation of plastic state. In this paper, the path normal to the yield surface is
chosen as the recovery path. For conservative design, the linear interactive equation Eq. (1) is used as the interactive equation for evaluating the cross-section plastic strength.

EFFECTS OF CONSTRUCTION SEQUENCE

The strength, stability and deflection checks in the conventional design are essentially based on the complete structure without consideration of construction process. In fact, the behaviour of the components or units in the erection process may be different from the ideal case because instability and excessive deflection may occur in the construction stage with limited propping. Furthermore, shortening and undesirable deformations of the incomplete structure under self-weight and construction loads are inevitable. Generally speaking, the structural self-weight, external loads, boundary conditions and materials are depended on stages during the construction process and their variations are easily overlooked in conventional design.

The deformation and stress level during construction are key considerations by structural engineers. For demonstration of the proposed method, a three-story building shown in Figure 2 is constructed through three stages in which one story is added in each stage. In each construction stage the building under construction is treated as an isolated structure based on the stress and deformation status in the last stage and therefore effects of construction sequence can be taken into account.

![Diagram](image)

Cons. Stage 1  Cons. Stage 2  Cons. Stage 3

Figure 2: The proposed approach for construction analysis

Unlike previous work using the first-order linear analysis which treats the limit states design and the construction analysis as two separate procedures with analysis by computers and design by charts, the proposed method integrates analysis to design and therefore the design and analysis models are consistent. Moreover, the P-∆ and P-δ effects, initial imperfections, pre-deformation and material yielding are all taken into account. The flow chart of the proposed procedure is shown in Figure 3.
Figure 3: The proposed approach for construction analysis

The steps in the flowchart (see Figure 3) are explained as follows.

(1) Define Construction Sequence
Engineers group structural elements in the design model into a number of construction units according to the construction scheme and assign each of them a unique “Construction ID”. Only the elements in the active groups are considered in analysis at a particular loading stage with other non-active structural elements ignored. New elements are added to the structure at initial stage.

(2) Apply Dead Loads (DL) and Construction Loads (CL)
Dead loads are mainly from the structural self-weight and a load factor of 1.2 is suggested. Construction loads include the live loads, self-weight of equipments and others.

(3) Pre-set Deformation
Excessive deflections can be reduced by pre-cambering through pre-set displacements.

(4) Construction Analysis
Stiffness matrix of the members in active groups is solved with the nonlinear incremental-iterative procedure adopted to improve convergence.
(5) **Strength and Instability Check**

With the consideration of initial imperfections and P-Δ and P-δ effects, a fast and accurate instability and strength checks can be conducted by the section capacity check in Eq. (1). Modification is needed when some members fail.

(6) **Deflection Check**

As checking for serviceability limit state, this step ensures deflection limit is not violated otherwise member re-sizing or pre-cambering is activated.

(7) **Permanent load design Check**

The conventional ultimate and serviceability limit state design load checks are carried out as the final safety and serviceability checks. The consideration of load sequence may affect the final force and moment distribution as some construction loads are present earlier than some members which do not share these construction loads.

The above procedure and the plastic hinge method presented in previous section have been coded into the program NIDA [10] which is used to study the following numerical examples.

**EXAMPLES**

*Example 1: Vogel six-story frame*

The two-bay six-story frame subjected to distributed gravity loads and concentrated lateral loads has been analyzed by Vogel [11]. The layout and the loads of the frame are shown in Figure 4.

The structure is first analyzed by the proposed method assuming that the vertical and horizontal forces are proportionally applied. Eq. (1) has been used in many design codes as conservative yield surface of beam-columns but a more economical yield surface in the paper by Vogel [11] is adopted here for direct comparison. The locations of the plastic hinges predicted by the proposed method are indicated in Figure 5.

This frame is further used to study the effects of non-proportional loads. Three loading cases have been considered and they are namely as (1) non-proportional case by applying vertical loads first and then the horizontal loads, i.e. V→H; (2) non-proportional case by applying the horizontal loads first and then the vertical loads, i.e. H→V; (3) proportional load case by applying simultaneously the horizontal and vertical loads, i.e. H + V. The results in different loading conditions by second-order elastic and inelastic analysis respectively are listed in TABLE 1. For demonstration, only the end moment of the indicated member and the horizontal displacement of the indicated node (see Figure 7) are tabulated in TABLE 1.

From TABLE 1, it can be seen that non-proportional load has no influence on the final structural response for second-order elastic analysis. However, the end moments and the displacements under three loading conditions by second-order inelastic analysis are slightly different. It is observed that some plastic hinges are formed at the beam ends after the full vertical loads in case 1 but no plastic hinges are found after the full horizontal loads in case 2. For inelastic analysis, the most unfavorable case was found to be non-proportional load case 1 but not the proportional load case 3.
Table 1: Comparison of Results under Three Loading Conditions

<table>
<thead>
<tr>
<th>Loading Conditions</th>
<th>Second-order elastic analysis</th>
<th>Second-order inelastic analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moment (kNm)</td>
<td>Disp-X (m)</td>
</tr>
<tr>
<td>Case 1: V → H</td>
<td>87.4</td>
<td>0.08</td>
</tr>
<tr>
<td>Case 2: H → V</td>
<td>87.4</td>
<td>0.08</td>
</tr>
<tr>
<td>Case 3: H + V</td>
<td>87.4</td>
<td>0.08</td>
</tr>
</tbody>
</table>

This example shows the effect of the non-proportional loads. It is recommended that load sequence analysis should be used for second-order plastic analysis.

Example 2: Three-story steel frame

In this example, a three-story steel frame is used to study the influence of the construction process. The layout and the member sections are shown in Figure 6. All steel members used are S355 with the design strength $p_0$ of 355 MPa and density of 7850 kg/m³. All member-to-member connections are assumed as rigidly connected and the steel columns are pinned to the foundation. Also, only the structural self-weight is considered and the construction and other design loads are ignored so that the construction effects can be lucidly identified.
Figure 6: Layout of the three-story steel frame

The structure is assumed to be constructed in three stages with one story added for each load stage. The positions of the new elements being added to the existing structure are assumed in their design positions. If pre-deformation is required, the new elements can be assigned with an initial deflections.

The deformed shapes in all construction stages are shown in Figure 7(a-c). For comparison, the deformed shape without consideration of construction effect is shown in Figure 7(d). It can be seen that the final deformed shape by construction analysis is different from that the one obtained from the conventional second-order analysis.

The end moments of the marked steel beams in Figure 6 with and without consideration of construction process are listed in TABLE 2. It is also noted that the end moment in the final construction stage is much higher than the case without consideration of construction effect.

**TABLE 2**

<table>
<thead>
<tr>
<th>Construction sequence</th>
<th>End Moment (kNm)</th>
<th>by NIDA [10]</th>
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<tr>
<td>1</td>
<td>19.60</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>33.90</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>42.40</td>
<td></td>
</tr>
<tr>
<td>No consideration of construction sequence</td>
<td>30.30</td>
<td></td>
</tr>
</tbody>
</table>

This example shows that the displacements and the load distributions can be varied for considering and ignoring the effects of construction process.
CONCLUSIONS

In this paper, a curved stability function element allowing for initial member curvature is proposed for modeling of steel members during construction. The element is further developed to consider plastic hinge by section spring model. By dividing the design model into several construction parts, the proposed second-order analysis has the capability to consider the construction process in its construction and permanent stages. The work further extends the second-order analysis method allowing for the P-Δ and P-δ effects, initial imperfections, non-proportional loading and construction process to practical design by a series of simulation. The proposed method allows design for permanent loads and during the construction stages and at the final stage. This represents a truer reflection of structural responses in practice and is believed to be a new generation design method.

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